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BEP analysis of OSTBC-OFDM systems with broadband PA and imperfect memory compensation

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***Abstract*— We analyze the performance of an OFDM system with diversity, in particular Orthogonal Space Time Block Code (OSTBC) systems, including a broadband nonlinear power amplifier (PA) with memory. Closed-form expressions for the BER are obtained for cases when PA memory is compensated at the transmitter or the receiver. The results confirm that not only clipping noise, but also imperfect PA memory compensation has a significant impact on the system performance.**

I. Introduction

The principal drawback of OFDM system performance is the high Peak-to-Average Power Ratio (PAPR). Real power amplifiers have a nonlinear response causing signal compression and clipping that result in signal distortion and adjacent channel interference. Furthermore, broadband PAs introduce memory which gives rise to intersymbol interference (ISI) [1]. Power backoff and PAPR reduction techniques reduce the nonlinear distortion level but do not compensate for the broadband PA memory effects which is necessary for achieving good performance in wireless systems. The PA memory effects can be removed by applying memory predistorter [1], memory precompensation at the transmitter [2], or postcompensation at the receiver [3]. The latter two approaches, which are of interest here, assume that a sufficient power backoff is applied so that the PA memory can be efficiently handled by a frequency-domain equalizer, i.e., any presence of nonlinear distortion is neglected. In addition, the pre- and postcompensation methods rely on a memory model (often FIR model) whose estimation is subject to errors due to, e.g., undermodeling or noise.

In the following, we derive BER expressions for an OSTBC-OFDM systems that include the effect of a broadband nonlinear PA and imperfect memory compensation. Our results are valid for the realistic case of low clipping levels [4] and when the PA is accurately represented by a Wiener model [1]. Expressions for other nonlinear PA models, e.g., the Hammerstein and Wiener-Hammerstein models [1], can be derived following similar steps as shown here. However, due to space limitations we restricted our analysis to the Wiener model which is widely used in literature for capturing the nonlinear behavior of a broadband PA. The results are valid for QPSK modulation, and they can be extended to M-QAM using the results presented in [5].

Notation: Small (capital) boldface letters denote vectors (matrices), and standard font (calligraphic) is used for time (frequency) domain variables.

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II. System model

The OFDM system under consideration has N subcarriers and employs Orthogonal Space Time Block Coding (OSTBC) with N_T transmit and N_R receive antennas. Each code word, comprising N_B consecutive OFDM symbols is transmitted from the N_T transmit antennas assuming that the channel does not change during the transmission of the block.

After the space-time encoder each signal is modulated and processed by the IDFT operator. The symbol block $\mathbf{x}_j(n) \in \mathbb{C}^{(N+v)\times 1}$ to be transmitted from antenna j at time n is

$$\mathbf{x}_j(n) = \mathbf{G}_{cp}\bar{\mathbf{x}}_j(n) = \mathbf{G}_{cp}\mathbf{Q}_N\boldsymbol{\mathcal{X}}_j(n), \;\; j=[1,2] \tag{1}$$

where $\mathbf{G}_{cp}$ is the $(N+v)\times N$ *cyclic prefix insertion matrix*, v is the length of the cyclic prefix, $\bar{\mathbf{x}}_j(n) = \mathbf{Q}_N\boldsymbol{\mathcal{X}}_j(n)$ is the IDFT of the modulated symbols $\boldsymbol{\mathcal{X}}_j(n) \in \mathbb{C}^{N\times 1}$, and $\mathbf{Q}_N$ is the $N\times N$ IDFT matrix.

The signal $\mathbf{x}_j(n)$ is then passed through a nonlinear PA with memory, here modeled using a Wiener structure [1]. The Wiener model is frequently used to model broadband PAs and is formed by a *linear filter* $\mathbf{c}$ followed by a *nonlinear static block* $g[\cdot]$. Here we model $\mathbf{c} = [c_1, \cdots, c_{L_c}]$ with an FIR filter with L_c taps. Under the assumption of low clipping levels [4], the multicarrier signal after the static nonlinearity $g[\cdot]$ is

$$\mathbf{u}_j(n) = g[\mathbf{v}_j(n)] = K_L\,\mathbf{v}_j(n) + \mathbf{d}_j(n) \tag{2}$$

where $\mathbf{v}_j(n)$ is the output of the linear filter $\mathbf{c}$ of the Wiener model, i.e., $\mathbf{v}_j(n) = c_n \otimes \mathbf{x}_j(n)$ ($\otimes$ denotes time domain convolution) and K_L is the gain of the linear part. The second term $\mathbf{d}_j(n)$ is the nonlinear distortion, modeled as an additive Gaussian noise (see [4]), which is a function of the modulated symbol vector $\boldsymbol{\mathcal{V}}_j(n)$ and the PA transfer function $g[\cdot]$.

