P. M. T. Ikonen, S. I. Maslovski, C. R. Simovski, and S. A. Tretyakov, On artificial magnetodielectric loading for improving the impedance bandwidth properties of microstrip antennas, IEEE Transactions on Antennas and Propagation, vol. 54, no. 6, pp. 1654-1662, 2006.

On Artificial Magnetodielectric Loading for Improving the Impedance Bandwidth Properties of Microstrip Antennas

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***Abstract*—The effect of artificial magnetodielectric substrates on the impedance bandwidth properties of microstrip antennas is discussed. We review the results found in the literature and then focus on practically realizable artificial magnetic media operating in the microwave regime. Next, a realistic dispersive behavior of a practically realizable artificial substrate is embedded into the model. It is shown that frequency dispersion of the substrate plays a very important role in the impedance bandwidth characteristics of the loaded antenna. The impedance bandwidths of reduced size patch antennas loaded with dispersive magnetodielectric substrates and high-permittivity substrates are compared. It is shown that unlike substrates with dispersion-free permeability, practically realizable artificial substrates with dispersive magnetic permeability are not advantageous in antenna miniaturization. This conclusion is experimentally validated.**

***Index Terms*—Artificial magnetic materials, frequency dispersion, miniaturization, magnetodielectric substrate, patch antenna, quality factor.**

I. INTRODUCTION

For a long time microstrip antennas have been miniaturized using different materials [1]–[3]. Most traditionally, high permittivity dielectrics have been used to decrease the physical dimensions of the radiator [4], [5]. Common problems encountered with high permittivity substrates include, e.g., the excitation of surface waves leading to lowered radiation efficiency and pattern degradation, and difficulties in impedance matching of the antenna. Recently, artificial high-permeability materials working in the microwave regime have gained increasing attention [6]–[13]. The possibility to create artificial magnetism at microwave frequencies has generated discussion on the possibility to enhance the impedance bandwidth properties of planar radiators using magnetodielectric substrates [14]–[22] or other electromagnetically exotic substrates [23]–[25].

According to the work of Hansen and Burke [26], inductive (magnetic) loading leads to an efficient size reduction of a microstrip antenna. When the material parameters of the antenna substrate are *dispersion-free*, and $\mu_{\rm eff} \gg \epsilon_{\rm eff}, \mu_{\rm eff} \gg 1$, a transmission-line (TL) model for a normal $\lambda/2$-patch antenna predicts that the impedance bandwidth is retained after miniaturization [26]. Edvardsson [27] derived a condition for the radiation quality factor $Q_{\rm r}$ of a planar inverted F-antenna (PIFA), and concluded that a high-permeability substrate allows size reduction without increasing $Q_{\rm r}$. However, to draw a general conclusion on the practical applicability of the magnetic loading there are three issues which need to be clarified:

i) The effect of frequency dispersion of the substrate. Practically realizable *artificial* magnetic media[1] operating at microwave frequencies are composed of electrically small resonating metal unit cells, thus, the magnetodielectric substrate obeys strong frequency dispersion. When the resonant substrate is coupled to the antenna element the overall performance is not obvious to predict. ii) The effect of high $\epsilon_{\rm eff}$ compared to high $\mu_{\rm eff}$ over the matching band of the antenna. With practically realizable artificial magnetodielectric substrates $\mathrm{Re}\{\mu_{\rm eff}\}$ is known to be rather moderate, thus in practice it seems very difficult to achieve condition $\mu_{\rm eff} \gg \epsilon_{\rm eff}, \mu_{\rm eff} \gg 1$ (outside the resonant region of the substrate). iii) The effect of losses. A realistic model representing the losses of the substrate is needed to correctly predict the behavior of the susceptance seen at the antenna terminal, and the quality factor.

Moreover, the numerical and experimental results found in the literature [14]–[22] do not offer a complete, quantitative comparison scheme applicable for practical antenna design. The impedance bandwidths (quality factors) of reduced size antennas utilizing practically realizable, artificial, dispersive magnetodielectric substrates have not been compared to the results obtained using high-permittivity dielectrics leading to the same size reduction. This quantitative comparison is clearly needed to make decisions whether or not the possibly enhanced impedance bandwidth outweighs the increased manufacturing cost and substrate weight.

The rest of the paper is organized in the following way: In Section II we briefly review the numerical and experimental results found in the literature for magnetodielectric loading of microstrip antennas. Section III presents a short discussion on practically realizable artificial magnetic media. The TL-model is presented in detail in Section IV. Section V presents the calcu-

Manuscript received August 21, 2005; revised January 2, 2006.

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Digital Object Identifier 10.1109/TAP.2006.875912

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[1] Paramagnetic response at microwave frequencies can also be achieved using hexaferrites or composite materials containing ferromagnetic inclusions [28], [29]. The utilization of these materials at microwave frequencies is, however, limited by either weak magnetic response or strong losses.

lated impedance bandwidth properties in different loading scenarios, and an experimental demonstration is presented in Section VI. The work is concluded in Section VII.

II. REVISION OF NUMERICAL AND EXPERIMENTAL RESULTS

A lot of numerical results can be found in the literature for antenna miniaturization using magnetodielectric substrates [14]–[17]. In most of the works the constant scalar permeability assumption is used [14]–[16] and the authors reproduce the qualitative result obtained by Hansen and Burke [26].

In [17] Kärkkäinen *et al.* numerically studied a PIFA with *dispersive* magnetic material filling. The authors pointed out the need to regard the loaded radiator as a system of two coupled resonators, rather than a radiator loaded with a static paramagnetic load. It was concluded that a material utilized considerably below its resonance enables size reduction while approximately retaining the fractional bandwidth. The $\epsilon_{\rm eff}$ of the substrate was assumed to be unity, and comparative simulations with conventional dielectric substrates offering the same size reduction were not conducted.

