Paper I

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Furnace Endoscope – Measuring fuel spray properties in hot and corrosive environments

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Abstract

A furnace endoscope was developed to carry out in-furnace measurements of black liquor sprays in order to discover the initial velocity, opening angle and trajectory of the spray and compare spray disintegration mechanisms and spray appearance with the ones measured in a spray chamber. An error analysis of the velocity measurement method was carried out, and the meaning of the optimum measurement distance from the optics to the observed object is discussed. Some details of the development process of the probe are also presented, especially the definition of the scale of the image and the cooling system of the protection tubes. The furnace endoscope can be used in difficult conditions, such as those found inside a chemical recovery boiler (~ 1200 °C, corrosive chemicals) with promising and accurate measurement results. The equipment has been tested in several furnaces.

Key words: Spray properties measurement technique endoscope

Nomenclature

d	Distance from endoscope lens to spray surface, m
d_{px}	Measured distance in a frame, pixels
d_{fr}	vertical size of the frame, pixels
s	Distance the spray has moved during t, m
t	Time (exposure time + delay), s
u	Velocity of a spray, m/s
α	Angle of view, °
Δd	Error in measuring d, m
Δd_{px}	Error in measuring d_{px}, pixels
Δt	Error in time, s
ΔT_e	Excess temperature, °C
ε	Absolute error (maximum), m/s
ε^*	Relative error, %

Introduction

When fuel is sprayed into the boiler furnace, the properties of a spray determine the combustion time and, in principle, the location where the reactions take place. To measure spray properties in a furnace environment, in this case in a chemical recovery boiler furnace where the combustion of black liquor and chemical recovery take place, is a challenging task. Not only is the temperature in the furnace about 900 – 1200 °C, but corrosive, sticky particles hit the measurement probe.

Measuring drop size of black liquor has so far been possible only in test-chamber conditions. The hypothesis is that if the droplet formation processes in a test chamber and a furnace are sufficiently similar, then the test-chamber measurement results can be applied to the furnace. Therefore, it is important to study the sheet properties in furnace conditions where the temperature and flow direction of surrounding gas differs from test chamber conditions (Miikkulainen *et al.* 2002b). In the research being reported here, a furnace endoscope was developed to carry out in-furnace measurements of black liquor sprays in order to discover the initial velocity, opening angle and trajectory of the spray and compare spray disintegration mechanisms and spray appearance with the ones measured in a spray chamber.

Black liquor plays an important role in heat and power generation worldwide. For example, in Finland and Sweden, black liquor combustion covers 7 and 4.5 % of the total heat and power generation, respectively. In Finland, 30 % of energy that is generated from biofuels, comes from black liquor combustion (Asplund 1997). Black liquor is a highly viscous by-product of the chemical pulping process: it contains water, organic matter separated during cooking, and inorganic cooking chemicals. The purpose of a recovery boiler is to recover the expensive cooking chemicals and the energy content of the organic matter. The dimensions of a modern (3000 tons dry solids / day) recovery boiler are ~12 m x 12 m x 60 m. Black liquor is commonly sprayed into a Kraft recovery boiler furnace by a set of splashplate nozzles, also called liquor guns. Splashplate nozzles produce a thin liquid sheet, which breaks up forming a wide distribution of large (diameter of 4 – 10 mm), odd-shaped drops. Char burning takes place on the surface of the char bed, located at the bottom of the boiler and during the flight of a droplet. The danger of a water-smelt explosion in the char bed makes it impossible to use water as a coolant for the measurement instruments in the lower part of a recovery boiler. In addition to high temperature, the harsh environment of flying sticky particles and falling large deposit pieces from upper parts of the furnace, and corrosive chemicals makes the measurement of spray properties difficult.

Commercial furnace control cameras are available and have been installed as part of some original boiler-equipment packages for almost half a century. Optical probes are used to monitor flames and especially flame failures in fossil-fuel-fired furnaces. For example, Lu *et al.* (2000) reported determining geometrical and luminous parameters from the images obtained of the entire flame in the furnace. Furnace cameras can also be used to control, for instance, NO_x-emissions by monitoring combustion-equipment changes in the boilers (McCarty and Lang, 2002). The cameras have been developed to obtain a view from a hole or window in the boiler wall; they are usually water cooled. None of these cameras are applicable to *spray* studies inside a recovery boiler.

In this paper, the measurement instrument - the furnace endoscope - and methods for measuring spray properties with it are studied in detail and some example results are presented. Some details of the development process of the probe are presented, especially the definition of the scale of the image and the cooling system of the protection tubes. The error analysis of the velocity measurement method is carried out, and the meaning of the optimum measurement distance from the optics to the black liquor spray is discussed.

