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Reducing Impact of Phase Noise on Accuracy of Measured MIMO Channel Capacity

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***Abstract*— Despite the fact that time-division multiplexed switching (TDMS) based multiple-input multiple-output (MIMO) wireless channel sounders are cost effective implementation techniques, they may result in significant measurement errors due to phase noise (PN) in the local oscillators. In this letter, the impact of PN on the accuracy of measured MIMO channel capacity is studied by considering its effect on both the spatial multiplexing gain and on the power gain. We show that in case of a low rank physical channel matrix the impact of PN is more pronounced on the spatial multiplexing gain than on the power gain. Based on that we propose an eigenvalue filtering (EVF) technique to improve the accuracy of measured channel capacity.**

***Index Terms*— Phase noise, Channel capacity, MIMO systems.**

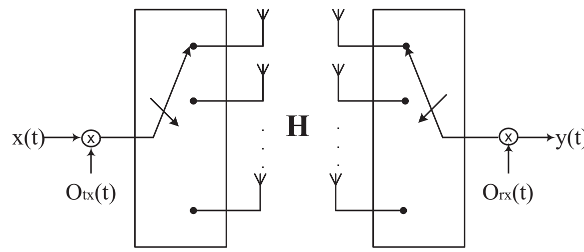

Fig. 1. Sketch of a TDMS based MIMO channel sounder.

I. Introduction

MULTIPLE-input multiple-output (MIMO) wireless technology promises significant channel capacity increase compared to the traditional single antenna transmission systems [1]. However, accurate MIMO channel capacity estimation based on measurement data is essential for successful MIMO system design and deployment. Time-division multiplexed switching (TDMS) of a single radio frequency chain systematically between the elements of transmit/recive antenna array is a widely used practical implementation technique of MIMO wireless channel measurement sounders [2]. Despite being a cost effective implementation technique, channel matrices measured with this kind of channel sounders are subject to significant measurement errors. In addition to the thermal additive white Gaussian noise, addressed in [3], phase noise (PN) in the local oscillators may result in significant channel measurement errors [4][5]. It was shown that this PN can result in deceptive channel capacity increase up to 100% [4]. In related studies the impact of PN on direction of arrival estimation was investigated in [6] for single-input multiple-output (SIMO) systems. In context of single antenna transmission schemes, the impact of measurement impairments was studied in [7].

This letter investigates the impact of PN on the accuracy of measured MIMO channel capacity by analyzing the error in the ergodic channel capacity due to PN impact. The impact of PN on the two MIMO gain mechanisms [8], namely, spatial multiplexing gain and power gain is considered. We show that under spatial multiplexing scheme the impact of PN is more pronounced on the spatial multiplexing gain than on the power gain and based on that we propose an eigenvalue filtering (EVF) technique to improve the accuracy of measured channel capacity. We support our theoretical observations with numerical results for known cases.

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II. MIMO channel measurement signal model

We consider a TDMS based MIMO channel sounder with N_t transmit antennas and N_r receive antennas, sketched in Figure 1. The sounder consists of a single radio frequency transmission chain with local oscillators at the transmitter and receiver each having an output signal given by: $O_{tx}(t) = \exp(j(2\pi f_{LO}t + \varphi_{tx}(t)))$ and $O_{rx}(t) = \exp(j(2\pi f_{LO}t + \varphi_{rx}(t)))$, respectively, where f_{LO} is the desired local frequency, φ_{tx} and φ_{rx} are the PN in the transmitter and receiver local oscillators, respectively. The single radio frequency chain is switched between the different transmit/receive antenna elements systematically during the measurement of one channel transfer matrix. Considering the case of a narrowband MIMO wireless system with synchronized frequency at both ends and assuming that the speed of the PN variations are faster than the measurement time of one physical channel matrix, the effective complex channel coefficient connecting j-th transmit antenna to i-th receive antenna can be written as [4]

$$\begin{aligned}\hat{h}(i,j) &= h(i,j)\exp(j(\varphi_{tx}(i,j) - \varphi_{rx}(i,j)) \\ &= h(i,j)\exp(j\varphi(i,j)) \qquad (1)\end{aligned}$$

where $h(i,j)$ is the complex physical channel coefficient and $\varphi(i,j)$ is the composite PN due to the transmitter and receiver local oscillators. In practice the PN depends on the characteristics of the phase locked loop, the switching speed and the number of transmit and receive antennas. However, the PN is widely modeled as a Gaussian wide sense stationary process with zero mean and σ_φ^2 variance [9].

