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Effect of Mutual Coupling on BER Performance of Alamouti Scheme

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1 Introduction

The advantages of Multiple-input multiple-output (MIMO) systems over traditional single-input single-output (SISO) systems come from the fact that there are different mechanisms contributing to the high performance achieved by MIMO systems. These mechanisms include spatial multiplexing gain and power gain which includes both the diversity gain and two ends array gains. The contribution of each mechanism depends on the employed coding scheme and the propagation scenario. For a specific coding scheme, the MIMO system performance is dominated by the the channel correlation properties and the target average receive signal to noise ratio (SNR). At a fixed SNR, low correlation propagation environment reveals high MIMO system performance in terms of spatial multiplexing gain and diversity gain and the vis versa [1].

One of the parameters that strongly affect the correlation properties is the mutual coupling between antenna elements. Due to the limitations in the available space in the handhold devices, antenna elements are closely spaced which make the signals transmitted or received by these antennas to couple. Available results in literature have drawn different conclusions concerning the impact of mutual coupling on the performance of MIMO systems. While positive impact is reported in [2], negative effect is shown in [3]. However, in this work we present a study on the effect of mutual coupling on the performance of Alamouti scheme [4] by studying the bit error rate (BER) performance under different propagation scenarios and for different coupling cases. We show that the presence of mutual coupling may have positive or negative impact on the achieved power gain depending on the correlation properties of the propagation environment in absence of coupling effect and on the mutual coupling matrix.

2 System model

We consider a MIMO wireless communication system with two transmit antennas and two receive antennas. The system employs the Alamouti transmit diversity scheme [4], where at each time instant signal s_1 is transmitted from transmit antenna 1 and signal s_2 is transmitted from transmit antenna 2. In the following time instant signal $-s_2^*$ is transmitted from transmit antenna 1 and signal $-s_1$ is transmitted from transmit antenna 2, simultaneously, where $(.)^*$ denotes complex conjugate. It is assumed that the channel is flat fading and it remains constant over two consecutive symbols. This is a realistic assumption in slow fading propagation environment where the channel does not change rapidly. Including the receiver mutual coupling effect, the received signals at each receive antenna in the first time instant can be

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written as:

$$y_1 = \sqrt{\frac{E_s}{2}} \underbrace{\underbrace{\begin{bmatrix} c_{1,1} & c_{1,2} \\ c_{2,1} & c_{2,2} \end{bmatrix}}_{\mathbf{C}_r} \underbrace{\begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix}}_{\mathbf{H}}}_{\hat{\mathbf{H}}} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{1}$$

and in the second time instant:

$$y_2 = \sqrt{\frac{E_s}{2}} \begin{bmatrix} c_{1,1} & c_{1,2} \\ c_{2,1} & c_{2,2} \end{bmatrix} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} -s_2^* \\ -s_1 \end{bmatrix} + \begin{bmatrix} n_3 \\ n_4 \end{bmatrix} \tag{2}$$

where $\mathbf{C}_r$ is the receiver mutual coupling matrix, $\mathbf{H}$ is the narrowband channel matrix obtained without receiver coupling effect, E_s is the total available energy in the transmitter side and n_1, n_2, n_3 and n_4 are uncorrelated zero mean complex Gaussian noise samples with variance $E\{|n_i|^2\} = N_o, i = 1, 2, ..., 4$. Using simple circuit theory analysis the receiver coupling matrix can be written as [5]:

$$\mathbf{C}_r = \mathbf{Z}_L(\mathbf{Z}_L + \mathbf{Z}_r)^{-1} \tag{3}$$

where $\mathbf{Z}_r$ is the receiver mutual impedance matrix and $\mathbf{Z}_L$ is the loading impedance matrix. The coupling matrix can be quantified using either numerical methods or measurement methods [6]. However, the numerical methods are more convenient.