Functions and design of furnace endoscope

The furnace endoscope is a measurement device developed for spray research in a hot and corrosive environment. Suitable material for an outer protection tube was researched and tested. The requirement that the material should be as stiff as possible without being subject to corrosion damage in temperatures as high as 1000 °C was difficult to meet. The most suitable material for the stiff structure of the furnace endoscope was found to be a titanium-stabilized stainless steel tube. It has been used in a number of experiments in which it showed good durability.

An air-cooled endoscope tube is 3 m long; there is a high shutter speed CCD-camera at one end of the tube and a furnace lens at the other, see Fig. 1. The diameter of the outer protection tube is 76 mm with 4 mm thick walls. The outer diameters of the second protection tube and the optics tube are 38 mm and 35.5 mm, respectively. The nozzle and the endoscope both enter the furnace from the same liquor gun hole, (Fig. 2) whose dimensions limit the outer diameter of the endoscope tube. The idea is to get a view from above the black liquor spray at different distances from the nozzle.

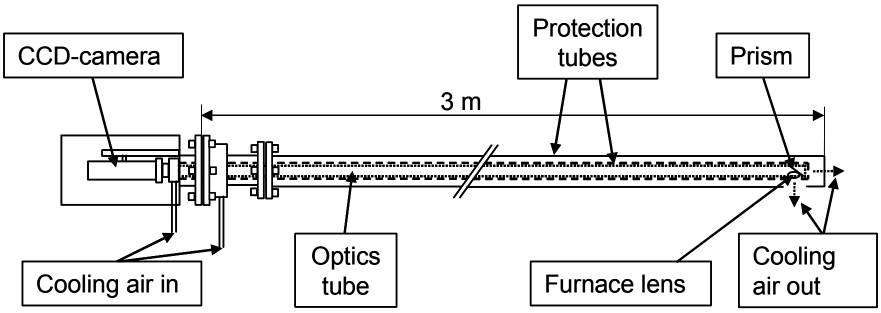


Figure 1. Furnace endoscope

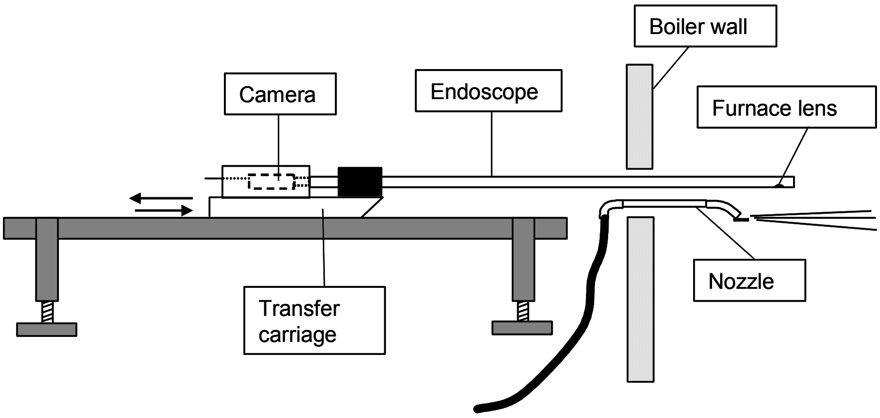
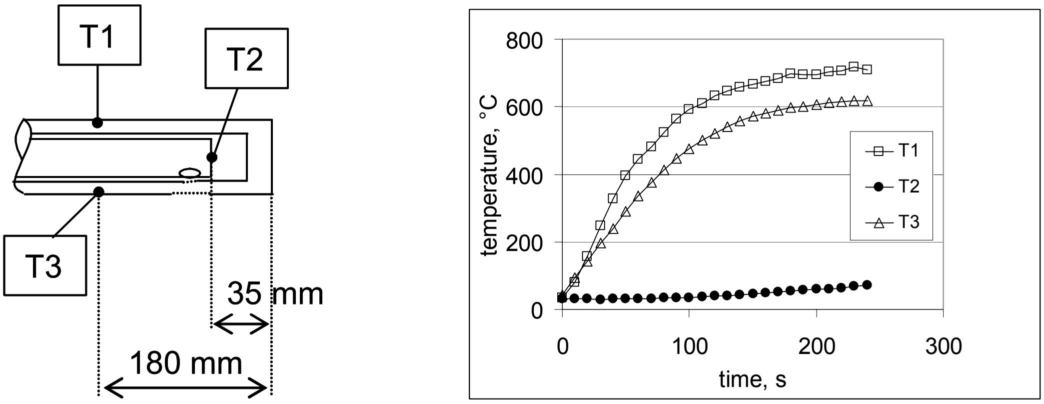