Using the measured effective channel matrix for link level studies, the received signal at each receive antenna can be written as

$$\begin{aligned} \mathbf{y} &= [\mathbf{H} \circ \exp(j\Phi)]\mathbf{x} + \mathbf{n} \\ &= \hat{\mathbf{H}}\mathbf{x} + \mathbf{n} \end{aligned} \tag{2}$$

where $\mathbf{y} \in C^{N_r \times 1}$ is the received signal vector, $[\mathbf{H}]_{i,j} = h(i,j)$ is the narrowband physical channel matrix normalized such that $\parallel \mathbf{H} \parallel_F^2 = N_r N_t$ where $\parallel . \parallel_F$ denotes matrix Frobenius norm, $\circ$ denotes Hadamard product, $[\Phi]_{i,j} = \varphi(i,j)$ is the composite phase noise matrix due to the transmitter and receiver local oscillators, $\mathbf{x} \in C^{N_t \times 1}$ is the transmitted signal vector, $\mathbf{n} \in C^{N_r \times 1}$ is zero mean complex Gaussian receiver noise vector with covariance matrix $E[\mathbf{n}\mathbf{n}^H] = \sigma_n^2 \mathbf{I}_{N_r}$ where $E[.]$ denotes expectation, σ_n^2 is the noise power at each receive antenna, $(.)^H$ denotes Hermitian transposition and $[\hat{\mathbf{H}}]_{i,j} = \hat{h}(i,j)$ is the effective channel matrix including the impact of PN.

III. MIMO channel Capacity

Channel capacity is one of the useful MIMO system performance measures. It maps a channel realization to a non negative scalar whose relative magnitude indicates channel quality. Under high data rate spatial multiplexing scheme and with the assumption of zero mean circularly symmetric complex Gaussian (ZMCSCG) transmitted signal vector with covariance matrix $E[\mathbf{x}\mathbf{x}^H] = \frac{\sigma_x^2}{N_t}\mathbf{I}_{N_t}$, where σ_x^2 is the total transmitted signal power, the ergodic capacity of the effective measured MIMO channel can be written as [1]

$$c_{meas} = E\left[\sum_{i=1}^{r(\hat{\mathbf{R}})} \log(1 + \frac{\rho}{N_t}\lambda_i(\hat{\mathbf{R}}))\right] \tag{3}$$

where $\rho = \frac{\sigma_x^2}{\sigma_n^2}$ is the average receive signal to noise ratio (SNR), $\hat{\mathbf{R}} = \hat{\mathbf{H}}\hat{\mathbf{H}}^H$ is the effective channel correlation matrix , $\lambda_i(\hat{\mathbf{R}})$ and $r(\hat{\mathbf{R}})$ are the i-th eigenvalue and the rank of the effective channel correlation matrix, respectively. The channel capacity in (3) reveals useful information about the MIMO system performance. It tells us that there are $r(\hat{\mathbf{R}})$ spatial parallel channels each has a SNR $\frac{\rho}{N_t}$ and a power gain of $\lambda_i(\hat{\mathbf{R}})$. Relative to single antenna transmission systems, the number of spatial parallel channels is usually referred to as spatial multiplexing gain and the power increase in each spatial channel is usually considered as power gain. These are the two mechanisms providing gain in MIMO wireless systems.

