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Global relaxation of superconducting qubits

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We consider coupled quantum two-state systems (qubits) exposed to a global relaxation process. The global relaxation refers to the assumption that qubits are coupled to the *same* quantum bath with approximately equal strengths, appropriate for long-wavelength environmental fluctuations. We show that interactions do not spoil the picture of Dicke's subradiant and super-radiant states where quantum interference effects lead to striking deviations from the independent relaxation picture. Remarkably, the system possess a stable entangled state and a state decaying faster than single qubit excitations. We propose a scheme for how these effects can be experimentally accessed in superconducting flux qubits and, possibly, used in constructing long-lived entangled states.

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A lot of experimental progress has been made in superconducting qubits recently including the achievement of several μs coherence times.[1–4] High visibilities[2,6,7] and even nondemolition[8] readout have been demonstrated. Several coupled-qubit experiments have been also carried out, see, e.g., Refs. 9–12. However, energy relaxation has proved to be a serious limitation to the coherence in quantum information applications. The origin and the detailed mechanism of relaxation has remained largely unknown.

We consider a two qubit system, where qubits feel the same fluctuating quantum bath. We concentrate on an interacting generalization of the well-known Dicke model,[13] which is relevant in the case of long-wavelength spontaneous emission induced by the environment. Dicke studied a spontaneous emission of an ensemble of noninteracting molecules coupled to a common bath and predicted large deviations from the independent relaxation picture. He showed that certain correlated states decay more rapidly (superradiance) or are more stable than uncorrelated excitations (subradiance). The existence of subradiant and super-radiant states was decisively observed much later in spontaneous emission of two nearby trapped atoms.[14] Since then, correlated decay of states has been studied experimentally and theoretically in quantum dot and double dot systems where the Dicke-type behavior has been observed.[15–19] Recently it was discovered that the subradiant states can be employed in optimizing multiqubit quantum algorithms in the presence of global relaxation.[20]

In this paper we demonstrate how the different energy states of interacting qubits may decay very differently under global relaxation due to quantum interference effects. As in the case of noninteracting molecules, there exist a stable entangled state and a state that decays faster than the uncorrelated excited state. Testing the validity of the correlated decay in the context of superconducting flux qubits is discussed in detail. By studying the decay of different two-qubit states, one can obtain information of the presently unknown relaxation mechanism that is inaccessible in single qubit experiments. Currently it is not understood whether the limiting intrinsic relaxation is caused by high-frequency flux noise or something else. See, e.g., Refs. 3–5 for some experimental data.

We consider a system consisting of qubits coupled to a relaxation-inducing quantum bath described by the Hamiltonian $H=H_q+H_{\mathrm{env}}+H_{\mathrm{i}}$, where

$$H_q=-\frac{\Delta}{2}\sum_i \sigma_z^{(i)}+J\sum_{i<j}\sigma_x^{(i)}\sigma_x^{(j)}, \quad H_{\mathrm{i}}=g\hat{x}\sum_i \sigma_x^{(i)}. \qquad (1)$$

Here $\hat{x}$ is a Hermitian operator in the environment part of the Hilbert space. The many-body Hamiltonian of the environment H_{env} does not need to be specified in detail, its effects enter through correlation functions of $\hat{x}$. It is assumed that qubits have equal energy splittings Δ, interaction strengths J, and bath coupling constants g. These features are realized in the case of similar qubits in the close proximity compared to the relevant length scale of environment fluctuations. Below we estimate effects due to detuning of parameters. The form of the coupling on Eq. (1) is assumed to be $\sigma_x\otimes\sigma_x$-type as this is natural for so-called optimally biased superconducting qubits as will become apparent below. Also the σ_x-type coupling to the environment is natural since the effect of longitudinal coupling is strongly suppressed. Moreover, we are focusing on the effect of relaxation which is not affected by longitudinal coupling.

The evolution of the total system obeys the von Neumann equation $\dot{\rho}_T(t)=-\frac{i}{\hbar}[H,\rho_T(t)]$, which is formally solved by $\rho_T(t)=U(t,t_0)\rho_T^0U^\dagger(t,t_0)$, where $U(t,t_0)=\exp[-iH(t-t_0)/\hbar]$. For a factorizable initial state $\rho_T^0=\rho^0\otimes\rho_{\mathrm{env}}^0$, the reduced density matrix for qubits can be written in terms of a propagator by $\rho_{ij}(t)=G(ij,t;kl,t_0)\rho_{kl}^0$ (summation over repeated indices), where

$$G(ij,t;kl,t_0) = \mathrm{Tr}_{\mathrm{env}}[\rho^0_{\mathrm{env}}\langle l,t_0|\tilde{T}e^{(i/\hbar)\int_{t_0}^{t}d\bar{t}H_{\mathrm{int}}(\bar{t})}|j,t\rangle \times \langle i,t|Te^{-(i/\hbar)\int_{t_0}^{t}d\bar{t}H_{\mathrm{int}}(\bar{t})}|k,t_0\rangle]. \quad (2)$$