Figure 2. Setup for in-furnace spray measurement with the furnace endoscope

The optical setup was designed to film a black liquor spray whose velocity is between 5 to 20 m/s. An air-cooled quartz lens faces the furnace. Next to the furnace lens there is a prism for a right-angle view. There are series of achromatic relay lenses between the furnace lens and the camera in order to bring the image to the CCD camera. The minimum distance from furnace lens to the object was designed to be 0.3 m. In addition to the optical system, the angle of view depends also on the camera used i.e. the size of the CCD-cell of the camera. The angle of view was 26° vertically, and 34° horizontally with the PCO-SensiCam black and white-camera, used in the tests. The hostile environment in the recovery boiler furnace, and air used as a coolant, forces the furnace lens diameter to be rather small, 10 mm. In addition to the small furnace lens, the long tube limits the light available for the camera. The long distance from the furnace lens to the camera causes the optical system to be sensitive to bending in a hot furnace. In the experiments, the optical system met the requirements to carry out measurements as planned. Optical filters could be used to get rid of disturbing wavelengths caused by water vapor in the flue gas. This was not needed in the study presented here.

At the liquor gun level in the recovery boiler (about 6 m from the bottom of the kraft recovery boiler) where measurements take place, the temperature is 900 - 1200 °C. When considering a suitable measurement tool to be used in the lower part of a kraft recovery boiler, the possibility of a water-smelt explosion must be taken into account. For this reason, water is not an option as a coolant for the furnace endoscope; compressed and filtered air is used instead. The furnace lens is air cooled internally through an inlet fitting in its flange. Filtration of the cooling air is necessary to avoid fouling the optics and housing of the CCD-camera. Extra cooling air (without filtration) is inducted between the outer protection tubes and carried away from the holes at the end of the endoscope. The air introduced by the endoscope is an insignificant disturbance to the recovery boiler operation. Also, it has been tested that cooling air emitted into the boiler does not disturb the measurement area of the spray. The flow direction of the exhaust cooling air is mostly forward. The cooling system can be monitored via thermocouples installed on the protection tube and optics. The location of the thermocouples is shown in Fig. 3a. The heat-up rate of the furnace endoscope varies significantly, depending on the boiler furnace temperature and the spraying practice of black liquor. A typical temperature heat-up curve is shown in Fig. 3b.

The protection tube is sensitive to temperature difference between different parts of the tube. A temperature difference (~100 °C) between T1 and T3 in Fig. 3 (upside and downside of protection tube) causes bending of the optics tube in high temperatures. Because of the rigid structure of the optics, the bending of the protection tube can cause the image to disappear; the endoscope has to be drawn out to cool it down before measurements can be continued. A typical measurement period is 5-15 minutes without a pause for cooling depending on the conditions in a furnace. The most important thermocouple is T2 in Fig. 3. It is assumed to measure the temperature of the furnace lens and the prism. The temperature limit for the lens is 260 °C.



a) b)

Figure 3. Location of the thermocouples at the end of the endoscope (T2: temperature of optics) and typical measured heating trend.

Depending on the flow field of the furnace, some unburned particles returning from the upper part of the boiler might hit the endoscope and stick on the protection tube, cooling it down locally. Spray from the liquor gun being measured or from the liquor guns next to it might hit the endoscope and cool it down very effectively. The relatively cold (~ 130 °C) and dense black liquor spray beneath the endoscope can effectively reduce the thermal radiation heat transfer rate from below the endoscope to its underside.

Measurement methods in furnace conditions

There is a great variety of possibilities for using the furnace endoscope as a spray-measurement tool. The endoscope has been tested by means of measurements to determine characteristics of different black liquor sprays such as velocity, opening angle, and spray disintegration mechanisms. The measurement method is based on a procedure whereby the spray appearance is videotaped or captured directly onto the hard disk of a computer. Images are processed with image analysis software to measure the required characteristics of a spray. (Miikkulainen *et al.* 2000)