In the receiver side, the receiver constructs the received signal vector that can be expressed as:

$$\underbrace{\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix}}_{\mathbf{y}} = \sqrt{\frac{E_s}{2}} \underbrace{\underbrace{\begin{bmatrix} c_{1,1} & c_{1,2} & 0 & 0 \\ c_{2,1} & c_{2,2} & 0 & 0 \\ 0 & 0 & c_{1,1}^* & c_{1,2}^* \\ 0 & 0 & c_{2,1}^* & c_{2,2}^* \end{bmatrix}}_{\mathbf{C}_{r,e}} \underbrace{\begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \\ h_{1,2}^* & -h_{1,1}^* \\ h_{2,2}^* & -h_{2,1}^* \end{bmatrix}}_{\mathbf{H}_e}}_{\hat{\mathbf{H}}_e} \underbrace{\begin{bmatrix} s_1 \\ s_2 \end{bmatrix}}_{\mathbf{s}} + \underbrace{\begin{bmatrix} n_1 \\ n_2 \\ n_3^* \\ n_4^* \end{bmatrix}}_{\mathbf{n}} \tag{4}$$

where $\mathbf{C}_{r,e}$ is a block diagonal matrix representing the effective receiver coupling matrix, $\mathbf{H}_e$ is the effective MIMO channel matrix in absence of coupling effect and $\hat{\mathbf{H}}_e$ is the effective MIMO channel matrix including receiver coupling effect. In compact matrix form, (4) can be written as:

$$\mathbf{y} = \sqrt{\frac{E_s}{2}} \mathbf{C}_{r,e} \mathbf{H}_e \mathbf{S} + \mathbf{n} = \sqrt{\frac{E_s}{2}} \hat{\mathbf{H}}_e \mathbf{S} + \mathbf{n} \tag{5}$$

Maximum ratio combining of the received signal vector with perfect channel state information results in:

$$\mathbf{z} = (\hat{\mathbf{H}}_e)^H \mathbf{y} = \sqrt{\frac{E_s}{2}} \hat{\mathbf{H}}_e^H \hat{\mathbf{H}}_e \mathbf{s} + \hat{\mathbf{H}}_e^H \mathbf{n} \tag{6}$$

where $(.)^H$ denotes Hermitian transposition. It can be noticed that the effective MIMO channel matrix including coupling effect is orthogonal irrespective to the receiver coupling matrix or the MIMO channel matrix in absence of coupling, i.e.,

$\hat{\mathbf{H}}_e^H \hat{\mathbf{H}}_e = \parallel \hat{\mathbf{H}} \parallel_F^2 \mathbf{I}_2$, where $\parallel . \parallel_F$ denotes matrix Frobenius norm and $\mathbf{I}$ is the identity matrix. Therefore, the received signal after maximum ratio combining can be further simplified to:

$$\mathbf{z} = \sqrt{\frac{E_s}{2}} \parallel \hat{\mathbf{H}} \parallel_F^2 \mathbf{s} + \hat{\mathbf{n}} = \sqrt{\frac{E_s}{2}} \parallel \mathbf{C}_r \mathbf{H} \parallel_F^2 \mathbf{I}_2 \mathbf{s} + \hat{\mathbf{H}}_e^H \mathbf{n} \tag{7}$$

Assuming maximum likelihood detection at the receiver side, the signal to noise ratio (SNR) at each receive antenna can be written as:

$$\eta = \frac{E_s}{2N_o} \parallel \mathbf{C}_r \mathbf{H} \parallel_F^2 = \frac{\rho}{2} \parallel \mathbf{C}_r \mathbf{H} \parallel_F^2 = \frac{\rho}{2} \sum_{i=1}^{R(\mathbf{C}_r \mathbf{H} \mathbf{H}^* \mathbf{C}_r^*)} \lambda_i(\mathbf{C}_r \mathbf{H} \mathbf{H}^* \mathbf{C}_r^*) \tag{8}$$

where ρ can be considered as the equivalent SNR of SISO system, $R(\mathbf{C}_r \mathbf{H} \mathbf{H}^* \mathbf{C}_r^*)$ and $\lambda_i(\mathbf{C}_r \mathbf{H} \mathbf{H}^* \mathbf{C}_r^*)$ are the rank and the i^{th} eigenvalue of the channel correlation matrix including coupling effect. It can be noticed that the coupling will affect both the correlation properties and the power of the new channel matrix. For accurate investigation we consider only the variations due correlation properties and compensate for the power variations by normalization. This is achieved by normalizing both the channel matrix in presence of coupling effect and the channel matrix with coupling effect as follows, $E\{|h_{i,j}|^2\} = 1$ and $E\{|\hat{h}_{i,j}|^2\} = 1$.

