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**MODELLING BIOMASS-FUELLED SMALL-SCALE CHP
PLANTS FOR PROCESS SYNTHESIS OPTIMISATION**

Doctoral Dissertation

Tuula Savola



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Dissertation for the degree of Doctor of Science in Technology to be presented with due permission of the Department of Mechanical Engineering for public examination and debate in Auditorium K216 at Helsinki University of Technology (Espoo, Finland) on the 15th of June, 2007, at 12 noon.

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HELSINKI UNIVERSITY OF TECHNOLOGY P. O. BOX 1000, FI-02015 TKK http://www.tkk.fi		ABSTRACT OF DOCTORAL DISSERTATION	
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Abstract <p>In this work possible process improvements for biomass-fuelled small-scale combined heat and power (CHP) plants are evaluated and a new mixed integer nonlinear programming (MINLP) model for process synthesis optimisation of these processes is presented. Small-scale (1-20 MW_e) CHP plants are of interest, as in Finland the potential to increase the CHP production is in converting small heating units to CHP production. However, the profitability of these small-scale CHP investments should be higher than today. The small-scale CHP plants are usually operated according to the heat demand of a district heating network or an industrial process. Thus, the possibility to increase the profitability of these plants is in improving their power-to-heat ratios (α) or electrical efficiencies (η_e).</p> <p>The possibilities to increase the power production of small-scale CHP plants are studied here with simulations and optimisation. Especially, a superstructure of the possible process improvements and a MINLP model including the special properties of small-scale CHP plants is developed for the process synthesis optimisation. Unlike previous models of CHP processes, the model includes the modelling of pressures and steam and water property functions that depend both on temperatures and pressures. Also, a new model for a back-pressure steam turbine is developed. This model takes into account the nonlinear efficiency changes in the regulation stage of the turbine, the changes caused by the exhaust losses at the end of the turbine, and the dependence of pressure on the steam mass flow through the turbine at part loads. The part load operation of the small-scale CHP plants is incorporated into the model with multiperiods.</p> <p>With the developed simulation and optimisation models the profitability of the process changes in small-scale CHP processes based on existing plants are evaluated. With the addition of a steam reheater, a feed water preheater, and a two-stage district heat exchanger the simulation and optimisation models found profitable processes where α is increased from 0.23-0.50 to 0.45-0.50, depending on the size of the plant. Similarly, η_e is increased in a profitable way from 0.17-0.30 to 0.28-0.30. If also natural gas is used as fuel and a gas engine integrated to the process, the efficiencies are further improved. In general, the process alternatives and model formulations presented here can be useful in the design and planning of new efficient small-scale CHP processes. Some of the model formulations can be utilised also in the modelling of other energy related processes with similar challenges as in the modelling of small-scale CHP plants.</p>			
Keywords Energy systems, CHP, MINLP, modelling, simulation, optimisation			
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<p>Tässä työssä arvioidaan mahdollisia sähköntuotantoa parantavia prosessimuutoksia biomassaa polttaville pienille yhdistetyn sähkön- ja lämmöntuotannon (CHP) laitoksille ja esitetään uusi jatkuvia ja kokonaislukumuuttujia sisältävä epälineaarinen (MINLP) malli näiden laitosten optimointiin. Suomessa potentiaalia CHP-tuotannon lisäämiseen on lähinnä pienen kokoluokan lämpölaitoksilla ja näiden muuntamisella yhdistettyyn sähkön- ja lämmöntuotantoon, joten pienet (1-20 MW_e) CHP-prosessit ovat tällä hetkellä erityisen kiinnostavia. Jotta tarvittavat investoinnit saataisiin nykyistä kannattavammiksi, tulisi näiden pienten CHP-laitosten tehokkuuden parantua. Kaukolämpöverkon tai teollisuusprosessin lämmöntarve määrää usein pienten CHP-laitosten ajotavan. Mahdollisuus pienten CHP-laitosten kannattavuuden parantamiseen on nostaa niiden sähköntuotannon hyötysuhdetta (η_e) ja/tai rakennusastetta (α) eli tuotetun sähkömäärän suhdetta tuotettuun lämpöenergiaan.</p> <p>Sähköntuotannon lisäämistä pienissä CHP-laitoksissa tutkitaan tässä työssä simuloimalla ja optimoimalla valittuja prosesseja. Erityisesti on kehitetty ns. superstruktuuri mahdollisista pienten CHP-laitosten sähköntuotantoa nostavista prosessimuutoksista ja siihen perustuva MINLP-malli laitosten prosessisynteesin optimointiin. Aikaisemmista malleista poiketen uusi MINLP-malli sisältää paineiden mallinnuksen ja standardin mukaiset vesihöyryn ja veden entalpian ja entropian funktiot, jotka riippuvat sekä lämpötilasta että paineesta. Lisäksi vastapainehöyryturbiinille on kehitetty uusi malli, joka huomioi säätövyöhykkeen ja höyryn ulosvirtaushäviöiden aiheuttamat epälineaariset turbiinin hyötysuhteen muutokset sekä turbiinin höyryn paineen ja höyryn massavirran riippuvuuden osakuormilla. Pienen CHP-laitoksen osakuorma-ajo on mallinnettu jakamalla laitoksen toiminta useaan eri ajanjaksoon.</p> <p>Pienten CHP-laitosten prosessimuutosten kannattavuutta arvioidaan kehitetyillä simulointi- ja optimointimalleilla. Höyryn välitulituksen, syöttöveden esilämmityksen ja kaksivaiheisen kaukolämmönvaihtimen lisäämiseen prosessiin on kannattavia mahdollisuuksia, joissa laitoksen koosta riippuen α nousee tasolta 0.23-0.50 tasolle 0.45-0.50 ja η_e tasolta 0.17-0.30 tasolle 0.28-0.30. Maakaasua käyttävän kaasumoottorin lisäys prosessiin nostaa hyötysuhteita edelleen. Kehitetyjä malleja voidaan hyödyntää erityisesti uusien pienten CHP-laitosten prosessien suunnittelussa. Useat mallin osat voivat olla hyödyllisiä myös muiden energiasysteemien optimoinnissa, joissa on ratkaistavana samankaltaisia mallinnuksen haasteita kuin pienten CHP-laitosten kohdalla.</p>			
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PREFACE

The work for this thesis was carried out between the years 2002 and 2006 at the Laboratory of Energy Engineering and Environmental Protection at the Helsinki University of Technology. Partly the research work was done also during the visits to the Kungliga Tekniska Högskolan (KTH) in the spring 2002 and to the University of California at Berkeley in the academic year 2002-2003.

The research work of this thesis was a part of the Tekes funded projects *Small-Scale CHP Plants and District Heating* (funded also by the Finnish District Heating Association) and *Multiobjective Optimization and Multidisciplinary Decision Support Systems*. Also, the Nordic Energy Research and the Graduate School for Energy Science and Technology provided financial support for this thesis. All the supporters are gratefully acknowledged for making this thesis possible.

The supervisor of this work, Professor Carl-Johan Fogelholm, is warmly thanked for the opportunity to conduct this thesis, for the advice and support during the work, and for the encouragement to visit universities and research groups abroad. I want to thank the instructor D.Sc Juha Aaltola for the comments and the great help in the beginning of the optimisation work and D.Sc Tor-Martin Tveit for the motivating academic discussions and co-operation in getting this work forward. Associate Professor Mats Söderström from Linköping University and Ph.D Jussi Manninen from VTT are thanked from very good comments on the thesis. Also, my thanks to many colleagues who have shared interesting comments and thoughts on this work and thus helped me to improve it. The whole ENY lab is thanked for the good working atmosphere and for the relaxing coffee breaks.

Special warm thanks belong to my parents and to my family, Risto and Antti.

Otaniemi May 8th, 2007

Tuula Savola

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LIST OF PUBLICATIONS

This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.

- I. Savola, T. and Keppo I. 2005. Off-design simulation and mathematical modeling of small-scale CHP plants at part loads. *Applied Thermal Engineering* 25(8-9):1219-1232.
- II. Savola, T. and Fogelholm, C-J. 2006. Increased power-to-heat ratio of small-scale CHP plants using biomass fuels and natural gas. *Energy Conversion and Management* 47(18-19):3105-3118.
- III. Savola, T. and Fogelholm, C-J. 2007. MINLP optimisation model for increased power production in small-scale CHP plants. *Applied Thermal Engineering* 27(1):89-99.
- IV. Tveit, T-M., Savola, T. and Fogelholm, C-J. 2005. Modelling of steam turbines for mixed integer nonlinear programming (MINLP) in design and off-design conditions. In: J. Amudsen, H. I. Andersson, E. Celledoni, T. Gravdahl, F. A. Michelsen, H. R. Nagel, T. Natvig (eds.), *Proc. SIMS2005 – 46th Conference on Simulation and Modeling*, 13-14 October 2005 Trondheim Norway. Trondheim: Tapir Academic Press:335-344.
- V. Savola, T., Tveit, T-M and Fogelholm, C-J. 2007. A MINLP model including the pressure levels and multiperiods for CHP process optimisation. *Applied Thermal Engineering* 27(11-12):1857-1867.

AUTHOR'S CONTRIBUTION

In *Publication I* the performance of small-scale CHP plants at part loads is modelled with simulations of four existing CHP plants and a mathematical model is constructed on the basis of the simulation results. The author wrote Sections 1, 2, 3 and 4 of the paper and participated in the writing of Section 5 with co-author Keppo. The author developed design and off-design simulation models of the CHP plants and performed a sensitivity analysis of the models. In addition, the author calculated the needed off-design simulation results for the regression model development and participated to the definition of the regression model coefficients. Co-author Keppo was responsible for the mathematical formulations and for the coefficient calculations of the new regression models.

Publication II presents a systematic simulation study of process improvements for increased power production in small-scale CHP plants. The author was the primary author of the paper and performed the work presented in it. Discussions with co-author Fogelholm helped to define the most relevant process improvement possibilities for increasing the power production in the simulated CHP plants.

In *Publication III* a mixed integer nonlinear programming model for small-scale CHP plant synthesis is presented. The objective of the model is to increase the power productions and the power-to-heat ratios of the optimised plants. The author was the primary author of the paper and performed the work presented in the paper. Co-author Fogelholm supervised the work.

Publication IV presents a new model formulation of a back-pressure steam turbine for design and off-design conditions. The developed model is compared to a much used Willans line model and to a simulation model. The author's contribution to the work was implementing the new steam turbine model formulation to GAMS optimisation program and performing the modelling and simulation runs and comparisons of the different models. The author was also responsible of writing the Section 4 in the paper describing the models and the comparison research, and she participated also to the writing of the rest of the article. The primary author Tveit provided the idea for the new steam turbine model and formulated the new equations used in the model. He was also the primary author of the parts describing the formulation of the new steam turbine model in the paper (Sections 1-3). The co-author Fogelholm supervised and commented the work.

In *Publication V* a new mixed integer nonlinear programming model for the synthesis of small-scale CHP plants is presented. The model includes the modelling of part load operation as multiperiods, the modelling of pressure levels, steam and water property functions depending both on temperatures and pressures, and a new back-pressure steam turbine model. The author performed all the work presented in the paper and was responsible for writing the paper. Co-author Tveit gave advice during the modelling work, co-author Fogelholm supervised the work, and both co-authors commented the paper.

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LIST OF ABBREVIATIONS

CHP	combined heat and power
CPU	central processing unit
DH	district heating
MINLP	mixed integer nonlinear programming
MILP	mixed integer linear programming
NLP	nonlinear programming
LP	linear programming
SQP	sequential quadratic programming

LIST OF SYMBOLS

Indices:

<i>i</i>	<i>units</i>
<i>j,k</i>	<i>streams</i>
<i>n</i>	<i>nodes</i>
<i>p</i>	<i>periods</i>
<i>steam</i>	<i>steam streams</i>
<i>water</i>	<i>water streams</i>
<i>cond</i>	<i>condensated water stream</i>

Sets:

<i>UNITS</i>	<i>(i: i is a unit)</i>
<i>STREAMS</i>	<i>(j: j is a stream)</i>
<i>NODES</i>	<i>(n: n is a selection node)</i>
<i>PERIODS</i>	<i>(p: p is a period)</i>
Subsets to units:	
<i>HEATEX</i>	<i>(i: i is a unit that is a heat exchanger)</i>
<i>SUPERHEATER</i>	<i>(i: i is a unit that is a superheater)</i>
<i>REHEATER</i>	<i>(i: i is a unit that is a reheater)</i>
<i>PREHEATER</i>	<i>(i: i is a unit that is a feed water preheater)</i>
<i>FEEDWT</i>	<i>(i: i is a unit that is a feed water tank)</i>
<i>PUMPS</i>	<i>(i: i is a unit that is a pump)</i>
<i>BLOWERS</i>	<i>(i: i is a unit that is a blower)</i>
<i>SPLITTERS</i>	<i>(i: i is a unit that is a stream splitter)</i>
<i>AIRPRE</i>	<i>(i: i is a unit that is an air preheater)</i>
<i>FUELDRY</i>	<i>(i: i is a unit that is a fuel dryer)</i>
<i>BURNERS</i>	<i>(i: i is a unit that is a burner)</i>
<i>DHEX</i>	<i>(i: i is a unit that is a district heat exchanger)</i>
<i>DHEX1</i>	<i>(i: i is a unit that is a first stage of a district heat exchanger)</i>
<i>DHEX2</i>	<i>(i: i is a unit that is a 2 stage district heat exchanger)</i>
<i>TURBINES</i>	<i>(i: i is a unit that is a turbine)</i>
<i>REGSTAG</i>	<i>(i: i is a unit that is a regulation stage of a turbine)</i>
<i>WORKSTAG</i>	<i>(i: i is a unit that is a working stage of a turbine)</i>
<i>EXHSTAG</i>	<i>(i: i is a unit that is a exhaust stage of a turbine)</i>
<i>GASTURB</i>	<i>(i: i is a unit that is a gas turbine)</i>
<i>GASENG</i>	<i>(i: i is a unit that is a gas engine)</i>

Subsets to streams:

<i>AIR</i>	(<i>j</i> : <i>j</i> is a stream that is air)
<i>FUEL</i>	(<i>j</i> : <i>j</i> is a stream that is fuel)
<i>EXHGAS</i>	(<i>j</i> : <i>j</i> is a stream that is gas engine/turbine exhaust gas)
<i>FLUEGAS</i>	(<i>j</i> : <i>j</i> is a stream that is flue gas)
<i>DHWATER</i>	(<i>j</i> : <i>j</i> is a stream that is district heating water)
<i>SATSTREAMS</i>	(<i>j</i> : <i>j</i> is a stream that is saturated)

Subsets to nodes:

<i>MIXNODES</i>	(<i>n</i> : <i>n</i> is a node with several incoming streams)
<i>SPLITNODES</i>	(<i>n</i> : <i>n</i> is a node with several outgoing streams)

Subsets to units or nodes and streams:

<i>IN</i>	(<i>i, j</i> : <i>j</i> is an incoming stream to unit <i>i</i>)
<i>OUT</i>	(<i>i, j</i> : <i>j</i> is an outgoing stream from unit <i>i</i>)
<i>NODEIN</i>	(<i>n, j</i> : <i>j</i> is an incoming stream to node <i>n</i>)
<i>NODEOUT</i>	(<i>n, j</i> : <i>j</i> is an outgoing stream from node <i>n</i>)
<i>HOTIN</i>	(<i>i, j</i> : <i>j</i> is an incoming hot stream to heat exchanger <i>i</i>)
<i>HOTOUT</i>	(<i>i, j</i> : <i>j</i> is an outgoing hot stream from heat exchanger <i>i</i>)
<i>COLDIN</i>	(<i>i, j</i> : <i>j</i> is an incoming cold stream to heat exchanger <i>i</i>)
<i>COLDOUT</i>	(<i>i, j</i> : <i>j</i> is an outgoing cold stream from heat exchanger <i>i</i>)

Parameters:		Units	Values	
			<u>Model I</u>	<u>Model II</u>
M_{FUEL}	biomass fuel flow	[kg/s]	Paper III	3.4
$M_{NG,i}$	natural gas input to unit <i>i</i>	[kg/s]	Paper III	-
$M_{AIR,i}$	air input to unit <i>i</i>	[kg/s]	Paper III	-
T_{sh}	temperature of superheated steam	[°C]	Paper III	510
P_{sh}	pressure of superheated steam	[bar]	Paper III	60
$b_{n,j}$	coefficients for stream <i>j</i>		Appendix	Table 6
$b_{n,i}$	coefficients for unit <i>i</i>		-	Table 7
AF_{ratio}	air-to-fuel ratio		3.0	3.1
cp_j	specific heat capacity of stream <i>j</i> ,	[kJ/kgK]		
	<i>j</i> = air		1.045	1.045
	<i>j</i> = flue gas		1.29	1.29
	<i>j</i> = exhaust gas		1.08	-
v	specific volume of water	[m ³ /kg]	0.105	-
$x_{O_2,j}$	oxygen content in stream <i>j</i>	[m-%]	Paper III	-

Parameters:		Units	Values	
			<u>Model I</u>	<u>Model II</u>
T_{return}	return temperature of DH water	[°C]	55	55
$T_{forward,p}$	minimum forward temperature of the DH water in period p , $p = 1$	[°C]	-	90
	$p = 2$		-	80
	$p = 3$		-	70
	$p = 4$		-	60
$T_{air, in}$	temperature of the incoming air	[°C]	10	10
$T_{EXH,i}$	exhaust gas temperature after the gas turbine or engine i	[°C]	Paper III	-
$P_{air,in}$	air inlet pressure	[bar]	1.013	1.013
$P_{air,blower}$	air outlet pressure from the blower	[bar]	1.08	1.08
$P_{flue\ gas}$	flue gas pressure	[bar]	1.013	1.013
ΔT	temperature difference in heat exchangers	[°C]	4	4
T_{ref}	reference temperature	[°C]	0	0
M	large value			
$F_{H_2O,j}$	moisture content of fuel stream j	[m-%]	55	55
LHV_{NG}	lower heating value of natural gas	[MJ/kg]	49.3	-
$LOAD_p$	fuel load of the process in period p ,	[%]		
	$p = 1$		-	100
	$p = 2$		-	91.7
	$p = 3$		-	75.0
	$p = 4$		-	51.7
$months_p$	duration of the loads	$p = 1$ [months/a]	-	2.08
		$p = 2$	-	1.81
		$p = 3$	-	2.22
		$p = 4$	-	2.22
t_p	duration of the loads	[h/a] = $months_p \cdot 30\ d/months \cdot 24\ h/d$		
t_{annual}	annual operation time	[h]	4000	-
U_i	overall heat transfer coefficient for unit i , $i =$ feed water preheater	[kW/m ² K]	4	4
	$i =$ DH exchanger		4	4
	$i =$ reheater		0.1	0.1

Parameters:		Units	Values	
			<u>Model I</u>	<u>Model II</u>
η_i	efficiency of unit i , $i = \text{pump}$ $i = \text{blower}$		0.75 0.90	- 0.90
$\eta_{e,i}$	electrical efficiency of unit i		Paper III	-
η_{ge}	efficiency of the generator		0.97	0.97
η_{coal}	efficiency of a condensing coal fired power plant	[%]	-	45
e_{coal}	specific CO_2 emissions for coal combustion	$[kgCO_2/$ $kWh_{coal}]$	-	0.328
$P_{base_case,p}$	power production at base case $p = 1$ $p = 2$ $p = 3$ $p = 4$	$[MW_e]$	Paper III - - - -	- 6.033 6.008 5.060 3.482
$W_{base\ case}$	annual power production at base case ($y_1=1$), $W_{base\ case} = \sum_{p \in \text{periods}} P_{base_case,p} \cdot \text{months}_p \cdot 30 \cdot 24h / \text{month}$	$[MW_e h/a]$	-	30518.7
c_{CO_2}	price of the CO_2 emission permit	$[\text{€}/tCO_2]$	-	22
c_{ng}	natural gas price	$[\text{€}/MWh]$	17	-
c_{el}	electricity price	$[\text{€}/MWh]$	20-50	40
a	annuity factor (15 years, 5 %)		0.0963	0.0963
Parameters only in Model I (variables in Model II):				
P_j	pressure level of stream j	$[bar]$	Paper III	-
Non-negative variables:				Units
$m_{j,p}$	mass flow of stream j in period p			$[kg/s]$
$p_{j,p}$	pressure of stream j in period p			$[bar]$
$T_{j,p}$	temperature of stream j in period p			$[^\circ C]$
$h_{j,p}$	enthalpy of stream j in period p			$[kJ/kg]$
$h_{isentropic,j,p}$	isentropic enthalpy of stream j in period p			$[kJ/kg]$
$S_{j,p}$	entropy of stream j in period p			$[kJ/kgK]$
$S_{isentropic,j,p}$	isentropic entropy of stream j in period p			$[kJ/kgK]$
$m_{0,j}$	mass flow of stream j at design load			$[kg/s]$
$p_{0,j}$	pressure of steam j at design load			$[bar]$

$\eta_{isotropic,i,p}$	<i>isentropic efficiency of a turbine stage i in period p</i>	
$W_{i,p}$	<i>work out from unit i in period p</i>	<i>[kW]</i>
$x_{i,p}$	<i>steam content after the turbine stage i in period p</i>	
$x_{is,i,p}$	<i>isentropic steam content after the turbine stage i in period p</i>	
$LHV_{j,p}$	<i>lower heating value of biomass fuel stream j in period p</i>	<i>[kJ/kg]</i>
$f_{H_2O,j}$	<i>moisture content of fuel stream j</i>	<i>[m-%]</i>
ΔT_{DH}	<i>temperature change of DH water in DH exchanger</i>	<i>[°C]</i>
$T_{SPLIT,i}$	<i>temperature in splitter i</i>	<i>[°C]</i>
$\Delta T_{lm,i,p}$	<i>logarithmic mean temperature difference of heat exchanger i in period p</i>	<i>[°C]</i>
$\Delta T_{1,i,p}, \Delta T_{2,i,p}$	<i>temperature differences in heat exchanger i in period p</i>	<i>[°C]</i>
$A_{hex,i}$	<i>heat exchanger area of heat exchanger i</i>	<i>[m²]</i>
c_i	<i>cost of unit i</i>	

Binary variables:

y_i	<i>binary variable for unit i</i>
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Variables:

z	<i>additional profit gained from process changes</i>	<i>[€/a]</i>
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1 INTRODUCTION

1.1 Combined Heat and Power (CHP) Production

Combined heat and power (CHP) production is simultaneous generation of usable heat and electricity in a single process. In CHP production the heat from steam condensing after the steam turbine can be used for heating, e.g. in district heating or cooling networks, instead of dissipating it with the cooling water into the environment. In the case of industrial CHP plants the steam can be also extracted in higher pressures from the turbine and used as process heat. The fuel consumption can be decreased approximately 25 % - 35 % with CHP production compared to the power and heat generation in separate processes (Cogen Europe et al. (2001)). Thus, the CO₂ emissions per produced heat and power are reduced and the total efficiency of the generation increases.

The fuel variation used in the CHP production is fairly large. Natural gas, coal, light fuel oils including diesel oil, solid and gaseous biomasses, and waste fuels have all been used for CHP production. Currently, the processes that are most often used in the CHP production with natural gas are a gas turbine and a heat recovery boiler processes, an internal combustion engine process including heat recovery, and a gas turbine process with a combined steam cycle. The CHP plants using biomass fuels are often steam Rankine cycle processes with fluidised bed boilers or grate furnaces, while coal is usually fired in pulverised coal-fired boilers. In addition to these processes, in the future there will be potential for special CHP technologies developed for micro-scale (under 500 kW_e) processes.

In the EU only 10.2 % (88.4 GW_e) of the electricity is generated in CHP plants, most of which are industrial plants (Cogen Europe et al. (2001), Eurostat (2004)). Thus, there exists potential to increase the efficiency of fossil fuel utilisation and to reduce fossil CO₂ emissions with more extensive CHP production. The European Directive of Cogeneration (Directive 2004/8/CE) was approved in the beginning of 2004 and it promotes the CHP production as a mean to increase the power plant efficiencies and to reduce the CO₂ emissions in the EU. There is potential to increase the electricity generation from CHP production in the EU to 135 GW_e of installed CHP capacity by 2010 and to 195 GW_e by 2020. Especially, for Central and Eastern Europe (Czech Republic, Slovak Republic, Poland, Hungary, Slovenia, Bulgaria, Romania, Estonia, Latvia and Lithuania) there has been estimated to be possibilities to increase the CHP production by 50 % from 25.6 GW_e to 38 GW_e (Cogen Europe et al. (2001), Eurostat (2004)).

In Finland a large amount of the electricity is currently produced with CHP (Finergy (2002)). Many of the large-scale power plants have already been converted to CHP plants, so the possibilities to increase the amount of CHP in Finland is in converting smaller regional heating plants to CHP production. These plants are often using domestic biofuels that may be available in nearby regions and that are considered to be neutral on CO₂ emissions.

1.2 Challenges of Biomass-Fuelled Small-Scale CHP Production

Biomass fuels are attracting much interest as renewable and CO₂ neutral fuel alternatives. With CHP production they offer potential to reduce the use of fossil fuels. To utilise the biofuel resources effectively, the distances of the fuel transportation should be minimised and the CHP production should be situated as near to the fuel source as possible. In CHP production also the end use of heat sets requirements for the plant location, as the CHP plants should either be connected to a district heating or cooling network or situated near to an industrial site.

In Finland there is potential to increase the CHP production especially in smaller heating plants corresponding the power production of 1-20 MWe. These plants are usually using biomass fuels. However, in the plants of this size range the ratio between produced power and produced heat is often lower than in larger plants. Furthermore, the power-to-heat ratios tend to decrease as the CHP plants get smaller as presented in Figure 1.

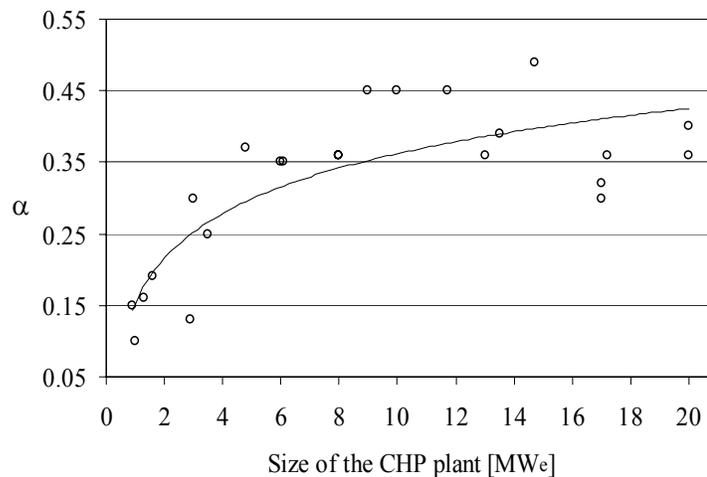


Figure 1. Power-to-heat ratios (α) versus the plant size of some Finnish and Swedish biomass-fuelled CHP plants producing 1-20 MWe and connected to a district heating network (Wahlund et al. (2000), Salomón et al. (2002), Kirjavainen et al. (2004)).

The decision to invest in a new CHP plant or to extend the current CHP production requires a corresponding heat demand. Therefore, to maximise the profit from the plant the power production of the plant should be as high as possible. To increase the economic feasibility of the small-scale CHP plant investments more electricity should be extracted from the process per produced heat unit. In addition to the higher electricity production, the increased power-to-heat ratio could also reduce the fuel consumption and the CO₂ emissions per produced electricity unit.