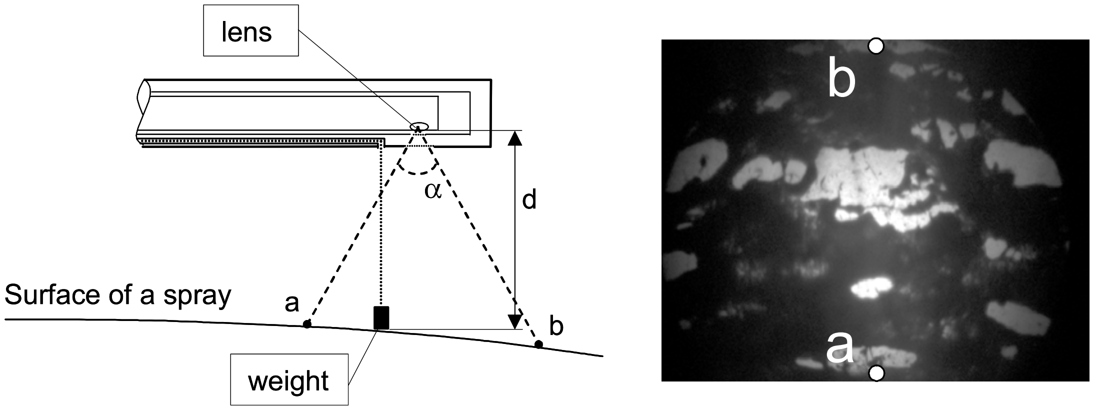


Figure 4. Defining the spray trajectory with wire rope and steel weight

To be able to measure distances from an image by image-analysis, one has to know the scale of the image. To determine the scale, the distance from the endoscope lens to the object (*d* in Fig. 4) must be known. A simple method was developed to measure this distance. A steel weight, controlled by means of a wire rope, is lowered until it reaches the spray; the length of the wire used is then measured. This can be observed from the monitor of the endoscope. The schematic picture of this method is presented in Fig. 5. The method measuring the distance gives surprisingly accurate results, which have been confirmed by comparing the weighted wire rope method to manual measurement in a laboratory environment. The maximum absolute error of the distance measured is estimated with total differential analysis to be approximately 7 mm, depending on the spray behavior and the existence of a flat spray. The error sources are two separate distance measurements at two locations and the movement of the endoscope between them. The distance measurement method is suitable only for flat sprays beneath the endoscope where the weight can touch the object. The time for in-furnace measurement is limited because the wire rope burns off and must be replaced. The wire rope is installed inside a tube, which is inside the cover tube of the endoscope. Compressed air is blown into the wire tube to keep it clear. When the distance between the black liquor sheet and the endoscope is measured in several places along the spray centerline, also a trajectory of the spray is achieved as a result. In Fig. 5, some example trajectories are presented. The spray trajectories are affected by gas flows in a furnace.

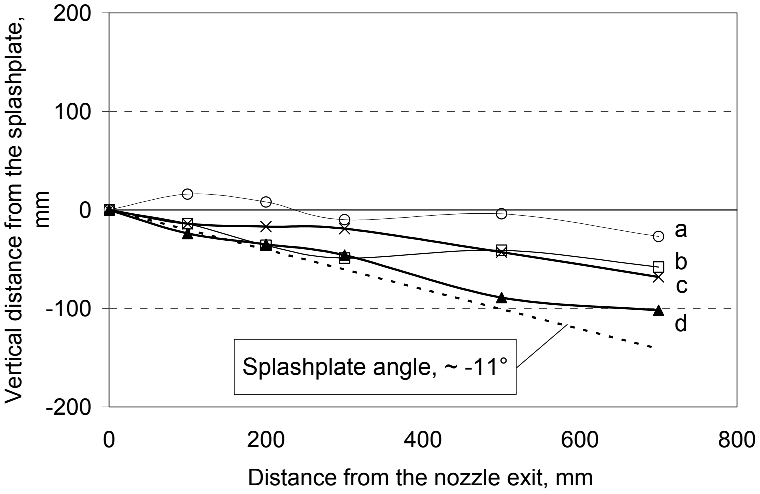
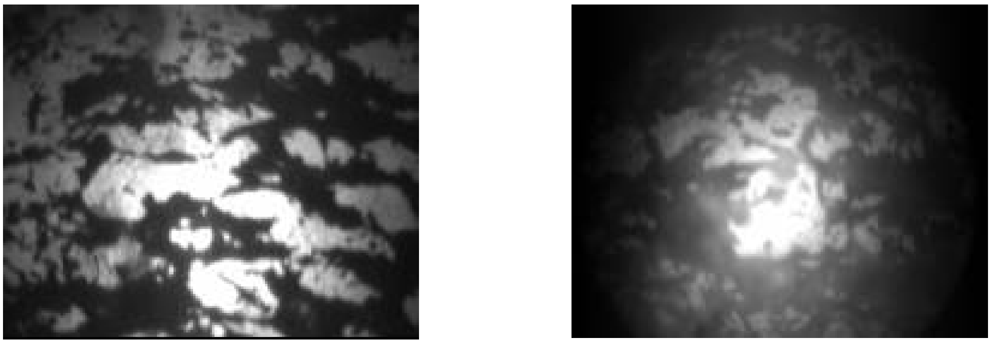


Figure 5. Measured trajectories at the centerline of four cases, a-d.

