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# BENDING RESPONSE OF LASER-WELDED WEB-CORE SANDWICH PLATES

**Doctoral Dissertation** 

Jani Romanoff



Helsinki University of Technology Department of Mechanical Engineering Ship Laboratory TKK Dissertations 88 Espoo 2007

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**Doctoral Dissertation** 

#### Jani Romanoff

Dissertation for the degree of Doctor of Science in Technology to be presented with due permission of the Department of Mechanical Engineering for public examination and debate in Auditorium K213 at Helsinki University of Technology (Espoo, Finland) on the 8th of November, 2007, at 12 noon.

Helsinki University of Technology Department of Mechanical Engineering Ship Laboratory

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Supervisor	Professor Petri Varsta	
(Instructor)	Professor Petri Varsta	
Abstract		
The thesis presents	a theory for the bending of laser-welde	d web-core sandwich plates. The sandwich plate theory and

The thesis presents a theory for the bending of laser-welded web-core sandwich plates. The sandwich plate theory and homogenization are utilised, enabling a similar, fairly coarse, FE mesh to be used for design alternatives with different cross-sectional dimensions. This is considered to be beneficial during the design, when different alternatives should be evaluated. The thesis also studies the main factors that contribute to the total bending response of laser-welded web-core sandwich plates.

The actual periodic structure is homogenized, giving equivalent stiffness properties for the sandwich plate. The differential equations for this homogenous sandwich plate, with thick face plates, consist of those for displacements of the well-known Reissner-Mindlin and Kirchoff plate theories. Therefore, the solution of these differential equations can be carried out using commercial Finite Element software. The periodic structure is reconsidered when the stress formulations are derived. The approach is validated with Finite Element calculations based on actual 3D topology.

The rotation stiffness of the laser stake welded T-joint between the web and face plate is derived experimentally. This is considered to be important since the rotation stiffness affects the shear stiffness in the opposite direction to that of the web plate. An experimental procedure for the determination of the T-joint rotation stiffness is developed and validated. The mechanics of the T-joint are investigated with Finite Element analyses.

The moment introduced by the web plate to the face plate is very important when deflections and normal stresses are considered. This moment is affected by the stiffnesses of the web plate and the T-joint. The deflections are significantly increased by a decrease in the web plate or T-joint rotation stiffness. The influence is greatest with cross-sections where the face plate thickness is large and the web plate spacing is small. Plates with a low aspect ratio under a uniform pressure load have the same maximum deflection, regardless of the T-joint rotation stiffness. Contrary to this, plates with a high aspect ratio or a patch load on a small area are very sensitive to rotation stiffness. The periodicity of the structure is found to have a significant influence on normal stresses in the face and web plates. Because of homogenization shear-induced, periodic, normal stresses vanish. In actual structures these stresses can be an order of magnitude higher than those induced by the bending moments of the sandwich plate. Therefore, the periodic structure as presented in this thesis should be taken into consideration when the normal stresses are calculated. Patch loads on a very small area can cause normal stresses, but in the measured range of T-joint rotation stiffness values, the stresses are found to be unaffected.

Keywords Web-Core; Sandwich Plate Theory; Stress Analysis; Laser Weld; Stake Weld; Rotation Stiffness		
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# VÄITÖSKIRJAN TIIVISTELMÄ

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Bending Response of Laser-Welded Web-Core Sandwich Plates (Laserhitsattujen kerroslevyjen taivutusvaste)

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Työn valvoja	Professori Petri Varsta	
(Työn ohjaaja)	Professori Petri Varsta	
Tiivistalmä		

Tiivistelmä

Tässä työssä kehitettiin analyysimenetelmä laserhitsattujen, I-tyypin kerroslevyjen taivutusvasteen laskentaa varten. Menetelmässä hyödynnetään kerroslevyjen laattateoriaa ja homogenisointia, minkä ansiosta rakenteen taivutusvaste voidaan määrittää eri poikkileikkausvaihtoehdoille käyttäen samaa, suhteellisen harvaa elementtiverkkoa. Tämä menetelmän ominaisuus on erittäin hyödyllinen, kun tarkastellaan ajan suhteen kriittistä laivarakenteiden konseptisuunnittelua. Lisäksi työssä pyritään löytämään tärkeimmät tekijät laatan vasteen kannalta.

Työn alussa kehitettiin teoria laatan taivutukselle. Rakenteen jaksollisuus keskiarvoistettiin, minkä tuloksena saatiin differentiaaliyhtälöt kerroslevylaatalle, jossa on paksut pintalevyt. Saadut differentiaaliyhtälöt koostuvat Reissner-Mindlinin ja Kirchoffin laattateorioiden mukaisista siirtymien differentiaaliyhtälöistä. Differentiaaliyhtälöt ratkaistaan käyttäen kaupallista, elementtimenetelmään, perustuvaa ohjelmistoa. Rakenteen jaksollisuus huomioidaan jännityslaskennassa. Kehitetty laattateoria validoitiin elementtimenetelmään avulla suoritetuilla laskelmilla todelliselle 3D rakenteelle.