The factors that are limiting the power-to-heat ratios in the small-scale CHP plants are mostly material properties and economical issues. For example, the superheated steam temperature in the process is limited mostly by the used materials in the superheaters and in the steam turbine, whereas many other process features that are commonly used in larger plants are considered to be too expensive for smaller size ranges. Thus, the trade-offs between costs, the complexity of the process, and the increased power production are important factors when defining the most profitable process for a CHP plant investment. Currently, the small-scale CHP processes are kept as simple as possible in new plant investments in order to maintain the economic feasibility of the plants. However, to improve the economic feasibility of the plant investments it would be important to know the CHP plants sizes, in which the different process changes increasing the power-to-heat ratio could become profitable.

1.3 Challenges of CHP Process Modelling and Optimisation

Usually, the objective of the optimisation of CHP process synthesis is to maximise the profits (or minimise the costs) of a new plant system either in design conditions or taking into account also the heat demand of a district heating network or of an industrial process. In small-scale CHP plants the demand of district or process heat is usually defining the operation of the plant and the electricity is an extra product that offers valuable addition to the profitability of the system. Thus, the part load operation is an important factor in the profitability of the small-scale CHP plants. This needs be taken into account also in the modelling of CHP processes by using for example multiperiod modelling for the discrete time and heat demand modelling and by ensuring that the power production has been modelled accurately enough also at part loads.

The profit of the modelled CHP process is heavily dependent on the capacity to produce power with the demanded heat loads. Thus, the modelling of the power production unit, e.g.

a back-pressure steam turbine, should correspond well with reality also at the part load operation. In small-scale units there are often significant efficiency differences to the larger ones and the part load operation may further have an effect to the performance of the power production equipment. Therefore, the power production process should be modelled with care and the effects of part loads should be considered in the model. This is challenging for the back-pressure steam turbines used commonly in the biomass-fuelled small-scale CHP plants based on the steam Rankine cycle.

One challenge of the process synthesis modelling is to choose the right level of complexity to the model as there often exists the trade-off between the complexity and the solvability of the model. In the case of CHP processes the complexity of the models have been reduced with thermodynamic analysis (Manninen and Zhu (1999b)), by fixing the pressures in the model as parameters (Manninen and Zhu (1999a, 1999b), Bruno et al. (1998)), or by concentrating in the single operating point optimisation and not including the part load operation of the CHP plant (Manninen (1999), Bruno et al. (1998)). However, especially for the small-scale CHP plants the part load operation is important. Also, fixing the pressures as parameters before optimisation requires extensive knowledge on the process and some important design decisions may need to be made before the optimisation.

The disadvantage of the more detailed part load and pressure modelling is the increased complexity of the model that may cause difficulties in the solving of the problem and decrease the model robustness. A process synthesis model of a CHP process is a nonlinear model containing both continuous and discrete variables (i.e. binary variables for the selection of units). There are often also constraints (e.g. bilinear energy balance equations) that cause the model to be nonconvex. This means that with the local solvers that are often used to solve these models it is not possible to guarantee a global optimum for the problem. For nonconvex problems only local solutions are found with local solvers and thus finding a good enough solution may require several optimisation runs. With a nonconvex problem selecting of a right solver to the model is crucial.

1.4 Objective of the Thesis

The objective of this thesis is to evaluate possible process improvements for small-scale (1-20 MW_e) CHP plants and to create an optimisation model of process design changes that could increase the power production and the power-to-heat ratio of these biomass-fuelled CHP plants. The model would be based on a superstructure including the process changes that could increase the power-to-heat ratio of small-scale CHP plants. The model should

include the part load behaviour of the plant for example by using multiperiod modelling for the discrete demand of district or process heat. Also, pressure modelling, steam and water property functions that depend both on temperature and pressure, and a detailed model of a back-pressure steam turbine suitable for small-scale CHP plants should be incorporated to the model. With the increased complexity of the CHP process synthesis model, the robustness and solvability of the model may suffer. Thus, convexities and nonconvexities of the model should be studied and a suitable solver for the problem selected. Selected small-scale CHP plant cases and their possible process improvements should be optimised with the developed optimisation model.

1.5 Outline of the Thesis

This thesis consists of five original articles and a summary of the research and its results. In Section 2 of the summary part the state-of-the-art of the biomass-fuelled CHP plants and the current state of their simulation and optimisation is presented. This part also describes the challenges of the CHP process modelling and the most important previous research done in this field. On the basis of this, the focus and the research problem of this work are formulated in Section 3. Selected process modifications and their simulations are presented in Section 4. The developed new MINLP optimisation models (Model I and Model II) are described in Section 5 and their robustness is discussed in Section 6. In Section 7 the most important results of this work are summarised and the significance of this work is evaluated. Finally, Section 7 gives also some main recommendations for the future work.

2 STATE-OF-THE-ART

2.1 Small-Scale CHP Plants Using Biomass

2.1.1 Combined Heat and Power (CHP) Processes Based on Steam Rankine Cycle

The biomass-fuelled CHP plants producing less than 20 MW_e are usually based on a Rankine cycle with steam superheating. The steam after the boiler is superheated at the constant pressure to a higher temperature than the saturation point. A flowsheet example of this process is presented in Figure 2. If the process is producing only power with a condensing steam turbine, the heat exchanger after the steam turbine uses cooling water to condensate the steam into water. The steam expansion in the steam turbine is limited by the moisture content of the steam after the turbine. The maximum value for moisture is around 12 %. The cooling water temperature after the condenser is 20-30°C, so the heat transferred to the cooling water in condensation usually cannot be utilised because of the low temperatures. If the process is used also for heat production, e.g. in a district heating (DH) network, the forward temperature of the heated water has to be higher, at least 85-110°C depending on the outdoor temperature. This defines the temperature and the corresponding pressure of the steam after the turbine. Thus the process is often called back-pressure process. The higher temperature and pressure after the back-pressure turbine reduces the power production as can be seen from Figures 3 and 4, where the temperature vs. entropy (T,s) -diagrams and the enthalpy vs. entropy (h,s) -diagrams of the condensating and back-pressure processes are compared. In the Figures 2-4 the letter A refers to the superheated steam, B is the steam after the back-pressure turbine, C is the steam after the condensing steam turbine, D is the water after the condenser in the condensing process and E in the back-pressure process, F is the feed water at the saturation temperature, and G is the saturated steam after the evaporator. In D and E two points are marked as also the pressure increase in the pump is taken into account. B' and C' are the corresponding isentropic steam values after the turbine.

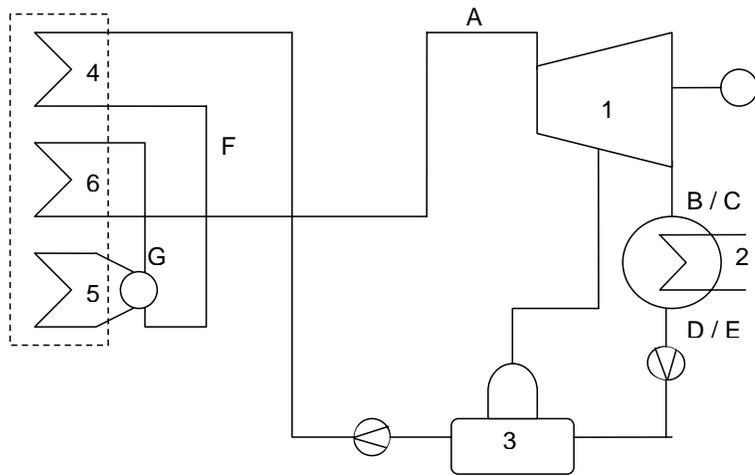


Figure 2. Simple power plant based on a steam Rankine cycle with steam superheating.
 1=Turbine, 2=Condenser/DH exchanger, 3=Feed water tank, 4=Economiser, 5=Evaporator,
 6=Superheater

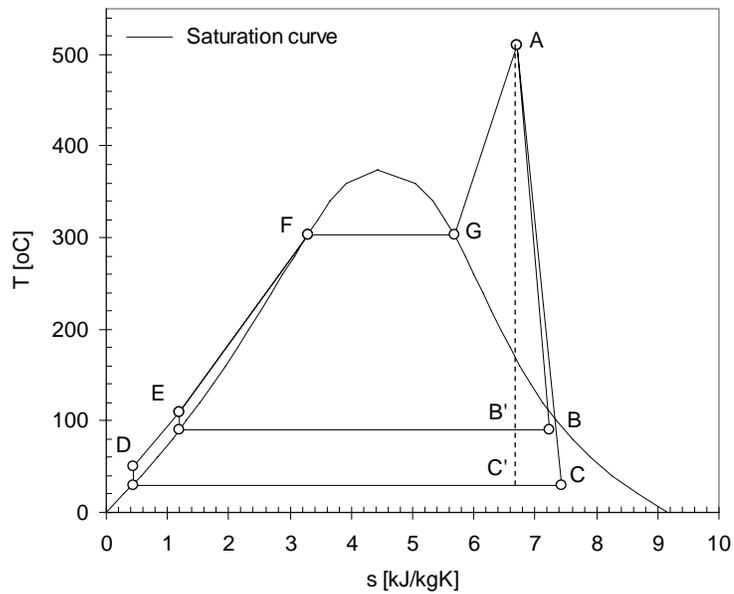


Figure 3. T,s -diagram comparison of a steam Rankine processes with steam superheating for a condensing power plant (ACDFG) and for a back-pressure CHP plant (ABEFG).

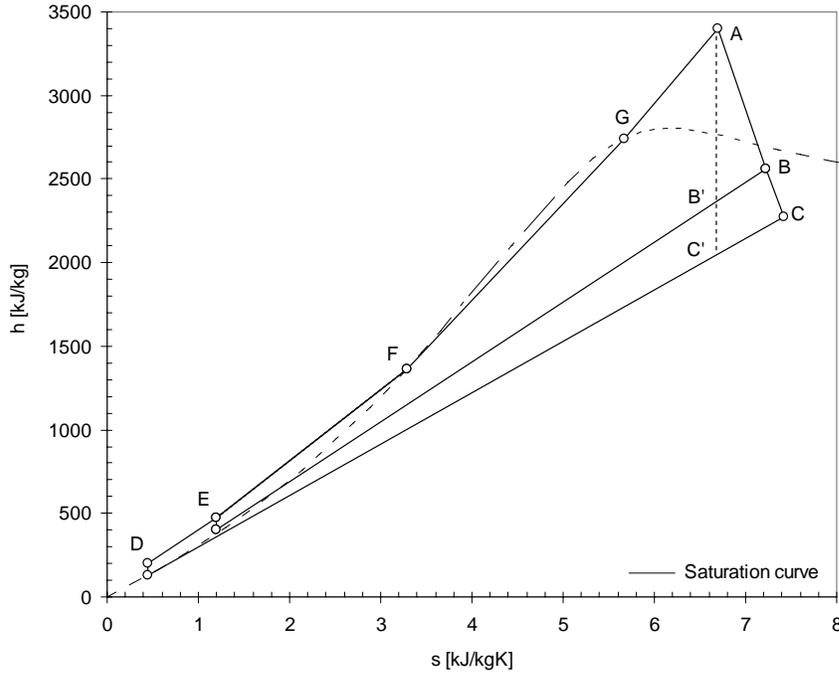


Figure 4. h,s -diagram comparison of a steam Rankine processes with steam superheating for a condensing power plant (ACDFG) and for a back-pressure CHP plant (ABEFG).

The reduction in the mechanical work from a steam turbine, when producing both district heat and power, equals the difference between AB and AC . When also the turbine losses, η_{loss} , and the generator efficiency, η_{gen} , are taken into account, the difference presented in Eq. (2.1.1) gives an estimation of the difference in the electricity production, ΔP_{gen} , between the condensing and the back-pressure processes.

$$\Delta P_{gen} = AB \cdot \eta_{loss} \cdot \eta_{gen} - AC \cdot \eta_{loss} \cdot \eta_{gen} \quad (2.1.1)$$

The electrical efficiency, η_e , of a power plant process can be defined

$$\eta_e = \frac{P_{net}}{Q_{fuel}}, \quad (2.1.2)$$

where P_{net} is the net power production of the plant and Q_{fuel} is the fuel input energy to the plant. The total efficiency, η_{tot} , of the back-pressure process producing power and district heat can be defined

$$\eta_{tot} = \frac{P_{net} + Q_{dh}}{Q_{fuel}}, \quad (2.1.3)$$

where Q_{dh} is the district heat produced in the plant. For a condensing power plant the $Q_{dh} = 0$. Though Eq. (2.1.3) is widely used, it is not completely logical in the energy technology point of view, as it considers the electricity and heat to be equally valuable products. In reality, electricity is a more valuable product as it can be transformed to any other energy form,

whereas the Carnot efficiency restricts the transformation of heat.

Although η_{tot} is higher in CHP plants than in condensing plants, η_e remains lower in CHP plants than in condensing plants because of the smaller power production as shown in Figures 3 and 4. For the 1-20 MW_e biomass-fuelled CHP plants η_e varies from 17 % to 29 % depending on the plant size (Wahlund et al. (2000), Salomón et al. (2002), Kirjavainen et al. (2004)), while it is 36-38 % for a condensing power plant and can be around 45 % for supercritical power plants. With the heat production η_{tot} is in the CHP plants over 90 % (LHV).

The CHP plants are characterized by a parameter indicating the produced power versus the produced heat called the power-to-heat ratio, α .

$$\alpha = \frac{P_{net}}{Q_{dh}} \quad (2.1.4)$$

α corresponds to the ratio

$$\alpha = \frac{AB \cdot \eta_{loss} \cdot \eta_{gen} - P_{process}}{BE}, \quad (2.1.5)$$

where AB and BE refer to the Figures 4 and 5 and $P_{process}$ corresponds to the power used in the CHP plant, e.g. in pumps and blowers. η_{loss} , η_{gen} and $P_{process}$ have usually only a small effect on AB , so a rough estimate, corresponding the upper bound of α , can be calculated with a ratio between AB and BE .

For an economical operation of the CHP plant a high α is preferred. The district heating CHP plants are usually operated according to the heat demand in the network, so a plant with a high α produces more electricity to the grid with the same heat demand than a plant with a low α . A special problem in the small-scale biomass-fuelled CHP plants is that α has remained fairly low compared to the larger plants. Currently, α is between 0.10 and 0.30 in the 1-5 MW_e CHP plants and between 0.35 and 0.45 in the 5-20 MW_e CHP plants (Wahlund et al. (2000), Salomón et al. (2002), Kirjavainen et al. (2004)). In larger back-pressure CHP plants producing district heat the α is usually 0.45, but with other competing CHP processes α can be much higher. In gas turbine processes with a simple recovery cycle α can be 0.55, in a gas turbine process with combined steam cycle 0.95, and in an internal combustion engine process with heat recovery 0.75 (Orispää (2000)). Improvement of α would increase the power production and could improve the economic feasibility of the new small-scale CHP plant investments.

2.1.2 Review of 1-20 MW_e Biomass-Fuelled CHP Plants in Finland and Sweden

In the year 2003 about 76 % of the district heating and 38 % (27.2 TWh_e) of the electricity in Finland was produced with CHP plants. The CHP was produced with natural gas (37 %), coal (27 %) and domestic fuels including peat, wood, and biogas (28 %) (Finergy, (2002)). Although there is a high share of CHP plants in Finland and many large-scale power plants have been converted to CHP plants, there are still possibilities to increase the amount of CHP by converting regional small-scale district heating plants to combined heat and power production (Kirjavainen et al. (2004)). The potential to increase the CHP production in Finland has been estimated to be around 5.5-7.5 TWh_e per year (Kirjavainen et al. (2004)). In Sweden only 10 % of the district heat (STEM (2001)) and 6 % (8.7 TWh_e) of the electricity (STEM (2004)) is produced in CHP processes. The main reason for these low shares is the abundance of affordable electricity from hydro and nuclear power plants. The future potential for CHP in Sweden is estimated to be 10-20 TWh_e per year. From this around 20 % could be small-scale CHP plants (Ambiente Italia srl et al. (2001)).

Table 1. Biomass-fuelled CHP plants producing 1-20 MW_e constructed in Finland from 1990 to 2004. (Kirjavainen et al. (2004))

Power plant	Power MW _e	Heat MW	Fuel MW	η_e %	η_{tot} %	α	Steam values bar/°C/kgs ⁻¹	Fuel	Technology	Start up
Kiuruvesi	0.9	6	8.1	11	85	0.15	25/350/2.8	bark, sawdust, wood chips	grate and steam engine	1999
Karstula	1	10	12.9	8	85	0.10	24/350/	bark, sawdust	grate and steam engine	2000
Renko	1.3	8	10.9	12	85	0.16		wood	grate	2004
Vilppula	2.9	22.5	29.9	10	85	0.13	50/450/	bark	grate	2004
Kuhmo	4.8	12.9	20.1	24	88	0.37	81/490/	wood residues	CFB	1992
Kuusamo	6.1	17.6	27.6	22	86	0.35	61/510/8	peat, wood chips, sawdust	BFB	1993
Kankaanpää	6	17	26.0	23	89	0.35	60/510/7.9	peat, wood	BFB	1992
Lieksa	8	22	33.9	24	89	0.36	61/510/8	peat, wood	BFB	1994
Ristiina ¹	10	64 ¹	86.0	12	86	0.16		wood	BFB	2002
Iisalmi	14.7	30	48.0	31	93	0.49	93/515/17.5	peat, wood, REF	BFB	2002
Kotka	17	56	81.1	21	90	0.30	62/480/21	bark, wood, peat	BFB	2003
Savonlinna	17	53	81.0	21	86	0.32	92/523/28	bark, wood, peat	BFB	2003
Forssa	17.2	48	71.7	24	91	0.36	62/510/22.8	wood, REF	BFB	1996
Kokkola	20	50	78.7	25	89	0.40	80/482/27	peat, wood	BFB	2002

¹Main product of the plant is process heat.

Table 2. Biomass-fuelled CHP plants producing 1-20 MW_e constructed in Sweden from 1990 to 2002 (Wahlund et al. (2000), Salomón et al. (2002), Kirjavainen et al. (2004)).

Power plant	Power MW _e	Heat MW	Fuel MW	η_e %	η_{tot} %	α	Steam values bar/°C/kg _s ⁻¹	Fuel	Tech- nology	Start up
Tranås	1.6	8,3+2,7 ²	11.5	14.5	104	0.19	16/345/3.4	sawdust, bark	grate,steam engine	2002
Malå	3	10	16.3	18	85	0.30	41/480/4.4	wood	BFB	1991
Lomma	3.5	14	18.3	19	93	0.25	60/510/5.7	wood, paper	FB	1995
Värnamo ¹	5.5	9	18.5	30	76	0.67	40/455/-	wood	IGCC	1994
Falun	8	22+8 ²	35	23	109	0.36	63/490/10.2	bark, wood	BFB	1993
Nässjö	9	20+6 ²	36	25	100	0.45	85/490/12	wood	CFB	1990
Sala	10	22	36	28	89	0.45	80/480/12.6	wood	BFB	2000
Härnösand	11.7	26+7 ²	42	28	106	0.45- 0.49	92/510/14	wood, bark, peat	BFB	2002
Hudiksvall	13	36	60	22	82	0.36	67/475/18	wood, peat	grate	1992
Kristianstad	13.5	35	55.5	24	87	0.39	65/510/17.5	wood	CFB	1994
Lycksele	14	28	50	32	84	0.51	88/520/17.5	wood	CFB	2001
Karlstad	20	55+20 ²	88	20	108	0.36	66.7/500/29	wood	CFB	1992

¹ Currently out of operation.

² Some heat is produced with flue gas condensing, and is considered when calculating η_{tot} but not when calculating α .

Summaries of the biomass-fuelled CHP plants producing less than 20 MW_e and built during the last fifteen years in Finland and Sweden are presented in Tables 1 and 2. Most of the plants producing 3-20 MW_e are based on a steam Rankine cycle with steam superheating as described in Section 2.1.1. The furnaces used in the processes are circulating fluidised beds (CFB), bubbling fluidised beds (BFB), and grate furnaces. In Kiuruvesi, Karstula, and Tranås the grate technology designed for biomass firing is used together with a steam engine. In addition to these technologies, an integrated gasification combined cycle process has been demonstrated in Värnamo with a 5.5 MW_e CHP plant. The plant was based on a pressurised biomass gasifier and on combustion of the cleaned gasification gas in a gas turbine. The flue gases of the gas turbine were utilised in a heat recovery boiler, which produced steam for a steam turbine. However, the plant is currently out of operation.

Many CHP plants in Sweden have also flue gas condensing which increases the heat production. The humidity of the flue gases is condensed and the heat is transferred to the returning district heating water. This process increases the η_{tot} of the plant often by 10-30 %. This can result in total efficiencies over 100 %, as the lower heating value of the fuel is used in the calculations. In Tables 1 and 2 α have been calculated without the heat from flue gas condensation, to make a comparison of the CHP plants possible. The flue gas condensing

requires that there are profitable possibilities to utilise the additional heat from the condenser. This has made the flue gas condensing unattractive for the CHP plants in the Finnish markets for the last ten years.

2.1.3 Possibilities to Improve the Power Production in Small-Scale CHP Plants

2.1.3.1 Temperature and Pressure of Superheated Steam

Important factors defining the power generation in Rankine cycles with steam superheating are the temperature and pressure of the superheated steam. Their current levels are mostly defined by the material limitations of the turbine. For example, the maximum temperature in high and medium pressure stages of the turbine is 600°C if high alloy steels are used and 550°C if low alloy steels are used. In low pressure parts of the turbine the non-alloy steels with maximum temperatures of 350°C are often used. A review by Fridh (2001) on the admission temperatures and pressures of 600 steam turbines producing from 1 to 25 MW_e noted that the admission temperatures to small-scale steam turbines are generally below 520°C-540°C, which is 40°C-60°C lower than the usual temperatures in large-scale plants. The economical feasibility of small-scale plants with higher steam temperatures depends on the possibilities to reduce the use of the more expensive materials in high temperature applications and on the successful scaling of these results from large-scale plants to smaller sizes.

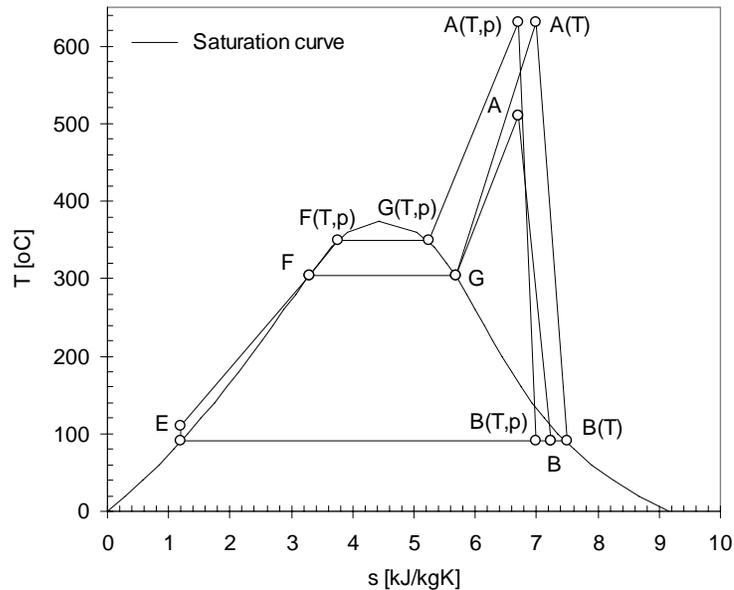


Figure 5. T,s -diagram comparison of the superheated steam temperature increase in constant pressure (A(T)) and of the simultaneous temperature and pressure increase (A(T,p), F(T,p), G(T,p)).

T,s -diagrams of the temperature increase at constant pressure and of the simultaneous temperature and pressure increases are presented in Figure 5. If only the temperature of the superheated steam is increased, the steam moisture content after the turbine decreases and the steam after the turbine may become even slightly superheated. If both the pressure and the temperature of the superheated steam are increased, the steam moisture content after the turbine increases. In back-pressure CHP processes, where the forward temperature of the DH water defines the pressure after the turbine, there may be potential for the steam moisture content to increase before the limitations of the turbine materials are met.

2.1.3.2 *Steam Reheating*

Another possibility to increase the power production is to extract the superheated steam from the steam turbine and to reheat it with flue gases to a higher temperature. In the case of a condensing plant, turbine materials limit the steam moisture content and thus the steam exit pressure from the turbine. Reheating makes it possible to have a lower exit pressure than without reheating. In CHP plants with back-pressure turbines the steam exit pressure from the turbine is defined by the forward temperature of the DH water. In the back-pressure turbine processes reheating improves power production, if the extraction pressure of the reheated steam is selected so that the reheating increases the average temperature of the incoming heat to the process. All the heat that is transferred to the process in reheating (A2-A1 in Figure 6) can be used for power generation in a steam turbine. Usually, the extraction pressure of the reheated steam is selected so that the steam can be reheated to the same temperature it had before entering the turbine. This makes it possible to use the same material both in a superheater and in a reheater. Reheating is common in larger power plants but in smaller plants it has not been used as its economical feasibility has been considered to be low.

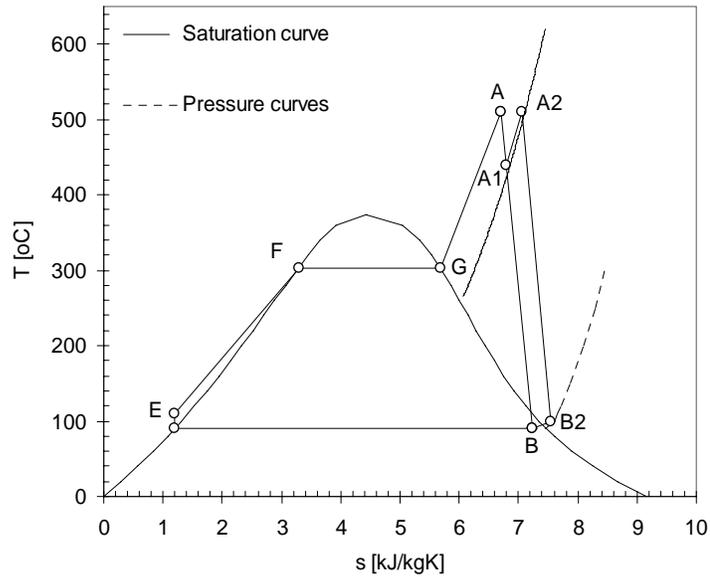


Figure 6. T,s -diagram of the steam reheater (A2) addition.

2.1.3.3 Feed Water Preheater

The efficiency of a power plant can be improved also by increasing the temperature of the feed water before the economiser. The temperature can be increased by extracting steam from the turbine before and after the feed water tank extraction and using this steam to preheat the feed water. A T,s -diagram of the feed water preheating is presented in Figure 7.

2.1.3.4 Two-Stage DH Exchanger

One possibility to increase the power generation of a back-pressure steam turbine is to reduce the steam exit pressure by dividing the DH exchanger into two or more stages, and by extracting the steam from the turbine to DH exchangers in several phases. The exit pressure of the saturated steam from the turbine is then defined by the DH water temperature after the additional stage of the DH exchanger, which may be significantly lower than the forward temperature as shown in Figure 8. The increase of power generation resulting from the two-stage DH exchanger compared to a single-stage corresponds to the difference between T_b and T_a . The two-stage DH exchangers have already been used in the CHP plants producing from 15 to 20 MW_e, but their economical feasibility in smaller plant sizes has not yet been demonstrated.