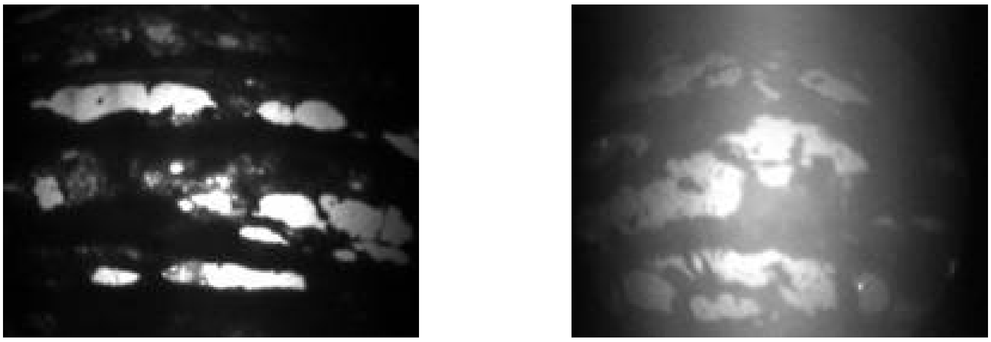
Spray appearance

The disintegration mechanism of a spray results from spraying parameters, liquid specific parameters and the spraying environment. Fraser (1956) suggested that three different disintegration mechanisms exist, namely rim disintegration, wavy sheet disintegration, and perforation. Spielbauer and Aidun (1992) found that the main disintegration mechanisms of black liquor sprays are wavy-sheet disintegration and perforation. In both cases, ligaments are formed at first and later drops are formed. Knowing the disintegration mechanism of a spray enables an estimation of the final drop size as Kankkunen and Nieminen (1997) demonstrated. Helpiö and Kankkunen (1996) discovered that flashing accelerated spray velocity, and higher temperatures decreased drop size. With the furnace endoscope, it is possible to observe the effect of furnace environments on spray disintegration mechanisms and on the velocity.

TEST CHAMBER, 80°C **FURNACE, 1000°C**



1. ΔT_e = 18°C, 211mm x 169mm **2.** ΔT_e = 18°C, 211mm x 169mm



3. ΔT_e = 14°C, 224mm x 179mm **4.** ΔT_e = 14°C, 224mm x 179mm

Figure 6. Part of a black liquor spray at the distance of 0.5 m from the exit of the nozzle. Two spraying temperatures and two environments.

The identification of sheet length, ratio of liquid and gas, and the shape and position of forming rims and ligaments is possible by analysis of endoscope images. The Fig. 6 shows images taken with the furnace endoscope at the location of 0.5 m from the nozzle exit in two different environments; cold spraying chamber on the left and hot recovery boiler furnace on the right. The direction of the flow in the images is upwards. These images show the possibilities that are available under in-furnace conditions. It is possible to see how the spray develops and observe differences and similarities in spray appearance with the test chamber environment, where more extensive measurements are available.

In Fig. 6, there are two different excess temperatures i.e. spraying temperatures above the atmospheric boiling point , ΔT_e = 14 and 18 °C, and the consequent disintegration mechanisms. The figures are comparable since all the spraying parameters are identical between the hot and cold environments. The main spraying parameters are: The nozzle is a splashplate nozzle with pipe diameter of 27 mm, black liquor dry solids content is 76 %, mass flow rate 5.2 kg/s, viscosity is 170 mPas with $\Delta T_e = 14°C$ and 120 mPas with $\Delta T_e = 18$ °C. The overall visual appearance of the specific part of spray centerline is similar in both environments. It can be observed that break-up process is in same stage and the size of the ligaments is about the same. At the lower temperature, when the flashing is not so strong, the alignment of the ligaments is more horizontal referring to wave formation. At the higher spraying temperature the orientation of the ligaments is more random and the sheet has already disintegrated.

Velocity measurement

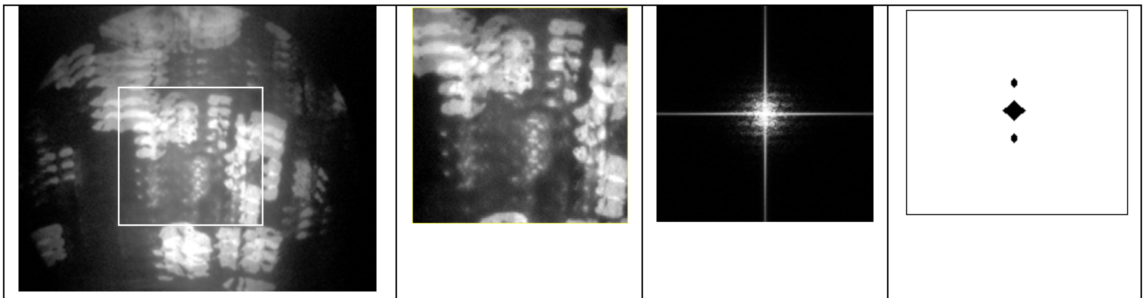
The velocity of a spray can be measured by the multi-exposure method (Merzkirch, 1987). The camera triggers several images in the same frame. When the exposure time and delay between the exposures are known, the velocity can be calculated on the basis of the distance the object has moved. The distance can be measured manually in each image using image analysis software. To achieve an accurate result, the number of images must be high; since manual measurement is very time consuming, this is problematical.