Most of the experimental results found in the literature do not contain proper comparison between different substrates. Results found in [18]–[20] indicate that artificial magnetodielectric substrates offer significant miniaturization factors while approximately retaining the impedance bandwidth. However, in [18]–[20] the impedance bandwidths were not compared to those obtained using conventional dielectric substrates offering the same size reduction. Moreover, in [18] the radiation efficiency was measured to be only 21.6%. Authors of [21] loaded a PIFA with an artificial magnetodielectric material and low-permittivity dielectrics, and concluded that artificial magnetodielectric material would behave as a paramagnetic load enabling efficient antenna miniaturization.[2] The authors of [22] loaded the volume under a $\lambda/2$-patch antenna with an artificial magnetodielectric substrate and compared the impedance bandwidth to that obtained using high-permittivity dielectrics. The result indicated that there is practically no advantage when using the magnetodielectric substrate.

Attempts have also been conducted to reduce the size of planar antennas by partially filling the volume under the radiating element with backward-wave materials [30]–[32]. With patch antennas, in practise, filling the volume with backward-wave material corresponds to inserting bulk inductors and capacitors to the antenna [32], thus the technique is very similar to the well-known technique of size miniaturization using reactive loads. Moreover, as expected, material obeying realistic dispersive behavior results in significantly narrower impedance bandwidth than the hypothetical material with $\epsilon_{\rm eff} = \mu_{\rm eff} = -1$ [32].

III. ON PRACTICALLY REALIZABLE ARTIFICIAL MAGNETIC MEDIA AT MICROWAVE FREQUENCIES

Resonant artificial magnetic media can be utilized with planar radiators in two ways [21]. First, if the resonance of the material lies inside the operational band of the (loaded) antenna, and can be combined with the antenna resonance, a multiresonant antenna is achieved. In the second approach the material is designed to resonate at a considerably higher frequency than the operational frequency of the loaded antenna. In the latter case it is important that the utilized material retains its effective magnetic properties over a wide frequency band, meaning that inside the operational band of the radiator $\mathrm{Re}\{\mu_{\rm eff}\} > 1$ (low losses are another desired feature).

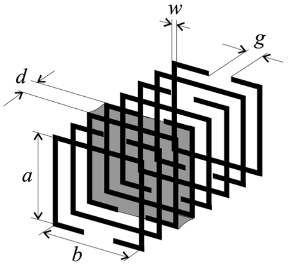

Fig. 1. Schematic illustration of the metasolenoid.

With most of the known designs for artificial magnetic media operating in the microwave regime [6]–[13] the effective magnetism rapidly vanishes as the frequency deviates from the particle resonance. Usually the maximum value obtained in practise (with realistic bulk concentration/volume filling ratio and loss factor) is $\mathrm{Re}\{\mu_{\rm eff}\} = 1.5 - 4$ at the resonance [6], [7], [10]. Thus, one should utilize particles rather close to their resonance to achieve condition $\mathrm{Re}\{\mu_{\rm eff}\} > 1$. This further increases the effect of frequency dispersion on the impedance bandwidth characteristics. Moreover, it might not be clear anymore if the loaded antenna should be considered as a system of two coupled resonators rather than a resonator filled with a homogenous, dispersion-free material.

Usually $\mathrm{Re}\{\epsilon_{\rm eff}\}$ of artificial magnetodielectric substrates is considerably higher than $\mathrm{Re}\{\mu_{\rm eff}\}$ (when the substrate is utilized well below its resonance). Thus, at microwave frequencies the condition considered in [26] seems unlikely to be achieved. For example, with the structure presented in [12] a realistic value for the real part of the effective permeability over the operational band of a patch antenna is $\mathrm{Re}\{\mu_{\rm eff}\} \simeq 1.5$. However, at the same time the real part of the effective permittivity can be as high as $\mathrm{Re}\{\epsilon_{\rm eff}\} \simeq 8.0$, even if the host substrate has a low value for the relative permittivity.

To give a link between the presented effective medium model and practical design for artificial magnetic media, we consider the metasolenoid [13] as an example particle constituting the medium. Fig. 1 introduces the structural geometry of the metasolenoid. The metasolenoid is chosen due to its simple structural geometry enabling rather straightforward, yet accurate theoretical analysis. Moreover, experimental results can be found in the literature for the utilization of the metasolenoid under microstrip antennas [21], [22], and the effective medium model has been experimentally validated and found to be accurate [13].

IV. TRANSMISSION-LINE MODEL

In this section we introduce the procedure for determining the impedance bandwidth characteristics of an arbitrary size strip

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[2] The figure of merit used to evaluate the performance of a PIFA with different material fillings is incorrectly calculated in [21]. When the figure of merit is calculated using the radiation quality factor $Q_{\rm r}$, air filling leads to the best figure of merit.

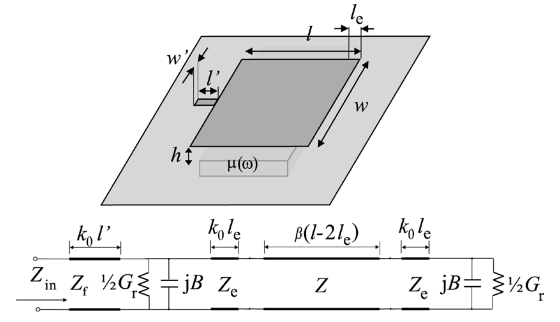

Fig. 2. Schematic illustration of the antenna geometry and the equivalent circuit of a strip fed antenna.