Työssä kehitettiin ja validoitiin laserhitsin kiertojäykkyysarvojen mittausmenetelmä. Laserhitsin kiertojäykkyydellä havaittiin olevan suuri merkitys laatan leikkausjäykkyyteen ydinlevyjä vastaan kohtisuorassa suunnassa. Kiertojäykkyyteen vaikuttavia tekijöitä tutkittiin elementtimenetelmän avulla.

Työn lopussa tunnistettiin laatan vasteen kannalta tärkeimmät tekijät. Laatan taipuma ja normaalijännitys ovat herkkiä ydinlevystä pintalevyyn välitetylle momentille. Tämä momentti pienenee, kun ydinlevyjen paksuus tai laserhitsin kiertojäykkyys pienenee. Tämän seurauksena laatan taipuma kasvaa. Vaikutus on suurin, kun laatan pintalevyt ovat paksut ja ydinlevyjen välinen etäisyys on pieni. Tasaisesti jakautuneen painekuormituksen alaisilla laatoilla, joilla laatan sivusuhte on pieni, taipuma ei riipu laserhitsin kiertojäykkyydestä. Pienelle alueelle kohdistuvan palstakuormituksen alaisilla, tai suurisivuhteisilla, laatoilla laserhitsin kiertojäykkyydellä on suuri merkitys taipumiin. Laatan jaksollisuuden huomioiminen on erittäin tärkeää, kun tarkastellaan laatassa vaikuttavia normaalijännityksiä. Suurin osuus aiheutuu ydinlevyjen tasoa vastaan kohtisuoraan olevasta leikkausvoimasta. Laatan keskitasossa vaikuttavien normaalivoimien ja taivutusmomenttien merkitys normaalijännityksiin on huomattavasti näitä osuuksia pienempi. Laserhitsin mitatuilla kiertojäykkyysarvoilla ei havaittu olevan merkitystä rakenteellisten normaalijännitysten kannalta.

Asiasanat Kerroslevy; laattateoria; jännitysanalyysi; laserhitsaus; kiertojäkkyys		
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# Preface

This thesis is based on work done at Helsinki University of Technology during 2001-2007. During the thesis process I was financed by the National Graduate School in Engineering Mechanics, the Finnish Maritime Foundation, and the EU-funded research projects SANDWICH, SANDCOR.e, LAPROMAT, MARSTRUCT, and DE-Light Transport. This financial help is gratefully acknowledged. Meyer Werft is thanked for producing the test specimens.

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Finally, I would like to thank my family and friends and, especially, my wife Riikka for their support during the thesis process.

Helsinki, April 2007

Jani Romanoff

# Contents

Abstract	1
Tiivistelmä	3
Preface	5
Contents	7
List of Symbols	8
List of Publications and Author's Contribution	10
Original Features	
1 Introduction	14
1.1 Background	14
1.2 State of the Art	16
1.3 Scope of Work	18
1.4 Limitations	19
2 Bending of Laser-Welded Web-Core Sandwich Plates	21
2.1 Periodic Response Transverse to Web-Plate Direction	21
2.1.1 Web-Induced Reactions	21
2.1.2 Homogenized Beam Theory	22
2.2 Bending Response for Plate	24
2.2.1 Theory for Plates under Uniform Pressure Load	24
2.2.2 Treatment of Patch Loads	25
2.3 T-Joint Rotation Stiffness	25
2.3.1 Experiments and Finite Element Analyses	25
2.3.2 Influence on the Structural Response	27
3 Conclusions	29
4 References	
5 Errata	35

#### Appendix

- [P1] Romanoff, J. and Varsta, P., "Bending Response of Web-core Sandwich Beams", Composite Structures, Vol. 73, No. 4, 2006, pp. 478-487.
- [P2] Romanoff, J., Varsta, P. and Klanac, A., "Stress Analysis of Homogenized Web-Core Sandwich Beams", Composite Structures, Vol. 79, No. 3, 2007, pp. 411-422.
- [P3] Romanoff, J., Remes, H., Socha, G., Jutila, M. and Varsta, P., "The Stiffness of Laser Stake Welded T-joints in Web-core Sandwich Structures", Thin- Walled Structures, Vol. 45, No. 4, 2007, pp. 453–462.
- [P4] Romanoff, J. and Varsta, P. "Bending response of web-core sandwich plates", Composite Structures, Vol. 81, No. 2, 2007, pp. 292-302.
- [P5] Romanoff, J., Varsta, P. And Remes, H., "Laser-Welded Web-Core Sandwich Plates under Patch-Loading", Marine Structures, Vol. 20, No. 1-2, 2007, pp. 25-48.