Fig. 4 presents the geometry of the measurement setup. The velocity of a spray (u) can be calculated from the expression shown in Eq. (1).

$$u = \frac{s}{t} = \frac{2\ d\ d_{px} \tan\alpha}{d_{fr}} \frac{1}{t} \qquad (1)$$

where s is the distance the spray moved during the exposure time and delay time, t, between images. The measured distance from the endoscope lens to the spray surface is d, and the distance in a frame measured in pixels, d_{px}. α is the angle of view. The vertical size of the frame d_{fr} depends on the sensitivity and size of the CCD-cell used. If there is a computer controlled (digital) video camera available, it is possible to achieve a higher light level in the images by merging pixels (and decreasing resolution) and still have the exposure time as short as possible. This is usually the case when measurements are taken in the furnace. The resolution of a frame in the studies presented in this paper is 1280 x 1024 pixels or 640 x 512 pixels depending on the amount of light.

The measurement procedure can be carried out semi-automatically by image-analysis software developed for this purpose. Fast Fourier Transformation method (FFT) is used to edit a grayscale image in its spectral form (Image Pro Plus Reference Guide, 1998) In order to find out displacement field in the image, which is formed from the three known exposures of the motion in the same frame, the FFT needs to be carried out twice (In Fig. 7, b to c, and c to d). The first FFT yields a fringe pattern where the fringe orientation is perpendicular to the direction of the displacement and the fringe spacing is inversely proportional to the magnitude of the displacement (Westerweel, 1997). The second two dimensional FFT produces the displacement field.



a) 3 exposures in the same frame. Area of interest is selected.

b) Area of interest

c) Fringe pattern after the first FFT

d) Result after the second FFT and threshold

Figure 7. Velocity measurement with an image analysis software and Double FFT procedure.

After the second FFT, some threshold for the image might be needed and, depending on the quality of the original frame, some noise might have to be removed before the object count. The filtration or threshold can be carried out as long as the coordinate of the center point of the dot of interest does not change. This is usually easy to carry out, because the area of the dots of disturbance is much smaller than the area of dots of interest. In the object count, the x-coordinate and y-coordinate of the result points are stored. The distance the spray has moved during the delay and the exposure time can be calculated from the coordinates.

The FFT method was tested also by video recording a rotating disk at constant speed through the furnace endoscope, see Fig. 8. Three exposures in the same frame were used. The velocity of the edge of the disk was determined carefully with a stroboscope to be 10 m/s. The results from several measurements varied between 9.98 – 10.06 m/s, confirming the method to be very accurate.

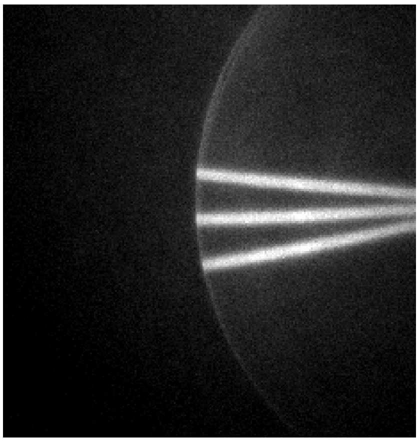


Figure 8. A rotating disk. Three exposures in the same frame. Velocity of the edge was 10 m/s.

Instead of FFT-method, the velocity can be measured manually from the raw image by measuring the distances of noticeably moved objects (holes in the sheet, ligaments, etc.) during the multiple exposures. In every test case, manual checking was carried out for few pictures, but because it is very time consuming, it could not be carried out with thousands of images. They both gave very similar results. The same image analysis based velocity measurement method has been used both in the furnaces and in the test chambers.

Error analysis

The velocity measurement is made up of a chain of components, each of which is subject to individual inaccuracy. The components of possible inaccuracy are the measured distance from the endoscope lens to the spray surface (d), i.e. the scale of the image, the measured distance in a frame (d_{px}) in pixels, and time (t) the camera used for the exposure and the delay. The absolute error ε of the measured velocity can be approximated using total differential analysis. (Doebelin, 1990)

$$\varepsilon = \left|\frac{\partial u}{\partial d}\right|\Delta d + \left|\frac{\partial u}{\partial d_{px}}\right|\Delta d_{px} + \left|\frac{\partial u}{\partial t}\right|\Delta t \qquad (2)$$

The effect of different components on the absolute error is shown in Fig. 9. The velocity of the spray is 15 m/s. As can be seen, the time used for exposure and delay has a negligible effect on the absolute error. The effect of any error in a distance measurement (*d*) decreases as distance increases. The error of distance measured (in pixels), d_{px}, is the main source of the absolute error when considering relevant measurement distances that are 0.3 m and over. The vertical size of a frame (d_{fr}), i.e. the number of usable pixels, intensifies the effect of error in d_{px}. The minimum of absolute error (0.63 m/s) in measuring the velocity of 15 m/s can be found at the distance of 0.35 m, when the vertical size of the frame is 512 pixels.