fed patch antenna. A schematic illustration of the antenna structure and the equivalent TL-model are presented in Fig. 2. The characteristic impedance of a wide microstrip line reads [33]

$$Z = \frac{\eta_0 h}{w}\sqrt{\frac{\mu_{\rm eff}}{\epsilon_{\rm eff}}} \tag{1}$$

where η_0 is the wave impedance in free space, h is the height of the substrate (assumed to be the same as the distance from the patch to the ground plane), w is the width of the patch, and $\mu_{\rm eff}$ and $\epsilon_{\rm eff}$ are the effective substrate material parameters. In the present model we neglect the effect of the material filling on the radiation conductance and the shunt susceptance. This is possible if the material filling leaves the ends of the patch free so that there is a short (about the patch height) section of the empty TL just near the radiating edges (in the present analysis the length of the empty section $l_{\rm e} = h/2$). With this assumption the radiation conductance and shunt susceptance read [1], [3]

$$G_{\rm r} = \frac{1}{90}\left(\frac{w}{\lambda_0}\right)^2, \quad w < 0.35\lambda_0$$

$$G_{\rm r} = \frac{1}{120}\frac{w}{\lambda_0} - \frac{1}{60\pi^2}, \quad 0.35\lambda_0 \le w \le 2\lambda_0 \tag{2}$$

$$B = 2\pi\frac{\Delta l}{Z\lambda_0} \tag{3}$$

where λ_0 is the wavelength in free space. In (3) Δl is the open end extension of the TL (with a wide microstrip $\Delta l \approx h/2$). The propagation factor in the antenna part of the TL reads

$$\beta = k_0\sqrt{\mu_{\rm eff}\epsilon_{\rm eff}} \tag{4}$$

where k_0 is the free space wave number. The impedance of the (narrow) feed line is [34]

$$Z_{\rm f} = \frac{42.5}{\sqrt{2}}\ln\left[1 + \frac{4h}{w'} \times \left(\frac{8h}{w'} + \sqrt{\left(\frac{8h}{w'}\right)^2 + \pi^2}\right)\right] \tag{5}$$

where w' is the width of the feed strip. The input impedance $Z_{\rm in}$, and the reflection coefficient ρ are calculated using the standard transmission-line equations.

To relate the unloaded quality factor to the return loss characteristics, we regard the antenna as a parallel-resonant RLC circuit in the vicinity of the fundamental resonant frequency [35]. Let us assume that the criterion for the maximum VSWR inside of the band of interest is

$$\text{VSWR} \le S. \tag{6}$$

In this case the fractional bandwidth can be expressed as [35]

$$BW = \frac{1}{Q_0}\sqrt{\frac{(TS-1)(S-T)}{S}}, \tag{7}$$

where Q_0 is the unloaded quality factor of the parallel resonator, and T is a coupling coefficient describing the quality of matching between the source and the resonator. Knowing the dispersive behavior of the input return loss, Q_0 can be solved from (7).

The dispersive behavior of the metasolenoid array (see Fig. 1) is of the following form [13]:

$$\mu_{\rm eff} = 1 - V_{\rm r}\frac{j\omega\mu_0\Lambda}{Z_{\rm tot}d} \tag{8}$$

where $\Lambda = ab$ is the cross-section area of the metasolenoid, and $V_{\rm r}$ is a coefficient taking into account realistic bulk concentration. The total impedance of the metasolenoid reads [13]:

$$Z_{\rm tot} = j\omega L_{\rm eff} + \frac{1}{j\omega C_{\rm eff}} + R_{\rm eff}. \tag{9}$$

The effective permittivity of the metasolenoid array does not equal to that of the host substrate relative permittivity. In the vicinity of the magnetic resonance the effective permittivity is weakly dispersive, but can be assumed to be constant.

V. IMPEDANCE BANDWIDTH PROPERTIES

A. *Dispersion-Free Material Parameters*

In the first case we load the volume between the patch element and the ground plane with hypothetical substrates having lossless and dispersion-free material parameters. The following substrate material parameters are considered: 1) $\epsilon_{\rm eff} = \mu_{\rm eff} = 1$, 2) $\epsilon_{\rm eff} = 1, \mu_{\rm eff} = 8$, 3) $\epsilon_{\rm eff} = 6.75, \mu_{\rm eff} = 1$, 4) $\epsilon_{\rm eff} = \mu_{\rm eff} = 2.65$. The empty antenna has dimensions (see Fig. 2) $l = w = 48.5$ mm, $h = 4$ mm, $l' = 15$ mm, $w' = 8.4$ mm, and it resonates at 3.0 GHz. The dimensions of the patch loaded with high-μ, high-ϵ, and magnetodielectric substrates are the following: $l = w = 22.5$ mm, $h = 4$ mm, $l' = 15$ mm, $w' = 1.5$ mm (high-μ substrate), $w' = 1.2$ mm (high-ϵ substrate and magnetodielectric substrate). Feed strip width w' is used to tune the quality of coupling in different loading scenarios. With all the loading scenarios T has been tuned to $T = T_{\rm opt} = 1/2(S + 1/S)$ [35] by varying w'.

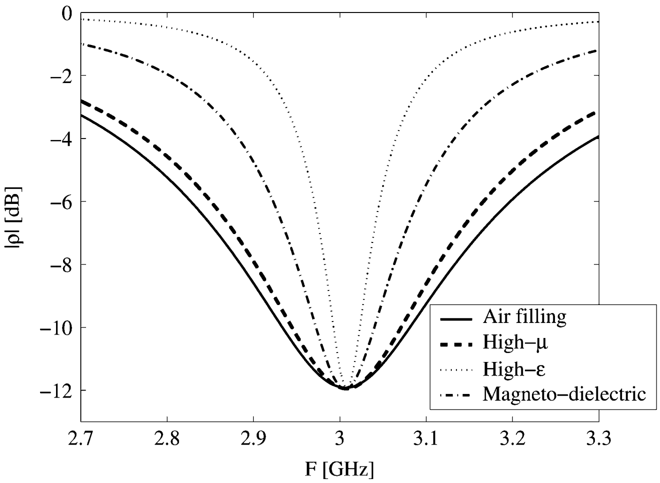

Fig. 3. Calculated reflection coefficient with different material fillings. Dispersion-free μ_{eff}.