Expression (2) is written in the interaction picture, where $H_{\mathrm{int}}(t)=e^{(i/\hbar)(H_q+H_{\mathrm{env}})t}H_{\mathrm{int}}e^{-(i/\hbar)(H_q+H_{\mathrm{env}})t}$ and T, $\tilde{T}$ are time and antitime ordering operators. In the case of two qubits the relevant Hilbert space is spanned by the vectors $|--\rangle\equiv|1\rangle,|-+\rangle\equiv|2\rangle,|+-\rangle\equiv|3\rangle,|++\rangle\equiv|4\rangle$. First we study the case $J=0$, so the basis vectors are also eigenstates of H_q. Supposing that the environment is at low temperature, excited states decay to the ground state $|1\rangle$. The transition rate $\Gamma_{\rho^0\to 1}$, defined as the linearly growing contribution to the probability ρ_{11} in the long-time evolution, can be calculated from Eq. (2) by $\Gamma_{\rho^0\to 1}=\lim_{T\to\infty}\frac{G(11,T/2;ij,-T/2)}{T}\rho^0_{ij}$, where ρ^0 corresponds to a stationary state. Expanding the propagator to the lowest nonvanishing order, one recovers the golden-rule results

$$\Gamma_{2\to 1} = \Gamma_{3\to 1} = \frac{g^2}{\hbar^2}S_x(\Delta/\hbar), \quad (3)$$

where $S_x(\omega)=\int_{-\infty}^{\infty}\langle\hat{x}(t)\hat{x}(0)\rangle e^{i\omega t}dt$. The transition rates are proportional to the noise power at frequency $\Delta/\hbar$. These results are structurally similar to the ones obtained in the case of independent baths for each qubit. Interference effects come into play in the decay of the correlated excited states $|\phi_s\rangle=(|+-\rangle+|-+\rangle)/\sqrt{2}$ and $|\phi_a\rangle=(|+-\rangle-|-+\rangle)/\sqrt{2}$. By performing an analogous calculation we obtain $\Gamma_{\phi_s\to 1}=2\Gamma_{2\to 1}$. The rate enhancement is a direct evidence of the global nature of the relaxation process. Interference effects have even more dramatic impact on the evolution of $|\phi_a\rangle$ since it does not decay at all. This statement does not rely on the perturbation theory and is an exact consequence of the dynamics generated by (1). This is in striking contrast to the case where the two qubits are exposed to independent environment fluctuations. In the case of finite interaction $J\neq 0$, the above described picture remains qualitatively the same. Now the system has four nondegenerate eigenstates $|d\rangle\equiv a|1\rangle+b|4\rangle$, $|\phi_s\rangle$, $|\phi_a\rangle$, and $|u\rangle\equiv -b|1\rangle+a|4\rangle$ with respective energies $-\sqrt{\Delta^2+J^2}$, J, $-J$, and $\sqrt{\Delta^2+J^2}$. The coefficients are given by $a=[1+\Delta/(2\sqrt{J^2+\Delta^2})]^{1/2}$ and $b=-[1-\Delta/(2\sqrt{J^2+\Delta^2})]^{1/2}$. The decay rate of the symmetric excitation is

$$\Gamma_{\phi_s\to d} = \frac{g^2}{\hbar^2}2(a+b)^2S_x\left(\frac{\sqrt{\Delta^2+J^2}+J}{\hbar}\right), \quad (4)$$

while $|\phi_a\rangle$ still remains exactly stable.

Contrary to what was assumed in Eq. (1), the bath couplings of qubits never coincide exactly in experimental realizations. Also when qubits are realized artificially, for example, by quantum dots or superconducting circuits, individual Hamiltonians are not identical but depend on material parameters and sample-specific geometries. These features lead to deviations from the model (1) and modify previous conclusions to some extent. Assuming the qubits are coupled to the bath with couplings g_1,g_2, the relaxation rates for ϕ_j $(j=s,a)$ become

$$\Gamma_{\phi_j\to d} = \frac{(g_1\pm g_2)^2}{2\hbar^2}(a+b)^2S_x\left(\frac{\sqrt{\Delta^2+J^2}\pm J}{\hbar}\right), \quad (5)$$

where upper signs correspond to $j=s$. The decay of subradiant state $|\phi_a\rangle$ vanishes as a square of the detuning g_1-g_2. In the case $J=0$ or when the noise is fairly insensitive to variations of magnitude J around $\sqrt{\Delta^2+J^2}$, the decay rates are related by $\Gamma_{\phi_s\to 1}/\Gamma_{\phi_a\to 1}=(g_1+g_2)^2/(g_1-g_2)^2$, clearly demonstrating a dramatic difference when $g_1\approx g_2$. Thus $|\phi_a\rangle$ is robust against fluctuations and $|\phi_s\rangle$ decays rapidly even when the bath couplings match only approximately. Let us assume now that the qubits have slightly different energies Δ_1 and Δ_2. To simplify the following expressions we define functions

$$a(x) = \frac{1}{\sqrt{2}}\left(1+\frac{x}{\sqrt{x^2+J^2}}\right)^{1/2}, \quad (6)$$

$$b(x) = -\frac{1}{\sqrt{2}}\left(1-\frac{x}{\sqrt{x^2+J^2}}\right)^{1/2}. \quad (7)$$