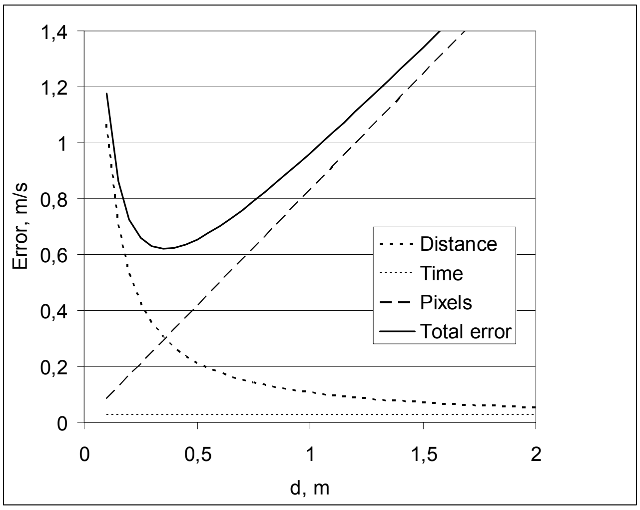


Figure 9. The effect of different components on the absolute error. Δd =7mm , Δd_{px}= 1 pixel, Δt=2μs, d_{fr}=512 pixels .

Relative maximum error can be expressed as $\varepsilon^* = \varepsilon / u$. (Råde and Westergren, 1998)

$$\varepsilon^* = \frac{\Delta d}{d} + \frac{\Delta d_{px}}{d_{px}} + \frac{\Delta t}{t} \qquad (3)$$

With the individual error assumptions presented in Fig. 9, the measurement distance can be optimized to minimize the relative error.

$$\frac{\partial \varepsilon^*}{\partial d} = -\frac{\Delta d}{d^2} + \frac{\Delta d_{px}\ 2\tan\alpha}{u\, d_{px}\, t} = 0 \qquad (4)$$

Fig. 10 presents the relative error as a function of distance d at different velocities and the minimum of the relative error when the vertical size of the frame is 1024 pixels. For example, in the case of a spray velocity of 10 m/s, the minimum of the relative error can be achieved at the distance of 0.35 m. The depth of focus of the optics is, in practice, from 300 mm to infinity (white area in Fig. 10) and the distance from the lens to the object is from 0.3 to 0.6 m due to the geometrical limitations of the measurement arrangement, i.e. a liquor gun hole in a boiler wall.

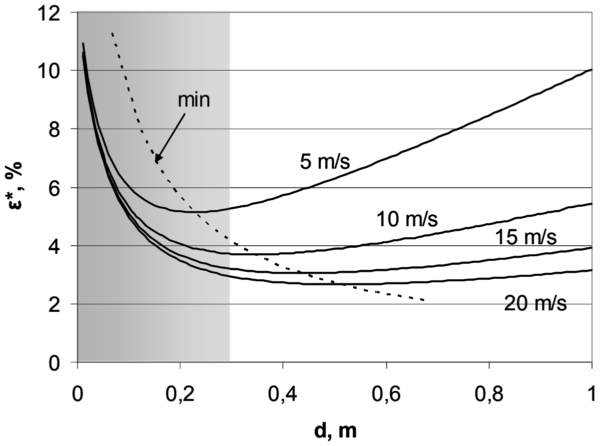


Figure 10. Relative error for different velocities as a function of distance from the lens to the object (d), and its minimum (dotted line). d_{fr}= 1024 pixels.

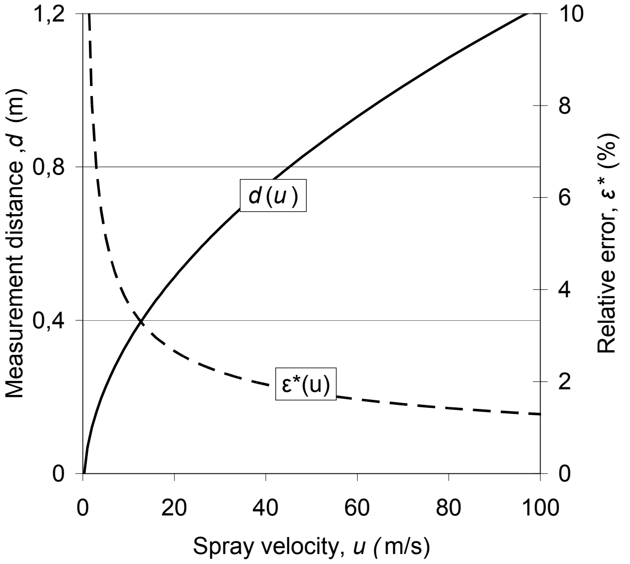


Figure 11. Optimum measurement distance (solid line) for different velocities for obtaining minimum relative error (dashed line). d_{fr}= 1024 pixels.