Similarly, in the absence of PN effect, the ergodic capacity of the physical MIMO channel can be written as

$$c_{phy} = E\left[\sum_{i=1}^{r(\mathbf{R})} \log(1 + \frac{\rho}{N_t}\lambda_i(\mathbf{R}))\right] \tag{4}$$

where $\mathbf{R} = \mathbf{H}\mathbf{H}^H$ is the physical channel correlation matrix, $\lambda_i(\mathbf{R})$ and $r(\mathbf{R})$ are the i-th eigenvalue and the rank of the physical channel correlation matrix, respectively. In the absence of PN effect there are $r(\mathbf{R})$ spatial parallel channels each has a power gain of $\lambda_i(\mathbf{R})$.

IV. Channel capacity error

In this section we analyze the channel capacity error due to the impact of PN. The purpose is to figure out how the PN affects the two MIMO gain mechanisms. The error in the ergodic channel capacity due to PN impact can be written as

$$\begin{aligned} e &= c_{meas} - c_{phy} = E\Bigg[\sum_{i=1}^{r(\hat{\mathbf{R}})} \log(1 + \frac{\rho}{N_t}\lambda_i(\hat{\mathbf{R}})) \\ &\quad - \sum_{i=1}^{r(\mathbf{R})} \log(1 + \frac{\rho}{N_t}\lambda_i(\mathbf{R}))\Bigg] \end{aligned} \tag{5}$$

With a low rank physical channel matrix, the presence of phase noise results in effective channel matrix with rank higher than that of the physical channel matrix [4], i.e. $r(\hat{\mathbf{R}}) > r(\mathbf{R})$, therefore, the channel capacity error in (5) can be decomposed as follows

$$\begin{aligned} e &= E\Bigg[\underbrace{\sum_{i=1}^{r(\mathbf{R})} \log(\frac{1 + \frac{\rho}{N_t}\lambda_i(\hat{\mathbf{R}})}{1 + \frac{\rho}{N_t}\lambda_i(\mathbf{R})})}_{e_1} \\ &\quad + \underbrace{\sum_{i=r(\mathbf{R})+1}^{r(\hat{\mathbf{R}})} \log(1 + \frac{\rho}{N_t}\lambda_i(\hat{\mathbf{R}}))}_{e_2}\Bigg] \end{aligned} \tag{6}$$

The first term in (6), e_1, represents the channel capacity error due to errors in the eigenvalues characteristics of the physical channel matrix. One can think of this error as the changes in the power gain of each spatial channel due to the impact of PN. On the other hand, the second term, e_2, represents the channel capacity error due to the newly introduced eigenvalues. In other words it represents the error in the spatial multiplexing gain. Instead of having $r(\mathbf{R})$ spatial parallel channels, the presence of PN deceptively results in $r(\hat{\mathbf{R}})$ spatial parallel channels.

The worst case scenario in terms of channel capacity estimation accuracy happens when the physical channel matrix is a rank one matrix, $r(\mathbf{R}) = 1$, and the presence of PN results in a full rank effective channel matrix, $r(\hat{\mathbf{R}}) = \min(N_r, N_t)$. Under this scenario the error in the ergodic channel capacity can be written as

$$\begin{aligned} e &= E\Bigg[\log(\frac{1 + \frac{\rho}{N_t}\lambda_1(\hat{\mathbf{R}})}{1 + \frac{\rho}{N_t} \parallel \mathbf{H} \parallel_F^2}) \\ &\quad + \sum_{i=2}^{\min(N_r,N_t)} \log(1 + \frac{\rho}{N_t}\lambda_i(\hat{\mathbf{R}}))\Bigg] \end{aligned} \tag{7}$$

In a high SNR scenario, (7) can be approximated as

$$e \approx E\left[\log(\frac{\lambda_1(\hat{\mathbf{R}})}{N_r N_t}) + \sum_{i=2}^{\min(N_r,N_t)} \log(\frac{\rho}{N_t}\lambda_i(\hat{\mathbf{R}}))\right] \tag{8}$$