TABLE I
CALCULATED IMPEDANCE BANDWIDTH CHARACTERISTICS. DISPERSION-FREE μ_{eff}

Loading	V cm^3	$BW\|_{-6\mathrm{dB}}$ percent	Q_0
Air filling	9.4	12.3	10.9
High-μ	2.0	10.5	12.8
High-ϵ	2.0	2.5	53.2
Magneto-dielectric	2.0	5.6	24.0

Fig. 3 shows the calculated reflection coefficients. The main calculated parameters are gathered in Table I. The unloaded quality factor obtained from (7) has been calculated using −6 dB matching criterion. We can observe that the impedance bandwidth properties behave as predicted in [26]. Dispersion-free high-μ materials allow size miniaturization while practically retaining the impedance bandwidth, whereas with high-ϵ materials the impedance bandwidth suffers significantly.

B. *Dispersive Material Parameters*

We first calculate the impedance bandwidth properties when the antenna is loaded with a substrate having practically realizable material parameter values. The dispersive behavior of μ_{eff} is shown in Fig. 4 [13]. We can see that $\mathrm{Re}\{\mu_{\mathrm{eff}}\} = 1.21$ at 3.0 GHz. The effective permittivity of the substrate is estimated to be $\epsilon_{\mathrm{eff}} = 8.5(1 - j0.001)$. For the reference dielectric substrate offering the same size reduction $\epsilon_{\mathrm{r}}^{\mathrm{ref}} = 10.1(1 - j.001)$, $\mu_{\mathrm{r}}^{\mathrm{ref}} = 1$. For comparison we also consider a loading scenario in which the dispersive magnetodielectric substrate is replaced with a substrate having *dispersion-free* material parameters $\mu_{\mathrm{eff}} = 1.21(1 - j0.0024)$ (picked up from the dispersion curve at the operational frequency of the loaded antenna), $\epsilon_{\mathrm{eff}} = 8.5(1 - j0.001)$. The dimensions of the loaded patches are the following: $l = w = 19.3$ mm, $h = 4$ mm, $l' = 15$ mm, $w' = 0.9$ mm (magnetodielectric substrates), $w' = 0.75$ mm (reference dielectric substrate).

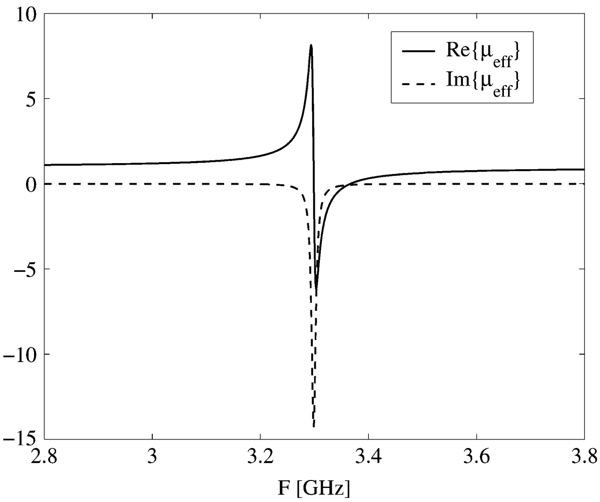

Fig. 4. Dispersive behavior of μ_{eff} of a practically realizable substrate.

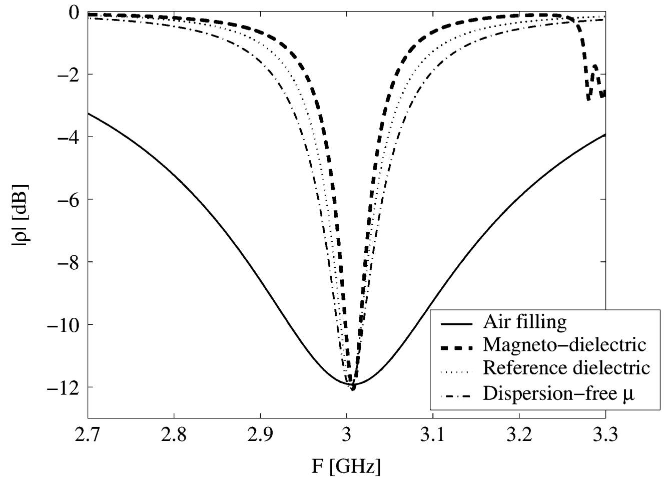

Fig. 5. Calculated reflection coefficient with different material fillings. Practically realizable example.

TABLE II
CALCULATED IMPEDANCE BANDWIDTH CHARACTERISTICS. PRACTICALLY REALIZABLE EXAMPLE

Loading	V cm^3	$BW\|_{-6\mathrm{dB}}$ percent	Q_0
Air filling	9.4	12.3	10.9
Magneto-dielectric	1.5	1.4	93.7
Reference dielectric	1.5	1.9	69.7
Dispersion-free μ	1.5	2.5	54.2

Fig. 5 shows the calculated reflection coefficient. The main calculated parameters are gathered in Table II. The obtained result indicates that the practically realizable magnetodielectric substrate offers no advantages over high-permittivity dielectrics, even if the dispersion is weak. If the scalar constant-permeability assumption is used, the TL-model predicts wider impedance bandwidth with magnetodielectrics than with pure high-permittivity dielectrics, also in the case when $\mathrm{Re}\{\mu_{\mathrm{eff}}\} \ll \mathrm{Re}\{\epsilon_{\mathrm{eff}}\}$. This result is in line with the general opinion in the literature (based on dispersion-free material parameters). However, when thinking of practical antenna design, the dispersion-free assumption gives a misleading impression on the applicability of artificial magnetodielectric substrates.

