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# Thermopower in mesoscopic normal–superconducting structures

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## Abstract

We examine the influence of the superconducting proximity effect on the thermoelectric response of hybrid mesoscopic normal metal – superconductor nanostructures. We demonstrate that Andreev scattering can break the well-known Mott relation between the thermopower and the logarithmic energy derivative of the conductance. We also consider the effect of superconductivity on the temperature dependence of the thermopower. © 2000 Elsevier Science B.V. All rights reserved.

*Keywords:* Mott relation; Proximity effect; Superconducting nanostructures; Thermopower

Electronic transport properties of phase-coherent normal–superconducting (NS) devices have been studied in great detail [1]. However, also other thermoelectric properties are expected to be affected by the presence of superconductivity [2,3]. In this paper, we apply the scattering approach to thermoelectric properties of NS structures, first developed in Ref. [2], to study the Andreev-reflection modified thermopower.

In Ref. [2], it was already shown that Andreev reflection breaks the Wiedemann–Franz law. Here we show that it may cause the thermopower to deviate from the celebrated Mott relation [4,5]

$$S = \frac{\pi^2 k_B^2 T}{3 e \varepsilon_F} \left. \frac{\partial \ln G(\varepsilon)}{\partial \varepsilon} \right|_{\varepsilon = \varepsilon_F}, \quad (1)$$

where  $G(\varepsilon)$  is the energy-dependent conductance through the diffusive sample.

Due to the energy derivative at  $\varepsilon_F$  in the Mott relation,  $S$  reflects the balance of energy current between the quasiparticle states slightly above (electrons) and slightly below (holes) the Fermi energy. In the presence of

Andreev reflection between the two, the linear-response formulae for the two-probe conductance  $G$  and thermopower  $S$  are given, in the limit  $T = 0$ , by Eqs. (20), (21) and (38) of Ref. [2]. It follows that

$$(\delta S/a)G \equiv (S/a)G - \frac{dG}{dE} = 2(R_a + T_a) \times \left[ \frac{(R_a + T_a)(\tilde{R}'_a + \tilde{T}'_a) - (R'_a + T_a)(\tilde{R}_a + \tilde{T}_a)}{(R_a + R'_a + T_a + T'_a)^2} \right], \quad (2)$$

indicating the deviation  $\delta S$  of the actual thermopower from the Mott-law prediction. Here  $a \equiv \pi^2 k_B^2 T / (3e)$  and the notation of Ref. [2] has been employed. Hence, to break the Mott relation, one requires a left–right asymmetric structure and energy-dependent transmission/reflection probabilities. To demonstrate this, Fig. 1 shows the computed (for details of the numerical calculations, see Ref. [6]) thermopower and its Mott-law prediction for the structure shown in the inset: a normal diffusive region with a superconducting inclusion strongly coupled to a quantum dot separated from the rest of the structure by barriers of height  $U_B$ . Varying  $U_B$  induces resonances into the transport coefficients and, consequently, strong variations of  $\delta S$ .

We have also considered previous experiments [3], probing the temperature dependence of the thermopower  $S_{NS}$  in NS hybrids. Therefore, we have applied Eqs.

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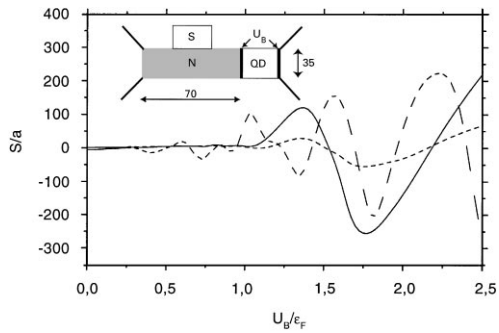


Fig. 1. Thermopower (solid) and Mott-law prediction (dotted) for structure in the inset as a function of the barrier height  $U_B$  of the dot. For comparison, the thermopower in the absence of the superconducting segment is also plotted (dashed). Inset: simulated structure comprising a diffusive normal wire (N) with a superconducting inclusion (S) in contact with a quantum dot (QD). Length scales are in units of the Fermi wavelength.

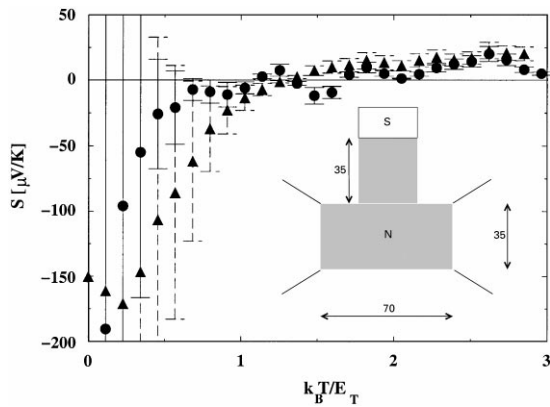


Fig. 2. Thermopower versus temperature for the normal diffusive wire (N) with a superconducting inclusion (S) (triangles), for the corresponding structure without the S segment (circles), and their error bars (dashed for  $S_{NS}$ , solid for  $S_N$  – at low  $T$ , due to strong fluctuations, they extend over the range of the plot). The results are averaged over 50 realizations of disorder. Inset: the simulated structure.

(9)–(13) of Ref. [2] to compute  $S(T)$  in the presence ( $S_{NS}$ ) and absence ( $S_N$ ) of superconductivity. Our preliminary results for the ensemble-averaged quantities are shown in Fig. 2 for the structure depicted in the inset.

For temperatures larger than the Thouless temperature  $E_T/k_B$ , both  $S_{NS}$  and  $S_N$  are positive, of the order of  $10 \mu\text{V/K}$ ,<sup>1</sup> and no large deviations between the two can be observed. For  $k_B T < E_T$ , the numerical results show a huge change in both quantities towards negative values and with an order of magnitude larger absolute value. However, this change is accompanied by fluctuations of the same order or even greater than the ensemble-averaged values. These strong fluctuations are typical for the conductance of mesoscopic wires. Since thermopower is a second-order quantity compared to conductance, its fluctuations are even larger.

In conclusion, we show that superconductivity may break the Mott relation for the thermopower  $S$ . We also consider the temperature dependence of  $S$  in the presence and absence of superconductivity. We find that below the Thouless energy, the sample-to-sample fluctuations in both cases are huge, and hence, no conclusive statements about the ensemble-averaged thermopower can be made.

#### Acknowledgements

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<sup>1</sup>This is larger than the experimental values below  $1 \mu\text{V/K}$  and may be due to the small size of the simulated structure.