The maximum effect of PN on the rank one physical channel matrix is to convert it to a diagonal matrix with equal eigenvalues each with power gain of $\frac{\|\mathbf{H}\|_F^2}{\min(N_r,N_t)} = \frac{N_r N_t}{\min(N_r,N_t)}$. For a MIMO system with equal number of transmit and receive antennas, $N_r = N_t = N$, the power gain of each eigenvalue is just the total power in the channel matrix divided by the MIMO system size, i.e. $\lambda_i = N$, $i = 1, 2, ..., N$. Substituting into (8) we get

$$e_{max} \approx -\log(N) + (N-1)\log(\rho) \quad (9)$$

It is clear that the maximum error in the ergodic capacity due to the errors in the power gain is a function of the MIMO system size and is independent on the SNR. On the other hand, the maximum error in the ergodic capacity due to the errors in the spatial multiplexing gain is a function of both the MIMO system size and the SNR. In a high SNR scenario $(N-1)\log(\rho) \gg \log(N)$ which implies that the maximum ergodic capacity error is dominated by the errors in the spatial multiplexing gain.

On the other side, the presence of PN results in minimum error in the ergodic capacity when the physical channel matrix is a full rank matrix, $r(\mathbf{R}) = \min(N_r, N_t)$. Similarly, for a MIMO system with equal number of transmit and receive antennas and diagonal physical channel matrix it can be easily shown that the minimum ergodic capacity error is zero.

V. Eigenvalues filtering

A. Spectrum of channel correlation matrix

A close look to the spectrum of the physical and effective channel correlation matrices reveals valuable information about the impact of PN. In order to gain deep understanding of the PN impact and also to motivate our later proposed solution we consider some numerical examples. We examine 8×8 MIMO wireless physical channel matrices with different ranks subject to PN effect. The elements of these physical channels are assumed to be zero mean complex Gaussian with unit variance. Following [4] we consider two PN cases, typical case with $\sigma_\varphi = 3.5°$ and worst case with $\sigma_\varphi = 7°$.

Figure 2 shows the spectrum of the 8×8 physical MIMO channel correlation matrices subject to typical and worst PN. With rank one, four and eight physical channel matrices there are one, four and eight spatial parallel channels available, respectively. The presence of PN results in eight spatial parallel channels regardless of the spatial multiplexing gain available from the physical channels in both typical and worst PN cases. The power gains of the newly introduced eigenvalues are small compared to the eigenvalues of the physical channel correlation matrix. For instance, in case of rank one physical channel matrix the first introduced eigenvalue, λ_2, is about 30 dB below the power gain of the physical channel correlation matrix in the typical PN case. In the worst PN case the power gain of the first introduced spatial channel is about 22 dB below the power gain of the physical channel correlation matrix.

It can be noticed that with low rank physical channel matrix the presence of PN effect turns the low rank physical channel matrix to a full rank one. On the other hand, in

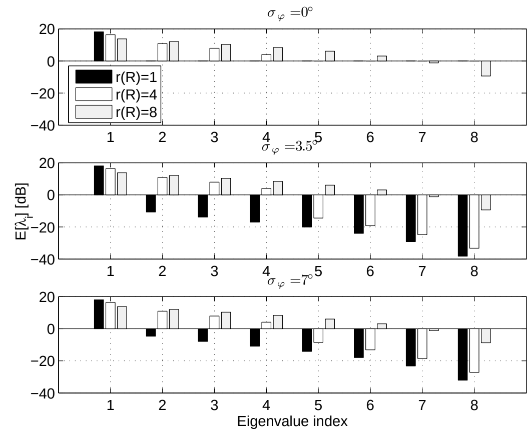

Fig. 2. Spectrum of 8×8 MIMO physical channel correlation matrix with different ranks subject to PN impact.

the case of full rank physical channel correlation matrix the impact of PN is not noticeable. This is simply because the presence of PN can not introduce more randomness to the physical channel matrix if it is already a full rank one. One general observation we can make is that the newly introduced spatial channels due to PN effect are in general small. These newly introduced spatial channels come as a price of small degradation in the power gain of the spatial channels of the physical channel correlation matrix. The impact of the newly created eigenvalues on the accuracy of the measured channel capacity will become significant only at a high SNR scenario.