# List of Symbols

a	Length of patch load
$a_{j}^{i}$	Location of load, type $i$ and location $j$
$\begin{bmatrix} A \end{bmatrix}$	Elongation stiffness matrix
b	Breadth of patch load
В	Breadth of sandwich plate
[B], [C]	Coupling stiffness matrixes
с	Constant
$d^{i}f/dx^{i}$	$i^{\text{th}}$ derivative of f with respect to x
D	Bending stiffness
$D_0$	Bending stiffness due to elongation of face plates
[D]	Bending stiffness matrix
$D_Q$	Shear stiffness
$\begin{bmatrix} D_Q \end{bmatrix}$	Shear stiffness matrix
d	Vertical distance
е	Horizontal position of the laser weld
Ε	Young's modulus
F	Force
$F_{-1}()$	Unit impulse function
$F_{-2}()$	Unit doublet function
G	Shear modulus
Н	Horizontal force
H()	Heaviside function
h	Height
$h_c$	Height of core
h <sub>rg</sub>	Height of root gap
k	Stiffness or stiffness parameter
$k_{\theta}$	T-joint rotation stiffness
L	Length of sandwich plate
М	Bending moment
Ν	Normal force
q	Distributed loading
Q	Shear force
R	Radius
S	Web plate spacing
t	Plate thickness
V	Vertical force
u, v, w	Displacements in x-, y- and z- directions

<i>x</i> , <i>y</i> , <i>z</i>	Global coordinates
$x_l, y_l, z_l$	Local coordinates
β	Angle
$\delta()$	Dirac's delta function
ε	Normal strain
γ	Shear strain
K	Curvature
$\theta$	Slope
$\theta_c$	Slope at laser-weld around <i>x</i> -axis
$\sigma$	Normal stress
Σ	Summation
τ	Shear stress

# List of Sub- and Superscripts

1,2,, <i>N</i>	Index
avg	Average
b	Bottom face plate
f	Face plates
g	Global
1	Local
m	Membrane
Q	Shear force
RM	Reissner-Mindlin
t	Top face plate
tf	Thick face plates effect
tot	Total
W	Web plate
<i>xy</i> , <i>xz</i> , <i>yz</i>	<i>xy</i> -, <i>xz</i> - and <i>yz</i> -planes

# Abbreviations

3D	Three dimensional
DNV	Det Norske Veritas
FE	Finite Element
FEA	Finite Element Analysis
FEM	Finite Element Method
FSM	Finite Strip Method
HB	Homogenized Beam
[P1],[P2],,[P5]	Paper 1, paper 2,, paper 5

# List of Publications and Author's Contribution

manuscript with valuable comments and suggestions.

This thesis consists of an introductory report and the following five papers:

- [P1] Romanoff, J. and Varsta, P., "Bending Response of Web-core Sandwich Beams", Composite Structures, Vol. 73, No. 4, 2006, pp. 478-487.
  The author developed the theory, carried out the validation, and was the main contributor to the manuscript. Varsta contributed to the
- [P2] Romanoff, J., Varsta, P. and Klanac, A., "Stress Analysis of Homogenized Web-Core Sandwich Beams", Composite Structures, Vol. 79, No. 3, 2007, pp. 411-422.

The author developed the theory, carried out the validation, and was the main contributor to the manuscript. Varsta and Klanac contributed to the manuscript with valuable comments and suggestions.

[P3] Romanoff, J., Remes, H., Socha, G., Jutila, M. and Varsta, P., "The Stiffness of Laser Stake Welded T-joints in Web-core Sandwich Structures", Thin- Walled Structures, Vol. 45, No. 4, 2007, pp. 453–462.

The author developed the theoretical basis for the measurements and carried out the design, validation, execution, and analysis of the stiffness tests. Remes, Socha, and Jutila took part in the design and validation of the measurement system. Jutila measured the geometrical properties of the welds. The calculations were carried out by the author. The manuscript was prepared by the author. Varsta, Remes, and Socha contributed to the manuscript with valuable comments and suggestions.

[P4] Romanoff, J. and Varsta, P. "Bending response of web-core sandwich plates", Composite Structures, Vol. 81, No. 2, 2007, pp. 292-302.

The author developed the theory, carried out the validation, and was the main contributor to the manuscript. Varsta contributed to the manuscript with valuable comments and suggestions. [P5] Romanoff, J., Varsta, P. And Remes, H., "Laser-Welded Web-Core Sandwich Plates under Patch-Loading", Marine Structures, Vol. 20, No. 1-2, 2007, pp. 25-48.

The author developed the theory, carried out the validation, and was the main contributor to the manuscript. Varsta contributed to the manuscript with valuable comments and suggestions. Remes, together with the author, carried out the FE analysis based on solid elements.

# **Original Features**

Laser-welded web-core sandwich plates have relatively complex structural behaviour as a result of the difference in scale between the laser weld and the plate itself. Up to now the only way to analyse these structures with acceptable accuracy has been 3D Finite Element or Finite Strip Analysis or experiments. The existing plate theories, based on homogenized stiffness, have given incomplete stress responses for these periodic structures. The following features of this thesis are believed to be original:

- 1. The periodic behaviour of the response was accurately derived for a web-core sandwich beam. The interaction between the local and global responses of the beam is presented accurately. [P1]
- 2. A theory for a homogenized web-core sandwich beam with thick face plates was developed. The theory describes the periodic stress components accurately, making it possible to obtain realistic stress estimates. [P2]
- 3. The shear stiffness transverse to the web plate direction, including the effect of laser-weld deformations, was derived. The elasticity of the laser weld was taken into account by the rotation stiffness of the T-joint. [P2]
- 4. Rotation stiffness was derived experimentally for the laser-welded Tjoint. The geometrical properties of the laser weld were studied and statistics on these were created. The main mechanics affecting this stiffness were identified by FE analyses. [P3]
- 5. The theory for unsymmetrical web-core sandwich plates with thick face plates under a uniform pressure load was developed in [P4]. The theory was extended to cover patch loads in [P5].
- 6. The different stress components contributing to the total stress response were identified. The interaction between these components is also described in [P2] and [P5].