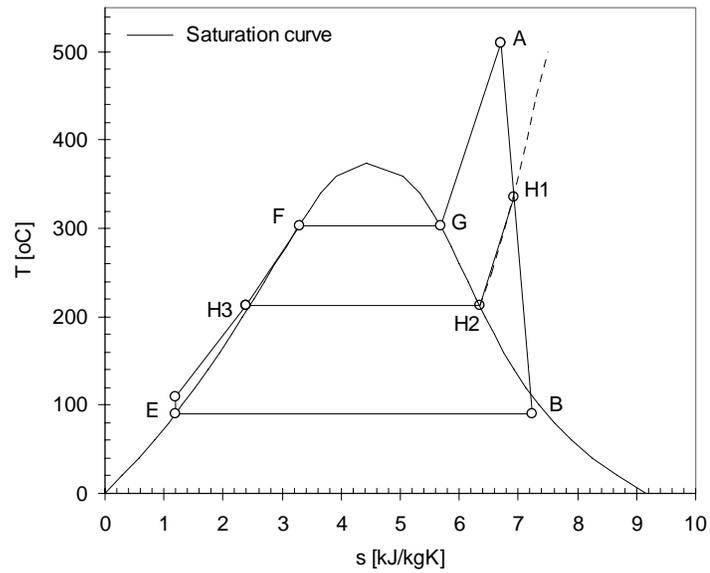


Figure 7. T,s -diagram of a feed water preheater (H1-3) addition.

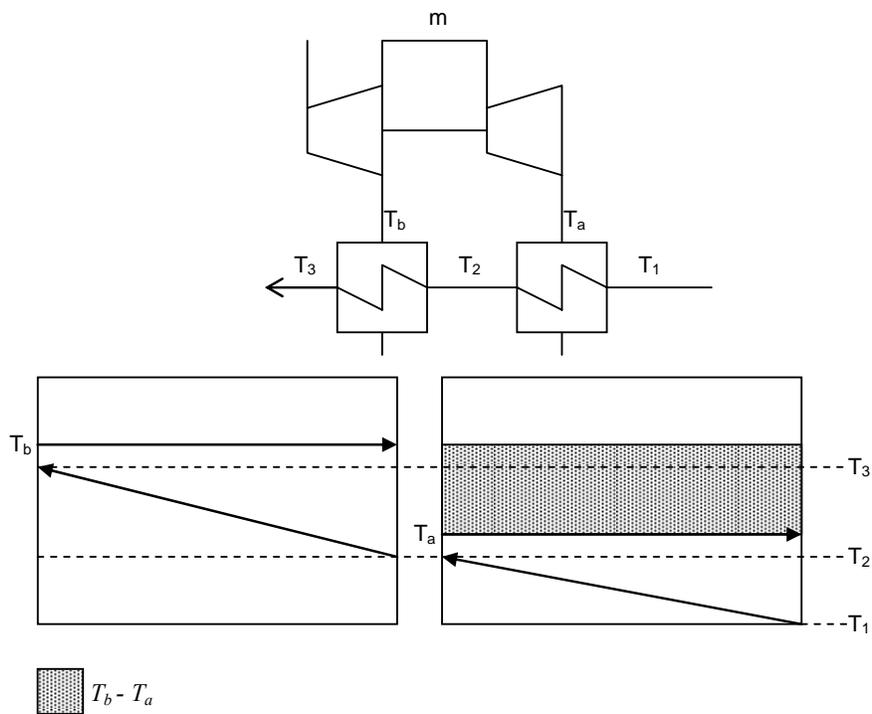


Figure 8. $m \cdot (h(T_b) - h(T_a))$ is the work transferred from district heat production to power generation by using a two-stage DH exchanger.

2.1.3.5 Fuel Drying

The moisture content of the biofuels can be up to 55 weight-%, so fuel drying with flue gases or steam has a good potential to increase the power production of a biomass-fired CHP plant. An overview of the current fuel drying technologies is presented by Wimmerstedt

(1999) and Brammer et al. (1999). In the case of a power plant situated near large biofuel resources it may be also profitable to dry the fuel for transportation to the more remote plants. Wahlund et al. (2002) describe a system configuration for a CHP plant, where a fuel dryer producing wood pellets is utilising steam during the low heat demand and thus increasing the annual power production of the plant.

2.1.3.6 *Gas Turbine or Gas Engine*

A gas turbine and a gas engine integration to the CHP plant is an efficient way to increase the electricity generation and the power-to-heat ratio of the plant. In larger CHP plants, a gas turbine is often connected directly to a heat recovery boiler, but the turbine can also be integrated to a solid fuel fired boiler by using the exhaust gas from the gas turbine in a feed water preheater, or using the gas turbine exhaust gas as combustion air in the boiler. Manninen and Zhu (1999a) have presented a method for finding the optimal integration of a gas turbine to the utility. Harvey et al. (2000) have studied gas turbine CHP plant performance including part loads and its effects on district heating costs and CO₂ emissions. Carcassi and Colitto Cormacchione (2001) presented a comparison of the gas turbine part load performances in the heat recovery boiler CHP application. In smaller plants especially the investment costs of a gas turbine or a gas engine integration may become critical, as the investment per unit of generated power increases when the size of the turbine or engine decreases.

2.2 Process Simulation of CHP Systems

Process simulation is a central part of the computer-aided process design both in the contemporary academic research and in the industrial applications. A comprehensive review of simulators and simulation methods has been presented by Biegler (1989, Biegler et al. (1997)).

2.2.1 Process Simulation Methods

Process simulator concepts have traditionally been classified into sequential modular methods and equation-oriented approaches. The sequential modular simulators are based on flowsheet topology of black-box unit modules and on the calculation of the mass and energy balances for each unit. The units and the thermodynamic properties are often divided into sub-programs or processes. In a sequential modular simulator the program sets up the flowsheet topology of the units, inserts the input data, and defines the calculation order of the unit modules in the process. Then the program calculates the mass and energy balances for each unit using the procedures defined in the unit operations library. Lastly, the physical

properties of the streams, e.g. the enthalpies and the steam properties, are calculated using the physical properties library in the simulator. The major differences among the modular simulators are in the libraries of unit operations and physical properties. The modular methods are widely used in process design work. The disadvantage of the modular simulators is that they are inflexible for a large variety of user specifications in flowsheet design.

In the simulators based on the equation-oriented approach the unit equations are assembled and solved simultaneously using general solution strategies (e.g. Newton-Raphson or quasi-Newton methods). The program sets up the flowsheet of the simulated process, organises the unit equations into one large set, and solves them with a general purpose equation solver. In the equation-oriented simulators there is almost no distinction between flowsheet or stream connection equations, unit operation equations, and physical property equations. The advantages of equation-oriented simulators include that they are flexible in the flowsheet design and that they allow the use of advanced optimisation strategies. On the other hand, the performance of the equation-oriented methods is limited by the capabilities of the equation solver. The equation-oriented simulators require also large-scale numerical algorithms, good initial values, and efficient strategies to prevent convergence failures. The main applications for the equation-oriented simulators are in the on-line modelling and optimisation fields.

The process simulation can be done either with a steady state or with a dynamic model. The steady state models are commonly used in process design simulation. In addition, the off-design simulations of the process at the loads differing from the design load are possible with steady state simulators. In order to include the time dependence of the load variations to the simulations a dynamic model is needed. Dynamic models are often used in on-line monitoring of the process, in planning of the optimal operation of the process, and in the diagnosis of the operational faults in the process. However, if time dependence of a process is not required, the performance of the steady state simulation programs is sufficient. An extensive comparison between some current programs for power plant simulations is presented in Giglmayr et al. (2001).

2.2.2 Process Simulation and Optimisation

A common approach to improve processes using simulation is to simulate a number of cases, and then select the best of these. As this is often very time consuming and it is difficult to efficiently handle the trade-offs, e.g. between the design and the costs, many commercial

simulation packages have integrated nonlinear programming (NLP) solvers for optimisation of constrained continuous variables. For example, both Aspen by AspenTech Inc. and Balas by VTT Technical Research Centre of Finland use the sequential quadratic programming approach (SQP), which is an algorithm for solving NLP problems that is relatively easy to implement into sequential modular simulators. The SQP-methods uses Newton's method to solve the Karush-Kuhn-Tucker optimality conditions for the NLP problem (Bazaraa (1993)). The resulting problem is a minimisation of a quadratic approximation of the Lagrange function where the constraints are linearised. For sequential modular simulation many of the variables and equations can be hidden from the SQP solver using a black-box approach, thus reducing the problem size. For equation-oriented simulators the variables and equations are incorporated into the optimisation problem, resulting in a larger optimisation problem. Since the computation time for the SQP approach increases cubically with the problem size, it is rarely used in commercial equation-oriented simulators (Biegler (1997)).

Solvers for mixed integer nonlinear programming (MINLP) problems have also been used in connection with commercial simulators. This is particularly useful in the cases where the optimisation would need to include discrete events (e.g. if-then logical statements). An example of an MINLP solver used together with Aspen is, for example, described in the work by Diwekar et al. (1992).

Benefit of the optimisation algorithms implemented into simulators is that the same model created for the simulator can be used in the optimisation and a construction of a new model for optimisation purposes is not necessary. Thus, it is possible to optimise process parameters of the simulated process without extensive additional work. Also, if an existing simulator offering detailed models for process modules can be used as a basis of the optimisation, the optimisation results may be more reliable than if a totally new mathematical model for the optimisation of the process would have been created. However, same problems related to nonlinearities, convexity, and combinatorial issues that are further discussed in Section 2.3.2 apply also to the optimisation algorithms implemented into simulators, and it is thus important to choose the correct algorithm for the optimisation. Equations that can be suitable for the simulators might have properties that are undesirable for optimisation algorithms. For these reasons it is often necessary to formulate the optimisation problem independent of the simulation model, in order to include equations with desirable properties and to be able to modify the optimisation model case-by-case to get solvers to produce good solutions in reasonable time.

2.2.3 Design and Off-Design Simulation of CHP Processes

The small-scale CHP plants connected to the DH network are usually operated according to the heat demand of the DH network. Thus the part load operation usually covers large periods of the total plant operation time and it is important to simulate the processes also at loads outside the design point, i.e. in off-design. In off-design the technical construction of the process modules is fixed in the model and the stream values as well as the process module properties depending on the stream values are simulated.

In off-design the steam turbine behaviour is an important factor defining the power production of the CHP plant. For power plants, where the steam flow to the turbine is regulated with a valve, the off-design operation of the steam turbine has often been modelled using the dependence between steam flow and the produced power (Mavromatis and Kokossis (1998), Manninen and Zhu (1999a)). This almost linear dependence is presented with Willans line (Church (1950), see Section 2.4.3). The Willans line is approximately a straight line between the smallest load, when the turbine is running but not yet producing power, and the load with the maximum efficiency. In the loads above the maximum efficiency the power production increases slower than in steam loads below the maximum efficiency. However, if the power plant is operating at partial loads long periods of its operating time, it may be more economical to control the output using a regulation stage before the normal turbine stages. This is typically the case with CHP plants. Then a nonlinear dependence between the steam flow and the turbine efficiency has to be defined case-by-case (Tveit (2004)).

The operation of the small-scale steam Rankine cycle CHP plants has previously been studied e.g. by Org'iro et al. (1996), who simulated the integration of a district heating production to an existing power plant. Harvey et al. (2000) used a simulation code developed by Carcasci et al. (1996) and created a simulation model on a CHP plant containing a gas turbine and a heat recovery boiler and discussed the optimal part load operation of the process. However, it is probable that most of the simulations of the existing small-scale CHP processes have been done in the industry and have not been reported publicly.

2.3 Process Synthesis Optimisation

2.3.1 Thermodynamic Approaches and Mathematical Programming

The methods used for finding the best process configurations in the energy systems and power plant optimisation can be divided into heuristic methods with thermodynamic targets

and into mathematical programming methods. Heuristic methods for process design are presented in Nishio et al. (1980) and for a gas turbine integration into a steam Rankine cycle in Chou and Shih (1987). The weakness of the heuristic methods is that even if the desired thermodynamic goal is reached in the process design, the capital costs of this solution may be too high as the economical factors are not included in the model. On the other hand, the mathematical programming methods are usually based on minimising the costs or maximising the profits of the system. Mathematical programming can efficiently take into account the trade-offs between different conflicting targets. It provides a possibility to use multiobjective optimisation for finding good solutions, when there are several conflicting targets that the optimal process should meet. It is also possible to include different operational conditions of the process to the model with multiperiod optimisation. A very extensive review on optimisation problems and methods is presented in Biegler and Grossmann (2004), and the future challenges and methods in the optimisation are summarised in Grossmann and Biegler (2004).

2.3.2 Mathematical Programming Methods for Process Synthesis

The first linear programming (LP) methods for the process synthesis were presented by Nishio and Johnson (1977) and by Petroulas and Reklaitis (1984). The LP models can optimise the operation of a process, which steam values and equipment behaviour is described with linear equations. To include noncontinuous variables in the models (e.g. for selecting the most profitable process equipment) Papoulias and Grossmann (1983) presented a mixed-integer linear (MILP) method, where the process alternatives are described as a superstructure and the binary variables include or exclude the modules in the process. However, the MILP method requires that the mathematical equations in the model are linear. This means, for example, that the steam temperature and pressure has to be parameters in the model to ensure that the energy balances are linear. The nonlinear programming (NLP) methods and the mixed-integer nonlinear (MINLP) method, presented e.g. by Kalitventzeff (1991), Grossmann and Kravanja (1995), and Grossmann and Daichendt (1996), allow the use of the nonlinear equations in the model so that the steam temperatures and pressures can be modelled as free variables.

For the solving of the mathematical programming problems, special algorithms for LP, NLP, MILP, and MINLP problems have been developed. For the LP problems there exist effective solvers, e.g. CPLEX by Ilog, that are able to find the global optimum of the problem. Many of these algorithms can be used also for solving the MILP problems to the global optimum. For the NLP and MINLP problems the algorithms often require the convexity of the problem

(see Section 2.5.2) in order to find the global optimum (e.g. the outer approximation method by Duran and Grossmann (1986)). However, the nonlinear problems have often nonconvex feasible regions and thus the optimal solution found by the traditional algorithms depends strongly on the initial point and may be optimal only locally (Tawarmalani and Sahindis (2002)). The more developed algorithms include different methods to decompose and convexify the problems in order to find the global optimum (Floudas (1999), Ryoo and Sahinidis (1995), Pörn et al. (1999), Westerlund (2006)). In process design nonlinear equations that may produce a nonconvex problem as well as combinations of continuous and discontinuous variables are often needed in order to achieve a realistic description of the system. Thus the formulation of the problem and the selection of the best possible algorithm are important for problem solving. For example in MINLP problems two very different solvers are used. DICOPT by Carnegie Mellon University works well with MINLP problems which have many discontinuous variables and large combinatorial problems but which do not include very complex nonlinear equations. On the other hand, SBB by ARKI Consulting and Development A/S is a more effective solver for problems with difficult nonlinearities but few discontinuous variables (GAMS (2004)). If the solver is not selected according to the special properties of the problem, it may be that not even a locally optimal solution can be found.

A process can be optimised with mathematical programming either using a single operating point or taking into account also the different operation conditions of the process. The different operational conditions can be presented in the problem as multiperiods. The benefit of the multiperiod optimisation is that the best process design or operational parameters when taking into account the whole operation of the process can be found with it. However, the multiperiod approach complexifies the model as it requires constraints for periods with different operational conditions. Multiperiod optimisation was first suggested by Hui and Natori (1996) for optimising the production of electricity and fuels in a utility plant. Iyer and Grossmann (1997, 1998) developed this further by proposing a two-stage decomposition approach for the multiperiod optimisation of utility systems. Applications of the multiperiod optimisation to industrial cases have been reported by Papalexandri et al. (1998).

In process design there can be several conflicting objectives, e.g. maximum profits and minimum emissions, that the optimal process should meet. In multiobjective optimisation several objective functions can be solved simultaneously resulting to a set of mathematically equal solutions that are optimal for the problem. This so called Pareto optimal set can be a useful tool in decision making as it can show how a change in one objective affects the values of the others. Multiobjective optimisation has been used e.g. by Chang and Hwang

(1996) for waste minimisation of utility plants and by Roosen et al. (2003) as a decision tool in combined cycle power system investment planning.

2.4 Optimisation of CHP Processes

MILP and MINLP models for a CHP and power plant process design optimisation have been reported previously by Bruno et al. (1998) and Manninen and Zhu (1999a, 1999b, Manninen (1999)). In the model by Manninen and Zhu (1999b) thermodynamic analysis is used to reduce the complexity of a power plant synthesis model. In a MINLP model presented for CHP process synthesis by Bruno et al. the complexity of the problem is reduced by giving the pressure levels as parameters in the model and by modelling the process only at design load. With the model by Bruno et al. the integration of several possible boilers, a gas turbine, and steam turbines to a utility plant process can be optimised.

2.4.1 Modelling of Pressures

In many previous models (e.g. Bruno et al (1998), Manninen and Zhu (1999a, 1999b)), the complexity of the model is reduced by fixing the pressure levels and modelling them as parameters. This enables the use of steam and water property functions (i.e. enthalpy and entropy functions) that are depending only on temperature (see e.g. Bruno et al. (1998)). Problems in finding detailed enough descriptions for the dependence of enthalpy and entropy on both temperature and pressure, and without deteriorating the solvability of the model, are avoided. However, some of the optimisation possibilities of the processes may be lost as the pressures are excluded from the optimised variables. In a single point optimisation of a power plant Manninen (1999) has modelled the pressures as variables and used approximated functions for the steam properties in needed temperature and pressure intervals. Modelling pressures as variables would be important also in multiperiod models of power plants, as the fixing of pressures in a multiperiod model requires extensive knowledge of the process behaviour at part loads and may limit the optimisation possibilities especially when comparing the different CHP process alternatives in MINLP modelling. On the other hand the detailed modelling of pressure makes the model more complex, and thus the solving of the model more difficult.

2.4.2 Modelling of Steam and Water Property Functions

Construction of enthalpy and entropy functions for steam and water is a major challenge in process optimisation and especially in the optimisation of power plant processes which include steam and water cycles with significantly varying pressures and temperatures.

Bruno et al. (1998) solved this problem by defining pressure levels for high, medium, and low pressure steams in the process and by creating separate enthalpy end entropy functions for each pressure level. Similar approach was used also by Manninen and Zhu (1999a, 1999b). However, this method requires that the pressure levels are parameters. If also the pressure is a free variable in the model, the enthalpy and entropy functions need to depend on both temperature and pressure. Manninen (1999) used approximated functions for the steam properties in needed temperature and pressure intervals in a single point optimisation of a power plant. The detailed and accurate enthalpy and entropy functions for steam and water are defined in the Industrial Standard IAPWS-IF97 (Wagner and Kruse (1998)). However, these functions are complex and may cause problems in mathematical programming. Creating more simple functions of enthalpy and entropy dependence on both temperature and pressure on the basis of these standard functions is challenging and may require the limiting of temperature and pressure ranges where the estimating function is valid. This problem is previously discussed in the work by Laukkanen and Tveit (2003), where an effort to generate estimators of the steam and water property functions according to the Industrial Standard IAPWS-IF97 (Wagner and Kruse (1998)) is presented.

2.4.3 Modelling of Steam Turbines

Modelling of a back-pressure steam turbine is an important part of the CHP process synthesis. Bruno et al. (1998) describe the efficiency of steam turbine with regression models according to the steam inlet pressure to the turbine. A full load operation is assumed for each turbine. A much used model (e.g. Mavromatis and Kokossis (1998), Manninen and Zhu (1999a)) for a back-pressure steam turbine is the Willans line (Church, 1950). It is mathematically a simple model of the dependence between the power output of the steam turbine and the steam flow through the turbine. In Figure 9 the line *GEL* is describing the Willans line plotted as steam flow [kg/s] in the vertical and power output [kW] in the horizontal axis. The point *L* in the Willans line describes the maximum load, the point *E* the load where the efficiency of the turbine is at its maximum and the point *G* the steam flow with no load, i.e. the steam flow that is required to keep the turbine running but that is not yet producing power. The curve *gel* illustrates the curve of steam rate with the specific steam rate [kg/kWs] presented in the vertical axis. This nonlinear dependence of the specific steam rate and the turbine efficiency at part loads is incorporated in the Willans line.

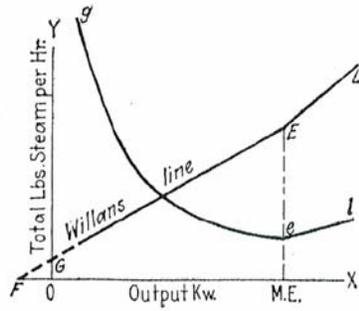


Figure 9. Willans line and the curve of steam rate (Church, 1950).

Between the most efficient load (E) and the point of no load (G) the Willans line is approximately a straight line. Thus this part of the Willans line can be described with Eq. (2.4.1)

$$W = n \cdot m - W_{loss} \quad (2.4.1)$$

where W is the produced power, m is the steam mass flow, n^{-1} is the slope of the Willans line and W_{loss} is the load lost between FG when the steam flow is needed to get the turbine running but is not yet producing power. In Varbanov et al. (2004, 2004a) the modelling of steam turbines on the basis of the Willans line is developed further using regression analysis to gain a better description of the part load performance of the steam turbine. Also, detailed methods for the calculation of the n and W_{loss} are presented.

2.4.4 Modelling of Part Loads

For a realistic CHP plant model, the part load behaviour of the process is crucial. The modelling of steam turbine performance at part loads is important and the changes of pressure levels during the part load operation may be significant. Especially, small-scale CHP plants are often operated according to the heat demand of a district heating network. Their load may vary from 100 % load to 50-30 % load (Marbe et al. (2004)) depending on the process and they may be operated long time periods at part load. Thus the variation of the heat demand in the network has a direct impact to the operation and the performance of the CHP plant and to its optimal design. In previous models the multiperiod modelling of CHP processes at part loads is presented (Manninen and Zhu (1999a, 1999b)) although some models have concentrated in the design modelling (Bruno et al. (1998)) without including the part loads in the optimisation model. Generally, in the process optimisation the multiperiod modelling is the common method to include the part load behaviour of the process to the model as discussed in Section 2.3.2.

2.4.5 Optimisation of Small-Scale CHP Processes

Currently, no models considering the process designs, efficiency improvements, and cost functions typical especially for the small-scale steam Rankine CHP processes have been presented. In general, the process design in small-scale plants has to be simpler than in larger processes because the costs of the process modules per produced power and heat tend to increase as the size of the process decreases. Also, long operation periods at part loads typical for small-scale CHP plants cause the need for including the heat demand and the part load modelling to the model. In addition, the efficiency changes of the process equipment at part load conditions should be incorporated in the model. Due to the high part load operation, the size of the process modules, and the superheated steam values that are often lower in small-scale plants than in larger plants (Fridh (2001)), many process changes that give good solutions for larger power plants may become unprofitable in smaller ones.

2.5 Robustness of CHP Process Synthesis Optimisation Models

2.5.1 Challenges for Robustness of CHP Process Synthesis Models

A robust model is not sensitive to initial values and is able to give the global optimum with all initial values within the bounds given for the variables. In practice, the commonly used local solvers can guarantee the global optimum only for the models that are convex. In the case of a nonconvex model the found optimum with a local solver may be a local optimum or the global one. There are some global solvers emerging to the optimisation use (e.g. BARON by Tawarmalani and Sahinidis). The solvers and local and global optimums are discussed more thoroughly in Section 2.3.2 and the concept of convexity is explained in Section 2.5.2.

The increase in the complexity of the CHP model often increases also the nonconvexity of the model and thus decreases its robustness. The formulation of the model is crucial for the solving of it and with a right formulation the nonconvexities also in complex models may be reduced or in some cases totally avoided. In a MINLP model of CHP process synthesis there are many nonconvex formulations that may reduce the robustness of the model and make it more sensitive to the initial values. Usually, temperatures of the process flows are free variables in the CHP process synthesis models (Bruno et al. (1998), Manninen and Zhu (1999a, 1999b)), which makes the energy balances in the model nonconvex. The pressure is often set as a parameter in the CHP models (Bruno et al. (1998), Manninen and Zhu (1999a, 1999b)) but if also the pressures are modelled as free variables the complexity of the steam and water property functions and the energy balances increase.

Steam turbine modelling is an important part of the CHP process synthesis but may cause nonconvexities to the problem. Willans line (Church, 1950) is a simple model of the dependence between the power output of the steam turbine and the steam flow through the turbine and it is often used in a CHP process synthesis modelling (Mavromatis and Kokossis (1998), Manninen and Zhu (1999a)). However, in complex steam turbine systems a model taking into account in more detail the nonlinearities related to the regulation stage and the exhaust losses of the turbine may be needed.

The CHP synthesis models often include also heat exchangers, which cause nonconvexities in the model with their logarithmic mean temperature difference functions and heat exchanger area formulations. Zamora and Grossmann (1997) have proposed convex bounding inequalities that can be used to replace the nonconvex logarithmic mean temperature difference in heat exchanger networks, and e.g Hashemi-Ahmedy et al. (1999) have used that approach in their convexification of a heat exchanger network model. In general, there are many phenomena in a CHP process that behave in a nonconvex way. Some of these nonconvexities can be avoided by modifying the model formulation, using less complex and convex functions or by fixing some free variables as parameters. However, some expressions in the model may have to be in their more complex form even if that formulation is nonconvex. In these cases, if a suitable convexification method is available, the convexification of these formulations may improve the model by making it more easily solvable and less sensitive to initial values.

2.5.2 Convexity of a Programming Model

A model is convex if the objective function is convex and the feasible region is a convex set. A set S is convex if for any elements x and $y \in S$

$$\alpha \cdot x + (1 - \alpha) \cdot y \in S \text{ for all } 0 \leq \alpha \leq 1 \quad (2.5.1)$$

This means that if $x, y \in S$, the line segment connecting the x and y must also be in S (Nash and Sofer (1996)). Examples of convex and nonconvex sets are presented in Figure 10.

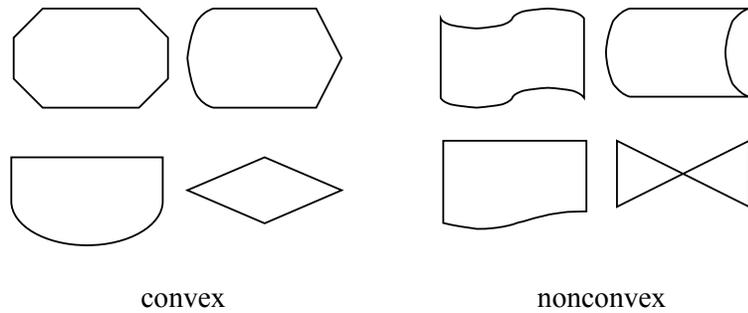


Figure 10. Examples of convex and nonconvex sets.

A function f is convex on a convex set S if it satisfies

$$f(\alpha \cdot x + (1 - \alpha) \cdot y) \leq \alpha \cdot f(x) + (1 - \alpha) \cdot f(y) \tag{2.5.2}$$

for all $0 \leq \alpha \leq 1$ and for all $x, y \in S$. This means that the line segment connecting the points $(x, f(x))$ and $(y, f(y))$ lies on or above the graph of the function f (Figure 11).

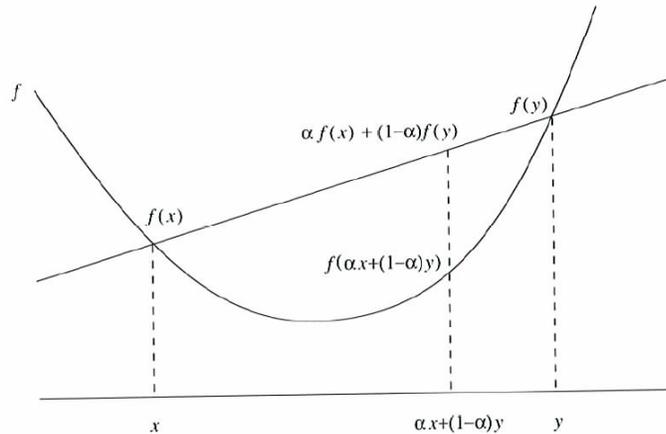


Figure 11. Convex function f and the line segment connecting the points $(x, f(x))$ and $(y, f(y))$ (Nash and Sofer (1996)).