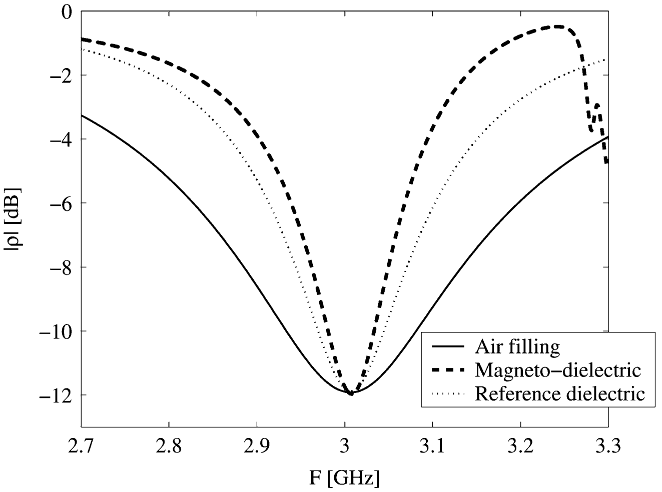

Fig. 6. Calculated reflection coefficient with different material fillings. $\epsilon_{\mathrm{eff}} = 1$ for the magnetodielectric substrate.

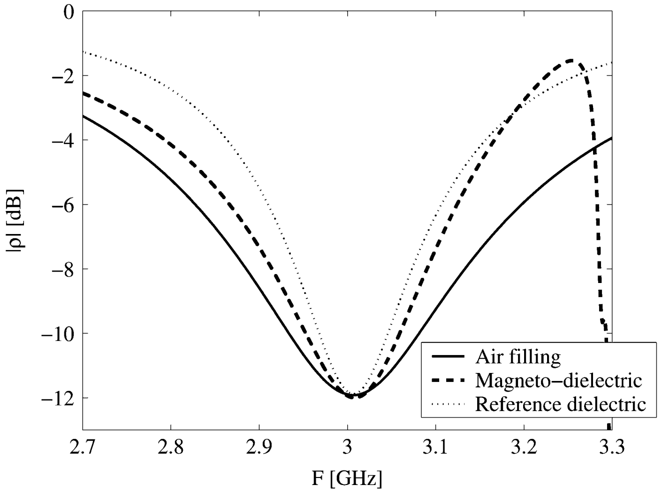

Fig. 7. Calculated reflection coefficient with different material fillings. Static value for $\mathrm{Re}\{\mu_{\mathrm{eff}}\} = 2$, $\epsilon_{\mathrm{eff}} = 1$ for the magnetodielectric substrate.

TABLE III
CALCULATED IMPEDANCE BANDWIDTH CHARACTERISTICS. $\epsilon_{\mathrm{eff}} = 1$ FOR THE MAGNETODIELECTRIC SUBSTRATE

Loading	V cm^3	$BW\vert_{-6\mathrm{dB}}$ percent	Q_0
Air filling	9.4	12.3	10.9
Magneto-dielectric	7.8	4.3	18.5
Reference dielectric	7.8	6.3	12.6

TABLE IV
CALCULATED IMPEDANCE BANDWIDTH CHARACTERISTICS. STATIC VALUE FOR $\mathrm{Re}\{\mu_{\mathrm{eff}}\} = 2$ FOR THE MAGNETODIELECTRIC SUBSTRATE

Loading	V cm^3	$BW\vert_{-6\mathrm{dB}}$ percent	Q_0
Air filling	9.4	12.3	10.9
Magneto-dielectric	5.0	8.6	15.5
Reference dielectric	5.0	6.5	20.5

Next, we consider the role of high $\mathrm{Re}\{\epsilon_{\mathrm{eff}}\}/\mathrm{Re}\{\mu_{\mathrm{eff}}\}$ added to dispersive μ_{eff}. The aim is to reveal if high ϵ_{eff} and dispersive μ_{eff} together lead to the undesired performance. We load the antenna with dispersive μ_{eff} (corresponding to a practically realizable substrate) and assume that $\mathrm{Re}\{\epsilon_{\mathrm{eff}}\}$ of the substrate is unity.[3] The size of the loaded antenna is $l = w = 44.2$ mm, the feed strip width in both loading scenarios (magnetodielectric and reference dielectric) is $w' = 2.0$ mm. The reference dielectric substrate has $\epsilon_{\mathrm{r}}^{\mathrm{ref}} = 1.19(1 - j.001)$.

Fig. 6 shows the calculated reflection coefficient. The main calculated parameters are gathered in Table III. The result indicates that there is no advantage in using magnetodielectric substrates obeying the modified Lorentzian type dispersion rule, even if $\mathrm{Re}\{\epsilon_{\mathrm{eff}}\} = 1$, and the dispersion in μ_{eff} is weak. Thus, a high value for $\mathrm{Re}\{\epsilon_{\mathrm{eff}}\}/\mathrm{Re}\{\mu_{\mathrm{eff}}\}$ is not the factor which eventually deteriorates the performance of the magnetodielectric substrate.

The above discussion holds for artificial magnetodielectric substrates whose static value for $\mathrm{Re}\{\mu_{\mathrm{eff}}\} = 1$. According to the Rozanov limit [38] for the thickness to bandwidth ratio of radar absorbers, the thickness of the absorber at microwave frequencies (with a given reflectivity level) is bounded by the *static* value of μ of the absorber. To reveal the influence of static μ on the impedance bandwidth of a microstrip antenna, we study a weakly dispersive magnetodielectric substrate with static value $\mathrm{Re}\{\mu_{\mathrm{eff}}\} > 1$. For example, the permeability of composite materials containing thin ferromagnetic films can be modeled using the Lorentzian type dispersion rule, while with these materials static $\mathrm{Re}\{\mu_{\mathrm{eff}}\} > 1$ is achievable [28], [29]. We assume that static $\mathrm{Re}\{\mu_{\mathrm{eff}}\} = 2$, otherwise the dispersive behavior of μ_{eff} is the same as depicted in Fig. 4. Moreover, $\epsilon_{\mathrm{eff}} = 1$.