7. It was found that the measured values of the T-joint rotation stiffness have a significant effect on the deflection of web-core sandwich beams and plates, but not on the normal stress. [P3]-[P5]

# **1** Introduction

# 1.1 Background

During recent decades shipbuilding has used stiffened plates as decks and bulkheads. These structures suffer from a large amount of material being positioned close to a neutral axis, giving them insufficient bending properties but also a relatively large structural height and weight. Therefore, in recent years the development of lightweight, space-saving, and modular ship structures has increased significantly. Sandwich plates offer an option to fulfil the requirement for these.

Sandwich plates are composed of face plates which are separated by core material. They are usually designed in such a way that the face plates carry the bending and in-plane loads; the face plates have relatively high stiffness and density. The core is designed to carry shear loads; it has relatively low density and stiffness. The face plates and core can be selected from various materials – metals, composites, plastics, and organic materials – but the core can also possess various topologies: a web, a honeycomb, and a cellular core; see Figure 1.

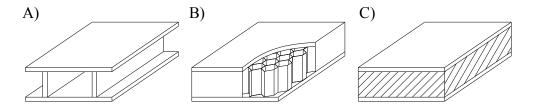


Figure 1. Various types of sandwich plates. A) web-core, B) honeycomb and cellular core.

A special sub-category of sandwich structures is composed of metal face plates and core stiffeners (corrugation, web, C, Z, etc.) positioned only in one direction; see Figure 2A. The connection between the core and the face plate can be achieved with adhesive bonding (Clark, 1987; Smith et al., 1992, Knox et al., 1998), mechanical joints (Fung et al., 1996; Fung et al., 1998), through extrusion (Seo et al., 1999), or with different welding methods such as spot welding (Norris, 1987; Wiernicki et al., 1991) or laser welding (Teasdale, 1988; Minamida et al., 1992; Denney et al., 1993; Roland and Reinert, 2000; Kujala and Roland, 2002; Bright and Smith, 2004). Thus, several different metal sandwich plate configurations can be obtained by varying the materials, core geometry, and production method.

In laser-welded web-core sandwich plates, the web plates are usually spaced at a distance 10-100 times the thickness of the face plate. The core provides continuous support to the face plates in the direction of the web plate and discrete support in the transverse direction. Thus, the web-core sandwich plate is very orthotropic. This orthotropy is further underlined by laser stake welding, since in marine applications the thickness of the laser weld is less than that of the face and web plates; see Figure 2B. Therefore, it is important to include the laser weld in the structural analysis.

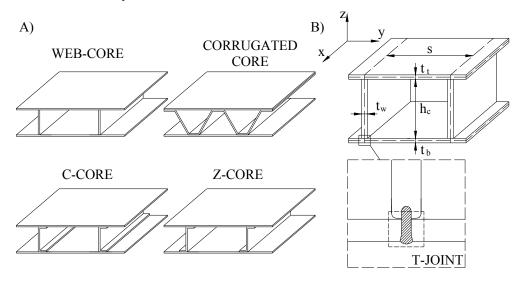


Figure 2. A) Different types of all-metal sandwich plate topologies and B) the cross-section dimensions and the laser weld of web-core sandwich plate.

Today, practical tools to estimate the structural response of web-core sandwich plates in bending are the 3D Finite Element and Finite Strip Methods and orthotropic sandwich plate theory. In 3D FEM and FSM the response of the sandwich plate is obtained by constructing an idealised mathematical model of the structure, where the laser-welded T-joint increases the modelling efforts remarkably. Therefore, these methods are time-consuming, mainly as a result of differences in scale between the dimensions of the laser weld and the plate itself; see Figure 2B. On the other hand, changing the cross-section dimensions during the design process means that the FEM or FSM model needs to be updated. Thus, these methods are not suitable for the concept design of ship structures, where several design alternatives should be evaluated in a short time.

An alternative approach is sandwich plate theory, where the periodic structure of the sandwich plate is homogenized with respect to stiffness. The benefit of the method is that the number of unknowns of the plate bending problem is significantly reduced, i.e. the computational efforts are reduced. Additionally, the plate bending solution only depends on the stiffness parameters, plate dimensions, loading, and boundary conditions. Thus, if the plate length, breadth, loading, and boundary conditions are the same, the same FE mesh can be used to solve the response of sandwich plates with different cross-sections. These properties make homogenized plate theory very interesting when the structural analysis of large structures, such as those that are present in ships, is being considered in the concept design phase.

#### 1.2 State of the Art

The use of sandwich plate theory requires homogenization where the average response of the orthotropic plate is the objective; see, for example, Libove and Batdorf (1948) and Libove and Hubka (1951).