A convex optimisation problem is of the form (Boyd and Vandenberghe, (2004))

$$\begin{aligned} &\text{minimise} && f(x) && (2.5.3) \\ &\text{subject to} && g_i(x) \leq 0, && i = 1, \dots, m \\ &&& h_i(x) = Ax - b = 0, && i = 1, \dots, n \end{aligned}$$

where the objective function f and the constraints g_i are convex functions, and the equality equations $h_i(x)$ are affine functions, i.e. sums of linear functions and a constant. In practice, the convexity of the objective function and the constraints can be tested using the knowledge that if the Hessian matrix of a function $f(x)$ for all $x \in S$ is positive semidefinite (i.e. the eigenvalues of the Hessian matrix are nonnegative) the function is convex. In the optimisation a global optimum of a programming problem can be guaranteed also with local

solvers if the model is convex. For a nonconvex model the local solvers are able to find only local solutions which may or may not include the global optimum.

2.5.3 Improving the Robustness of Process Synthesis Models

Improving the robustness of the model often means that its complexity has to be reduced and thus a trade-off between the correspondance of the model to the real process and the solvability of the model has to be made. If the lack of robustness is caused by the nonconvexity of the model it may be possible to convexify the model without changes in the model accuracy or without significant inaccuracies in the results. On the other hand some convexification methods bring estimations to the model and may thus reduce the accuracy of the model significantly.

There are many strategies to convexify a nonconvex optimisation problem. A review of the current status of the global optimisation and convexification is presented in Floudas et al. (2005). Global optimisation and different convexification methods are discussed in more detail e.g. in Tawarmalani and Sahindis (2002), and some recent developments in the convexifying methods are described in Björk and Westerlund (2002), Pörn et al. (1999), Pörn (2000), and Westerlund (2006). Some of the most important nonconvexities in the CHP process synthesis modelling are caused by bilinear terms (e.g. the energy balances and the dependence of the heat exchanger area on the logarithmic mean temperature difference). These bilinear terms can be convexified by using convex envelopes for bilinear terms developed by McCormick (1976) and Al-Khayyal and Falks (1983), and described for example in Floudas (1999). A convex envelope of a nonconvex function can be defined to be the tightest under-estimator of the function over the region of interest. The convex envelopes are used e.g. by Hashemi-Ahmady et al. (1999) to convexify bilinear heat balance constraints in the optimal synthesis of heat exchanger networks.

3 FOCUS OF THE THESIS

As described in Section 2.1.2 there are several new biomass-fuelled CHP plants producing less than 20 MW_e in Finland and Sweden. Many of these biomass plants can be considered to represent the best current technology of small-scale CHP processes. Although the recently built biomass-fired CHP plants are of a high standard, there still exists a significant difference in the efficiencies of the small-scale and the large-scale CHP plants.

The objective of this thesis is to evaluate possibilities to improve power production of small-scale (1-20 MW_e) CHP plants and to create an optimisation model of process design changes that could increase the power production and the power-to-heat ratio of these biomass-fuelled CHP plants. The small-scale CHP plants are often operated according to a heat demand of a district heating network or an industrial process, so the model should include the part load operation of the CHP process. Prior to constructing the optimisation model some profitable process improvements that could raise the efficiencies of the small-scale CHP plants closer to the the large-scale ones need to be selected with simulation tools. The process designs should be based on the current biomass-fired CHP processes.

The first part of the research problem is to find process improvements that could increase the power production and power-to-heat ratio of biomass-fuelled small-scale CHP plants and to construct a superstructure including these possible process improvements. Second part of the reasearch problem is to develop a mixed integer nonlinear programming (MINLP) model that would be able to optimise the most profitable process change combinations for the CHP plants and to optimise the selected case plants with the developed model. The optimisation model should take into account the part load operation of the CHP plant caused by the heat demand of the district heating network or an industrial process, and the trade-offs between the increased power production, the investment costs, and the additional fuel costs. Also, the changes in the fossil CO₂ emissions should be evaluated. It would be preferred if both temperatures and pressures could be optimised in the model, instead of fixing the pressures as parameters like in many previous models. However, this requires more complex steam and water property functions in the model as well as a steam turbine model which takes into account the pressure and efficiency changes at part loads.

The research tasks listed below consist of data collection (task 1), mapping and simulating the possible process improvements (tasks 2-5), construction of the superstructure and the MINLP model (tasks 6, 9 and 10), analysing the developed models (tasks 7 and 11), and optimising the process synthesis of the selected CHP plant cases with the developed models

(tasks 8 and 12).

Research tasks:

1. Mapping and collecting data of the existing small-scale CHP plants in Finland and Sweden and selecting good CHP case plants representing different sizes between 1 and 20 MW_e (Publication I).
2. Constructing simulation models of the case plants and analysing the sensitivities of the models (Publication I).
3. Simulating the power production of the case plants at part load (off-design) operation (Publication I).
4. Mapping the possible process improvements that could increase the power production and the power-to-heat ratio in small-scale CHP plants (Publication II).
5. Simulating the changes by applying them to the case plant models and analysing the effects of the process improvements on production, costs, and fossil CO₂ emissions (Publication II).
6. Constructing a superstructure of the considered process changes and a design case MINLP model suitable for process synthesis optimisation of small-scale CHP plants based on steam Rankine cycle with steam superheating (Publication III).
7. Analysing the sensitivity of the design case MINLP model (Publication III).
8. Optimising the most profitable process changes for four selected CHP plant cases (1.8 MW_e, 6 MW_e, 11 MW_e and 14.7 MW_e) with the developed design case model.
9. Developing a new model of a back-pressure steam turbine which includes the modelling of pressures and the effect of the part load operation on the turbine (Publication IV).
10. Developing a multiperiod MINLP model for the process synthesis optimisation of small-scale CHP plants. The model includes the modelling of pressures, steam and water property functions that depend both on temperatures and pressures, the part load modelling with multiperiods, and a new back-pressure steam turbine model. (Publication V).
11. Analysing the sensitivity of the multiperiod MINLP model (Publication V).
12. Optimising the most profitable process synthesis for the selected CHP plant case (6 MW_e) (Publication V).

As the main results of this work a superstructure of the possible process improvements is created, a multiperiod MINLP model for the process synthesis optimisation of small-scale CHP plants is constructed, and the profitable process improvements for the case plants are optimised. The formulation of the mathematical model is partly based on the models

presented by Bruno et al. (1998) and Manninen and Zhu (1999a, 1999b). However, in the developed model also the pressure levels can be optimised. The modelling of pressures requires also steam and water property functions that depend both on temperature and pressure and a back-pressure steam turbine model that can take into account the changes caused by the part loads. Special for the developed model is also that its superstructure is based on a typical process design used in 1-20 MW_e biomass-fuelled CHP plants, and that it includes alternatives for process design changes relevant to these plants, e.g. the selection of a single- or two-stage district heat exchanger to the process. Furthermore, the cost functions and cost coefficients have been modified to correspond to the conditions in these smaller plants. For example, the cost functions of gas engines and gas turbines are here specifically developed for small sizes. Therefore, unlike the other corresponding models, this model includes the design configurations and regression coefficients specifically suitable for small-scale CHP plants. In Table 3 this work is compared to the most relevant earlier CHP plant modelling research.

Table 3. Earlier key developments in power plant process modelling compared to this work.

Considers:	Iyer and Grossmann (1997, 1998)	Bruno et. al (1998)	Manninen (1999, model in Chapter 3)	Manninen and Zhu 1999a/b		Varbanov et al. (2004, 2004a)	This work
small-scale (< 20 MW _e) CHP plants							x
process improvements	x	x	x	x	x	x	x
thermodynamic analysis					x		
LP modelling				x			
MILP modelling	x			x		x	
MINLP modelling		x	x		x		x
fixed pressure levels	x	x		x	x	x	
pressure modelling			x				x
h and s parameters	x					x	
$h(T)$ and $s(T)$ functions for p levels		x		x	x		
$h(T,p)$ and $s(T,p)$ functions			x				x
part loads (multiperiods)	x			x	x	x	x
steam turbine:	x	x	x	x	x	x	x
- back-pressure steam turbine		x				x	x
- part load modelling				x	x	x	x
- pressure modelling at part loads							x
- detailed modelling of regulation stage efficiency and exhaust losses							x

4 SIMULATIONS OF SMALL-SCALE CHP PROCESS IMPROVEMENTS

4.1 Simulation Method

A simulation program Prosim by Endat Oy used in this research is a steady state simulation program. A steady state simulation program is sufficient for this study because although the CHP processes needed to be simulated at part loads, no detailed time dependent behaviour of the process was required in the simulations.

The process flowsheets in the used simulation program are constructed from modules and streams. The modules can be chosen from a module library, which contains most of the typical power plant equipment (e.g. burners, boilers, turbines, and heat exchangers). Stream analyses (e.g. fuel analysis) are available from an analysis library. The needed process modules for the model are selected and connected together with appropriate streams. The relevant parameters for the modelling case are inserted to the modules and the design simulation of the process model can be run. In a design simulation the energy and mass balances of the process modules are calculated. In addition, the physical properties of the plant equipment modules are calculated according to the given data. For each module the program uses an iterative Newton-Raphson method for the balance calculations, which are performed in the order of the user defined numbering of the modules. After the design model of the process has been created the technical construction of the modules can be fixed for off-design calculations. In off-design mode the operation of the power plant at part loads and in different external conditions can be calculated.

4.2 Design and Off-Design Simulation Models of the Case Plants

Four existing CHP plants producing 1.8 MW, 6 MW, 11 MW, and 14.7 MW electricity were selected to present the state-of-the-art steam Rankine processes with steam superheating in the CHP production from biomass. Some basic data of these processes at full load design conditions is presented in Table 4. All of the selected plants were quite new and three of them had started their operation in the year 2002, when also the process data of these CHP plants was collected.

The simulation models were constructed following the basic properties of the simulation program described in Section 4.1. The fuel used in the processes was wood, the lower heating value of which was 6.24 MJ/kg. The excess combustion air factor, λ , was 1.1 and the temperature of the combustion air after the air preheater was adjusted in design case to be

between 200°C and 300°C. In the processes with bubbling fluidised bed part of the superheating was done after the bed zone and part in the bed. The evaporator and the superheater in the bed were considered to be placed parallel to each other in the fluidised bed. This means that the bed temperature (850-870°C) indicated the flue gas exit temperature from both the evaporator and the superheater in the bed. Therefore the bed temperature defined the steam mass flow in the fluidised bed design cases, if the fuel input and thus the flue gas flow remained constant. In the grate boiler case a radiant furnace model with an evaporator was used and the flue gas exit temperature from the radiant furnace defined the steam flow in design case with the constant fuel flow.

Table 4. Basic data of the modelled CHP processes at full load conditions.

Process	MW _e	1.8	6	11	14.7
Boiler type		grate	BFB	BFB	BFB
Wood fuel input	MW	11.5	26	42	48
Temperature after boiler / in bed	°C	650	850	870	850
Steam values	°C / bar	355/16.5	510/60	510/92	515/93
	kg/s	3.8	7.9	13.8	16.1
Condenser pressure	bar	0.68	0.68	0.68	0.68/0.37
District heat in /out temperature	°C	55/85	55/85	55/85	55/70/85
Feed water tank temperature	°C	105	120	160	158
Flue gas exit temperature	°C	174	176	172	174
Net electricity production	MW _e	1.8	6.2	11	13.6
District heat production	MW	8.3	16.5	25.8	28.4
Electrical efficiency (η_e)		0.16	0.24	0.26	0.28
Power-to-heat ratio (α)		0.22	0.38	0.43	0.48
Total efficiency (η_{tot})		0.88	0.87	0.88	0.88

The operation of a CHP plant depends usually on the district or process heat demand. The minimum load of a general biofuelled steam power plant CHP system is in previous studies mentioned to be 30 % of the full load (Marbe et al. (2004)). The part load operation of the modelled CHP plants was here simulated by changing the fuel mass flow into the fluidised bed or grate boiler in the off-design mode. The minimum heat load used in off-design simulations depended on the plant size and was 35 % for the larger 11 MW_e and 14.7 MW_e plants and 45 % for the smaller 1.8 MW_e and 6 MW_e plants. This heat demand is converted to fuel input and the part loads are simulated according to the fuel input to the process.

A steam turbine was modelled as several turbine modules each corresponding to a group of one to five turbine stages (i.e. the expansion between two steam extractions). A similar

decomposition principle of steam turbines for modelling purposes has been presented by Chou and Shih (1987). The forward temperature of the district heating water defines the back-pressures in the steam turbine both at full and at part loads. The pressures of the other turbine stages are defined by the turbine constant, which can be derived from the cone rule (e.g. Traupel (2001)), and is calculated from the data given in the design point. To keep the pressure of the superheated steam constant also at the partial steam loads, the first turbine stage is a regulating stage, which adjusts the steam flow so that the required constant pressure in the boiler is obtained. The friction losses in blading and the relative efficiency changes of the regulation valves adjusting the steam flow decrease the efficiency of the regulation stage at part loads. The efficiency of the regulation stage is typically designed to be at its maximum at partial steam load (90 % load).

Here the part load performance of the regulation stage is defined in the turbine module as user defined second order polynomial function. The function describes the dependence of the efficiency on the isentropic enthalpy difference over the regulation stage and is valid only for a certain isentropic enthalpy difference range. Thus, the function is calculated for each CHP plant case separately. The functions are based on the estimation that the maximum efficiency of the regulation stage, 0.80, is gained at around 90 % steam load. At full steam load the efficiency is 0.75 and as the steam load decreases towards 10 % the efficiency goes to zero. This estimation, where the maximum efficiency of the regulation stage is gained at part load, corresponds the usual conditions in a CHP plant. With these estimations the efficiency of the regulation stage starts to decrease rapidly, when the steam load is less than 80-70 %.

The efficiencies, η_{ts} , of the working turbine stages at design conditions are calculated by Prosim using Eq. (4.2.1), which is based on the turbine design specifications from the late 1990's.

$$\eta_{ts} = 0.023521 \cdot \ln(\nu) + 0.749538 \quad (4.2.1)$$

The average volume flow, ν , is calculated as

$$\nu = \frac{m \cdot \Delta h_s}{P_{in} - P_{out}} \quad (4.2.2)$$

where m is the mass flow of the steam, Δh_s is the isentropic enthalpy change and p is the pressure.

When the mass flow of steam decreases during the part load operation, the inlet pressure of the turbine stage decreases accordingly. Thus the average volume flow and therefore the

isentropic efficiency of the turbine stage are not affected by the load changes. Overall, the part load operation affects the efficiency of the whole turbine system by changing the efficiency of the regulation stage and the exhaust losses at the turbine exit. The exhaust losses of the last turbine stage were calculated according to the reference data of exhaust loss versus relative volumetric flow. A detailed description of the models, their sensitivity analysis, and the off-design simulations of the CHP plant models are presented in Publication I. Graphs and discussion on the efficiencies of the steam turbine stages are also presented in Publication IV.

4.3 Process Improvement Simulations of the Case Plants

The simulation models of the base case CHP plants were modified according to the selected process improvements. The process improvements were divided to those using only biomass fuels and to those including a gas turbine or a gas engine and requiring the use of natural gas as additional fuel.

4.3.1 Process Improvements Using Biomass Fuels

The process improvements of the CHP plants considered here were higher superheated steam temperature and pressure before the turbine (the current maximum of 540°C / 92 bar and a long-term goal of 600°C / 170 bars), steam reheating and feed water preheating, and the division of the DH exchanger into two stages. The flowsheets of the process changes are presented in Publication II. For the smallest plant, producing 1.8 MW_e no simulations of superheated steam temperature and pressure increase were conducted as they would have required substantial changes to the process.

The power and heat productions of the process were simulated using the same wood fuel input flow as in the models without the changes. The part load behaviour of the changed processes was simulated by varying the fuel flow in the off-design models from 100 % to 45 % in 1.8 MW_e and 6 MW_e processes and from 100 % to 35 % in 11 MW_e and 14.7 MW_e processes.

The off-design simulation results for the process change cases are presented in Table 5 and in Publication II. For the 1.8 MW_e process the power production increase after the addition of a reheater and a two-stage DH exchanger were close to each other. For the 6 MW_e and 11 MW_e plants the superheated steam temperature and pressure increase to 600°C and 170 bars offered the highest power production. For the 14.7 MW_e process, in which the base case already included the two-stage DH exchanger, the highest power production was gained with

a steam reheater and a feed water preheater.

Table 5. Simulated process changes and their effect on power-to-heat ratios (α) and electrical efficiencies (η_e).

Process change	Efficiencies		Biomass-to-natural gas ratio
	α	η_e	
<u>1.8 MW_e</u>			
Base case	0.23	0.17	-
Reheat and high pressure preheat	0.25	0.18	-
Two-stage DH exchanger	0.26	0.19	-
Gas engine / turbine integration	up to 0.35	up to 0.22	5.1 / 3.4
<u>6 MW_e</u>			
Base case	0.34	0.23	-
Superheated steam T = 540°C, p = 92 bar	0.40	0.26	-
Superheated steam T = 600°C, p = 170 bar	0.45	0.28	-
Reheat and high pressure preheat	0.42	0.26	-
Two-stage DH exchanger	0.39	0.25	-
Gas engine / turbine integration	up to 0.55	up to 0.30	3.8 / 2.2
<u>11 MW_e</u>			
Base case	0.43	0.27	-
Superheated steam T = 540°C, p = 92 bar	0.44	0.27	-
Superheated steam T = 600°C, p = 170 bar	0.50	0.30	-
Reheat and high pressure preheat	0.48	0.29	-
Two-stage DH exchanger	0.45	0.28	-
Gas engine / turbine integration	up to 0.65	up to 0.33	3.2 / 2.4
<u>14.7 MW_e</u>			
Base case	0.48	0.29	-
Superheated steam T = 540°C, p = 92 bar	0.49	0.29	-
Superheated steam T = 600°C, p = 170 bar	0.55	0.32	-
Reheat and high pressure preheat	0.56	0.32	-
Gas engine / turbine integration	up to 0.63	up to 0.32	3.0 / 1.9

4.3.2 Process Improvements Using Biomass Fuels and Natural Gas

The use of natural gas to increase the power-to-heat ratio was considered by integrating a gas turbine and a gas engine to the case plants. The concept for the gas turbine or engine integration was to use the flue gas flow from the turbine or engine, containing about 15 % and 12.5 % oxygen, respectively, as combustion air in the biomass boiler. Furthermore, some additional combustion air was injected into the boiler to cover the total oxygen need.

However, the effect of the flue gas flow increase on the size of the furnace or the fluidised bed was not included to the analysis. The flowsheets of these process changes are presented in Publication II. Also in the CHP plants with process improvements using biomass fuels and natural gas the biomass should remain as the main fuel and the natural gas should be an additional fuel. Thus, the gas turbine and engine sizes used in the simulations were selected so that they corresponded to 10, 25, and 50 % of the power production of the base case plants. The excess heat in the flue gases of the biomass boiler after the gas turbine or engine integration was utilised in a steam reheater, in a feed water preheater, and in a fuel dryer.

In the processes, using both natural gas and biomass fuel, the input of both fuels was reduced at the same rate during the off-design simulation. Usually, the gas turbines or engines integrated into the CHP plants are operated at full load until the outdoor temperature has reached a certain limiting value, while the CHP boiler load is reduced according to the heat demand. When the limiting outdoor temperature is reached, the load in the gas turbine is reduced until the gas turbine load is at the minimum level of 50 % - 30 % (Harvey et al. (2000), Carcasci et al. (2001)). The result is an increased α at part loads. In this study the goal was to simulate the design of a process that produces electricity with biomass and to avoid fossil CO₂ emissions. Thus the gas turbines and engines were operated in a same way as the boiler at part loads during the lower heat demand. This means that throughout the part load operation the loads of the gas turbine or the engine and the boiler were reduced at the same rate. If only the optimal operation of the CHP process would have been the goal of the optimisation, the gas turbines and gas engines would have been operated at full load as much as possible regardless of the load reduction of the biomass boiler. However, then the biomass-to-natural gas ratio would have decreased in the process at part loads leading to increasing power-to-heat loads with higher natural gas proportion during the part load operation. The off-design simulation results of the addition of gas turbines and gas engines are presented in Table 5 and in Publication II.

At full load the α of the CHP processes with gas engines were slightly higher than in the processes with gas turbines but the η_e of the gas engine and turbine processes were almost the same. The increase in the α and in the η_e given by the integration of a gas turbine or engine depended on the size of the plant and of the gas turbine or engine selected. The α that can be gained with gas turbine or engine integration in the 6 MW_e, 11 MW_e, and 14.7 MW_e cases are presented as functions of natural gas usage in the processes in Figure 12. The lower the biomass-to-natural gas ratio is, the larger the integrated gas turbine or engine. The larger the gas engine or turbine integrated into the process, the more it increases the α and η_e of the

whole CHP process. The curves in Figure 12 thus indicate the size of the gas turbine or engine that has to be selected for integration in order to gain a certain level of α in the process.

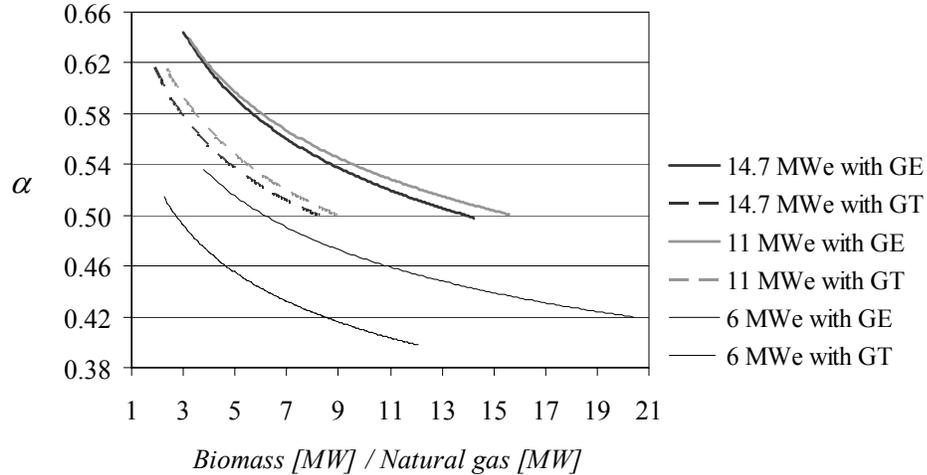


Figure 12. Effect of the biomass-to-natural gas ratio on the α of the CHP processes with gas engine (GE) and gas turbine (GT) integration.

4.4 Production, Cost, and CO₂ Analysis

The economic feasibility of the simulated changes in the differently sized CHP processes was evaluated by analysing the improvements in the power and heat production, the income and the investment costs of the process changes, and the CO₂ emissions after the changes. The investment cost analysis was based on the best available data of the power plant equipment prices, so the analysis presents the level of the economic feasibility of the process change rather than the exact amount of the profit gained. Summaries of the profitable changes for each case process and the increases in the efficiencies are presented in Table 6. The detailed calculation principles of the investment costs and profits as well as a more extensive presentation of the results can be found from Publication II.

The profit in Table 6 refers to the additional profit gained with the changes compared to the base case process. So the profit is calculated by subtracting the investment costs of the changes and the cost of the natural gas, if used in the process, from the additional power production resulting from the changes. The costs of the biomass fuel can be excluded from the analysis, as the amount of biomass fuel in the processes remains the same regardless of the process changes. The changes in the income from the DH production were not included in the cost analysis. The district heat was here considered to be a less profitable product than electricity, so changing from heat production to power production was considered to be

always beneficial in these cases. The electricity price used in the analysis was 30 €/MWh and the natural gas price 15 €/MWh. The interest rate used when calculating the annuity was 5 % and the lifetime of the plant 15 years.

Table 6. Profitable process changes according to the simulations. The profits with saved CO₂ emissions are presented in parentheses.

Process change	Profit (with CO ₂) [k€/a]	CO ₂ saved [t/a]	Efficiencies		
			α	η_e	η_{tot}
<u>1.8 MW_e</u>					
Base case	0	0	0.23	0.17	0.90
Two-stage DH exchanger	19.7 (23.8)	520	0.26	0.19	0.90
Reheat and high pressure preheat	7.7 (9.6)	235	0.25	0.18	0.90
<u>6 MW_e</u>					
Base case	0	0	0.34	0.23	0.90
Two-stage DH exchanger	33.2 (40.2)	876	0.39	0.25	0.89
Reheat and high pressure preheat	73.7 (88.6)	1 864	0.42	0.26	0.90
GE 0.5 MW _e , reheat, preheat and fuel dryer	98.0 (137.0)	4 873	0.41	0.24	0.84
GE 1.5 MW _e reheat, preheat and fuel dryer	79.7 (140.0)	7 538	0.42	0.25	0.84
GE 3.0 MW _e reheat, preheat and fuel dryer	44.2 (129.2)	10 623	0.53	0.30	0.83
<u>11 MW_e</u>					
Base case	0	0	0.43	0.27	0.89
Two-stage DH exchanger	70.2 (84.5)	1 793	0.45	0.28	0.90
Reheat and high pressure preheat	134.5 (161.7)	3 394	0.48	0.29	0.90
GE 1.1 MW _e , reheat, preheat and fuel dryer	126.5 (186.7)	7 524	0.50	0.27	0.79
GE 2.75 MW _e , reheat, preheat and fuel dryer	96.0 (188.1)	11 517	0.57	0.29	0.80
GE 5.5 MW _e , reheat, preheat and fuel dryer	32.4 (174.7)	17 781	0.65	0.32	0.81
<u>14.7 MW_e</u>					
Base case	0	0	0.48	0.29	0.90
Reheat and high pressure preheat	201.8 (243.5)	5 210	0.56	0.32	0.88
GT 1.4 MWe, reheat, preheat and fuel dryer	270.7 (362.4)	11 582	0.50	0.28	0.83
GE 3.5 MWe reheat, preheat and fuel dryer	9.3 (97.6)	11 036	0.56	0.30	0.82

A second estimate of the profit, including the saved CO₂ emissions due to the improvements, was also calculated and is presented in parentheses in Table 6. The CO₂ emissions were included in the analysis by calculating how much fossil CO₂ can be saved in comparison of the situation where the additional electricity production resulting from the process improvements were to be produced in a coal-fired condensing power plant with 45 % electrical efficiency. In the cases where the process changes include the addition of the natural gas to the process, the fossil CO₂ emissions resulting from the use of natural gas are

subtracted from the CO₂ benefits of the changes. The price used for a tonne of CO₂ was 8 €, which was the price estimation for CO₂ emissions given by some Finnish power and heat producers in 2004.

The investment costs of the increased temperature and pressure of the superheated steam depend on the used heat exchanger and turbine materials and are difficult to estimate. Therefore the profits from the higher superheated steam temperatures and pressures could not be estimated reliable enough to include them in the analysis.

Overall, it seems that both adding a reheater and a feed water preheater and adding a two-stage DH exchanger, can be profitable ways to increase the power production and the power-to-heat ratio in the 1-20 MW_e CHP plants. Additionally, the increase in the superheated steam temperature and pressure provides substantial income in all cases but the investment costs are difficult to estimate reliably enough as they depend on novel material solutions. The feasibility of the addition of a gas engine improves as the CHP plant size increases from 1.8 MW_e to 11 MW_e. For the 14.7 MW_e plant the addition of a gas engine is not as feasible as in other cases, as this plant has already a high power-to-heat ratio at the base case, because of the two-stage DH exchanger. In these 1-20 MW_e CHP plants, and with the selected electricity and natural gas prices, the addition of a gas turbine seems to be economically feasible only for the largest 14.7 MW_e case. The factors that strongly affect the profitability of the addition of a gas turbine and engine are the price of the turbine or the engine, the price of the electricity, and, especially, the price of natural gas.