The transmitted signal in frequency domain is expressed as

$$\mathcal{U}_j(n,k) = K_L\mathcal{C}(n,k)\mathcal{X}_j(n,k) + \mathcal{D}_j(n,k) \tag{3}$$

If the cyclic prefix is chosen larger than the effective channel length $L_{eff} = L_h + L_c$, where L_h is the length of the time-varying wireless channel impulse response. Then, the received signal at subcarrier k, at time n by the ith receive antenna, can be expressed as

$$\begin{aligned}\mathcal{Y}_i(n,k) \;\; &= \;\; \sum_{j=1}^{N_T}\mathcal{H}_{i,j}(n,k)\left[K_L\mathcal{C}(n,k)\mathcal{X}_j(n,k) + \mathcal{D}_j(n,k)\right] \\ &\quad +\mathcal{N}_i(n,k)\end{aligned} \tag{4}$$

where $\mathcal{N}_i(n,k)$ is the additive Gaussian noise assumed to be i.i.d. with zero mean and variance σ_n^2 and the frequency-domain channel coefficients $\mathcal{H}_{i,j}(n,k)$ are assumed to be independent stationary, zero-mean and unit variance circular complex Gaussian distributed processes

A. Space time decoding

It is well known that the decision variable for the transmitted symbol $x_j(n,k)$ at the output of the OSTBC-decoder, assuming perfect channel knowledge in the receiver, gives the same results as a Maximum Ratio Combining (MRC), i.e.,

$$\hat{x}_j(n,k) = \alpha(n,k)(K_L c(n,k) x_j(n,k) + d_j(n,k)) + w(n,k)$$
$$\alpha(n,k) = \sum_{j=1}^{N_T}\sum_{i=1}^{N_R} |h_{i,j}(n,k)|^2 \tag{5}$$

where $w(n,k)$ is a zero-mean Complex Gaussian variable with variance $\alpha(n,k)\sigma_n^2$ due to the noise from the N_R receive antennas at different time instants.

We see from Eq. (5) that the nonlinear distortion $d_j(n,k)$ affects the decision variable in an additive way and that the memory $c(n,k)$ scales the received signal. Both these effects may cause errors in the detection process.

III. Imperfect Memory compensation

Consider that our unbiased memory estimate is modeled as

$$\hat{c}(n,k) = c(n,k) + \Delta_c(n,k) \tag{6}$$

where $\Delta_c(n,k)$ is the modeling error assumed to be a complex Gaussian variable with $CN(0,\sigma_c^2)$ independent of $c(n,k)$. Precompensation and postcompensation apply the inverse of Eq. (6) to Eq. (3) and Eq. (4), respectively.

A. Memory precompensation

In the following time index n and subcarriers index k are dropped to simplify the notation. After precompensation, the transmitted signal at subcarrier k by Antenna j becomes

$$u_j = K_L \frac{c}{c+\Delta_c} x_j + \frac{1}{c+\Delta_c} d_j \tag{7}$$

The decision variable after ST decoding is

$$\hat{x}_j = K_L \left(1 - \frac{\Delta_c}{c+\Delta_c}\right)\alpha x_j + \left(\frac{1}{c+\Delta_c}\right)\alpha d_j + w \tag{8}$$

The signal power P_s of the decision variable becomes $P_s = K_L^2\alpha^2\sigma_x^2$ where $\sigma_x^2 = E[x_j x_j^*]$. The interference noise power is now given by

$$P_n = K_L^2\alpha^2\epsilon_1\sigma_x^2 + \alpha^2\epsilon_2\sigma_d^2 + \alpha\sigma_n^2 \tag{9}$$

where $\sigma_d^2 = E[d_j d_j^*]$ is the nonlinear distortion, which for a limiter PA model is given by

$$\sigma_d^2 = \sigma_x^2|c|^2\left[1 - \exp(-\nu^2) - K_L^2\right]$$
$$K_L = 1 - \exp(-\nu^2) + \frac{1}{2}\sqrt{\pi}\nu\,\mathrm{erfc}(\nu) \tag{10}$$

and ν is the clipping level (see [4] for details) defined as $\nu = A_s/\sqrt{E\{|x(n)|^2\}}$. Furthermore, A_s denotes the amplifier input saturation, $\sqrt{E\{|x(n)|^2\}}$ is the RMS value of the OFDM signal, and erfc is the complementary error function [6]. The variables associated with the imperfect memory compensation, ϵ_1 and ϵ_2, in (9) are given by

$$\epsilon_1 = E\left[\left|\frac{\Delta_c}{c+\Delta_c}\right|^2\right] = \sum_{n=1}^{\infty} n!(\sigma_c^2/|c|)^n$$
$$\epsilon_2 = E\left[\left|\frac{c}{c+\Delta_c}\right|^2\right] = |c|\sum_{n=0}^{\infty} n!(\sigma_c^2/|c|)^n \tag{11}$$

To evaluate ϵ_1 we employed the series expansion $1 - \left(\frac{1}{1+\Delta_c/c}\right) = 1 - \sum_{n=0}^{\infty}(-1)^n(\Delta_c/|c|)^n$. Thereafter, the moments of Δ_c are directly given by standard formulas [7]. The same procedure was used to solve for ϵ_2.