B. Eigenvalue filtering technique

It is clear that while the error due to the newly introduced spatial channels can be treated using some sort of filtering technique, the error due to changes in the power gain is more difficult to deal with. Fortunately, as we have seen that the PN impact is more pronounced on the spatial multiplexing gain than on the power gain. Bearing this in mind, one possible solution for more accurate MIMO channel capacity estimation in presence of PN is to use EVF technique. Eigenvalues with power gain less than predefined threshold are filtered out and are not used for the channel capacity calculations. Therefore, the ergodic channel capacity with the EVF can be written as

$$c_{EVF} = E\left[\sum_{i=1}^{m} \log(1 + \frac{\rho}{N_t}\hat{\lambda}_i)\right] \quad (10)$$

where $\hat{\lambda}_i$ is the i-th filtered eigenvalue of the effective channel correlation matrix that can be obtained as

$$\hat{\lambda}_i = \begin{cases} \lambda_i(\hat{\mathbf{R}}), & \forall \quad \lambda_i(\hat{\mathbf{R}}) \geq \lambda_{th} \\ 0, \quad \forall & \lambda_i(\hat{\mathbf{R}}) < \lambda_{th} \end{cases} \quad (11)$$

where λ_{th} is the threshold value and m is the number of eigenvalues of the effective channel correlation matrix passing the EVF. An immediate question one may ask is that how to choose the threshold value. In order to answer this question we study the percentage of relative ergodic capacity error, $100 \times \frac{c_{EVF} - c_{phy}}{c_{phy}}$, in presence of PN with EVF at different thresholds. We choose to relate the threshold value to the

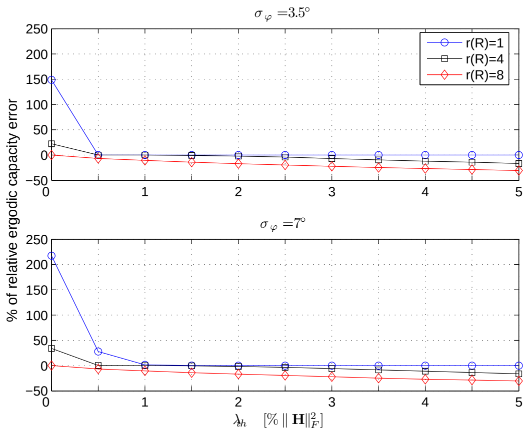

Fig. 3. Percentage of relative ergodic capacity error with EVF at different threshold of 8×8 MIMO system subject to PN.

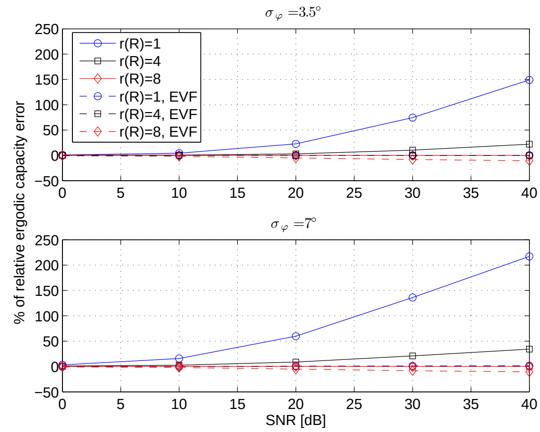

Fig. 4. Percentage of relative ergodic capacity error without and with EVF $\lambda_{th} = 0.01 \parallel \mathbf{H} \parallel_F^2$ of 8×8 MIMO system.

MIMO system size to make the threshold value sensible as follows

$$\lambda_{th} = x \parallel \mathbf{H} \parallel_F^2 \tag{12}$$

where x is a percentage ranging from 0 % to 100%. With this choice we sacrifice a fraction of the power in the channel matrix for sake of more accurate channel capacity estimation. Figure 3 shows the percentage of relative ergodic capacity error with EVF at different threshold for the 8×8 MIMO system described above at 40 dB SNR. A threshold ranging from 1% to 5% of the power in the channel matrix is considered. We can clearly see that sacrificing 1% of the physical channel power and keeping 99% of the power is a good choice to reduce the channel capacity error significantly. In the case of low rank physical channel matrix filtering out up to 5% of the channel power still results in very small channel capacity error. However, in the case of full rank physical channel matrix the EVF results in channel capacity errors that grow with the threshold value. At $\lambda_{th} = 0.05 \parallel \mathbf{H} \parallel_F^2$ the error in the ergodic channel capacity is up to -30%.