4.5 Discussion

The simulation results show that the addition of a steam reheater and a feed water preheater and the addition of a two-stage DH exchanger are economically feasible alternatives for all CHP plants between 1-20 MW_e. The increase of the superheated steam temperature and pressure to the high values of 600°C and 170 bars offered, in most cases, the best opportunity to increase the power-to-heat ratio. However, it was not possible to evaluate the investment costs and thus the economic feasibility of this change. The addition of a steam reheater provided high power production but the mere addition of a two-stage DH exchanger also increased power production significantly in comparison to the base cases.

The integration of an engine to the CHP process offered profitable solutions in the 6 MW_e and 11 MW_e plants and also in the 14.7 MW_e plant if the CO₂ savings were included in the analysis. However, increases in the power-to-heat ratio and the economic feasibility are

heavily dependent on the size of the integrated gas engine.

The simulations and cost analysis show that there is potential to increase the power-to-heat ratio and electrical efficiency of small-scale biomass CHP plants in an economically feasible way. However, a simulation study of all the interesting process improvement combinations for finding the most profitable one would be very work consuming. The optimisation tools should be used to include the trade-offs between the power production increase, the investment costs, and the additional fuel costs in the selection of the process improvements. Although simulation is a good tool for the preliminary studies of the process change possibilities, optimisation makes it easier to find the optimal combinations of the possible process changes and to include the economic considerations in the selection of the best process changes. The simulations showed that the changes selected for a CHP process should be chosen according to the plant size and the plant properties, which affect strongly the overall investments, profits, and CO₂ savings.

5 MULTIPERIOD MINLP MODEL FOR CHP PLANT SYNTHESIS

A multiperiod mixed integer nonlinear programming (MINLP) model for the synthesis of small-scale CHP plants was developed in two phases. First, a superstructure including the possible process improvements was created, a MINLP model for a design case of a CHP plant was formulated (Model I), and syntheses of four example plants were optimised without taking into account the effects of part load operation. In this first model the pressures were defined as parameters and linear steam and water enthalpy and entropy functions were defined for different pressure levels and temperature ranges. Secondly, the superstructure of the model was slightly simplified and the part load operation of the plant was included to the model by adding one period for each selected heat demand level in the district heating network (Model II). Also, the pressures were modelled as free variables and steam and water property functions depending both on temperatures and pressures were defined according to the Industrial Standard IAPSW-IF97 (Wagner and Kruse (1998)). In addition, a more detailed modelling of steam turbine efficiencies at part loads was incorporated in the model. Model I is discussed in detail in Publication III and Model II in Publication V. The pressure and steam turbine modelling used in Model II is described in Publication IV.

5.1 Optimisation Method

The MINLP method was used for optimising the best process improvements for the CHP processes. Integer programming was needed to have a possibility to include and exclude modules in the process. Nonlinear optimisation was required for example in order to formulate the energy balances without fixing the temperatures and pressures in the model. There were also nonconvexities in the models, which made the finding of the global optimum challenging with the available solvers.

The MINLP models were implemented in the General Algebraic Modeling System (GAMS) by GAMS Development Corporation. GAMS offers two local solvers for MINLP problems. Since nonconvex MINLP problems are often difficult to solve and sensitive to the initial values both solvers were tested for the models. Standard Branch and Bound (SBB) solver by ARKI Consulting & Development A/S is especially suitable for problems with complex nonlinearities but fairly small combinatorial problem. The SBB solver uses CONOPT3 (GAMS (2004)) as the solver for the integer relaxed nonlinear programming (NLP) part of the model. The Discrete and Continuous Optimizer (DICOPT) by Viswanathan and Grossmann at Carnegie Mellon University should perform better than SBB if the model has

a significant and difficult combinatorial part (GAMS (2004)). DICOPT is used here with CONOPT3 as a NLP solver and CPLEX (GAMS (2004)) as a mixed integer programming (MIP) solver. There are only 13 binary variables in Model I and 4 binary variables in Model II connected to the combinatorial possibilities but the nonlinear functions in the models are quite complex and nonconvex. As SBB and DICOPT are local solvers, the global optimality of the solution can not be guaranteed to these nonconvex MINLP models and the solutions may be sensitive to the selected initial values. The models were run on an HPnw8000 with 512 GB of RAM and a 1.7 GHz Intel Pentium M processor.

5.2 The Design Case MINLP Model (Model I)

5.2.1 Problem Statement

The objective of the optimisation was to increase the power production of the small-scale biomass-fuelled CHP plant by changing the process design of the plant, and to maximise the additional profit from these changes. The superstructure of Model I is presented in Figure 13. The model should find the process change combination that increases the power production of a selected CHP plant and results in the highest possible profit with the selected prices of electricity and natural gas. The biomass fuel input to the model is fixed but it is possible to dry the biomass fuel in a fuel dryer utilising the excess heat of the flue gases. Three different gas turbines and gas engines can be integrated to the CHP process using the oxygen content of their exhaust gases, 15 % and 12.5 %, respectively, as combustion air in the biomass boiler. Additional combustion air can be preheated and injected to the boiler. The steam turbine construction is divided into separate stages in the superstructure as suggested by Chou and Shih (1987). Four different steam turbine configurations can be selected for the process: the base case steam turbine, a steam turbine with a steam reheater, a steam turbine with a feed water preheater, and a steam turbine with both a steam reheater and a feed water preheater. The DH exchanger used in the process can be either a single-stage or a two-stage heat exchanger.

The different process options presented in the superstructure are listed below and presented in Figure 13.

- y_1 : Base case steam turbine system consisting of a regulation stage, a group of working stages, and an exhaust stage .
- y_2 : Steam turbine system with an addition of a steam reheater.
- y_3 : Steam turbine system with additions of a steam reheater and a high pressure feed water preheater.
- y_4 : Steam turbine system with an addition of a high pressure feed water preheater.

- $y_{2stageDH}$: Addition of the second stage of a DH exchanger to the process.
- y_{airpre} : Addition of an air preheater to the process.
- $y_{fueldry}$: Addition of a fuel dryer to the process.
- y_{gt1-3} : Addition of a gas turbine (three different size options) to the process.
- y_{ge1-3} : Addition of a gas engine (three different size options) to the process.

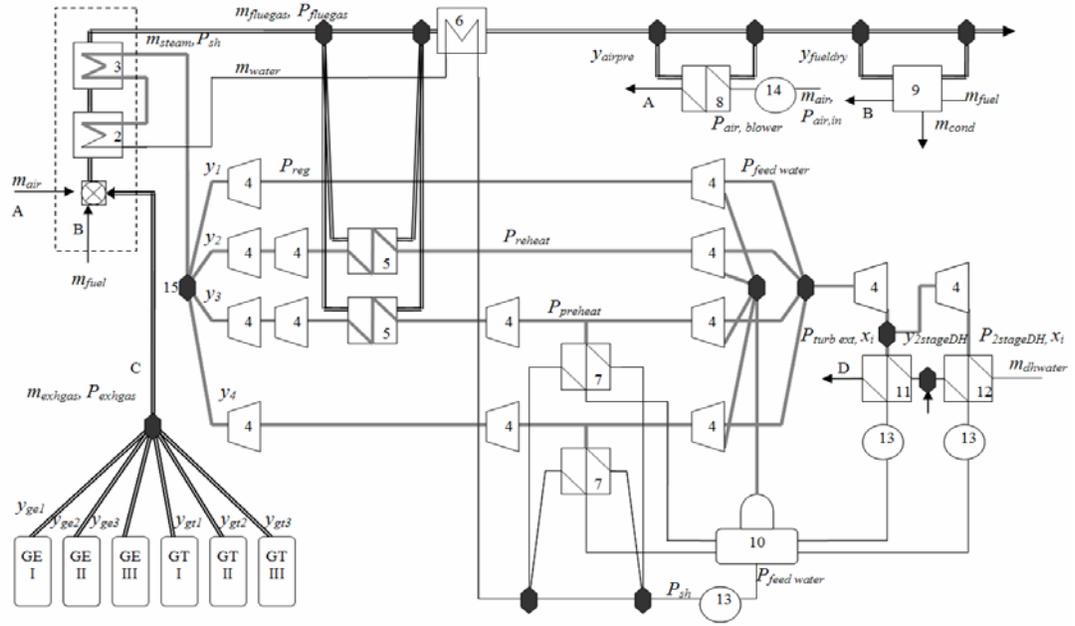


Figure 13. The superstructure of Model I.

1. Burner, 2. Evaporator, 3. Superheater, 4. Turbine stage, 5. Reheater, 6. Economiser, 7. Feed water preheater, 8. Air preheater, 9. Fuel dryer, 10. Feed water tank, 11. DH exchanger, 12. 2nd stage of DH exchanger, 13. Pump, 14. Blower, 15. Selection node, GE I-III: Gas engine alternatives, GT I-III: Gas turbine alternatives, A=Air, B=Fuel, C=Exhaust gas, D=DH water.

The DH production in the CHP plant can be slightly lower after the process changes than in the base case but the total efficiency of the plant should remain close to 0.90. Primarily the process improvements should increase the power-to-heat ratios and electrical efficiencies without degrading the heat production.

5.2.2 Model I Formulation

The objective function of the model is a sum of additional incomes and costs resulting from the process improvements. The constraints for the Model I are constructed by creating the mass and energy balances for the process modules and by connecting the binary variables to the selection of modules with Big-M constraints. The streams are directed to different modules depending which one is included into the model by using selection nodes and Big-M constraints. The pressures were modelled as parameters in the model and linear regression

functions for steam and water enthalpies and entropies were calculated with a separate optimisation program for fixed pressures and for the needed temperature ranges on the basis of the equations in Wagner and Kruse (1998). Linear regression functions of the dependence between the temperature and pressure of the saturated steam were also constructed.

In Model I the CHP plants were modelled only at the design load. In reality the small-scale biomass-fuelled CHP plants can be very dependent on the district heat load especially if they are situated in small municipal areas with no additional heat users. Thus the small-scale CHP plants may be operating long periods at part loads. In Model I the annual heat production was estimated from a heat load duration curve adopted from Harvey et al. (2000). The curve was scaled for different cases so that at the full load the CHP plant would produce 65 % of the annual peak load of the DH network, and that the plant would totally produce about 80 % of the annual energy generation of the network.

The size of the CHP plant was defined in Model I by giving the biomass input flow to the boiler and the maximum temperature of the superheated steam. In addition, the net power production of the base case process was given, so that the changes in the process can be compared to that. The resulting district heat production was calculated by the model according to the optimal process change and parameters. It was possible to change the sizes of the integrated gas turbines and engines by defining their natural gas and air input flow, exhaust gas temperature, and the electrical efficiency. The annual income from the process changes included the additional production of electricity. The cost of natural gas was subtracted from this income, if gas was used as an additional fuel in the process. Similarly to the simulations, the changes in the income from the district heat production were not included in the cost optimisation because changing from the heat production to the power production, when the heat demand is covered, was considered to be always beneficial in these cases. The general and case related parameters used in the four example cases in Model I are presented in Publication III.

The objective function of the additional profit gained with the process changes

$$\begin{aligned} \max z = c_{el} \cdot & \left(\left(\sum_{i \in \text{TURBINES}} \eta_{ge} \cdot W_i + \sum_{\substack{i \in \text{GASENG} \\ \cup \text{GASTURB}}} W_i - \sum_{\substack{i \in \text{PUMPS} \\ \cup \text{BLOWERS}}} W_i \right) / 1000 \frac{kW_e}{MW_e} - P_{base_case} \right) - \\ & c_{ng} \cdot \left(\sum_{\substack{i \in \text{GASTURB} \cup \text{GASENG} \\ j \in \text{IN}(i,j) \cap \text{NATGAS}(j)}} y_i \cdot m_j \cdot LHV_{NG} \right) / 1000 \frac{kW_e}{MW_e} \cdot t_{annual} - a \cdot \sum_{\substack{i \in \text{REHEATER} \cup \text{PREHEATER} \\ \cup \text{DHEX2} \cup \text{FUELDRY} \\ \cup \text{GASTURB} \cup \text{GASENG}}} c_i \cdot y_i \end{aligned} \quad (5.2.1)$$

Mass balance equations (linear constraints for the optimisation model) :

$$\sum_{j \in IN(i,j)} m_j = \sum_{j \in OUT(i,j)} m_j \quad \forall i \in UNITS \quad (5.2.2)$$

Energy balance equations (nonlinear and nonconvex constraints) :

$$\sum_{j \in IN(i,j)} m_j \cdot h_j = \sum_{j \in OUT(i,j)} m_j \cdot h_j + W_i \quad (5.2.3)$$

$$\forall i \in UNITS \cap \{HEATEX, FEEDWT, TURBINES, SPLITTERS, FUELDRY\}$$

where the work done by some of the units is assigned (parameters):

$$W_i = 0 \quad \forall i \in UNITS \cap \{HEATEX, FEEDWT, SPLITTERS, FUELDRY\} \quad (5.2.4)$$

Enthalpy and entropy equations are presented with linear regression models calculated according to the Industrial Standard IAPWS-IF97 (Wagner and Kruse (1998)) with fixed pressures and needed temperature ranges for each optimisation case (linear constraints):

$$h_j(T_j), s_j(T_j), h_{isentropic,j}(T_{isentropic,j}), s_{isentropic,j}(T_{isentropic,j}) = b_{1,j} \cdot T + b_{2,j} \quad (5.2.5)$$

The coefficients b_1 and b_2 for each case are presented in Appendix.

Steam pressures (parameters):

$$p_j = P_j \quad \forall j \in STREAMS \quad (5.2.6)$$

Temperature of superheated steam (parameters):

$$T_j = T_{sh} \quad \forall i \in SUPERHEATER, j \in COLDOUT(i,j) \quad (5.2.7)$$

Temperature dependence on pressures for saturated streams is calculated for every case (linear constraint):

$$T_j = b_{3,j} \cdot p_j + b_{4,j} \quad \forall j \in SATSTREAMS \quad (5.2.8)$$

The coefficients b_3 and b_4 for each case are presented in Appendix.

Enthalpies for flue gases, exhaust gases and air (linear constraint):

$$h_j = cp_j \cdot (T_j - T_{ref}) \quad \forall j \in FLUEGAS \cup AIR \cup EXHGAS \quad (5.2.9)$$

Enthalpy and the lower heating value of moist biomass are calculated according to data from Alakangas (2000) (linear constraint):

$$h_j = LHV_j = -0.221 \cdot f_{H20,j} + 19.8 \quad \forall j \in FUEL \quad (5.2.10)$$

Biomass fuel moisture and input to the process (parameters):

$$m_j = M_{FUEL} \quad \forall i \in FUELDRY, j \in IN(i,j) \cap FUEL(j) \quad (5.2.11)$$

$$f_{H20,j} = F_{H20,j} \quad \forall i \in FUELDRY, j \in IN(i,j) \cap FUEL(j) \quad (5.2.12)$$

Burner behaviour is defined with an energy balance equation (nonlinear and nonconvex constraint):

$$\sum_{j \in IN(i,j) \cap FUEL(j)} m_j \cdot LHV_j + \sum_{j \in IN(i,j) \cap (AIR(j) \cup EXHGAS(j))} m_j \cdot cp_j \cdot (T_j - T_{ref}) = \sum_{j \in OUT(i,j) \cap FLUEGAS(j)} m_j \cdot cp_j \cdot (T_j - T_{ref}) \quad \forall i \in BURNERS \quad (5.2.13)$$

The air flow to the burner (linear constraint) :

$$\sum_{j \in IN(i,j) \cap AIR(j)} m_j + \sum_{j \in IN(i,j) \cap EXHGAS(j)} m_j \cdot x_{02,j} = AF_{ratio} \cdot \sum_{j \in IN(i,j) \cap FUEL(j)} m_j \quad \forall i \in BURNERS \quad (5.2.14)$$

The temperature of incoming air (parameter):

$$T_j = T_{air, in} \quad \forall i \in AIRPRE, j \in IN(i,j) \cap AIR(j) \quad (5.2.15)$$

Steam turbine entropy equation for turbine stages (linear constraint):

$$S_j = S_{isentropic,k} \quad \forall i \in TURBINES, j \in IN(i,j), k \in OUT(i,k) \quad (5.2.16)$$

Isentropic enthalpy equation for steam turbine stages (nonlinear and nonconvex constraint):

$$h_j - h_k = \eta_{isentropic,i} (h_j - h_{isentropic,k}) \quad \forall i \in TURBINES, j \in IN(i,j), k \in OUT(i,k) \quad (5.2.17)$$

The calculation of the steam content of the moist steam after the exhaust stages of the turbine is based on Bruno et al. (1998) (nonlinear and nonconvex constraint):

$$x_i \cdot (S_{steam,k} - S_{water,k}) = (S_j - S_{water,k}) \quad \forall i \in TURBINES, j \in IN(i,j), k \in OUT(i,k) \quad (5.2.18)$$

However, here the x_i should rather be the isentropic steam content similarly to Model II.

The steam and moisture mass flows after the exhaust stages (nonlinear and nonconvex constraint):

$$x_i \cdot (m_{steam,j} + m_{water,j}) = m_{steam,j} \quad \forall i \in TURBINES, j \in OUT(i,j) \quad (5.2.19)$$

Heat exchanger mass balances for the hot and cold sides of the heat exchangers (linear constraints):

$$\sum_{j \in COLDIN(i,j)} m_j = \sum_{j \in COLDOUT(i,j)} m_j \quad \forall i \in HEATEX \quad (5.2.20)$$

$$\sum_{j \in HOTIN(i,j)} m_j = \sum_{j \in HOTOUT(i,j)} m_j \quad \forall i \in HEATEX \quad (5.2.21)$$

Temperature difference between hot and cold streams (linear constraints):

$$T_j - T_k \geq \Delta T \quad \forall i \in HEATEX, j \in HOTIN(i,j), k \in COLDOUT(i,k) \quad (5.2.22)$$

$$T_j - T_k \geq \Delta T \quad \forall i \in HEATEX, j \in HOTOUT(i,j), k \in COLDIN(i,k) \quad (5.2.23)$$

Equal load distribution in both DH exchangers (linear constraint):

$$T_j - T_k = \Delta T_{DH} \quad \forall i \in DHEX, j \in IN(i,j) \cap DHWATER(j), k \in OUT(i,k) \cap DHWATER(k) \quad (5.2.24)$$

Return temperature of district heating water (parameter):

$$T_j = T_{return} \quad \forall i \in DHEX2, j \in IN(i,j) \cap DHWATER(j) \quad (5.2.25)$$

Logical constraints for the temperature of district heating water in a process without two-stage district heating (Big-M constraints):

$$T_j - T_{return} \leq M \cdot y_{2stageDH} \quad (5.2.26)$$

$$T_j - T_{return} \geq -M \cdot y_{2stageDH} \quad \forall i \in DHEX, j \in IN(i,j) \cap DHWATER(j) \quad (5.2.27)$$

The heat exchanger area (nonlinear and nonconvex constraint):

$$\sum_{j \in HOTIN(i,j)} m_j \cdot h_j - \sum_{j \in HOTOUT(i,j)} m_j \cdot h_j = A_{hex,i} \cdot U_i \cdot \Delta T_{lm,i} \quad \forall i \in HEATEX \quad (5.2.28)$$

Paterson approximation for the logarithmic mean temperature difference (nonlinear and nonconvex constraint):

$$\Delta T_{lm,i} = \frac{2}{3} (\Delta T_{1,i} \cdot \Delta T_{2,i})^{\frac{1}{2}} + \frac{1}{3} \left(\frac{\Delta T_{1,i} + \Delta T_{2,i}}{2} \right) \quad \forall i \in HEATEX \quad (5.2.29)$$

$$\Delta T_{1,i} = T_j - T_k \quad \forall i \in HEATEX, j \in HOTIN(i,j), k \in COLDOUT(i,k) \quad (5.2.30)$$

$$\Delta T_{2,i} = T_j - T_k \quad \forall i \in HEATEX, j \in HOTOUT(i,j), k \in COLDIN(i,k) \quad (5.2.31)$$

Heat exchanger cost (linear constraint):

$$c_i = 8000 \text{ € /unit} + 100 \text{ €/m}^2 \cdot A_{hex,i} \quad \forall i \in HEATEX \quad (5.2.32)$$

Pump and blower equations (linear constraints):

$$-W_i \cdot \eta_i = v(m_j \cdot p_j - m_k \cdot p_k) \quad \forall i \in PUMPS, j \in IN(i, j), k \in OUT(i, k) \quad (5.2.33)$$

$$-W_i \cdot \eta_i = (m_j \cdot p_j - m_k \cdot p_k) \quad \forall i \in BLOWER, j \in IN(i, j), k \in OUT(i, k) \quad (5.2.34)$$

Fuel dryer with condensated water from the dryer (nonlinear and nonconvex constraint):

$$m_j \cdot f_{H2O,j} - m_k \cdot f_{H2O,k} = m_{cond} \quad (5.2.35)$$

$$\forall i \in FUELDRY, j \in IN(i, j) \cap FUEL(j), k \in OUT(i, k) \cap FUEL(k)$$

Fuel dryer cost (fixed variable):

$$c_i = 300 \text{ k € /unit} \quad \forall i \in FUELDRY \quad (5.2.36)$$

The fuel dryer investment cost is evaluated on the basis of an unpublished Finnish research on the economical feasibility of the dryers and a constant cost for the fuel dryer in a small-scale CHP plant is used.

Gas turbine and gas engine equations (parameters and linear constraints):

$$m_j = M_{FUEL,i} \cdot y_i \quad \forall i \in GASTURB \cup GASENG, j \in IN(i, j) \cap NATGAS(j) \quad (5.2.37)$$

$$m_j = M_{AIR,i} \cdot y_i \quad \forall i \in GASTURB \cup GASENG, j \in IN(i, j) \cap AIR(j) \quad (5.2.38)$$

$$m_j \cdot LHV_{NG} \cdot \eta_{e,i} = W_i \quad \forall i \in GASTURB \cup GASENG, j \in IN(i, j) \cap NATGAS(j) \quad (5.2.39)$$

$$T_j = T_{EXH,i} \quad \forall i \in GASTURB \cup GASENG, j \in OUT(i, j) \cap EXHGAS(j) \quad (5.2.40)$$

Gas turbine and gas engine costs (nonlinear but convex constraints):

$$c_i = 1251.1 \cdot W_i^{-0.3925} \quad (0.5 \text{ MW}_e \leq W_i \leq 20 \text{ MW}_e) \quad \forall i \in \text{GASTURB} \quad (5.2.41)$$

$$c_i = 514.2 \cdot W_i^{-0.0760} \quad (0.5 \text{ MW}_e \leq W_i \leq 20 \text{ MW}_e) \quad \forall i \in \text{GASENG} \quad (5.2.42)$$

The gas turbine price data is adopted from GTW (2003) and the gas engine price data is based on the manufacturer information. In both cost equations (5.2.41) and (5.2.42) the price of the gas turbine and gas engine has been multiplied by a factor of 1.7 to include the additional installation etc. costs of the gas turbine and gas engine integration. Also this factor is based on the manufacturer information.

Splitter temperature conditions (linear constraints):

$$T_j - T_{\text{SPLIT},i} = 0 \quad \forall i \in \text{SPLITTERS}, j \in \text{IN}(i, j) \quad (5.2.42)$$

$$T_j - T_{\text{SPLIT},i} = 0 \quad \forall i \in \text{SPLITTERS}, j \in \text{OUT}(i, j) \quad (5.2.43)$$

Selection node mass balance (linear constraints):

$$\sum_{j \in \text{IN}(n, j)} m_j = \sum_{j \in \text{OUT}(n, j)} m_j \quad \forall n \in \text{NODES}, j \in \text{NODEIN}(n, j) \quad (5.2.44)$$

Logical constraints for temperatures in the selection nodes (Big-M constraints):

$$T_j - T_k \leq M \cdot (1 - y_i) \quad (5.2.45)$$

$$T_j - T_k \geq -M \cdot (1 - y_i) \quad (5.2.46)$$

$$\forall n \in \text{MIXNODES}, j \in \text{NODEIN}(n, j) \cap \text{OUT}(i, j), k \in \text{NODEOUT}(n, k)$$

$$T_j - T_k \leq M \cdot (1 - y_i) \quad (5.2.47)$$

$$T_j - T_k \geq -M \cdot (1 - y_i) \quad (5.2.48)$$

$$\forall n \in \text{SPLITNODES}, j \in \text{NODEIN}(n, j), k \in \text{NODEOUT}(n, k) \cap \text{IN}(i, j)$$

Binary variable equations. Selection of a turbine (logical constraint for binary variables):

$$\sum_{i \in \text{TURBINES}} y_i = 1 \quad y \in \{0, 1\} \quad (5.2.49)$$

Selection of a gas turbine or gas engine (logical constraint for binary variables):

$$\sum_{i \in \text{GASENG} \cup \text{GASTURB}} y_i \leq 1 \quad y \in \{0, 1\} \quad (5.2.50)$$

Logical constraints for mass flows in two-stage DH exchanger (Big-M constraint):

$$m_j \leq y_i \cdot M \quad y \in \{0, 1\} \quad \forall i \in \text{TURBINES} \cup \text{DHEX2}, j \in \text{IN}(i, j) \quad (5.2.51)$$

Logical constraints for temperatures in air preheater and fuel dryer (Big-M constraint):

$$T_j - T_k \leq y_i \cdot M \quad y \in \{0, 1\} \quad \forall i \in \text{AIRPRE} \cup \text{FUELDYR}, j \in \text{IN}(i, j), k \in \text{OUT}(i, k) \quad (5.2.52)$$

5.2.3 Model I Analysis

Model I includes 537 equations and 653 variables. The formulation of Model I includes both convex and nonconvex equations identified in Section 5.2.2. Most of the nonconvexities in the model result from bilinear terms in the constraints, for example the $m_j \cdot h_j$ in the energy balances. In addition to the bilinear terms, the Paterson approximation for logarithmic mean temperature difference is nonconvex. The nonconvexity of the model means that no global optimum for the problem can be guaranteed with local solvers. The nonconvexity and the robustness of the model are discussed more in Section 6.

A problem with nonconvex models is that they are sensitive to the initial values and may often end up to local optimums instead of the global one. The sensitivity of the model to the initial values was tested using both SBB and DICOPT solvers. From these only SBB was able to solve the whole MINLP problem, as DICOPT was only able to find the NLP solution but not the integer solutions for the 13 binary variables. Thus SBB was selected for a solver in Model I.

The initial points for Model I in optimisation were chosen according to the best available information on the optimal point in order to avoid problems with local optimums. However, for the sensitivity analysis runs the initial values were chosen randomly between the lower and higher bounds of the variables. The bounds were selected in the construction of the model with the help of the simulation model results. The solution time for one optimisation run with SBB solver of the 6 MW_e case plant model was 1.5 CPU seconds.

Sensitivity analysis of Model I with the case of 6 MW_e CHP plant is presented in Figure 14. Although SBB solves the problem, the solution is sensitive to the initial values. SBB is able to find the best local solution ($y_3=1, y_{airpre}=1, y_{2stageDH}=1, y_{ge3}=1, profit = 303 \text{ k€}/a$) in 32 % of the runs but finds the second best local optimum even more often, in 36 % of the runs. However, the difference between the best and the second best local solutions is in this case only 2.6 %, while the other local solutions are from 8 % to 36 % lower than the best local solution.