The dimensions of the loaded patches are the following: $l = w = 33.5$ mm, $h = 4$ mm, $l' = 15$ mm, $w' = 4.4$ mm (for both substrates). The reference dielectric substrate has $\epsilon_{\mathrm{r}}^{\mathrm{ref}} = 2.14(1 - j.001)$. Fig. 7 shows the calculated reflection coefficient. The main calculated parameters are gathered in Table IV. We can observe, that enough below the resonance of the substrate the negative effect of the frequency dispersion in μ_{eff} is outweighed by the positive effect of increased static value for $\mathrm{Re}\{\mu_{\mathrm{eff}}\}$. Thus it seems that weak frequency dispersion modulating a static value $\mathrm{Re}\{\mu_{\mathrm{eff}}\} > 1$ leads qualitatively to the desired performance of hypothetical, high-permeability dispersion-free substrates.

C. Relative Radiation Quality Factor

Consider a $\lambda/2$ long section of a transmission line (modeling a resonant patch element) filled with a certain low-loss material having material parameters μ, ϵ, in general dispersive. To find the energy stored under the patch we use the known relation for the volume density of electromagnetic field energy

$$w_{\mathrm{em}} = \frac{\epsilon_0 \partial(\omega\epsilon)}{\partial\omega}\frac{E_{\mathrm{m}}^2}{4} + \frac{\mu_0 \partial(\omega\mu)}{\partial\omega}\frac{H_{\mathrm{m}}^2}{4}. \qquad (10)$$

Above E_{m} and H_{m} are the amplitudes (real) of the electric and magnetic fields. The total energy stored under the patch can be

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[3] In principle condition $\mathrm{Re}\{\epsilon_{\mathrm{eff}}\} = 1$ can be realized, e.g., using the wire medium [36], [37]. However, condition $\mathrm{Re}\{\epsilon_{\mathrm{eff}}\}$ is fulfilled only at one frequency, and the realized effective permittivity is necessarily dispersive.

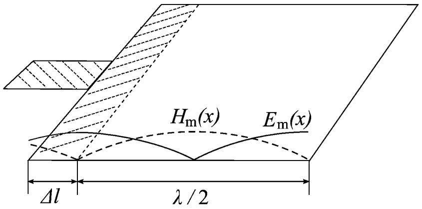

Fig. 8. Patch operating slightly above its parallel resonant frequency. The stroked areas represent the areas where additional reactive energy is stored.

found by integrating $w_{\rm em}$ over the volume under the patch. For a resonant patch this results in

$$W_{\lambda/2} = \frac{\pi Y}{16\omega}\left(\frac{1}{\mu}\frac{\partial(\omega\mu)}{\partial\omega} + \frac{1}{\epsilon}\frac{\partial(\omega\epsilon)}{\partial\omega}\right)U_{\rm max}^2. \qquad (11)$$

Here $U_{\rm max}$ is the voltage amplitude (real) at the voltage maximum of the standing wave, and Y is the characteristic admittance of the patch [given by (1)].

Typically a patch antenna is fed by a narrow microstrip line which can be considered as an additional series inductance for the equivalent circuit. Thus, the whole system becomes double-resonant and usually the antenna operates somewhere in between the series and parallel resonance. The second, series resonance appears at a bit higher frequency. This situation is outlined in Fig. 8. The total energy stored in the whole system at the series resonance is

$$W_{\rm tot} = W_{\lambda/2} + W_{\Delta l} + W_L \geq W_{\lambda/2} \qquad (12)$$

where $W_{\Delta l}$ is the additional energy stored under the part of the patch exceeding $\lambda/2$ length (parallel resonance), and W_L is the amount of energy stored in the effective inductor. If $\Delta l \ll \lambda/4$ the magnetic energy stored in the "additional" patch segment can be neglected. With this assumption we get

$$W_{\Delta l} \approx \frac{\pi Y}{2\omega\epsilon}\frac{\partial(\omega\epsilon)}{\partial\omega}\left(\frac{\Delta l}{\lambda}\right)U_{\rm max}^2. \qquad (13)$$

If the patch susceptance dominates over the radiation conductance the energy stored in the inductor reads

$$W_L = \frac{\pi Y}{2\omega}\left(\frac{\Delta l}{\lambda}\right)U_{\rm max}^2. \qquad (14)$$

From above the quality factor of a resonant $\lambda/2$ patch [which is the lowest possible quality factor in view of (12)] can be found as

$$Q_{\rm r} = \frac{\omega W_{\lambda/2}}{P_{\rm r}} = \frac{\pi Y}{8G_{\rm r}}\left(\frac{1}{\mu}\frac{\partial(\omega\mu)}{\partial\omega} + \frac{1}{\epsilon}\frac{\partial(\omega\epsilon)}{\partial\omega}\right). \qquad (15)$$

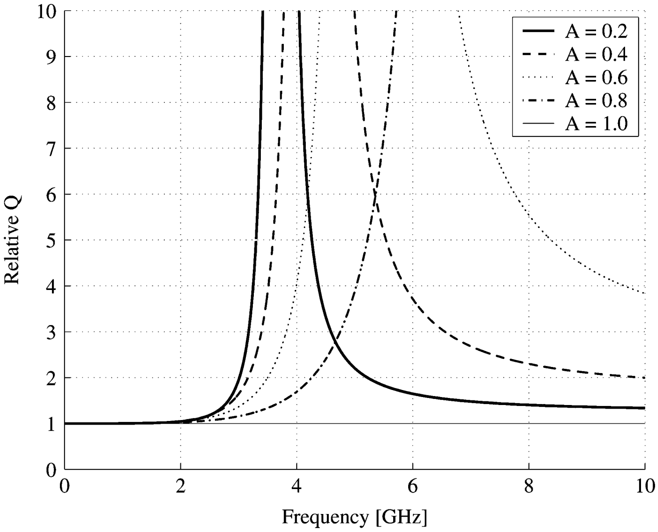

Fig. 9. Relative quality factor. Static $\mathrm{Re}\{\mu\} = 1$.