Libove and Batdorf (1948) assumed that a corrugated core sandwich plate follows the well-known Reissner-Mindlin kinematics (Reissner, 1948 and Mindlin, 1951). This means that the bending and shear deformations are assumed to be uncoupled and the normal strain through the thickness of the sandwich is linear. On the basis of this assumption, several extensions to this plate-bending model have been presented for various other types of stiffener topologies using updated stiffness expressions: web-core (Holmberg, 1950) and C- and Z-profiles (Fung et al., 1993, 1994). Since these early works, several modified stiffness formulations have been presented; for web-core sandwich plates see Chen et al. (1971), Lok et al. (1999), Kolsters and Zenkert (2002), and Kujala and Klanac (2002).

An idea of shear-induced secondary bending moment, which the homogenized plate theory cannot predict fully, has been presented for corrugated and Z-core sandwich plates (Wiernicki et al., 1991; Smith et al., 1992; Fung et al., 1994; Knox et al., 1998). Such a moment component is also very likely to occur in web-core sandwich plates. Wiernicki et al. (1991) proposed the idea that this shear-induced bending moment is derived from the shear force distribution and the resulting stresses are then added to those caused by the global bending moment. This approach is justified if the local and global scales of dimensions differ remarkably. However, the order of magnitude of web plate spacing can be similar to that of the breadth of the sandwich plate. Therefore, the local bending moment component causing secondary stresses can have a remarkable influence on the global moment. In this case the direct summation of these two components can give erroneous results.

Application of loads directly onto the face plates can cause significant stresses on these which the homogenized sandwich plate theory cannot predict (Tan et al., 1991). The influence of this stress component has been taken into account by a separate response analysis of the face plate; the results of this analysis are added to those given by the bending of the homogenized sandwich plate; see Seo et al. (1999), Romanoff and Kujala (2001), and Klanac and Kujala (2004). However, none of these papers considers the influence of shear-induced secondary bending stresses. Thus, an approach that estimates all stress components is needed.

The derivation of the homogenized stiffness properties is, in general, a straightforward task, apart from the determination of the shear stiffness perpendicular to the web plates. The value of this shear stiffness is very low, while in the direction of the web plates the shear stiffness is practically infinite. This difference in shear stiffness is further emphasised by laser-welding, since the assumption that is generally made of a non-deforming T-joint is no longer valid (Romanoff and Kujala, 2003). Therefore, the rotation at the laser weld should be

included in the stiffness formulation. Romanoff and Kujala (2003) showed with FE simulations that the weld thickness is a very important parameter when deformations in the laser weld are concerned. Fung at al. (1996) and Fung and Tan (1998) showed, with C- and Z-core sandwich plates, that the shear stiffness can vary considerably, depending on the contact mechanism that occurs between the flanges of core stiffeners and the face plates. Kozak (2004) showed experimentally that an outside weld contact occurs in laser-welded web-core sandwich plates. Therefore, investigation of the contact mechanism and its influence on the response in web-core sandwich plates is needed.

Kujala and Klanac (2002) showed that with web-core sandwich beams the Reissner-Mindlin kinematics are not always valid, especially with beams that have very low shear stiffness. Similar findings were also presented by Knox et al. (1998) for corrugated core sandwich plates. Therefore, the sandwich theory with the thick face plate assumption should be used; using purely the Reissner-Mindlin kinematics can lead to significant overestimation in the deflections. An extensive review of plate theories with the thick face plate assumption for homogenous plates is presented by Carrera (2003, 2004).

# 1.3 Scope of Work

The present work concentrates on the development of a theory for the bending response of laser-welded web-core sandwich plates. The basic requirements for the theory are:

- The solution for the bending response should depend only on the equivalent stiffness parameters, beam or plate dimensions, loading, and boundary conditions; the influence of changing the cross-section dimensions is modelled through the equivalent stiffness parameters.
- 2. The theory should take into account the periodic response occurring between the web plates.
- 3. The theory should include the local stresses arising from patch loads.

- 4. The theory should include the rotation stiffness of the laser weld, which may have a strong influence on the bending response.
- 5. In order to study the feasibility of sandwich plates for given application cases, the plate response should be evaluated with commercial FE codes.

The first requirement means that the structural response should be calculated with a sandwich theory that is based on homogenized stiffness properties. This makes it possible to apply the same element mesh for different cross-sections of the sandwich plate, making possible a relatively short analysis time in the concept design phase of ship structures.

The second requirement indicates that a method to calculate the periodic stresses from the averaged internal forces should be developed. Although the structure is homogenized to solve the averaged response, the periodicity of the structure should be considered in stress derivation.

The third requirement demands an approach to the evaluation of the local stress response arising from patch loads.

The fourth requirement implies that the theory should allow deformations at the laser welds which reduce the equivalent shear stiffness transverse to the web plate direction. Because of the low shear stiffness, the Reissner-Mindlin kinematics are not necessarily valid; the normal strain can be piecewise linear through the thickness of the sandwich plate. Therefore, a sandwich plate theory that includes the thick face plate effect should be developed.

The fifth requirement demands that the response analysis method presented in this thesis should be exploited in commercial FE code.

#### 1.4 Limitations

In this thesis only web-core sandwich plates are considered in attempting to reveal the kinematics of steel sandwich structures. There, the thickness of the laser weld is relatively small compared to that of the face and web plates.

