A Errata to [I]

The main result of [**I**] is the following claim:

> If ε, μ are simultaneously diagonalizable with eigenvalues ε_i, μ_j, the induced geometry is Riemannian if and only if $\varepsilon_i \mu_j = \varepsilon_j \mu_i$ for some $i \neq j$.

The "only if" direction of this claim does not follow from [**I**]: If $\Delta_{ij} = 0$ in some open set, the *given* Hamiltonians in [**I**] need not be smooth with respect to the location variable. The aim of this errata is to prove that with a slight modification of the Hamiltonian functions, the claim holds.

Let us start with a counter-example. Suppose the media is defined by $m_i = 1$ for $i = 1, 2, 3$, $e_1 = 1 + x$, and $e_2 = e_3 = 1 - x$. In this case, $\Delta_{23} = 0$, $\Delta_{12} = \Delta_{13} = 4x$, and by Lemma 4.1 *(ii)*, the Hamiltonians are not smooth at $x = 0$.

To illustrate the underlying problem, suppose $\Delta_{23} = 0$. Then $\operatorname{sign} \Delta_{12} = \operatorname{sign} \Delta_{13}$ by Lemma 4.1. If $\Delta_{12} \geq 0$, the Hamiltonians are given by

$$\begin{aligned} h_+ &= \|\operatorname{diag}\,(e_2 m_3, e_1 m_3, e_1 m_2) \cdot \xi\|, \\ h_- &= \|\operatorname{diag}\,(e_3 m_2, e_3 m_1, e_2 m_1) \cdot \xi\|. \end{aligned}$$

Similarly, if $\Delta_{12} \leq 0$, the same expressions hold, but h_+ and h_- exchange places. (This follows from the definition of $h_\pm$ and using Lemma 4.1 *(i)–(ii)*.) Thus, if the two expressions for h_+ are not compatible near $\Delta_{12} = 0$, then h_+ will not be smooth.

To address this problem, one need to pointwise redefine the Hamiltonian functions as described in Section 4.1: If $\Delta_{ij} = 0$ on an open set U, and i, j, k are distinct, let use introduce new Hamiltonians $\tilde{h}_\pm$ as follows

$$\tilde{h}_\pm = h_{\pm \operatorname{sign} \Delta_{kj}}. \tag{29}$$

Suppose $\Delta_{23} = 0$ in U. Then, by the above properties for $h_\pm$, we have

$$\begin{aligned} \tilde{h}_+ &= \|\operatorname{diag}\,(e_2 m_3, e_1 m_3, e_1 m_2) \cdot \xi\|, \\ \tilde{h}_- &= \|\operatorname{diag}\,(e_3 m_2, e_3 m_1, e_2 m_1) \cdot \xi\|, \end{aligned}$$

and these expressions are independent of $\operatorname{sign} \Delta_{12}$. Since $\tilde{h}_\pm$ satisfy the geometrization assumptions on the Hamiltonians, the result follows for $i = 2$, $j = 3$. The other combinations are analogous.

With these modified Hamiltonians, one can remove the assumption Δ_{12}, $\Delta_{13} > 0$ from Example 5.2 and 5.4. Equation (29) can be used to translate Proposition 4.4.