Here $G_{\rm r}$ is the radiation conductance and $P_{\rm r} = G_{\rm r}U_{\rm max}^2/2$ is the radiated power. For a patch antenna having the same dimensions and loaded with a reference, dispersion-free magnetodielectric material we have

$$Q_{\rm r}^{\rm ref} = \frac{\pi Y^{\rm ref}}{4G_{\rm r}} \qquad (16)$$

where $Y^{\rm ref}$ is the characteristic admittance of the reference antenna. For now on we only consider resonant $\lambda/2$ patches. Moreover, we load the patches with a dispersive magnetodielectric material (μ is dispersive, $\epsilon \approx$ constant), and with a conventional dielectric material offering the same size reduction ($\mu^{\rm ref}\epsilon^{\rm ref} = \epsilon^{\rm ref} = \mu\epsilon$). With these assumptions the ratio between the radiation quality factors can be expressed as

$$\frac{Q_{\rm r}}{Q_{\rm r}^{\rm ref}} = \frac{1}{2\mu}\left(\frac{1}{\mu}\frac{\partial(\omega\mu)}{\partial\omega} + 1\right). \qquad (17)$$

Furthermore, we assume the dispersion of μ to be covered by the modified (lossless) Lorentzian type dispersion rule

$$\mu = 1 + \frac{A\omega^2}{\omega_0^2 - \omega^2}, \qquad (18)$$

where A is the amplitude factor ($0 < A < 1$) and ω_0 is the undamped frequency of the zeroth pole pair.

The relative radiation quality factor [(17)] is presented with different amplitude factors in Fig. 9 ($f_0 = 3.3$ GHz). We can observe that if the static $\mathrm{Re}\{\mu\} = 1$, a substrate with modified Lorentzian type dispersion in μ leads always to larger $Q_{\rm r}$ than pure dielectrics offering the same size reduction (except in the limiting case with $A = 1$). However, if static $\mathrm{Re}\{\mu\} = 2$, there exist frequency bands over which $Q_{\rm r}$ obtained with magnetodielectric substrates is smaller than $Q_{\rm r}$ obtained using dielectrics, Fig. 10. Only close to ω_0 the strong frequency dispersion outweighs the effect of increased static $\mathrm{Re}\{\mu_{\rm eff}\}$.

Physical understanding of the effect of frequency dispersion can be achieved also by inspecting the input impedance of loaded antennas. Fig. 11 presents the calculated input

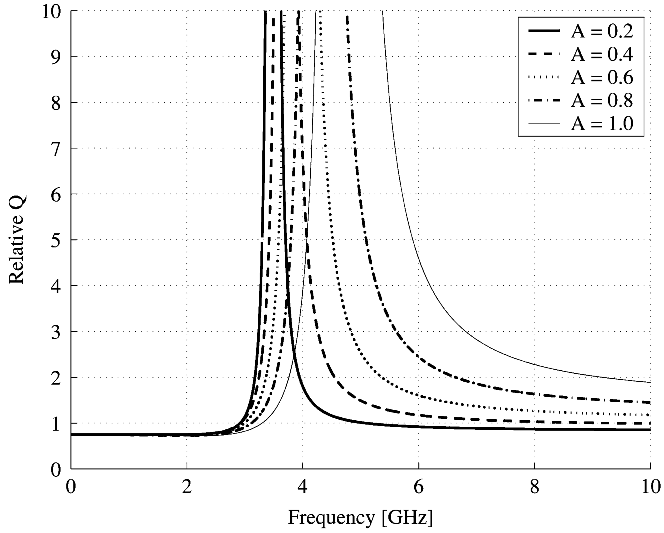

Fig. 10. Relative quality factor. Static $\mathrm{Re}\{\mu\} = 2$.

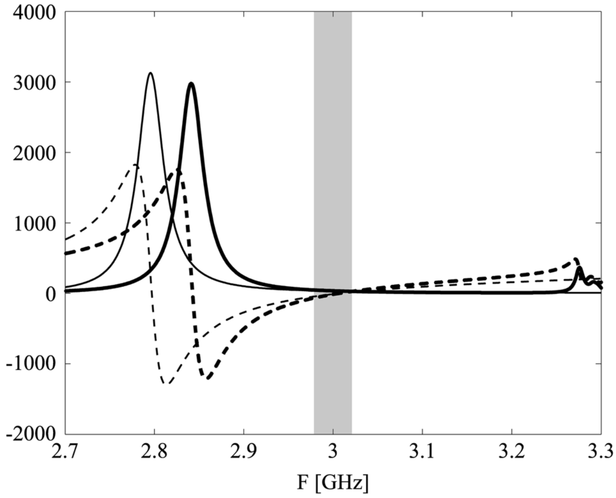

Fig. 11. Calculated input impedance for the practically realizable case. Thick lines are for the magnetodielectric substrate, thin lines for high-permittivity substrate. The grey region denotes the -6 dB matching band of the antennas.

impedance corresponding to the input return loss presented in Fig. 5. The known definition for the quality factor of a resonator near its parallel resonance reads

$$Q = \frac{\omega W}{P} = \frac{\omega}{2G}\frac{\partial B}{\partial \omega}. \tag{19}$$

Above, ω is the angular frequency, W denotes the amount of stored energy in the volume defined by the near fields of the antenna, P is the power dissipated during one cycle, B is the susceptance of the resonator, and G represents loss conductance. In the case of a series resonance, B is replaced by the reactance X, and G by the loss resistance R. Physically the behavior of the magnetodielectric substrate is understandable from Fig. 11 and (19): The input reactance changes more strongly at the resonance when using the magnetodielectric substrate. This is due to the material resonance appearing at 3.3 GHz. In terms of electromagnetic energy this rapid change in the input reactance corresponds to increased effect of reactive near fields leading to increased amount of stored energy, thus to increased quality factor.