3 Numerical results and discussions

Figure 1 shows the BER performance of Alamouti scheme with QPSK constellation in two propagation environments modeled as independent identical distributed (iid) zero mean complex Gaussian random variables with two correlation values, 0.1 and 0.9. The first correlation value represents rich scattering scenario while the second one represents highly correlated scenario. The BER performance in presence of coupling effect on these propagation environment is also shown where we consider two half wavelength dipole antennas spacing 0.1 wavelength and operating at 2 GHz carrier frequency. The dipole antennas are connected to loading network matched to the antenna mutual impedance matrix, i.e., $\mathbf{Z}_L = \mathbf{Z}_r^*$. Under this matching condition the power delivered to the loading network is maximized. However, because normalization is performed after including coupling effect only BER variations due to changes in correlation properties are considered. The BER performance of SISO system is also shown for sake of comparison. As it can be seen at low correlation value deploying Alamouti scheme provides significant improvement in the BER performance compared to the SISO case. This improvement in BER performance is due to both the diversity gain and the receiver array gain. However, the presence of mutual coupling degrades the BER performance by amount of 2 dB at 3×10^{-2} BER. In the high correlation propagation scenario the power gain of Alamouti scheme is less than that in rich scattering scenario. This is due to the fact that the signal paths become correlated which increases the probability that they fade together and consequently the diversity gain is reduced. However, it can be seen that the presence of mutual coupling in this propagation scenario has transformed the highly correlated scenario to less correlated one and the same power gain of Alamouti scheme in rich scattering environment is achieved. The presence of mutual coupling in this case has decorrelated the signal paths which results in increase in the diversity gain.

The BER performance of Alamouti scheme in the same propagation scenarios but with 0.5 wavelength antenna spacing is shown in Figure 2. One can notice that the effect mutual coupling on BER performance is not significant in both propagation scenarios. This is due to the fact that the mutual coupling between dipoles at 0.5 wavelength spacing is not significant and therefore, the impact on BER is negligible. This result confirms the common understating in literature that antenna spacing large than or equal 0.5 wavelength is sufficient to provide decorrelated paths.

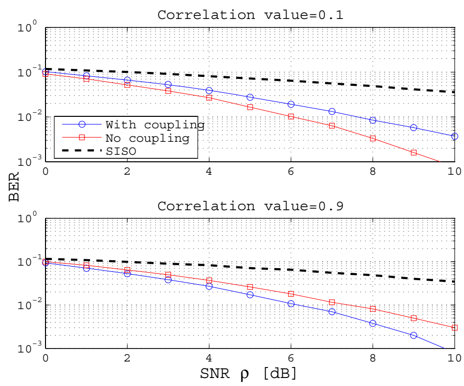

Figure 1: BER at 0.1 wavelength antenna spacing.

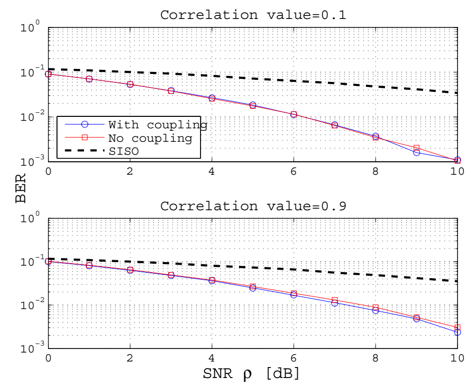

Figure 2: BER at 0.5 wavelength antenna spacing.

4 Conclusions

We can conclude that the effect of mutual coupling on BER performance of Alamouti scheme is not always negative. Depending on the correlation properties of the channel matrix in absence of coupling and on the coupling matrix, the effect of mutual coupling can be negative or positive.

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