The theory is based on the assumption of a linear elastic response, where the material is isotropic and follows Hooke's law. The assumption of isotropic material is acceptable, since web-core sandwich plates are usually made from metals. The linear material behaviour is justified, since the limiting stress value in design should always be below the material yield strength. The geometrical linearity is justified, as the relative plate thicknesses in marine applications are such that initial deformations are not present after manufacturing. The face and web plates are assumed to have no stresses acting in their thickness direction. Thus, the basic beam and plate theories can be applied when describing their bending behaviour. The stiffness of the laser weld is modelled through a linear rotation spring.

# 2 Bending of Laser-Welded Web-Core Sandwich Plates

# 2.1 Periodic Response Transverse to Web-Plate Direction

### 2.1.1 Web-Induced Reactions

Web-induced reactions are most easily studied with a beam structure. Thus, a web-core sandwich beam was investigated in [P1] by applying plane frame analysis. It was assumed that the external loading, located at the web plates, and internal web-induced forces induce both local and global deflection of the sandwich beam, the total response being the sum of these; see Figure 3A.

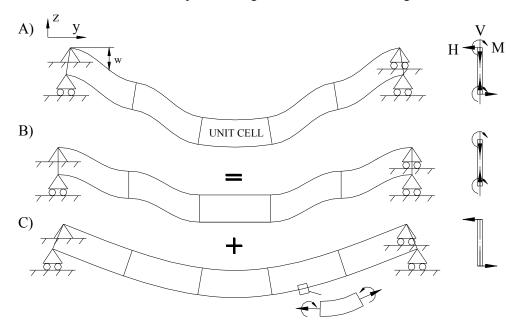


Figure 3. A) The total deflection of the frame and web-induced reactions; B) local deflection and corresponding web-induced moment and out-of-plane reactions, and C) global deflection and web-induced in-plane reactions.

The local deflection is caused by the external loading and web-induced outof-plane forces and bending moments; see Figure 3B. In order to balance the webinduced bending moments, in-plane forces are introduced into the face plates; see Figure 3C. The equilibrium equations of the web plate reveal that the in-plane forces are constant inside a unit cell, i.e. between consecutive web plates, and have opposite signs in the top and bottom plates. This causes elongation of the face plates in the opposite directions, and as a result of this, global bending deflection of the plane frame occurs. Therefore, the web-induced moments have an influence on the total deflection of the frame. These moments depend on the stiffness of the web plates and the web-to-face plate connection.

The plane frame method that was developed was validated with 3D FE analyses based on shell elements and compared to the homogenized beam method, i.e. the HB method. The plane frame analysis was found to give exactly the same results as the FE analyses. An especially important result is the consistency of the stresses in the face plates, where the values of web-induced bending stresses are very high when compared to those of membrane stresses. This was also found experimentally and with FE analysis by Smith et al. (1992) and Knox et al. (1998). The homogenized beam method or HB method gives continuous stress distribution for the membrane and bending stress in the face plates, while the plane frame and FE analyses reveal that the stress distributions are actually discrete; the bending stress is significantly underestimated with the HB method. However, the membrane stress in the face plates calculated with the HB method is accurate in the middle of the span between two web plates. This was also found, for example, by Norris (1987) and Tan (1989).

#### 2.1.2 Homogenized Beam Theory

Although the theory presented in [P1] gives an accurate response for web-core sandwich beams, it has little value in practical applications. This follows, for example, from the fact that real structures are usually composed of numerous web plates. Therefore the plane frame analysis can become tedious because of the increasing number of equations to be solved.

The purpose of [P2] is to develop the HB method further so as to reap the benefits of homogenization while attaining the capability to estimate the normal stresses in the face plates accurately. The main simplification made in this paper is that the loads are located at the web plates.

The total response consists of the global and local responses; see [P1] and Figure 4. In [P2], the global response is divided into two parts, elongation and bending of the face plates. The local response is also divided into two parts; these describe the unsymmetric and symmetric slopes of the face plates at the ends of the unit cells. The parts of the global and local response that correspond to the elongation of the face plates and the symmetric slope are averaged, i.e. homogenized, over the unit cell. This process makes it possible to model the average sandwich beam response according to the thick face plate sandwich theory for homogenous beams presented by Allen (1969). This theory considers the effects of the elongation of the face plates and shear with Timoshenko beam theory, which is analogous to the Reissner-Mindlin plate theory. The thick face plate effect is modelled with the Euler-Bernoulli beam theory, which is analogous to the Kirchhoff plate theory. Details of the homogenization process are given in Figure 4.

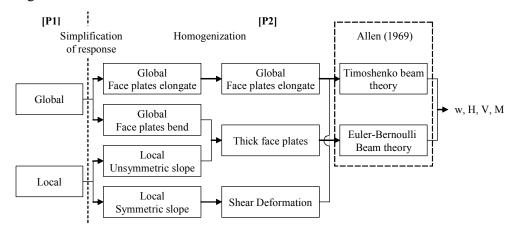


Figure 4. The description of the periodic response with homogenized beam theory.