In a real measurement scenario we do not know whether the measured channel matrix is a low or a full rank matrix. Therefore, sacrificing 1% of the physical channel power and keeping 99% of the power seems to be a good choice for unknown rank measured channel matrix in order to reduce any possible PN impact significantly. Figure 4 shows the percentage of relative ergodic capacity error for the 8×8 MIMO system without and with EVF at $\lambda_{th} = 0.01 \parallel \mathbf{H} \parallel_F^2$. With the full rank physical channel matrix the presence of PN has no impact on the estimated ergodic channel capacity regardless of the SNR. On the other hand, with the rank one physical channel matrix the presence of PN results in 150% and 220% deceptive channel capacity increase at 40 dB SNR for the typical and worst PN cases, respectively. With the rank four physical channel matrix the error in the channel capacity is about 25% and 35% for the typical and worst PN cases, respectively, at the same SNR. Using the EVF reduces the channel capacity error dramatically. In the case of full rank physical channel matrix, the EVF results in slight underestimation of the channel capacity. However, this underestimation is less than 10% regardless of the amount of PN and the SNR.

VI. Conclusions

The impact of PN on the accuracy of measured MIMO channel capacity is significant in a high SNR and with a low rank physical channel matrix. However, we have shown that in presence of PN more accurate ergodic channel capacity estimation can be obtained by filtering the eigenvalues of the measured channel matrix. One possible choice for the filter threshold is to sacrifice a fraction of the power in the channel matrix for sake of more accurate channel capacity estimation. For unknown rank measured channel matrix, filtering out 1% of the power of the channel matrix has shown to be a good choice in terms of channel capacity accuracy.

References

[1] G.J. Foschini, and M.J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, no. 3, pp. 311−335, Mar. 1998.

[2] J.D. Parsons, D.A. Demery, and A.M.D. Turkmani,"Sounding techniques for wideband mobile radio channels: A review," *IEE Proc. Commun., Speech, Vision* , vol. 138, no. 5, pp. 437−446, Oct. 1991.

[3] P. Kyritsi, R.A. Valenzuela, and D.C. Cox,"Channel and capacity estimation errors," *IEEE Commun. Letters*, vol. 6, no. 12, pp. 517−519, Dec. 2002.

[4] D.S. Baum, and H. Bölcskei,"Impact of phase noise on MIMO channel measurement accuracy," *in proc. of IEEE Veh. Tech. Conf.*, pp. 34−38 , Sep. 2004.

[5] P. Almers, S. Wyne, F. Tufvesson, and A.F. Molisch,"Effect of random walk phase noise on MIMO measurements ," *in proc. of IEEE Veh. Tech. Conf.*, pp. 141−145, Jun. 2005.

[6] J. Kivinen, and P. Vainikainen,"Calibration scheme for synthesizer phase fluctuations in virtual antenna array measurements ," *IEEE Microwave and Optical Tech. Letters*, vol. 26, no. 3, pp. 183−187, Aug. 2000.

[7] G. Matz, A.F. Molisch, F. Hlawatsch, M. Steinbauer, and I. Gaspard,"On the systematic measurement error of correlative mobile radio channel sounders," *IEEE Trans. on Commun.*, vol. 50, no. 5, pp. 808−821, May 2002.

[8] J.B. Andersen,"Array gain and capacity for known random channels with multiple element arrays at both ends", *IEEE J. on Selected Areas in Commun.*, vol. 18, pp. 2172−2178, Nov. 2000.

[9] J. Rutman,"Characterization of phase and frequency instabilities in precision frequency: Fifteen years of progress," *in Proc. of the IEEE* , vol. 66, no. 9 , pp. 1048−1075, 1978.