It can be seen that the measurements have been carried out within the range of minimum error. The relative error of the measurements has been from 3 to 7 % in the experiments when assuming that the individual errors are as presented in Fig. 9, depending on the vertical frame size. Fig. 11 presents the distance *d* in which the minimum error can be achieved and the corresponding relative error as a function of spray velocity. For example, a minimum error of measuring spray velocity of 30 m/s can be achieved at the distance of 0.6 m with ~ 2% of relative error.

Discussion and conclusions

An optical air-cooled probe for on-line visualization of a fuel spray, in this case black liquor spray, was developed and tested. The inflexible structure of the optics limited the measurement time and location in the boiler furnace. Heat-induced bending requires extra cooling breaks during the measurement period.

The furnace endoscope has been used successfully in several measurements and some endoscope improvements have been carried out, especially in the choice of the material of the protecting tube and in making the cooling air system more effective. A distance measurement system was integrated into the protecting tubes. Further development is needed in optics to increase the amount of light transmitted from the lens to the CCD-sensor. In the future work it would be worthwhile to study the usefulness of the different optics with filters in order to remove water vapor or flue gases disturbing the images.

As a measurement tool, the furnace endoscope is unique and enables the in-furnace measurement of spray velocity, shape of the spray, trajectory and spray disintegration mechanisms. The velocity measurement method has been analyzed further and the optimal distance from the optics to the object was determined. Also, the double-FFT method was tested against the velocity measurement of a rotating disk with known speed. The maximum relative error of velocity measurements was between 4 and 7%, which makes the method of measuring the spray properties with the furnace endoscope highly accurate. The accuracy depends mainly on the definition of the scale of the image. The wire-weight method for defining the scale is accurate only when the spray or the object beneath the furnace endoscope has an unambiguous surface.

The furnace endoscope is a useful and relatively inexpensive tool for improving the techniques for spraying black liquor into a recovery boiler. The spraying experiments in a spraying chamber can be compared to measurements in real-boiler conditions. There are several industrial applications available for the furnace endoscope. It has been tested with an integrated light supply system in the videotaping of the fouling of super heater tubes in the upper parts of a recovery boiler. Other possible applications may be to determine the local flow fields of furnaces and detect smelt spouts at the bottom of boilers.

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References

Asplund, D. (1997) The Status and Development Possibilities of Bioenergy in Energy Industry, in Finnish, Ministry of Trade and Industry, Energy Department, Finland

McCarty, B.C. and Lang, B. (2002) Furnace Cameras Assist in NOx Reduction, Power Engineering 106: 138-144.
Dobelin, E.O., (1990) Measurement Systems Application and Design (4th Edition), McGraw – Hill, Singapore, pp 58-59.

Fraser, R.P. (1956) Liquid Fuel Atomization, 6th Symposium on Combustion, Aug. 19-24, New Haven, Connecticut, pp. 687-701.

Helpiö, T., Kankkunen, A. (1996) The Effect of Black Liquor Firing Temperature on Atomization Performance, Tappi Journal Vol. 79, No 9, pp. 158-163.

Image Pro Plus Reference Guide for Windows (1998), FFT, Media Cybernetics, pp 209-219.

Kankkunen, A. and Nieminen, K. (1997) Black Liquor Sheet Breakup Mechanisms and the Effect on the Droplet Size, ICLASS ’97, 18.-22. Aug 1997, Korea.

Lu, G., Yan, Y. And Ward, D.D. (2000) Advanced Monitoring, Characterisation and evaluation of gas-fired flames in a utility boiler. Journal of the Institute of Energy 73: 43-49.

Merzkirch, W. (1987) Flow visualization, Academic Press, Inc. Orlando, Florida, USA, p. 51.

Miikkulainen, P., Järvinen, M. and Kankkunen A. (2000) Black Liquor Spray Properties in Operating Recovery Boiler Furnaces, INFUB 5th European Conference on Industrial Furnaces and Boilers, Porto – Portugal, 11-14 April.

Miikkulainen, P., Kankkunen, A. and Järvinen (2002a) M. The Effect of Excess Temperature and Flashing on Black Liquor Spray Properties, Finnish - Swedish Flame Days 2002, Vaasa, Finland, 24-25 September 2002.

Miikkulainen, P., Järvinen, M. and Kankkunen, A. (2002b) The Effect of a Furnace Environment on Black Liquor Spray Properties, Pulp&Paper Canada 103(9):T238-242 pp.34-38

Råde, L., Westergren, B. (1998) Beta, Mathematics Handbook for Science and Engineering, Studentlitteratur, Lund, Sweden, p. 347.

Spielbauer, T.M. and Aidun, C.K. (1992) Mechanism of Liquid Sheet Breakup and the Resulting Drop Size Distributions, Part 2: Strand Breakup and Experimental Observations. Tappi Journal. Vol 73 no 3, pp. 195-200.

Westerweel, J. (1997) Fundamentals of digital particle image velocimetry, Meas. Sci. Technol. 8 (1997) 1379-1392.