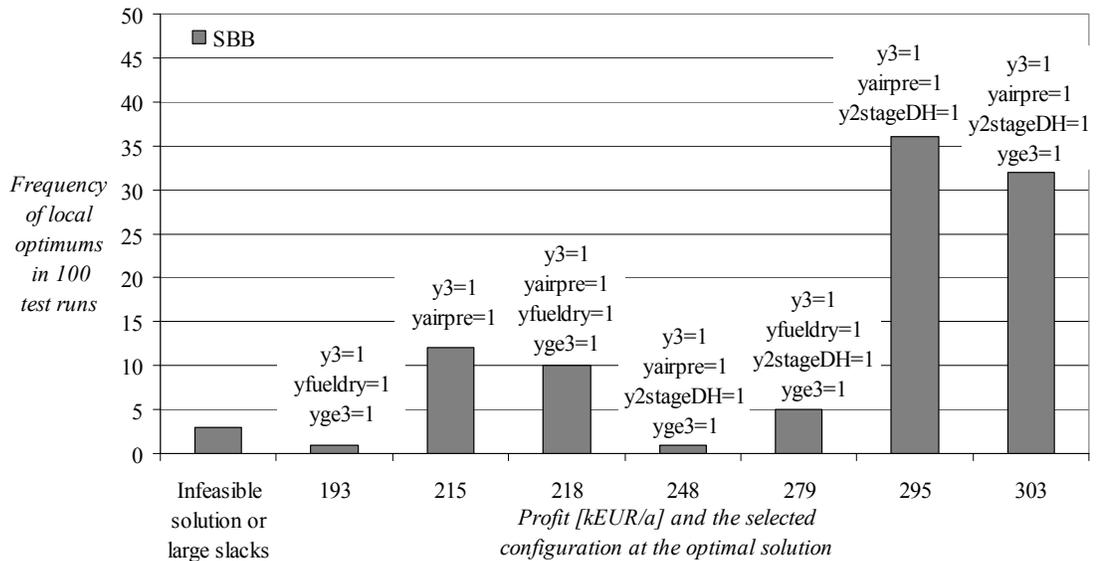


Figure 14. Sensitivity of the optimal solution to the initial values of variables when Model I of a case plant with 6 MW electricity production is solved with SBB.

5.2.4 Results and Discussion

Model I was used to optimise the configuration of four existing CHP plants producing 1.8 MW_e, 6 MW_e, 11 MW_e, and 14.7 MW_e. The base case models and data of these processes are presented in Section 4.2 and in Publication I. The goal for the optimisations was to increase the power-to-heat ratios and the electrical efficiencies of the processes in the most profitable way.

The results of the optimisation cases are presented in Figure 15, where the additional profit gained with the process improvements is presented as a function of the electricity price-to-natural gas price ratio. For optimisation, the ratio between the electricity and natural gas prices is important, so when electricity price was varied the natural gas price could be held constant at 17 €/MWh, which was an estimate of the current price of natural gas. In all optimised cases there was a break point in profit between the ratios 2.4-2.7 when a gas engine was added to the process. At ratios below this the selected process improvement was the addition of a steam reheater, a feed water preheater, and a two-stage DH exchanger. The saved CO₂ emissions and efficiencies of these profitable process improvements are summarised in Table 7.

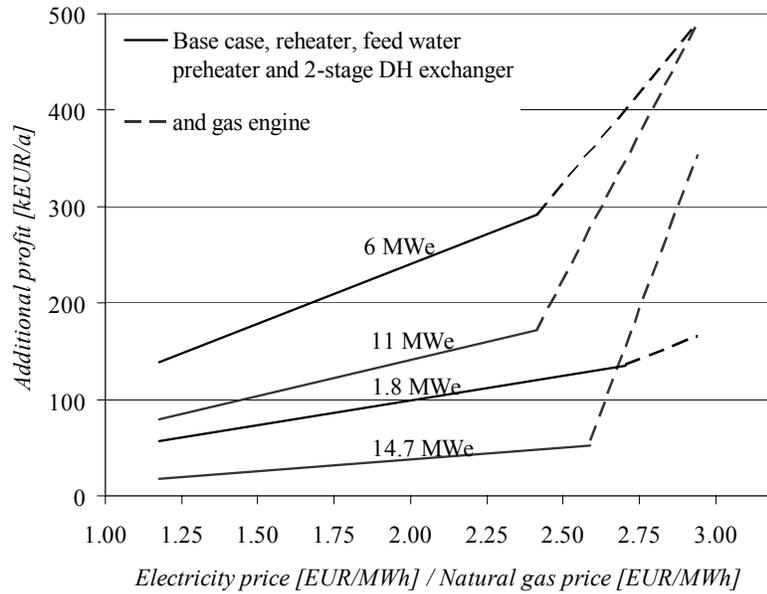


Figure 15. Results of the optimal process changes for the 1.8 MWe, 6 MWe, 11 MWe, and 14.7 MWe plants. A fuel dryer was added in the 6 MWe case at the electricity price-to-natural gas price ratios 1.2 - 2.8 and in the 11 MWe case at ratios 2.4-2.8.

Table 7. The saved CO₂ emissions and the efficiencies of the most profitable process changes in the optimisations with Model I.

Process change	CO ₂ saved [t/a]	Efficiencies		
		α	η_e	η_{tot}
<u>1.8 MWe</u>				
Base case	0	0.23	0.17	0.90
Feed water preheater, reheater, and 2-stage DH exchanger...	2237	0.45	0.28	0.90
... and gas engine	5087	0.59	0.31	0.85
<u>6 MWe</u>				
Base case	0	0.34	0.23	0.90
Feed water preheater, reheater, 2-stage DH exchanger, and fuel dryer...	5381	0.49	0.29	0.90
... and gas engine	16 252	0.66	0.35	0.87
<u>11 MWe</u>				
Base case	0	0.41	0.27	0.94
Feed water preheater, reheater, and 2-stage DH exchanger...	3269	0.47	0.29	0.90
... and gas engine and fuel dryer	24 958	0.66	0.35	0.87
<u>14.7 MWe</u>				
Base case (with feed water preheater and 2-stage DH exchanger)	0	0.50	0.30	0.90
Reheater...	1079	0.50	0.30	0.90
...and gas engine	34 943	0.78	0.36	0.81

For the 1.8 MW_e CHP process the addition of a feed water preheater, a steam reheater, and a two-stage DH exchanger gave some additional profit when the price of the electricity varied from 20 €/MWh to 45 €/MWh. When the price of the electricity was higher than 45 €/MWh, the addition of a gas engine became profitable. In this smallest CHP plant case the fuel dryer was not included in the process with a gas engine.

For the 6 MW_e CHP process the addition of a steam reheater, a feed water preheater, a two-stage DH exchanger, and a fuel dryer was profitable between the electricity prices of 20 €/MWh and 42 €/MWh. When the price of the electricity increased from 42 €/MWh, it became profitable to add a gas engine to the process.

Similarly to the 6 MW_e process, the breaking point for the process changes for the 11 MW_e CHP process was the electricity price of 42 €/MWh. Below that price the profitable change was to add a feed water preheater, a steam reheater, and a two-stage DH exchanger to the process. Above this price the addition of a gas engine and a fuel dryer was the profitable process change.

In the 14.7 MW_e case there were a two-stage DH exchanger and a feed water preheater already at the base case plant. Therefore, the efficiency increase and the additional profits from the process improvements that were calculated in comparison to the base case results remained lower than in the other cases. For the electricity prices between 20 €/MWh and 44 €/MWh the addition of a steam reheater was slightly profitable. No fuel dryer was added to the process in this case with a steam reheater. When the price of the electricity increased, the addition of a gas engine became profitable.

The integration of a gas turbine was not profitable to any of the case plants with the used natural gas price and the electricity price range. The ratio of the electricity price and the natural gas price should have been much higher, at least 3.5-4.0, before the gas turbine integration would have become profitable. As high ratio as this between the prices of the electricity and the natural gas is very unlikely, as it would require over the double price for the electricity while the price of the natural gas should remain the same. This is not probable as the fuel prices tend to increase when the price of the electricity increases.

The effects of the process improvements on the total investment costs of the case plants are presented in Figure 16. According to the optimisation results some increase in the power production and efficiencies can be gained with fairly low investment costs. The addition of a feed water preheater, a two-stage DH exchanger, and a steam reheater increased the CHP

investment costs only by 0.5-0.7 %. In the 6 MW_e case also a fuel dryer was added with the reheater, the feed water preheater, and the two-stage DH exchanger. This increased the investment costs by 4.5 %. However, the investment costs of a two-stage DH exchanger did not include the costs of an additional extraction in a steam turbine, which increases the total investment costs of a two-stage DH exchanger. The integration of a gas engine increased the investment costs by about one fifth except in the smallest CHP process where the increase was below 10 %. Because injecting the exhaust gases from a gas engine or a gas turbine to a biomass boiler is a novel technology, there are still many uncertainties in the investment cost evaluation. Changes may be required, for example, to the boiler structure that increase the costs of this change. Overall, it may be concluded that the addition of a two-stage DH exchanger, a feed water preheater, a steam reheater, and a fuel dryer provide good solutions for a moderate increase in the power-to-heat ratio and the electrical efficiency of a small-scale CHP plant. Higher improvements may be gained with the integration of a gas engine, but more work should be done to find detailed process modification needs and cost estimates for this new process.

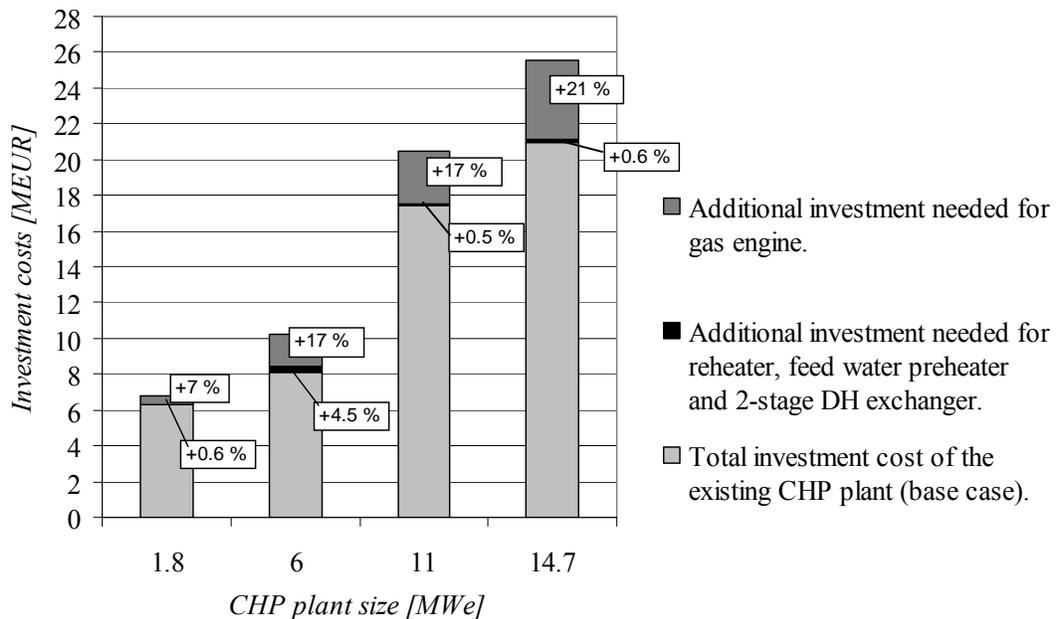


Figure 16. Effects of the process improvements on the total investment costs of the 1.8 MW_e, 6 MW_e, 11 MW_e, and 14.7 MW_e plants. In the 6 MW_e case also a fuel dryer was added with a reheater, a feed water preheater, and a two-stage DH exchanger. In the 11 MW_e case a fuel dryer was added with a gas engine. In the 14.7 MW_e case the investment costs of the two-stage DH exchanger were included already in the total investment costs of the base case.

5.3 The Multiperiod MINLP Model (Model II)

5.3.1 Problem Statement

The objective of the optimisation is to find process designs for the small-scale (1-20 MW_e) CHP plants that would increase the power production and the power-to-heat ratios from their current levels. The superstructure of Model II (Figure 17) is a simplified version of the Model I presented in Section 5.2 and in Publication III. The case plant selected for optimisation is the 6 MW_e CHP plant described in Table 4 in Section 4.2 and in Publication I. In Model II the formulation of Model I is developed further by dividing the DH demand into multiperiods and by including the modelling of the pressure levels in the problem. The steam and water property functions depending on both temperatures and pressures are formulated according to the Industrial Standard IAPWS-IF97 (Wagner and Kruse (1998)). In addition, the more detailed steam turbine modelling presented in Publication IV is used in the model. Model II is described and discussed in detail in Publication V.

The different process options in the superstructure of Model II include three turbine configurations and two DH exchanger configurations. The binary variables y_i connected to the process options are listed below and presented in Figure 17.

y_1 : Base case steam turbine system consisting of a regulation stage, a group of working stages, and an exhaust stage.

y_2 : Steam turbine system with an addition of a steam reheater.

y_3 : Steam turbine system with additions of a steam reheater and a high pressure feed water preheater.

y_4 : Addition of the second stage of a DH exchanger to the process.

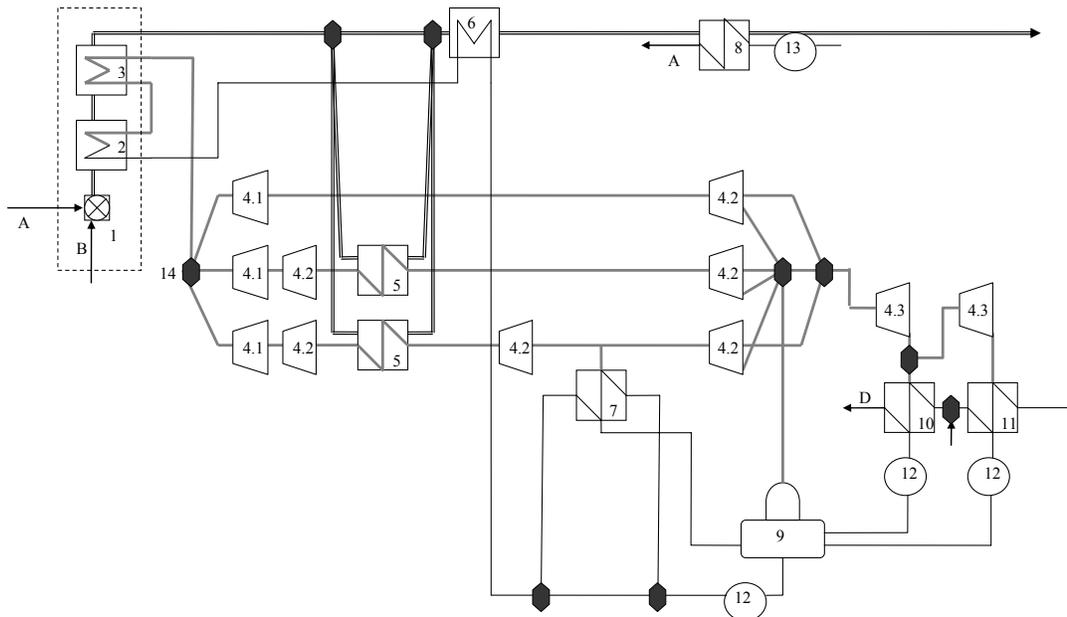


Figure 17. The superstructure of Model II.

1. Burner, 2. Evaporator, 3. Superheater, 4.1 Regulation stage of a turbine, 4.2 Working stage of a turbine, 4.3 Exhaust stage of a turbine, 5. Reheater, 6. Economizer, 7. Feed water preheater, 8. Air preheater, 9. Feed water tank, 10. DH exchanger, 11. 2nd stage of DH exchanger, 12. Pump, 13. Blower, 14. Selection node, A=Air, B=Fuel, C=DH water

5.3.2 Modelling of Pressure Levels

In Model II the pressures are modelled as free variables both at design load and at part loads and not fixed to parameters as in Model I. This enables a more thorough optimisation of the process compared to the previous model but makes the model also more complex. Especially, the enthalpy and entropy functions for steam and water become more complex and the linear fittings used in Model I can not be used. Instead, in Model II the steam and water property functions of the Industrial Standard IAPWS-IF97 presented in Wagner and Kruse (1998) are used. Modelling of pressures changes also the formulation of the steam turbine model that should now describe the part load behaviour of the turbine, the efficiencies of the turbine stages, and the dependence of the steam turbine pressures on the steam mass flow rate.

5.3.3 Modelling of Steam and Water Property Functions

The enthalpy and entropy functions for steam and water are modelled according the Industrial Standard IAPWS-IF97 (Wagner and Kruse (1998)). These functions depend both on temperature and pressure and their complexity may cause problems in the solving of the model. However, in this model these functions are used rather than creating simpler and less

accurate functions that require limiting the temperature and pressure ranges where each estimating function is valid and thus bring discontinuities to the model.

5.3.4 Modelling of a Back-Pressure Steam Turbine

The development and the basis of the new steam turbine model used in Model II is described and discussed thoroughly in Publication IV. The purpose of the developed back-pressure steam turbine model is to include the part load behaviour of the regulation stage efficiency and the exhaust losses in the turbine outlet to the model. These changes are especially important in the models of small-scale CHP plants as they are operated long time periods at part loads. The efficiencies of turbine stages are modelled with polynomials described in Equation (5.3.16) and in Table 9. The enthalpy and entropy changes over the turbine stage are calculated with same principles as in Model I. The dependence of pressure on the steam mass flow through the turbine at part loads is modelled with Equation (5.3.24) that is developed from the relationship between the mass flow of steam and the inlet and outlet pressures of a steam turbine stage with a fixed blade construction presented for example in Traupel (2001). The modification of this relationship to the form of Equation (5.3.24) is described in Publication IV.

The new steam turbine model was tested by modelling a selected steam turbine case in three different ways:

1. A simulation model of a back-pressure steam turbine including the part load behaviour presented in Publication IV.
2. A multiperiod NLP model of the steam turbine constructed using the Willans line (see Section 2.4.3) as a description of the total steam turbine efficiency.
3. A multiperiod NLP model where the efficiencies of the regulation stage, the working stage, and the exhaust stage are modelled as described in Publication IV.

The steam turbine case selected for the simulation and NLP modelling is a back-pressure turbine producing 16.5 MW district heat and 6.1 MW electricity. The more detailed description of the case turbine can be found in Publication IV.

The simulation program used was Prosim by Endat Oy and the optimisation problem was implemented in GAMS and solved with CONOPT3. The number of equations and variables and the solving times for the three optimisation models are presented in Publication IV. The new more detailed steam turbine model is done at the costs of more variables and equations in the model and thus also a longer solution time.

The detailed results for each separate turbine stage are presented in Publication IV. In Figure 18 the results for the whole turbine system are presented for different models. The results show that when the nonlinear behaviour of the steam turbine efficiency is included in the optimisation model, it describes the behaviour of the simulated case plant more accurately than the linear description estimated with the Willans line. The new NLP model gives slightly higher relative power productions than the simulation model. The reason for this may be that in the optimisation models the power production as an objective function was maximised and thus the models may find more optimal process values than the simulated one.

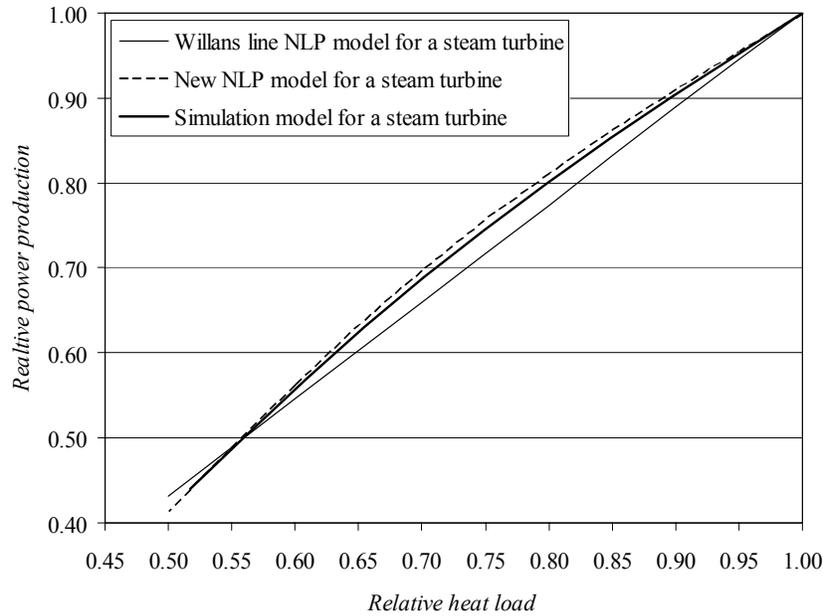


Figure 18. Relative power production of the steam turbine as a function of the relative heat demand according to the tested models.

5.3.5 Modelling of Part Loads

The demand of district heating is included to the model by adding four heat demand periods according to the heat load duration curve adopted from Harvey et al. (2000) and presented in Figure 19. The modelled CHP plant is producing 60 % of the highest annual heat load in the DH network. The lowest load at which the plant is operating is 50 % heat load (30 % of the highest annual heat load in the DH network), which is a common lower bound for the small-scale CHP plant operation. This heat demand is converted to fuel input and the part loads are presented according to the changes in the fuel input to the process.

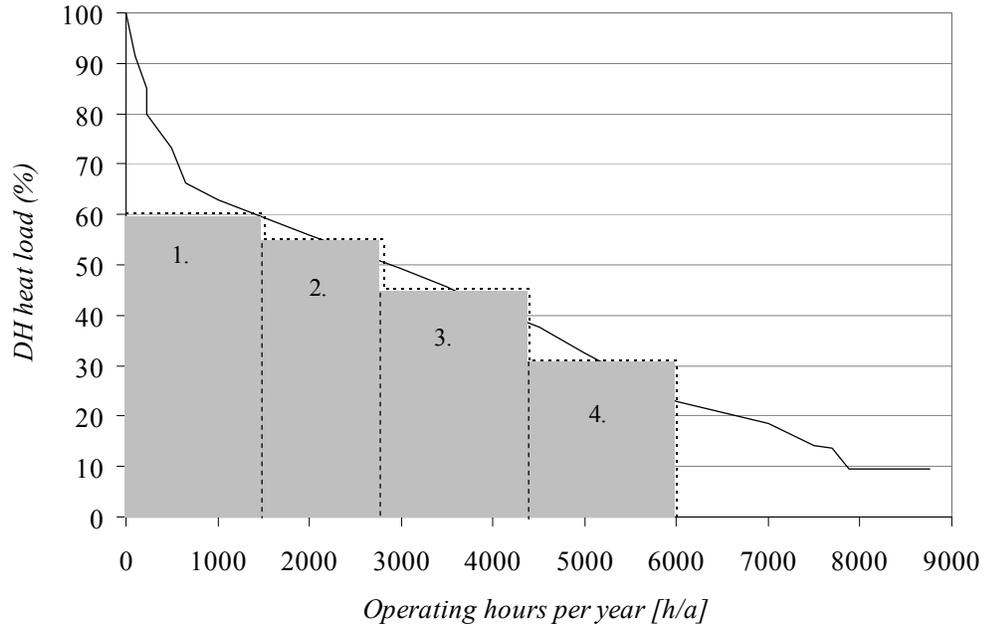


Figure 19. The district heat load duration curve and the multiperiods 1- 4 used for the modelling of the heat demand.

5.3.6 Model II Formulation

The objective function of the additional profit gained with the process changes

$$\begin{aligned} \max z = & c_{el} \cdot \left(\sum_{p \in PERIODS} \left(\sum_{i \in TURBINES} \eta_{ge} \cdot W_{i,p} \cdot t_p - \sum_{\substack{i \in PUMPS \\ \cup BLOWERS}} W_{i,p} \cdot t_p \right) / 1000 \frac{kW_e}{MW_e} - W_{base\ case} \right) \\ & - a \cdot \sum_{\substack{i \in REHEATER \\ \cup PREHEATER \\ \cup DHEX2}} c_i \cdot y_i \end{aligned} \quad (5.3.1)$$

$$+ c_{CO2} \cdot \left(\sum_{p \in PERIODS} \left(\sum_{i \in TURBINES} \eta_{ge} \cdot W_{i,p} \cdot t_p - \sum_{\substack{i \in PUMPS \\ \cup BLOWERS}} W_{i,p} \cdot t_p \right) - W_{base\ case} \right) \cdot 1000 \frac{kW_e}{MW_e} \cdot \frac{e_{coal}}{\eta_{coal}} / 1000 \frac{kg}{t}$$

Mass balance equations (linear constraints for the optimisation model) :

$$\sum_{j \in IN(i,j)} m_{j,p} = \sum_{k \in OUT(i,k)} m_{k,p} \quad \forall i \in UNITS, p \in PERIODS \quad (5.3.2)$$

Energy balance equations (nonlinear and nonconvex constraints) :

$$\begin{aligned} \sum_{j \in IN(i,j)} m_{j,p} \cdot h_{j,p} &= \sum_{k \in OUT(i,k)} m_{k,p} \cdot h_{k,p} + W_{i,p} \\ \forall i \in UNITS \setminus \{HEATEX, FEEDWT, SPLITTERS, BURNERS, TURBINES\}, p \in PERIODS \end{aligned} \quad (5.3.3)$$

where the work done by some of the units is assigned (parameters):

$$W_{i,p} = 0 \quad \forall i \in \text{UNITS} \setminus \{\text{HEATEX}, \text{FEEDWT}, \text{SPLITTERS}, \text{BURNERS}\}, p \in \text{PERIODS} \quad (5.3.4)$$

$$\sum_{j \in \text{IN}(i,j)} m_{j,p} \cdot h_{j,p} + W_{i,p} = \sum_{k \in \text{OUT}(i,k)} m_{k,p} \cdot h_{k,p} \quad (5.3.5)$$

$$\forall i \in \text{UNITS} \setminus \{\text{PUMPS}\}, p \in \text{PERIODS}$$

Enthalpies and entropies for steam and water as functions of pressures and temperatures are formulated to the optimisation model according to the functions presented in the Industrial Standard IAPWS-IF97 (Wagner and Kruse (1998)).

Temperature and pressure of the superheated steam (parameters):

$$T_{j,p} = T_{sh} \quad \forall i \in \text{SUPERHEATER}, j \in \text{COLDOUT}(i,j), p \in \text{PERIODS} \quad (5.3.6)$$

$$p_{j,p} = P_{sh}$$

Temperature dependences on pressures for saturated streams and used pressure ranges:

$$T_{j,p} = \sum_{n=0}^{n=4} b_{n,j} \cdot p_{j,p}^n \quad \forall j \in \text{SATSTREAMS}, p \in \text{PERIODS} \quad (5.3.7)$$

Table 8. Coefficients $b_{n,j}$ for Eq. (5.3.7) in needed pressure ranges and the equation types.