In the homogenization process, equivalent bending and shear stiffness properties for the beam were derived that allow relative rotation between the web and the face plates; this rotation can occur as a result of the elasticity of the laser weld.

The periodic structure is considered again when the stress response is calculated. The unknown web-induced bending moments are determined from the

shear deformation of the homogenized sandwich beam. Considering the total beam, the web-induced and the thick face plate bending moments give the bending moment for the global response; this was found to be stepwise constant, as in the plane frame analysis.

The agreement of the results between the updated HB method and the 3D-FEM based on shell elements is excellent for web-core sandwich beams with various cross-section dimensions. The method presented makes it possible to define the stress response in the face and web plates with better accuracy than the existing methods. It was shown that in order to consider the beam response accurately all deformation modes have to be considered in the analysis: elongation of the face plates; shear deformation, and thick face plate behaviour. This result is in agreement with the findings of Kujala and Klanac (2002).

## 2.2 Bending Response for Plate

#### 2.2.1 Theory for Plates under Uniform Pressure Load

Beam analysis is valid for plates in cylindrical bending. For structures such as ship decks, beam analysis is not sufficient because of the possibility of there being a low aspect ratio and local loading. Therefore, a homogenized plate theory is needed that deals with the thick face plate kinematics and takes into consideration the shear-induced secondary bending stresses. The aim of [P4] is to develop a plate theory for unsymmetric web-core sandwich plates, which considers the effect of thick face plates and predicts all stress components accurately; the beam theory presented in [P2] was extended to cover plates.

The main simplification that was made was the assumption that the local bending of the face plates as a result of direct loading is negligible. Analogously to [P2], the elongation and the shear deflection of the face plates is averaged over the unit cell and described with the Reissner-Mindlin plate theory. The thick face plate behaviour is described with the Kirchhoff plate theory. Thereafter, a set of differential equations for the displacement of a homogenous sandwich plate is obtained. These equations were solved with a commercial FE code that gives the stiffness-dependent internal forces derived from the displacements of the homogenized plate. Then it is possible to reconsider the periodic structure of the sandwich plate, making it possible to define the exact stresses in the structure. The theory was validated with 3D-FEM analysis based on shell elements for plates under uniform pressure loading. The agreement is found to be excellent in plate deflection and also in the normal stresses of the face and web plates and in the shear stresses in the web plates.

#### 2.2.2 Treatment of Patch Loads

The main limitation of the sandwich plate theory presented in [P4] is the assumption that the response of the face plates as a result of the direct application of loads to these is negligible. This assumption can lead to non-conservative stress estimates in the case of plates which are locally loaded, such as wheel-loaded car decks, where vehicle wheel pressure can be considered as a local patch load.

[P5] presents an approach that can also capture the local stress component on the face plates resulting from a patch load. First, the bending theory of [P4] for web-core sandwich plates was extended to cover patch loads of various sizes. Then, an approach to evaluate the local bending of the face plate as a result of patch loads was developed. The total response of the sandwich plate was obtained as a sum of these two analyses. The proposed approach was validated with 3D-FE analyses based on solid or shell elements and the agreement was found to be very good. A case study on a wheel-loaded car-deck showed that the response of patchloaded web-core sandwich plates was dominated by patch load-induced local stresses, followed by the shear-induced secondary bending stresses; the normal forces and bending moments of the homogenized sandwich plate were found to have only a minor influence on the total stress response.

# 2.3 T-Joint Rotation Stiffness

#### 2.3.1 Experiments and Finite Element Analyses

The rotation stiffness of the T-joint models the influence of elasticity on the laser weld. The purpose of [P3] is to determine experimentally the rotation stiffness and

also the geometry of the joint. Thus, the aim of this study is the quantitative understanding of the stiffness, as compared to the qualitative one presented by Kozak (2004). The mechanics associated with the stiffness were further identified with the FEA; a deeper analysis of these topics was given than in Romanoff and Kujala (2003).

A special test set-up was built to measure the T-joint rotation stiffness. Test specimens with face plate thicknesses of 2.5 mm and 3.0 mm were considered. The measured values were implemented in the shear stiffness of the HB models and experiments on a beam subjected to four-point bending were also carried out; a comparison of the measured and calculated stresses and deflections indicates that the rotation stiffness measurements were successful. Macroscopic analyses were carried out for a total of 80 specimens in order to identify the weld geometry of the specimens. The weld thickness, root gap, and weld centricity vary quite significantly.

The FE analyses were performed to study the effect of these geometrical variations on the rotation stiffness. The influences of weld thickness and contact on the T-joint rotation stiffness are significant, especially when the plate thickness increases. Depending on the type of contact, the slope of the force-displacement curve may change drastically and, because of contact, the effective thickness of the weld may be greater than the real thickness. The measured values were also found to vary, even in the same specimen, when opposite rotation directions are considered. A similar conclusion was presented by Fung at al. (1996) for C-core plates and Fung and Tan (1998) for Z-core plates where screws were used instead of laser-welding; the differences were found to occur as a result of the contact.