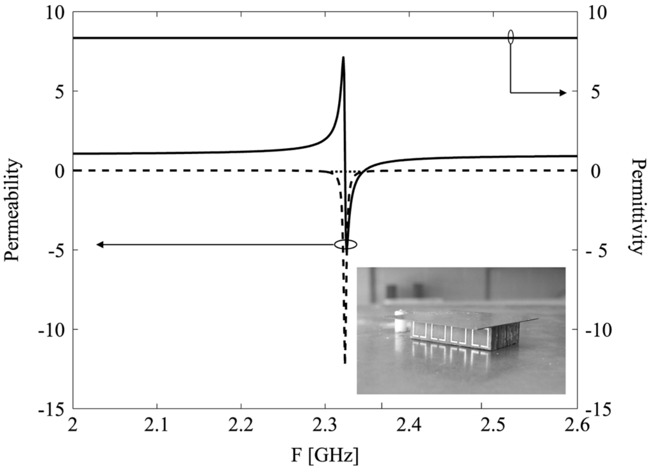

Fig. 12. Estimated material parameters for the implemented magnetodielectric substrate and the manufactured prototype antenna. Solid lines for the real parts, dashed lines for the imaginary parts.

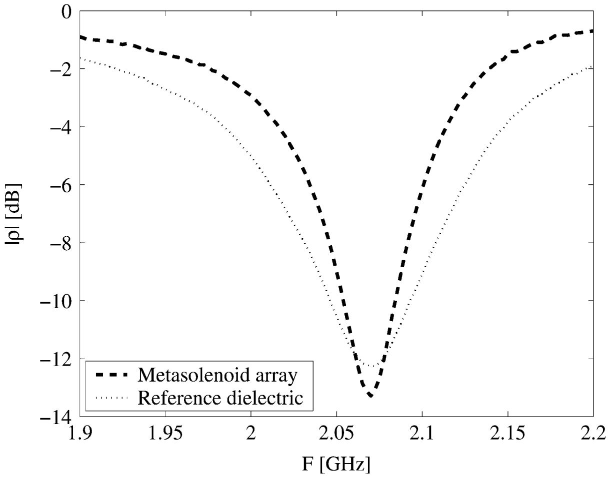

Fig. 13. Measured reflection coefficient with different material fillings.

VI. EXPERIMENTAL DEMONSTRATION

We manufacture a prototype antenna and load the volume under the antenna with an array of metasolenoids. A reference measurement (dielectrics) is also conducted. The estimated dispersive behavior of the manufactured metasolenoid array and a photograph of the prototype antenna are shown in Fig. 12. At the operational frequency of the loaded antenna ($F = 2.07$ GHz) we estimate that $\mathrm{Re}\{\mu_{\mathrm{eff}}\} = 1.25$ and $\mathrm{Re}\{\epsilon_{\mathrm{eff}}\} = 8.5$ (the host substrate for the metasolenoids is Rogers R/T Duroid 5870). The dimensions of the loaded antenna are $l = w = 35$ mm, $h = 7.5$ mm, $l' = 10$ mm, $w' = 3$ mm. The relative permittivity for the reference dielectric is $\epsilon_{\mathrm{r}}^{\mathrm{ref}} = 10.8(1 - j.0037)$.

Fig. 13 and Table V show the measured reflection coefficient and gather the main measured parameters. The radiation efficiency η_{rad} has been measured using the Wheeler cap method. The unloaded quality factor obtained from (7) has been calculated using -6 dB matching criterion.

TABLE V
MAIN MEASURED PARAMETERS

Loading	V cm^3	$BW\big\vert_{-6\mathrm{dB}}$ percent	Q_0	η_{rad} percent
Metasol. array	9.2	3.2	41.5	89
Reference dielectric	9.2	5.5	24.3	92

VII. CONCLUSION

In the present paper we have systematically studied the effect of artificial magnetodielectric substrates on the impedance bandwidth properties of microstrip antennas. It has been shown that with artificial magnetodielectric substrates obeying the modified Lorentzian type dispersion for μ_{eff}, frequency dispersion can not be neglected in the analysis. A relation has been derived for the ratio between radiation quality factors of ideally shaped antennas loaded with dispersive magnetodielectrics and dispersion-free reference dielectrics. The result shows that artificial magnetodielectric substrates lead always to larger radiation quality factor if static $\mathrm{Re}\{\mu_{\mathrm{eff}}\} = 1$. The main observation on the negative effect of frequency dispersion on the impedance bandwidth properties has been experimentally validated.

ACKNOWLEDGMENT

Inspiring discussions with Dr. M. Ermutlu, Dr. J. Ollikainen, and Prof. P. Vainikainen are warmly acknowledged. The authors thank Dr. M. Lapine for useful comments related to the manuscript.

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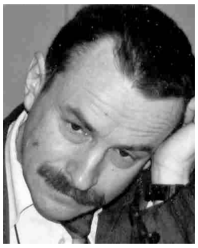

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