Pressure range	b_4	b_3	b_2	b_1	b_0	Eq. type
$55 \text{ bar} < p_{j,p} < 65 \text{ bar}$				1.0964	209.57	convex
$0.15 \text{ bar} < p_{j,p} < 1.45 \text{ bar}$	-30.583	121.39	-187.26	161.700	34.220	concave
$0.68 \text{ bar} < p_{j,p} < 3.6 \text{ bar}$	-0.45910	5.0949	-22.756	60.161	57.494	concave
$1 \text{ bar} < p_{j,p} < 5 \text{ bar}$	-0.1225	1.9080	-12.000	44.950	65.044	concave

Enthalpies for flue gases and air (linear constraint):

$$h_{j,p} = cp_j \cdot (T_{j,p} - T_{ref}) \quad \forall j \in \text{FLUEGAS} \cup \text{AIR}, p \in \text{PERIODS} \quad (5.3.8)$$

Enthalpy and the lower heating value of moist biomass are fixed according to data from Alakangas (2000) (parameter):

$$h_{j,p} = LHV_{j,p} = -0.221 \cdot F_{H20,j} + 19.8 \quad \forall j \in \text{FUEL}, p \in \text{PERIODS} \quad (5.3.9)$$

Biomass fuel input to the process (parameter):

$$m_{j,p} = \text{LOAD}_p \cdot M_{\text{FUEL}} \quad \forall i \in \text{BURNERS}, j \in \text{IN}(i,j) \cap \text{FUEL}(j), p \in \text{PERIODS} \quad (5.3.10)$$

The air flow to the burner (linear constraint) :

$$\sum_{j \in \text{IN}(i,j) \cap \text{AIR}(j)} m_{j,p} = AF_{ratio} \cdot \sum_{j \in \text{IN}(i,j) \cap \text{FUEL}(j)} m_{j,p} \quad \forall i \in \text{BURNERS}, p \in \text{PERIODS} \quad (5.3.11)$$

Air temperature and pressure at the inlet (parameter):

$$T_{j,p} = T_{air, in} \quad \forall i \in BLOWERS, j \in IN(i, j), p \in PERIODS \quad (5.3.12)$$

$$p_{j,p} = P_{air, in} \quad \forall i \in BLOWERS, j \in IN(i, j), p \in PERIODS \quad (5.3.13)$$

Air pressure after the blower (parameter):

$$p_{k,p} = P_{air, blower} \quad \forall i \in BLOWERS, k \in OUT(i, k), p \in PERIODS \quad (5.3.14)$$

Flue gas pressure (parameter):

$$p_{i,p} = P_{flue\ gas} \quad \forall j \in FLUEGAS, p \in PERIODS \quad (5.3.15)$$

Steam turbine efficiency for each turbine stage:

$$\eta_{isentropic, i, p} = \sum_{n=0}^{n=6} b_{n, i} \cdot LOAD_p^n \quad \forall i \in TURBINES, p \in PERIODS \quad (5.3.16)$$

Table 9. Coefficients $b_{n,i}$ of different turbine stages for Eq. (5.3.16).

Efficiency curves of steam turbine stages are discussed in more detail in Publication IV.

Turbine stage	b_6	b_5	b_4	b_3	b_2	b_1	b_0	Eq. type
Regulation stage				3.8508	-13.020	13.807	-3.8914	concave
Working stage							0.88	fixed*
Exhaust stage	-2.638	7.206	-3.123	-8.042	10.010	-3.621	1.056	concave

* The efficiency of a working stage depends mainly on the pressure ratio of the turbine stage and on the volumetric flow of the steam. Due to the conical rule (e.g. Traupel (2001)) both of them are almost constant at the loads from 100 % to 50 % typical for small-scale CHP plants. Thus, the efficiency of a working stage can be approximated as a fixed value.

Entropy equation for steam turbine stages (linear constraint):

$$s_{j,p} = s_{isentropic, k, p} \quad \forall i \in TURBINES, j \in IN(i, j), k \in OUT(i, k), p \in PERIODS \quad (5.3.17)$$

Isentropic enthalpy equation for steam turbine stages (nonlinear and nonconvex constraint):

$$h_{j,p} - h_{k,p} = \eta_{isentropic, i, p} \cdot (h_{j,p} - h_{isentropic, k, p}) \quad (5.3.18)$$

$$\forall i \in TURBINES, j \in IN(i, j), k \in OUT(i, k), p \in PERIODS$$

The steam contents of the moist steam after the exhaust stages of the turbine are calculated by using the enthalpies and entropies of the moist steam in Eqs. (5.3.18) - (5.3.21) (nonlinear

and nonconvex constraints):

$$S_{isotropic,k,p} = x_{is,i,p} \cdot S_{steam,k,p} + (1 - x_{is,i,p}) \cdot S_{water,k,p} \quad (5.3.19)$$

$$S_{k,p} = x_{i,p} \cdot S_{steam,k,p} + (1 - x_{i,p}) \cdot S_{water,k,p} \quad \forall i \in EXHSTAG, k \in OUT(i, k), p \in PERIODS \quad (5.3.20)$$

$$h_{isotropic,k,p} = x_{is,i,p} \cdot h_{steam,k,p} + (1 - x_{is,i,p}) \cdot h_{water,k,p} \quad (5.3.21)$$

$$h_{k,p} = x_{i,p} \cdot h_{steam,k,p} + (1 - x_{i,p}) \cdot h_{water,k,p} \quad \forall i \in EXHSTAG, k \in OUT(i, k), p \in PERIODS \quad (5.3.22)$$

The steam and moisture mass flows after the exhaust stages (nonlinear and nonconvex constraint):

$$x_{i,p} \cdot (m_{steam,j,p} + m_{water,j,p}) = m_{steam,j,p} \quad \forall i \in EXHSTAG, j \in OUT(i, j), p \in PERIODS \quad (5.3.23)$$

Dependence of pressure on the steam mass flow in the working and exhaust stages of the turbine at part loads (nonlinear and nonconvex constraint):

$$p_{j,p}^2 \cdot m_{0,j}^2 - m_{j,p}^2 \cdot p_{0,j}^2 = p_{k,p}^2 \cdot m_{0,k}^2 - m_{k,p}^2 \cdot p_{0,k}^2 \quad (5.3.24)$$

$$\forall i \in WORKSTAG \cup EXHSTAG, j \in IN(i, j), k \in OUT(i, k), p \in PERIODS$$

Heat exchanger mass balances for the hot and cold sides (linear constraints):

$$\sum_{j \in COLDIN(i,j)} m_{j,p} = \sum_{k \in COLDOUT(i,k)} m_{k,p} \quad \forall i \in HEATEX, p \in PERIODS \quad (5.3.25)$$

$$\sum_{j \in HOTIN(i,j)} m_{j,p} = \sum_{k \in HOTOUT(i,k)} m_{k,p} \quad \forall i \in HEATEX, p \in PERIODS \quad (5.3.26)$$

Temperature difference between hot and cold streams at design load (linear constraints):

$$T_{j,p=1} - T_{k,p=1} \geq \Delta T \quad \forall i \in HEATEX, j \in HOTIN(i,j), k \in COLDOUT(i,k), p \in PERIODS \quad (5.3.27)$$

$$T_{j,p=1} - T_{k,p=1} \geq \Delta T \quad \forall i \in HEATEX, j \in HOTOUT(i,j), k \in COLDIN(i,k), p \in PERIODS \quad (5.3.28)$$

Pressure conditions for heat exchangers (linear constraints):

$$p_{j,p} = p_{k,p} \quad \forall i \in HEATEX, j \in HOTIN(i,j), k \in HOTOUT(i,k), p \in PERIODS \quad (5.3.29)$$

$$p_{j,p} = p_{k,p} \quad \forall i \in HEATEX, j \in COLDIN(i,j), k \in COLDOUT(i,k), p \in PERIODS \quad (5.3.30)$$

The areas of heat exchangers (nonlinear and nonconvex constraint):

$$\sum_{j \in HOTIN(i,j)} m_{j,p} \cdot h_{j,p} - \sum_{k \in HOTOUT(i,k)} m_{k,p} \cdot h_{k,p} = A_{hex,i} \cdot U_i \cdot \Delta T_{lm,i,p} \quad (5.3.31)$$

$$\forall i \in HEATEX, p \in PERIODS$$

Heat exchanger cost (linear constraint):

$$c_i = 8000 \text{ €/unit} + 100 \text{ €/m}^2 \cdot A_{hex,i} \quad \forall i \in HEATEX \quad (5.3.32)$$

Chen approximation for the logarithmic mean temperature difference (nonlinear and concave constraint):

$$\Delta T_{lm,i,p} = \left(\frac{1}{2} \cdot \Delta T_{1,i,p} \cdot \Delta T_{2,i,p} \cdot (\Delta T_{1,i,p} + \Delta T_{2,i,p}) \right)^{\frac{1}{3}} \quad \forall i \in HEATEX, p \in PERIODS \quad (5.3.33)$$

$$\Delta T_{1,i,p} = T_{j,p} - T_{k,p} \quad \forall i \in HEATEX, j \in HOTIN(i,j), k \in COLDOUT(i,k), p \in PERIODS \quad (5.3.34)$$

$$\Delta T_{2,i,p} = T_{j,p} - T_{k,p} \quad \forall i \in HEATEX, j \in HOTOUT(i,j), k \in COLDIN(i,k), p \in PERIODS \quad (5.3.35)$$

Equal mass flow distribution in both DH exchangers at design load (nonlinear constraint):

$$m_{k,p=1} = y_i \cdot m_{j,p=1} \quad \forall i \in DHEX2, j \in HOTIN(i=DHEX1,j), k \in HOTIN(i,k), p \in PERIODS \quad (5.3.36)$$

Return temperature of DH water (parameter):

$$T_{j,p} = T_{return} \quad \forall i \in DHEX2, j \in IN(i,j) \cap DHWATER(j), p \in PERIODS \quad (5.3.37)$$

Forward temperature of DH water (linear constraint):

$$T_{k,p} \geq T_{forward} \quad \forall i \in DHEX1, k \in OUT(i,k) \cap DHWATER(k), p \in PERIODS \quad (5.3.38)$$

Blower equations (linear constraints):

$$-W_{i,p} \cdot \eta_i = (m_{j,p} \cdot h_{j,p} - m_{k,p} \cdot h_{k,p}) \quad (5.3.39)$$

$$\forall i \in BLOWERS, j \in IN(i,j), k \in OUT(i,k), p \in PERIODS$$

Splitter temperature and pressure conditions (linear constraints):

$$T_{j,p} = T_{k,p} \quad \forall i \in SPLITTERS, j \in IN(i,j), k \in OUT(i,k), p \in PERIODS \quad (5.3.40)$$

$$p_{j,p} = p_{k,p} \quad \forall i \in SPLITTERS, j \in IN(i,j), k \in OUT(i,k), p \in PERIODS \quad (5.3.41)$$

Selection node mass balances (linear constraints):

$$\sum_{j \in NODEIN(n,j)} m_{j,p} = \sum_{k \in NODEOUT(n,k)} m_{k,p} \quad \forall n \in NODES, p \in PERIODS \quad (5.3.42)$$

Logical constraints for temperatures in the selection nodes (Big-M constraints):

$$T_j - T_k \leq M \cdot (1 - y_i) \quad (5.3.43)$$

$$T_j - T_k \geq -M \cdot (1 - y_i) \quad (5.3.44)$$

$$\forall n \in MIXNODES, j \in NODEIN(n,j) \cap OUT(i,j), k \in NODEOUT(n,k), p \in PERIODS$$

$$T_j - T_k \leq M \cdot (1 - y_i) \quad (5.3.45)$$

$$T_j - T_k \geq -M \cdot (1 - y_i) \quad (5.3.46)$$

$$\forall n \in SPLITNODES, j \in NODEIN(n,j), k \in NODEOUT(n,k) \cap IN(i,j), p \in PERIODS$$

Constraints for pressures in selection nodes (linear constraints):

$$p_{j,p} = p_{k,p} \quad (5.3.47)$$

$$\forall n \in MIXNODES, j \in NODEIN(n,j) \cap OUT(i,j), k \in NODEOUT(n,k), p \in PERIODS$$

$$p_{j,p} = p_{k,p} \quad (5.3.48)$$

$$\forall n \in SPLITNODES, j \in NODEIN(n,j), k \in NODEOUT(n,k) \cap IN(i,j), p \in PERIODS$$

Binary variable equation for the selection of a turbine system:

$$\sum_{i \in REGSTAG} y_i = 1 \quad y \in \{0,1\} \quad (5.3.49)$$

Logical constraints for mass flows in the two-stage DH exchanger (Big-M constraint):

$$m_{j,p} \leq y_i \cdot M \quad y \in \{0,1\} \quad \forall i \in \text{REGSTAG} \cup \text{DHEX2}, j \in \text{IN}(i,j), p \in \text{PERIODS} \quad (5.3.50)$$

5.3.7 Model II Analysis

Model II includes 2030 equations and 2936 variables. The solution time of the model with SBB solver is 771 CPU seconds and with DICOPT solver 630 CPU seconds. The initial points for the Model II were chosen according to the best available information on the optimal point in order to avoid problems with local optimums. For the sensitivity analysis runs the initial values were chosen randomly between the lower and higher bounds of the variables. The bounds were selected very carefully in the construction of the model with the help of the simulation model results.

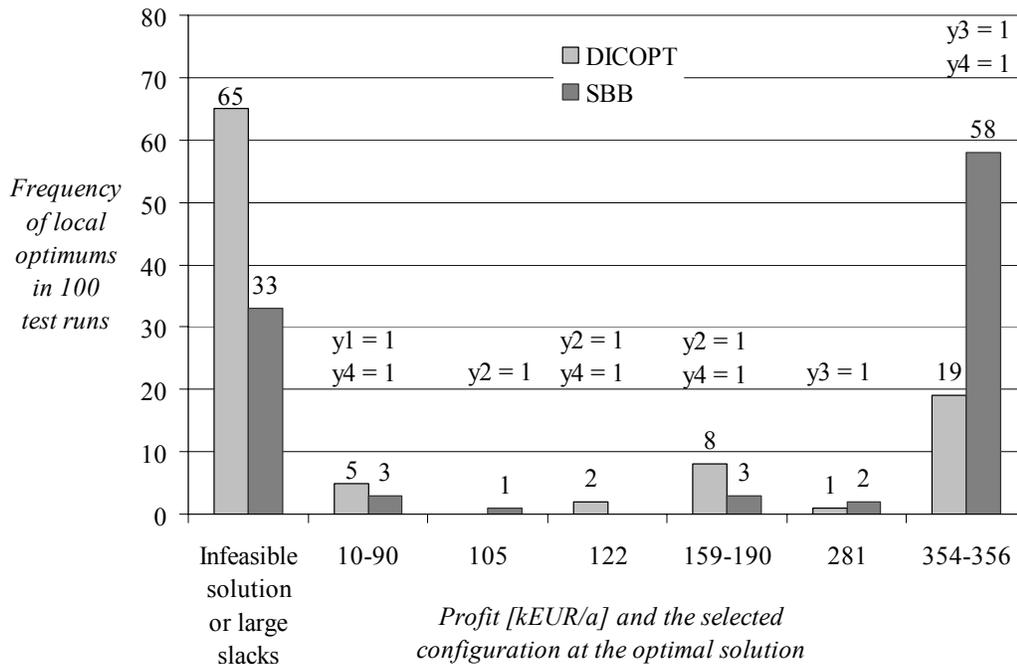


Figure 20. Sensitivity of the optimal solution to the initial values of the variables when Model II is solved with SBB and DICOPT.

The sensitivity analysis for 100 test runs is presented in Figure 20. Although DICOPT is a faster solver for the problem the results show that it is also much more sensitive to the selected initial values and gives the best found solution only in 19 % of the runs. On the other hand SBB was able to find the best solution in the 58 % of the runs. With both DICOPT and SBB solvers the second best local solution is 21 % lower than the best local solution ($y_3=1, y_4=1, profit = 354-356$ k€/a). The other local solutions are from 50 % to 86 %

worse than the best solution found so the difference of local solutions in the model is significant. Although SBB is slightly slower to solve the model, it gives the best found solution to the problem more often and seems to be less sensitive to the selected initial values than DICOPT. Thus SBB is used as a solver with the model.

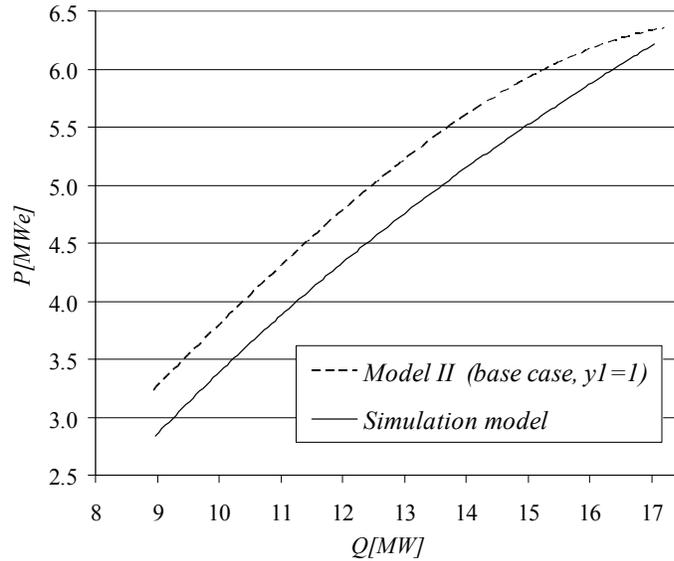


Figure 21. Comparison of the 6 MW_e CHP plant base case ($y_l=1$) power production at part loads calculated with Model II and the simulation model.

The correspondence of Model II to the actual CHP processes is evaluated by comparing the base case ($y_l = 1$) results of the model with the simulation results of an existing CHP plant producing 6 MW_e. The extraction of the complete operational data from a power plant is often very difficult because all the needed data may not be available and the data that is available may be ambiguous. Therefore, the complete reference data is here created with a simulation model. The simulation model is discussed in more detail in Publication I, where it is evaluated to describe quite realistically the behaviour of a CHP plant. The comparison of the new steam turbine model to the actual behaviour of a steam turbine is presented in Publication IV. The correspondence of the power and heat production of Model II and the simulation model at part loads is presented in Figure 21. Model II is a good description of the simulation model, although the power production of the model is higher at all loads. The trend between the simulation and the model results is the same but higher absolute power productions are gained with the model. In Model II the temperatures and pressures may vary as the power production is optimised. In the simulation model they are fixed and not necessary optimal for the process. The percentual difference of the Model II results and the simulation model increases as the load decreases. The average difference of the model

results from the simulation results is 9 % with the standard deviation of 5 %. The coefficient of determination, R^2 , of Model II at base case in comparison with the simulation model is 0.9044.

5.3.8 Results and Discussion

The effect of the electricity price to the optimal process improvement in the modelled 6 MW_e CHP plant case and the profits of the improvement was evaluated by changing the electricity prices from 10 to 60 €/MWh. Similarly, the effect of the fossil CO₂ emission permit price to the profitability of the process improvements were evaluated by changing the permit prices from 6 to 45 €/tCO₂. The results are presented in Figure 22. With all the prices the addition of a steam reheater, a high pressure feed water preheater, and a two-stage DH exchanger was the most profitable process change compared to the base case. This process change increased the power-to-heat ratio (α) of the process at design load from 0.37 in the base case to 0.55 in the improved process. Similarly, the electrical efficiency (η_e) increased from 0.24 to 0.30 but the total efficiency (η_{tot}) was reduced from 0.90 to 0.82. The dependence of the efficiency of the improved process on the load is presented in Table 10.

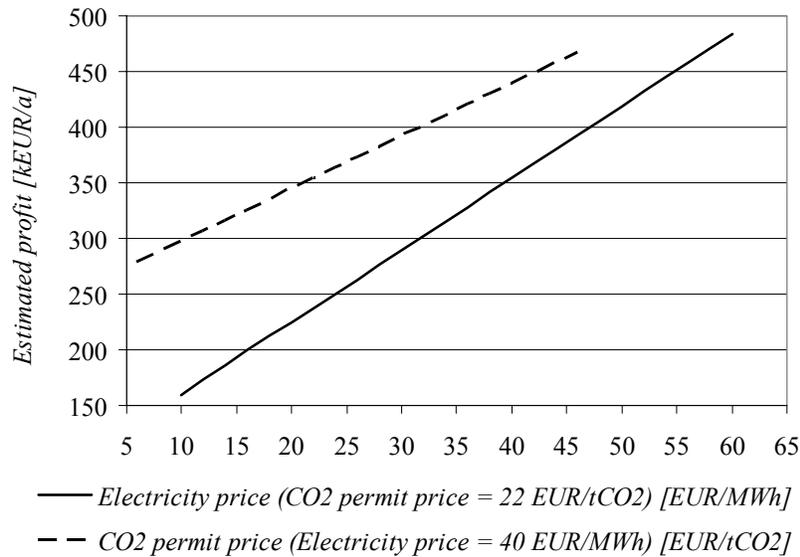


Figure 22. Effect of the electricity and CO₂ permit prices on the profitability of the process changes according to Model II for the 6 MW_e case plant.

Table 10. Efficiencies of the improved new process ($y_3 = 1, y_4 = 1$) according to Model II at different loads.

Load (%)	α (%)	η_e (%)	η_{tot} (%)
100	55	30	82
92	54	30	80
75	52	30	78
52	48	29	90

5.4 Discussion

The results of Model I showed that it is possible to find profitable solutions for increasing the electrical efficiency and the power-to-heat ratio in the case plants with optimisation tools. In the case plants producing 1.8 MW_e, 6 MW_e, 11 MW_e, and 14.7 MW_e the electrical efficiency increased up to 0.28-0.30 and the power-to-heat ratio up to 0.45-0.50 with additions of a two-stage DH exchanger, a feed water preheater, and a steam reheater. A gas engine and a fuel dryer integration increased the electrical efficiency further to 0.31-0.36 and the power-to-heat ratio to 0.59-0.78. The effect of a two-stage DH exchanger, a feed water preheater, a steam reheater, and a fuel dryer addition on the CHP plant investment costs was small, but the integration of a gas engine increased the investment costs in most cases by one fifth.

Model I showed its capability to handle the trade-offs between efficiency and costs, when different process alternatives were compared and when profitable process improvements for current CHP process designs were searched for. However, Model I can be improved by including the part load operation of a CHP plant in a DH network to the model. Also, the more accurate enthalpy and entropy functions with the modelling of pressure levels can improve the model and enable the pressure optimisation of the process changes, for example in the steam turbine extractions for the steam reheat and the feed water preheat.

These modelling challenges were included in Model II, where the part load behaviour of the CHP plant was modelled with multiperiods, the pressure modelling was included to the model, the steam and water property functions were modelled according to the Industrial Standard IAPWS-IF97 (Wagner and Kruse (1998)), and a more detailed steam turbine model was used. Previously in CHP problems the behaviour of the steam turbine system is often described with Willans line and the pressure variables are often fixed to reduce the complexity of the problem. In the developed NLP model for the back-pressure steam turbine

the Willans line is substituted with more detailed efficiency curves describing the regulation stage efficiency and the exhaust losses at the end of the turbine and with an equation describing the dependence of pressure on the steam mass flow through the turbine at part loads. Also, the pressures are modelled as free variables for both design and off-design optimisation of the steam turbine. When the results of the new NLP model for the steam turbine are compared with the results from the similar NLP optimisation model based on the Willans line and with a simulation model of an existing steam turbine, the new NLP model gives the best description of the total simulated steam turbine as well as of the behaviour of the separate steam turbine stages.

The superstructure of Model II is a simplified version of Model I and the optimisation was limited to one CHP plant case producing 6 MW_e. For this case the addition of a steam reheater, a high pressure feed water preheater, and a two-stage DH exchanger was the most profitable process improvement and increased the power-to-heat ratio at design load from 0.37 in the base case to 0.55 and the electrical efficiency from 0.24 to 0.30. At part loads the power-to-heat ratio reduced and was 0.48 when the load was approximately half of the full load.

For both models the solver selected according to a sensitivity analysis was SBB. The optimisation run of Model I was much faster than in Model II, as in Model I the pressures were parameters, enthalpy and entropy functions were linear, and there were no multiperiods. However, according to the sensitivity analysis the results of Model I were more sensitive to the initial values than in Model II at least when using the SBB solver. This may be due to a more complex superstructure of Model I. On the other hand, the complexity of the equations in Model II can be a reason for the large proportion of infeasible solutions in the sensitivity analysis. The defining of initial values was important for finding a feasible solution in Model II but when the feasible solution was found it was also very often the best local solution.

Model II is a highly nonconvex model and in the future modelling the reduction of these nonconvexities could improve the model. If it would be possible to formulate the model in a convex way, the global solution of the optimisation problem could be found also with local solvers. Model II describes the modelled CHP process well but it would be possible to improve the results with more detailed and comprehensive cost data and cost functions of the different process units. In general, Model II can be useful in the design and planning of new efficient CHP plant processes and its formulations can also be implemented to other processes with similar kind of modelling problems.

6 DISCUSSION ON THE ROBUSTNESS OF THE DEVELOPED MODELS

The sensitivity analyses for the models are presented in Figures 14 and 20. Both analyses show that the models are sensitive to the initial values of the variables. However, in Model I a feasible solution is found almost always and the two best local solutions that are close to each other are found in over 60 % of the runs. The complexity of Model II formulation can be seen also in the sensitivity analysis. With SBB solver over 30 % of the runs resulted in an infeasible solution. However, again in almost 60 % of the runs the best local solution was found. In these models the increased complexity affected mostly the frequency of infeasible solutions in the optimisation runs, while the best local solution was found almost as often regardless of the complexity of the model. In these sensitivity analyses the initial values were selected randomly from the feasible regions of the variables. In reality, the initial values are selected by the user, who should have fairly good knowledge on the reasonable process values that should be set as initial values. Thus, when the knowledge of the user is included to the optimisation process, the rate at which the best local solution is found with the model can be improved.

A factor affecting the sensitivity of the model results and to the robustness of the model is the convexity (see Section 2.5.2). If the model is convex, the global solution for the problem can be found also with local solvers. In both Model I and II there are many functions that cause nonconvexities to the model. The analysis of the convexity or nonconvexity of the equations is included in the model formulations in Sections 5.2.2 and 5.3.6. In general, the nonconvex equations in the models are the bilinear terms in the:

- energy balances, Eqs. (5.2.3), (5.2.13), (5.3.3), and (5.3.5)
- isentropic enthalpy equations for steam turbine stages, Eqs. (5.2.17) and (5.3.18)
- enthalpy and entropy calculations of the moist steam after the exhaust stage of the steam turbine, Eqs. (5.2.18) and (5.3.19)-(5.3.22)
- moisture and steam mass flow definitions after the exhaust stage of the turbine, Eqs. (5.2.19) and (5.3.23)
- heat exchanger area equations, Eqs. (5.2.28) and (5.3.31)

and the concave functions in Model II:

- the temperature and pressure dependence functions for saturated steams Eq. (5.3.7)
- the efficiency functions for steam turbine stages Eq. (5.3.16)
- the Chen approximation for the logarithmic mean temperature difference (for concavity see Publication IV) Eq. (5.3.33).

In addition the Paterson approximation (5.2.29) used for the logarithmic temperature difference in Model I and the equations of the pressure dependence on the mass flow rate in steam turbine stages (5.3.24) used in Model II are nonconvex. The Paterson approximation can be changed to the Chen approximation, which can then be convexified e.g. by using the convex bounding inequalities proposed by Zamora and Grossmann (1997). The bilinear terms can be modified to the convex form by using convex envelopes developed by McCormick (1976) and Al-Khayyal and Falks (1983), and described for example in Floudas (1999). For other nonconvex functions the best convexification method should be evaluated case-by-case. The equality equations in the model should also be affine (i.e. a sum of a linear function and a constant) in order to the whole model to be convex. In Model II there are equality equations that are not affine, e.g. the dependence of saturated temperature and pressure in Eq. (5.3.7) and the entropy and enthalpy change equations for steam turbine stages in Eqs. (5.3.18)-(5.3.22). The requirements for the convexity of an optimisation model are summarised in Section 2.5.2.

If the mathematical convexification of a function can not be used, the nonconvex function describing the process phenomenon could be replaced with another function that models the process in a convex way. However, this often means that a more detailed description of a process is changed to a simpler and a less accurate one. This may have significant impact on the results, and the choice between the accuracy of the results and the sensitivity of the model should be considered and evaluated by the user.

In general, with complex models there are often trade-offs between the sensitivity of the model and its accuracy. The accuracy of a large model may be improved with more detailed and thus complex sub-models describing the different parts of the problem. However, this may increase the sensitivity of the model solutions compared to a previously used simpler model. If no methods are available for convexifying the model without losing its accuracy, the decision between the importance of the model accuracy and the globality of the solution has to be made. The finding of this good enough level for the solution accuracy and robustness of the model requires knowledge both on the mathematical programming possibilities and on the modelled phenomena and the needs of the modelled process. Again, the user's experience on the commonly used values of the modelled process as well as on the behaviour of the developed optimisation model are vital for the reliability of the optimisation results.