In [P5] a 3D topology of the actual structure was constructed from solid elements, excluding the non-linear contact calculation from the analysis. The influence of contact increasing the thickness of the laser weld was taken into account. Even then the analysis proved to be extremely time-consuming; both preand post-processing took about two days, while the analysis itself took about 9 hours in a super-computer. The agreement between the proposed method and 3D solid FEA was fairly good, even though some shear locking was present in the solid element model. Shear locking was observed especially in the deflection distribution. However, the mesh size was considered to be at a maximum since the number of parabolic solid elements was 359,280. Modelling the 3D topology with shell elements would require a method to describe the laser weld rotation stiffness. This could be done using spring elements, but this would increase the size of the analysis model. In the proposed method, the laser weld rotation stiffness is included analytically in the shear and rotation spring stiffnesses of the global and local analyses respectively. This led to a calculation model of 3040 elements, which took about 15 seconds to solve with a laptop. This clearly shows that in practice the proposed method is the only realistic way to evaluate the response of laser-welded web-core sandwich plates in the concept design stage of ship structures.

#### **2.3.2** Influence on the Structural Response

The influence of T-joint rotation stiffness on the deflections of beams, [P3], and of plates, [P4]-[P5] was found to be significant. The comparison of stresses shows that the maximum stress is not sensitive to rotation stiffness in the range of values measured. However, if the rotation stiffness is reduced below this range then the maximum stress in the face plates of the beams starts to increase. This fact is not necessarily valid for plates, since different load-carrying mechanisms can occur, depending on the shear stiffness. However, it is seen that the rotation stiffness affects the contents of the total moment causing the stresses; small rotation stiffness increases the thick face plate effect, while at the same time reducing the shear-induced stresses.

It was clearly demonstrated that the T-joint rotation stiffness, which affects the shear stiffness, has a significant influence on the maximum deflection of the plate, especially with plates that have a high aspect ratio and a patch load on a small area. Plates with a low aspect ratio under a uniform pressure load have the same maximum deflection, regardless of the shear stiffness value used. This indicates that a simple beam theory, considering plate length and the breadth of a unit cell, could be used to analyse maximum plate deflection with low aspect ratios. However, with patch-loaded plates different shear stiffness values lead to different maximum deflection values with all aspect ratios. Thus, the use of simple beam analysis would require a modification of the effective beam breadth according to the shear stiffness. The influence of the T-joint rotation stiffness was found to be significant for those cross-sections that have a small stiffener spacing and thick web and face plates.

# 3 Conclusions

A theory for the bending response of laser-welded web-core sandwich plates was developed in this thesis. The theory is suitable for the concept design of ship structures. It considers the periodic structure when the stiffness properties of a homogenized plate are derived. The plate-bending problem is solved for the homogenized plate. As a result the deflections and internal forces are obtained for the homogenized plate. When the internal forces are known, the periodic structure is reconsidered and the discrete stresses are calculated. In addition, a method to evaluate the local stress response resulting from the application of loads directly onto the face plates was developed. When these two analyses are considered together, the total response of the web-core sandwich panel is known. It was seen that the most important stress component is caused by local patch loads. This is followed by shear-induced stresses. The normal forces and bending moments of a homogenized sandwich plate are found to have a slight influence on the total stress response.

It was seen that the rigidity of the connection between the face and web plate is very important when the response is considered. This influence of the laser weld rotation stiffness was included in the formulation of the shear stiffness. The laser-welded connection transfers out-of-plane, in-plane, and moment loads to the face plates. Depending on the stiffness of the web plate and the laser weld, the moment that rotates the laser weld is transferred to the face plate. This same moment creates in-plane forces on the face plates. When the moment has a value close to zero, the web plate is not capable of transferring in-plane loads to the face plates; the thick face plate effect carries most of the load. Such a situation can occur when the web plates are very thin or the rotation stiffness of the laser weld is very small.

The rotation stiffness of the T-joint was derived experimentally, as was the geometry of the connecting welds. FE simulations show that the thickness of the laser weld and possible contact between the web and face plate affect the T-joint rotation stiffness the most. The contact can increase the effective thickness of the

laser weld. The influence of the T-joint rotation stiffness is significant for those cross-sections that have a small stiffener spacing and thick web and face plates. The case studies show that with the present rotation stiffness values only the deflection of the web-core sandwich beam or plate is affected; normal stress is found to be practically unaffected.

The main limitation of this study is that it considers only one sandwich topology, i.e. the web-core type. The theory should be expandable for various other types of cross-sections, such as corrugated and C- and Z-core. However, this work needs to be carried out following the principles presented in this thesis in order to guarantee the reliability of the results. The study is also limited to covering only the linear elastic strength of isotropic materials. The theory could be expanded to cover material and geometrical non-linearity, as well as material orthotropy. However, this requires knowledge of the non-linear behaviour of the structural elements of the sandwich, such as the face and web plates and the laser weld. To include orthotropic material behaviour in the theory means that the relations between different responses need to be updated.

The method presented in this thesis is very attractive when the optimisation of the sandwich structures is considered. This follows from the fact that the plate-bending solution can be solved for several cross-sections using the same FE mesh. However, the optimisation constraints need to be developed; such criteria include buckling, fatigue, ultimate strength etc. Therefore, the implementation of the method presented here into an optimisation algorithm is left for future work.

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