The effective SNR at subcarrier k for a system with memory precompensation can now be written as

$$\gamma_{PRE} = \frac{P_s}{P_n} = \frac{K_L^2\gamma}{[K_L^2\epsilon_1 + \sigma_d^2/\sigma_x^2\epsilon_2]\gamma + 1} \tag{12}$$

where $\gamma = \alpha\sigma_x^2/\sigma_n^2$ is a chi-square variable with $2N_TN_R$ degrees of freedom.

B. Memory postcompensation

The effective SNR for the case of postcompensation is obtained in similar manner as above. The decision variable is given by

$$\hat{x}_1 = K_L\left(1 - \frac{\Delta_c}{c+\Delta_c}\right)\alpha x_1 + \left(\frac{1}{c+\Delta_c}\right)(\alpha d_1 + w) \tag{13}$$

Eq. (13) differs from Eq. (8) in the additive noise term. The effective SNR becomes

$$\gamma_{POS} = \frac{K_L^2\gamma}{[K_L^2\epsilon_1 + \sigma_d^2/\sigma_x^2\epsilon_2]\gamma + \epsilon_2} \tag{14}$$

with ϵ_1 and ϵ_2 given by Eq. (11).

IV. Error probability derivation

Considering a system with QPSK modulation,[1] the conditional error probability is given by

$$P_e(k) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma_{eff}}\right) \tag{15}$$

where γ_{eff} is the effective SNR. The average bit error probability is obtained by averaging P_e in (15), over the fading channel as

$$\bar{P}_e(k) = \int_0^{\infty} P_e P(\gamma)d\gamma = \int_0^{\infty} P_e \frac{\gamma^{L-1}}{(L-1)!\bar{\gamma}^L}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right)d\gamma \tag{16}$$

where $P(\gamma)$ is a chi-square PDF with $2L$ degrees of freedom, where $L = N_R \times N_T$ and $\bar{\gamma} = E[\gamma]$ is the average SNR.

The effective SNR γ_{eff} is given by Eq. (12) and Eq. (14) for transmitter precompensation and receiver postcompesation, respectively. These equation can be written in a generic form in order to evaluate the integral as

$$\gamma_{eff} = \frac{K_L^2\gamma}{(K_L^2\epsilon_1 + \sigma_d^2/\sigma_x^2\epsilon_2)\gamma + K} \tag{17}$$

[1] BEP expressions for M-QAM modulation can be obtained by replacing Eq. (15) with $P_e = \sum_j A_{M,j}\mathrm{erfc}(\sqrt{B_{M,j}\gamma_{eff}})$ [5, Eq. (4)] where $A_{M,j}$ and $B_{M,j}$ are constants depending on the constellation size M.

where the constant $K = 1$ for memory precompensation and $K = \epsilon_2$ for postcompensation.

The erfc function can be expressed as $\text{erfc}\left(\sqrt{\gamma_{eff}}\right) = \frac{\Gamma(1/2,\gamma_{eff})}{\sqrt{\pi}}$, where $\Gamma(\cdot,\cdot)$ is the incomplete gamma function. The incomplete gamma function is then replaced by the series expansion [6, Sect. 8.354, Eq. (2)] given by $\Gamma(\alpha, x) = \Gamma(\alpha) - \sum_{n=0}^{\infty} \frac{(-1)^n x^{\alpha+n}}{n!(\alpha+n)}$. Combining Eq. (17) and Eq. (16) with the series expansion for $\Gamma(\alpha, x)$ gives

$$\bar{P}_e(k) = \frac{1}{2(L-1)!\bar{\gamma}^L\sqrt{\pi}} \left[\Gamma\left(\frac{1}{2}\right)\int_0^{\infty} \gamma^{L-1}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right)d\gamma \right.$$
$$-\sum_{n=0}^{\infty}\int_0^{\infty}\frac{2}{n!(1+2n)}\left(\frac{K_L^2\gamma}{(K_L^2\epsilon_1+\sigma_d^2/\sigma_x^2\epsilon_2)\gamma+K}\right)^{1/2+n}$$
$$\left.\times\exp\left(-\frac{\gamma}{\bar{\gamma}}\right)\gamma^{L-1}d\gamma\right] \qquad (18)$$