7 CONCLUSIONS

In this work possible process improvements that could increase the power production of small-scale (<20 MW_e) CHP plants were evaluated and a MINLP model for the process synthesis optimisation of a biomass-fuelled small-scale CHP plants was developed. The model is based on a superstructure including the possible process improvements and it includes multiperiod modelling of the part load operation, modelling of pressures as free variables, steam and water property functions depending both on temperatures and pressures, and new detailed modelling of a back-pressure steam turbine system. The sensitivity of the developed models were studied and analysed. The possibilities to increase the power production and the power-to-heat ratio of the small-scale CHP plants were studied with the developed simulation and optimisation models.

The basis for the simulations and optimisations was the data collected from Finnish and Swedish small-scale CHP (1-20 MW_e) plants between 2002 and 2004. The process changes that could increase power production evaluated in this research were:

- (i) high temperature and pressure of superheated steam (only in simulations)
- (ii) steam reheater
- (iii) high pressure feed water preheater
- (iv) two-stage district heat exchanger
- (v) fuel dryer
- (vi) gas engine or gas turbine integration

In this case a novel concept for the addition of a gas engine and a gas turbine to the small-scale CHP plant was suggested, in which the exhaust gases, with 12.5 % and 15 % oxygen content, from a gas engine or a turbine were used as combustion air in a fluidised bed or grate furnace firing biomass.

In order to test the considered changes four existing CHP plants from Finland and Sweden were selected to represent the different size CHP processes. Simulation models of these case plants were constructed, and the process changes were added to these models. The simulation results and the cost analysis showed that there were possibilities to increase the power production of the CHP plants with the selected process changes. With the current electricity price of 30 €/MWh, the addition of a two-stage district heat exchanger, the steam reheater and a feed water preheater had potential to be profitable changes for all the case plants. Furthermore, the integration of a gas engine gave profit in some cases, but the costs of a gas engine and a gas turbine integration were quite uncertain. The addition of a fuel dryer was connected to the integration of a gas engine and a gas turbine, as this provided

extra heat to the flue gases that could be used for fuel drying.

Testing all possible profitable combinations with the simulation models would have been a very time consuming task. To include the simultaneous trade-off between power production, increased efficiencies and the resulting costs in the evaluation of the process changes, as well as the optimisation of the process variables, an optimisation model (Model I) for the improvement of a small-scale CHP process was constructed. In Model I the pressures were parameters, enthalpy and entropy functions for steam and water were linear, and there were no multiperiods for part load modelling. The model was solved with a local solver (SBB). In 32 % of the sensitivity test runs, when the initial values were selected randomly, the best local solution was found and in 36 % of the runs the found solution was the next best one. Thus, the efficient use of this nonconvex model requires good initial values for the variables. The optimisation results of Model I showed that it was possible to find profitable solutions for increasing the electrical efficiency and the power-to-heat ratio in the case plants. In the case plants producing 1.8 MW_e, 6 MW_e, 11 MW_e, and 14.7 MW_e the electrical efficiencies increased from 0.17-0.30 up to 0.28-0.30 and the power-to-heat ratios from 0.23-0.50 up to 0.45-0.50 with additions of a two-stage DH exchanger, a feed water preheater, and a steam reheater. A gas engine and a fuel dryer integration increased the electrical efficiency further to 0.31-0.36 and the power-to-heat ratio to 0.59-0.78. The effect of a two-stage DH exchanger, a feed water preheater, a steam reheater, and a fuel dryer addition on the CHP plant investment costs was small, but the integration of a gas engine increased the investment costs in most cases by one fifth.

Model I was improved by including the part load operation of a CHP plant in a district heating network to the model. The modelling of pressure levels were added to the model, and more accurate steam and water property functions depending on both temperatures and pressures were used. Also, in this improved model (Model II) the part load behaviour of the CHP plant was modelled with multiperiods and a more detailed steam turbine model was used. Two local solvers (DICOPT and SBB) were tested for the model. In test runs, where the initial values were selected randomly, SBB performed better finding the best local solution in 58 % of the runs when DICOPT was able to find it only in 19 % of the runs. Also with Model II, it was important to select the initial values of the variables carefully during the optimisations.

The superstructure of Model II was a simplified version of Model I and the modelling was limited to one CHP plant case producing 6 MW_e. For this case the addition of a steam reheater, a high pressure feed water preheater and a two-stage DH exchanger was the most

profitable process improvement and increased the power-to-heat ratio at design load from 0.37 in the base case to 0.55 and the electrical efficiency from 0.24 to 0.30. At part loads the power-to-heat ratio reduced and was 0.48 when the load was approximately half of the full load. In general, the developed model can be useful in the design and planning of new efficient CHP plant processes and its formulations can be utilised also in other processes with similar modelling problems.

7.1 Contribution and Significance of the Work

Prior to this work the possibilities to improve the power production by modifying the currently used process design of 1-20 MW_e biomass-fuelled CHP plants were not studied systematically nor reported publicly. In the industry there was knowledge of the currently used best practices but the potentials of the process improvements in increasing the power-to-heat ratios and power production in the plants were not accurately known. Also, in the industry the new process possibilities are usually tested with case studies and the superstructure approach and optimisation is seldom used for finding the most profitable process improvements. However, the knowledge of the possibilities to increase the power production of small-scale CHP plants is vitally important especially in Finland, where the potential for increasing the CHP production is in the conversion of small heating plants into combined heat and power production. If the economical feasibility of these CHP investments could be improved by using process designs with higher power production capabilities and higher power-to-heat ratios, the new CHP investments could become more lucrative and the biomass fuels used in the heating plants could be utilised more efficiently. Furthermore, the fossil CO₂ emissions could be decreased, if the power produced in the CHP plants could replace coal-fired condensing power.

In this work process data of the state-of-the-art biomass-fuelled 1-20 MW_e CHP plants was summarised and some possible process changes that could improve the power production of the small-scale CHP plants were selected and a superstructure including these process changes was created. The simulation and optimisation models constructed in this work provided knowledge on the efficiencies, profitability, and CO₂ emission savings of the different changes. The results were based on four existing small-scale CHP case plants from Finland and Sweden. The results from these cases give good basis for the future case-by-case considerations on the process designs of the biomass-fuelled CHP plants.

For the case-by-case evaluations of the process changes a MINLP optimisation model (Model I) including the most promising process change possibilities was developed. The

model was based on a well established mathematical formulation presented for example by Bruno et al. (1998). Unlike the other corresponding models, the model included the design configurations specifically suitable for small-scale CHP plants. For example, the selection of a single- or two-stage district heat exchanger and the integration of the exhaust gas from a gas engine or a gas turbine as combustion air in the biomass boiler have not been included in the previous optimisation models. In addition, the cost functions of gas engines and gas turbines used here are specifically developed for small sizes.

Similarly to the many previously reported models for CHP process optimisation (e.g. Bruno et al. (1998), Manninen and Zhu (1999a, 1999b)), also Model I required that the pressures in the modelled CHP process are parameters. This is undesirable when the process changes are optimised, as the selected pressure may have significant effect on the profitability of the changes. Especially, for the profitability of a steam reheater and a feed water preheater the pressure in which the steam is extracted from the steam turbine is very important. The model was developed further (Model II) by modelling the pressures as free variables, and by adding more detailed enthalpy and entropy functions for steam and water according to the Industrial Standard IAPWS-IF97 to the model. Also, the part load behaviour of the CHP plant was included in the model using multiperiods, and a new model for a back-pressure steam turbine behaviour was incorporated in the model. With these modifications a new MINLP model including the part load modelling of heat demands was presented for a CHP process synthesis. The comparison of the work done in this thesis and the earlier key developments in the CHP process modelling is summarised in Table 3 in Section 3. Unlike in many previous models of CHP process synthesis, in this model the pressures were modelled as free variables also at multiperiods and thus the optimisation of the pressure levels was included to the model. This causes challenges to the solvability of the model and requires new approaches for the modelling of the steam and water property functions as well as on the modelling of steam turbine behaviour at part loads. Although the developed model is sensitive to the initial values of the variables, it is able to give good solutions for the modelled CHP plant case. The model can be useful in the CHP process design evaluations and the formulations presented in the model can be used also with other process synthesis models.

7.2 Recommendations for Future Work

Currently, the developed MINLP model for CHP process synthesis is nonconvex and sensitive to initial values of the variables. In the future possibilities to increase the convexity of the model could be tested. One possibility would be to use convex envelopes to convexify

the bilinear terms that cause most of the nonconvexities of the model but a detailed analysis should be made on the possibilities to modify also the other nonconvex parts of the model to a convex form. Especially, the convexity of the steam and water property functions should be analysed, as they may have a significant influence on the robustness of the model.

The inclusion of more process improvement options to the superstructure of the model could be interesting for the process synthesis modelling of small-scale CHP plants. However, unnecessary increase in the complexity of the model should be avoided. Also, if it would be possible to get more accurate and case related cost functions for the process changes the results of the model could be improved.

In reality, the small-scale CHP plants operate often as a part of a larger energy system with several producers and users of power and heat. Thus, it would be interesting to integrate the process synthesis model of a CHP plant with a larger model of district heating network or with an industrial process where the heat is utilised. In a larger district heating network there may be other CHP plants or for example thermal storages that have an effect on the operation of the modelled CHP plant. If the CHP plant is located in an industrial site there may be an industrial process which needs are setting constraints for the CHP plant construction and operation. This would enable the utilisation of the developed model already in the planning of the new CHP plant investment to the district heating network or to the industrial site.

REFERENCES

- Alakangas E. 2000. *Properties of fuels used in Finland*. VTT Research Notes 2045. Finland: Technical Research Centre of Finland. (in Finnish)
- Al-Khayyal FA and Falk JE. 1983. Jointly constrained biconvex programming. *Mathematics of Operations Research* 8(2):273-286.
- Ambiente Italia srl, Italy, Kraftwärmeeanlagen GmbH, Germany, Eicher + Pauli AG, Switzerland, and CIT Energy Management AB, Sweden. 2001. *Risks and chances for small scale combined heat and power in the liberalised energy market*. Final project report, European Commission SAVE contract XVII/4.1031/Z/99-063. Brussels, Belgium. [Available at <http://www.cogen.org>, last visited July 17th 2002]
- Bazaraa MS, Shetty CM, and Sherali HD. 1993. *Non-linear Programming: Theory and Algorithms*. New York: John Wiley and Sons.
- Biegler LT. 1989. Chemical process simulation. *Chemical Engineering Progress* 85(10):50-61.
- Biegler LT and Grossmann IE. 2004. Retrospective on optimization. *Computers and Chemical Engineering* 28(8):1169-1192.
- Biegler LT, Grossmann IE, and Westerberg AW. 1997. *Systematic methods of Chemical Process Design*. New Jersey: Prentice Hall PTR.
- Boyd S and Vandenberghe L. 2004. *Convex optimisation*. New York: Cambridge University Press.
- Brammer JG and Bridgewater AV. 1999. Drying technologies for an integrated gasification bio-energy plant. *Renewable and Sustainable Energy Review* 3(4):243-289.
- Bruno JC, Fernandez F, Castells F, and Grossmann IE. 1998. A rigorous MINLP model for the optimal synthesis and operation of utility plants. *Transactions of the Institute of Chemical Engineers Part A* 76(A3):246-258.
- Björk KM and Westerlund T. 2002. Global optimization of heat exchanger network synthesis problems with and without the isothermal mixing assumption. *Computers and Chemical Engineering* 26(11):1581-1593.

- Carcasci C and Colitto Cormacchione NA. 2001. Part load operating strategies for gas turbines in district heating CHP applications. *Proceedings of the Institution of Mechanical Engineers Part A: Journal of Power and Energy* 215(5):529-544.
- Carcasci C and Facchini B. 1996. A numerical method for power plant simulations. *Journal of Energy Resources Technology* 118(1):36-43.
- Chang CT and Hwang JR. 1996. A multiobjective programming approach to waste minimization in the utility systems of chemical processes. *Chemical Engineering Science* 51(16):3951-3965.
- Chou CC and Shih YS. 1987. A thermodynamic approach to the design and synthesis of plant utility systems. *Industrial & Engineering Chemical Research* 26(6):1100-1108.
- Church EF. 1950. *Steam turbines*. New York: McGraw-Hill.
- Cogen Europe, Energy for Sustainable Development Ltd, ETSU – AEA Technology plc, KAPE S.A., VTT, and Sigma Elektroteknisk. 2001. *The future of CHP in the European market – The European cogeneration study*. European Commission SAVE XVII/4.1031/P/99-169: Brussels. [Available at <http://www.cogen.org>, last visited February 10th 2005]
- Diwekar UM, Grossmann IE, and Rubin ES. 1992. An MINLP process synthesizer for a sequential modular simulator. *Industrial Engineering and Chemical Research* 31(1):313-322.
- Duran MA and Grossmann IE. 1986. An outer approximation algorithm for a class of mixed integer nonlinear programs. *Mathematical Programming* 36(3):307-339.
- Eurostat. 2004. Combined heat and power (CHP) in the EU, Bulgaria, Romania and Turkey – 2004 data. [Available at <http://epp.eurostat.ec.europa.eu>, last visited April 26th 2007]
- Finergy. 2002. *Electricity and district heating 2002 yearbook*. Helsinki: Adato Energia Oy. [Available at <http://www.energia.fi>, last visited January 20th 2005]
- Floudas CA. 1999. *Deterministic Global Optimization: Theory, methods and applications*. Vol. 37 of nonconvex optimization and its applications. Boston: Kluwer Academic Publishers.

- Floudas CA, Akrotirianakis IG, Caratzoulas S, Meyer CA, and Kallrath J. 2005. Global optimization in the 21st century: Advances and challenges. *Computers and Chemical Engineering* 29(6):1185-1202.
- Fridh J. 2001. *Efficient steam turbines for small-scale energy conversion plants*. Technical report KTH/HPT 01-14. Royal Institute of Technology, Sweden. [Available in <http://www.energy.kth.se/>, last visited September 27th, 2004]
- GAMS. 2004. *GAMS - The Solver Manuals*. GAMS Development Corporation.
- Giglmayr I, Nixdorf M, and Pogoreutz M. 2001. Comparison of software for thermodynamic process calculations. *VGB PowerTech* 81(2):44-51.
- Grossmann IE and Biegler LT. 2004. Part II. Future perspective on optimization. *Computers and Chemical Engineering* 28(8):1193-1218.
- Grossmann IE and Daichendt MM. 1996. New trends in optimization-based approaches to process synthesis. *Computers and Chemical Engineering* 20(6-7):665-683.
- Grossmann IE and Kravanja Z. 1995. Mixed-integer nonlinear programming techniques for process systems engineering. *Computers and Chemical Engineering* 19(Suppl.): S189-S204.
- GTW. 2003. *Gas Turbine World 2003 GTW Handbook*. Fairfield, CT: Pequot Publishing Inc.
- Harvey S, Carcasci C, and Berntsson T. 2000. Gas turbines in district heating combined heat and power systems: influence of performance on heating costs and emissions. *Applied Thermal Engineering* 20(12):1075-1103.
- Hashemi-Ahmedy A, Zamora JM, and Gundersen T. 1999. A sequential framework for optimal synthesis of industrial size heat exchanger networks. In: *Proc. 2nd Conference on Process Integration, Modelling and Optimisation for Energy Saving and Pollution Reduction PRES-99*, Budapest, Hungary:329-334.
- Hui CW and Natori Y. 1996. Industrial application using mixed-integer programming technique: A multi-period utility system model. *Computers and Chemical Engineering* 20(Suppl.):S1577-S1582.
- Iyer RR and Grossmann IE. 1997. Optimal multiperiod operational planning for utility systems. *Computers and Chemical Engineering* 21(8):787-800.

- Iyer RR and Grossmann IE. 1998. Synthesis and operational planning of utility systems for multiperiod operation. *Computers and Chemical Engineering* 22(7-8):979-993.
- Kalitventzeff B. 1991. Mixed integer non-linear programming and its applications to the management of utility networks. *Engineering Optimization* 18:183-207.
- Kirjavainen M, Sipilä K, Savola T, Salomón M, and Alakangas E. 2004. *Small-scale biomass CHP technologies: Situation in Finland, Denmark and Sweden*. OPET Report 12, VTT Processes and Finnish District Heating Association. [Available at www.opet-chp.net, last visited September 21st 2004]
- Laukkanen T and Tveit TM. 2003. Steam property functions suitable for mathematical programming. In: Haubak N, Elmegaard B, Qvale B and Moean M (editors). *Proc. 16th International Conference on Efficiency, Costs, Optimization, Simulation and Environmental Impact of Energy Systems (ECOS)*. Copenhagen, Denmark:1735-1741.
- Manninen J. 1999. *Flowsheet synthesis and optimisation of power plants*. Ph.D. Thesis. University of Manchester Institute of Science and Technology.
- Manninen J and Zhu XX. 1999a. Optimal gas turbine integration to the process industries. *Industrial Engineering and Chemical Research* 38(11):4317-4329.
- Manninen J and Zhu XX. 1999b. Optimal flowsheet synthesis for power station design considering overall integration. *Energy* 24(6):451-478.
- Marbe Å, Harvey S, and Berntsson T. 2004. Biofuel gasification combined heat and power – new implementation opportunities resulting from combined supply of process steam and district heating. *Energy* 29(8):1117-1137.
- Mavromatis SF and Kokossis AC. 1998. Hardware composites: A new conceptual tool for the analysis and optimisation of steam turbine networks in chemical process industries. *Chemical Engineering Science* 53(7):1405-1434.
- McCormick GP. 1976. Computability of global solutions to factorable nonconvex programs: Part I – Convex underestimating problems. *Mathematical Programming* 10(2):147-175
- Nash SG and Sofer A. 1996. *Linear and Nonlinear Programming*. New York: McGraw-Hill.

- Nishio M, Itoh J, Shiroko K, and Umeda T. 1980. A thermodynamic approach to steam-power system design. *Industrial & Engineering Chemistry Process Design and Development* 19(2):306-312.
- Nishio M and Johnson AI. 1977. The optimal synthesis of steam and power plants. in: *Proc. 2nd Pacific Chemical Engineering Congress (PACHC'77)* Denver, USA 2:726-729.
- Ong'iro A, Ugursal VI, Al Tawee AM, and Lajeunesse G. 1996. Thermodynamic simulation and evaluation of a steam CHP plant using ASPEN Plus. *Applied Thermal Engineering* 16(3):263-271.
- Orispää Y. 2000. *Manual for calculating CHP electricity and heat*. Finnish District Heating Association. Helsinki: Protermo Oy.
- Papalexandri KP, Pistikopoulos EN, and Kalitventzeff B. 1998. Modelling and optimization aspects in energy management and plant operation with variable energy demands - application to industrial problems. *Computers and Chemical Engineering* 22(9):1319-1333.
- Papoulias SA and Grossmann IE. 1983. A structural optimization approach in process synthesis-I: Utility systems. *Computers and Chemical Engineering* 7(6):695-706.
- Petroulas T and Reklaitis GV. 1984. Computer-aided synthesis and design of plant utility systems. *AiChE Journal* 30(1):69-78.
- Pörn R, Harjunkoski I, and Westerlund T. 1999. Convexification of different classes of non-convex MINLP problems. *Computers and Chemical Engineering* 23(3):439-448.
- Pörn R. 2000. *Mixed integer non-linear programming: Convexification techniques and algorithm development*. Ph.D. Thesis. Åbo Akademi University.
- Roosen P, Uhlenbruck S, and Lucas K. 2003. Pareto optimization of a combined cycle power system as a decision support tool for trading off investment vs. operation costs. *International Journal of Thermal Sciences* 42(6):553-560.
- Ryoo S and Sahindis NV. 1995. Global optimization of nonconvex NLPs and MINLPs with applications in process design. *Computers and Chemical Engineering* 19(5):551-566.

- Salomón M, Savola T, Kirjavainen M, Martin AR, and Fogelholm CJ. 2002. Distributed combined heat and power generation with small-scale biomass plants – state of the art review, in: *Proc. The Second International Symposium on Distributed Generation: Power System and Market Aspects*. Stockholm, Sweden.
- STEM (Swedish Energy Administration). 2004. *Energy in Sweden, Facts and figures 2004*. [Available at <http://www.stem.se>, last visited January 20th, 2005]
- STEM (Swedish Energy Administration). 2001. *Electricity market 2001*. [Available at <http://www.stem.se>, last visited January 20th, 2005]
- Tawarmalani M and Sahindis NV. 2002. *Convexification and Global Optimization in Continuous and Mixed-Integer Nonlinear Programming*. The Netherlands: Kluwer Academic Publishers.
- Traupel W. 2001. *Thermische Turbomaschinen*. Zweiter Band. 4. Auflage. Heidelberg: Springer.
- Tveit TM. 2004. Steam turbine modelling for optimisation of CHP power plants. In: Savola T, editor. *Proc. The 9th International Symposium on District Heating and Cooling*. Espoo, Finland: 125-132.
- Varbanov P, Doyle S and Smith R. 2004. Modelling and optimization of utility systems. *Chemical Engineering Research and Design* 82(5):561-578.
- Varbanov P, Smith R and Perry S. 2004a. Optimisation and top-level analysis of industrial utility systems. *Proc. 2004 AIChE Spring National Meeting* New Orleans, LA, USA: 2156-2162.
- Wagner W and Kruse A. 1998. *Properties of water and steam, The industrial standard IAPWS-IF97 for the thermodynamic properties and supplementary equations for other properties*. Germany: Springer.
- Wahlund B, Yan J, and Westermark M. 2000. Comparative assessment of biofuel-based combined heat and power generation plants in Sweden. In: Kyritsis S, Beenackers A, Helm P, Grassi A, Chiaramonti D, editors. *Proc. 1st World Conference on Biomass for Energy and Industry*, Seville, Spain: 1852-1855.

- Wahlund B, Yan J, and Westermark M. 2002. A total energy system of fuel upgrading by drying biomass feedstock for cogeneration: a case study of Skellefteå bioenergy combine. *Biomass and Bioenergy* 23(4):271-281.
- Westerlund T. 2006. Some transformation techniques in global optimization, in: L. Liberti and N. Maculan ed., *Global Optimization, from Theory to Implementation. Nonconvex Optimization and Its Applications Series*, vol 84. Berlin: Springer.
- Wimmerstedt R. 1999. Recent advances in biofuel drying. *Chemical Engineering Processes* 38(4-6):441-447.
- Zamora and Grossmann. 1997. A comprehensive global optimization approach for the synthesis of heat exchanger networks with no stream splits. *Computers and Chemical Engineering* 21(Suppl.):S65-S70.

APPENDIX

Table A1. Coefficients for enthalpy and entropy functions in Model I.

p [bars]	T [°C]	$h[\text{kJ/kg}] = b_1 \cdot T + b_2$		$s[\text{kJ/kgK}] = b_1 \cdot T + b_2$	
		b_1	b_2	b_1	b_2
1.8 MW _e					
16.5 / steam	$360 \geq T \geq 300$	2.209	2371.903	0.003	5.887
	$300 \geq T \geq 150$	2.799	2212.519	0.004	5.694
16.5 / water	$300 \geq T \geq 100$	4.530	-46.249	0.010	0.393
	$250 \geq T \geq 100$	4.397	-25.079	0.010	0.360
12.4 / steam	$360 \geq T \geq 200$	2.221	2378.025	0.004	5.941
8 / steam	$360 \geq T \geq 300$	2.104	2425.902	0.003	6.241
	$360 \geq T \geq 200$	2.127	2418.308	0.004	6.156
3 / steam	$360 \geq T \geq 130$	2.041	2457.586	0.004	6.578
3 / water	$300 \geq T \geq 120$	4.368	-22.546	0.010	0.352
1.2 / steam	$360 \geq T \geq 100$	1.998	2474.978	0.004	6.960
1.2 / water	$170 \geq T \geq 105$	4.283	-10.202	0.010	0.276
	$150 \geq T \geq 60$	4.225	-3.067	0.011	0.182
1.013 / water	$100 \geq T \geq 10$	4.183	0.303	0.013	0.056
6 MW _e					
60 / steam	$520 \geq T \geq 500$	2.372	2236.989	0.003	5.515
	$300 \geq T \geq 250$	5.245	1329.152	0.004	5.012
60 / water	$300 \geq T \geq 150$	4.646	-72.192	0.009	0.442
	$300 \geq T \geq 40$	4.384	-15.410	0.010	0.312
45 / steam	$520 \geq T \geq 450$	2.317	2281.683	0.003	5.624
30 / steam	$520 \geq T \geq 400$	2.248	2332.897	0.003	5.870
	$500 \geq T \geq 400$	2.252	2331.245	0.003	5.776
3 / steam	$500 \geq T \geq 200$	2.083	2442.974	0.003	6.802
3 / water	$400 \geq T \geq 190$	4.890	-131.776	0.009	0.441
1.2 / steam	$500 \geq T \geq 100$	2.019	2470.918	0.004	7.049
1.2 / water	$170 \geq T \geq 105$	4.283	-10.202	0.010	0.276
	$100 \geq T \geq 60$	4.196	-0.616	0.012	0.123
1.013 / water	$100 \geq T \geq 10$	4.183	0.303	0.013	0.056
11 MW _e and 14.7 MW _e					
92 / steam	$520 \geq T \geq 500$	2.522	2124.110	0.003	5.277
	$310 \geq T \geq 300$	5.615	721.788	0.004	4.747
92 / water	$500 \geq T \geq 40$	4.390	-22.537	0.010	0.403
	$310 \geq T \geq 150$	5.911	-429.553	0.010	0.184
69 / steam	$520 \geq T \geq 450$	2.440	2192.250	0.003	5.393
45 / steam	$520 \geq T \geq 500$	2.308	2286.042	0.003	5.665
	$500 \geq T \geq 400$	2.350	2266.24	0.003	5.549
20 / steam	$500 \geq T \geq 300$	2.207	2364.906	0.003	5.881
20 / water	$500 \geq T \geq 190$	4.618	-71.524	0.009	0.458
6.18 / steam	$500 \geq T \geq 180$	2.138	2422.457	0.004	6.204
6.18 / water	$170 \geq T \geq 105$	4.281	-9.677	0.010	0.276
	$100 \geq T \geq 60$	4.204	-0.950	0.012	0.150
1.013 / water	$100 \geq T \geq 20$	4.184	0.257	0.013	0.069
All cases					
2 / water	$100 \geq T \geq 50$	4.192	-0.227	0.012	0.109
0.6755 / steam	$100 \geq T \geq 80$	2.036	2476.679	0.005	7.046
0.6755 / water	$100 \geq T \geq 80$	4.205	-1.487	0.012	0.150
0.31 / steam	$80 \geq T \geq 55$	2.003	2485.862	0.006	7.383
0.31 / water	$80 \geq T \geq 55$	4.187	-0.052	0.012	0.094

Table A2. Temperature and pressure dependence of a saturated stream in Model I.

$T [^{\circ}\text{C}]$	$p[\text{bars}] = b_3 \cdot T + b_4$	
	b_3	b_4
1.8 MW _e		
$300 \geq T \geq 150$	3.080	151.023
$300 \geq T \geq 120$	11.186	99.741
6 MW _e		
$400 \geq T \geq 190$	10.998	100.070
$300 \geq T \geq 250$	1.079	210.602
11 MW _e and 14.7 MW _e		
$500 \geq T \geq 190$	2.491	162.09
$310 \geq T \geq 300$	0.792	231.952
All cases		
$170 \geq T \geq 105$	21.01	76.87
$100 \geq T \geq 55$	41.365	58.892



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