The first integral is solved using [6, Sect. 3.351, Eq. (3)] and the second integral using [6, Sect. 3.383, Eq. (5)]. Finally, the error probability for a OSTBC-OFDM system with nonlinear PA and imperfect memory compensation becomes

$$\bar{P}_e(k) = \frac{1}{2K}\left\{1 - \frac{1}{(L+1)!\bar{\gamma}^L\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{K_L^{1+2n}}{n!(1/2+n)}\right.$$
$$\times\left(\frac{K_L^2\epsilon_1+\sigma_d^2/\sigma_x^2\epsilon_2}{K}\right)^{n+L-1/2}\times\Gamma(n+L+1/2)$$
$$\left.\times U_{n+L+1/2,L+1}\left(\frac{K}{\bar{\gamma}(K_L^2\epsilon_1+\sigma_d^2/\sigma_x^2\epsilon_2)}\right)\right\} \qquad (19)$$

where $U_{a,b}(z)$ is the *Confluent Hypergeometric Function* [6, Sect. 9.211, Eq. (4)] defined as $U_{a,b}(z) = \frac{1}{\Gamma(a)}\int_0^{\infty}\exp(-zt)t^{a-1}(1+t)^{b-a-1}dt$. The BER for an OFDM system with N subcarriers is obtained as $\frac{1}{N}\sum_{k=0}^{N-1}\bar{P}_e(k)$. The asymptotic BER, that provides us with the error floor, is obtained by evaluating integral (16) at high SNR levels and is given by

$$\bar{P}_e^{\infty}(k) = \text{erfc}\left(\sqrt{\frac{K_L^2}{(K_L^2\epsilon_1+\sigma_d^2/\sigma_x^2\epsilon_2)}}\right) \qquad (20)$$

V. Numerical results

The BER expressions are validated for a 2×2 OSTBC-OFDM system (Alamouti) with QPSK modulation on $N = 512$ subcarriers without error correction coding. The channel is Rayleigh fading with four independent paths generated according to a Jakes' Doppler spectrum with a mobile speed $s = 10$ km/h. Our simulations employed a Wiener model of the HMC409LP4 PA suitable for WLAN and WiMAX implementations, and an FIR filter $\mathbf{c} = [1, -0.011, -0.101, -0.022, -0.035, -0.038, 0.002]$ for the linear block and a limiter model for the nonlinear block $g[\cdot]$.

Figure 1 shows the BER curves obtained by simulations and from Eq. (19) employing the first 30 terms for different values of memory compensation errors σ_c^2. The top figure shows the results without nonlinear distortion (clipping level $\nu = \infty$), i.e., a sufficiently large power backoff is applied to the input signal to force it into the linear region of the PA. The bottom figure illustrates a more power efficient solution where the clipping level is set to $\nu = 0$ dB. We see that the theoretical curves can accurately predict the performance degradation due to imperfect PA memory compensation and clipping noise.

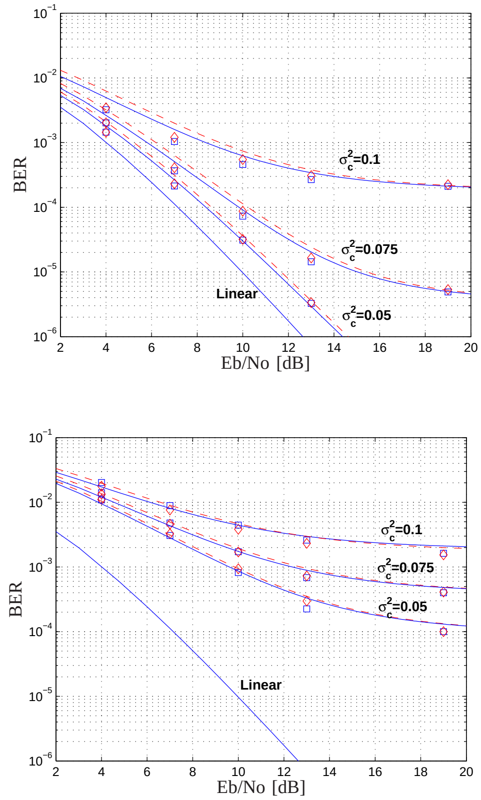

Fig. 1. BER versus Eb/No in a 2 x 2 OSTBC-OFDM (Alamouti) system with QPSK for different memory modeling error σ_c with a Wiener-type nonlinear PA. Clipping level $\nu = \infty$ (top) and $\nu = 0$ dB (bottom). Precompensation: theoretical (solid line) and simulation ('□'). Postcompensation: theoretical (dashed line) and simulation ('⋄').

VI. Conclusions

Closed form BER expressions were derived for an OSTBC-OFDM system, impaired by wideband nonlinear power amplifier (PA) for the case of imperfect memory compensation at the transmitter or the receiver. The expressions are valid when the PA is represented by the commonly used Wiener model, and can accurately predict the performance degradation due to PA memory estimation errors and clipping noise.

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