The eigenstates become $|\tilde{d}\rangle\equiv a(\Delta_s)|1\rangle+b(\Delta_s)|4\rangle$, $|\tilde{\phi}_s\rangle\equiv -b(\Delta_a)|2\rangle+a(\Delta_a)|3\rangle$, $|\tilde{\phi}_a\rangle\equiv a(\Delta_a)|2\rangle+b(\Delta_a)|3\rangle$, and $|\tilde{u}\rangle\equiv -b(\Delta_s)|1\rangle+a(\Delta_s)|4\rangle$, where $\Delta_s=(\Delta_1+\Delta_2)/2$ and $\Delta_a=(\Delta_1-\Delta_2)/2$. The states have respective energies $E_{\tilde{d}/\tilde{u}}=\mp\sqrt{\Delta_s^2+J^2}$, $E_{\tilde{\phi}_s/\tilde{\phi}_a}=\pm\sqrt{\Delta_a^2+J^2}$. In the limit of vanishing bath coupling detuning the rates become

$$\Gamma_{\tilde{\phi}_j\to\tilde{d}} = \frac{g^2}{\hbar^2}[a(\Delta_s)+b(\Delta_s)]^2 \times [a(\Delta_a)\mp b(\Delta_a)]^2S_x\left(\frac{E_{\tilde{\phi}_j}-E_{\tilde{d}}}{\hbar}\right), \quad (8)$$

where the minus sign corresponds to $j=s$. In the regime $|\Delta_a|/J\ll 1$ these expressions can be estimated by

$$\Gamma_{\tilde{\phi}_s\to\tilde{d}} = \frac{2g^2}{\hbar^2}[a(\Delta_s)+b(\Delta_s)]^2S_x\left(\frac{E_{\tilde{\phi}_s}-E_{\tilde{d}}}{\hbar}\right),$$

$$\Gamma_{\tilde{\phi}_a\to\tilde{d}} = \frac{2g^2}{\hbar^2}[a(\Delta_s)+b(\Delta_s)]^2\left(\frac{\Delta_a}{2J}\right)^2S_x\left(\frac{E_{\tilde{\phi}_a}-E_{\tilde{d}}}{\hbar}\right),$$

implying that $|\tilde{\phi}_a\rangle$ maintains its subradiant nature when detuning is small compared to the interqubit coupling.

To study the nature of the relaxation process we suggest a system of two flux qubits[21,22] with as indentical parameters as possible coupled to a high-Q cavity,[23] see Fig. 1. We will now discuss a numerical example to show that the phenomenon is indeed very spectacular even in the presence of imperfections provided the assumption of globality of the noise holds. As shown above, using a large coupling energy protects against any parameter fluctuations and therefore the assumption of identical qubits is quite realistic. The qubit j $(j=1,2)$ subspace when biased at the half-flux quantum point $\Phi_0/2$ consists of two circulating current states carrying a current of $\pm I^j_{\mathrm{p}}$. Tunneling between the states happens at a rate

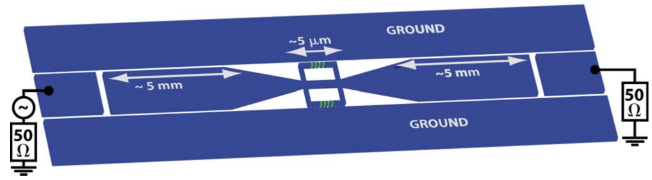

FIG. 1. (Color online) Schematic of the suggested experiment. The dimensions are exaggerated for clarity. The disconnected section in the middle forms a coplanar resonator whose resonant frequency is modified depending on the qubit state thus allowing for dispersive readout. The chirality is such that the control microwave input via the same port as the readout couples antisymmetrically to the qubit.

of $\Delta_j/\hbar$. Neglecting the off-resonant coupling to the cavity (used for dispersive readout), the qubits are described explicitly by the Hamiltonian

$$H_q = -\sum_{j=1}^{2}\left(\frac{\Delta_j}{2}\sigma_z^{(j)} - \frac{\epsilon_j}{2}\sigma_x^{(j)}\right) + J\sigma_x^{(1)}\sigma_x^{(2)}. \tag{9}$$

At the optimal point $\varepsilon_j = 2I_{\mathrm{p}}^j(\Phi - \Phi_0/2) = 0$ dephasing due to low-frequency flux fluctuations is minimized. To achieve symmetry and to optimize coherence we assume $\varepsilon_1 \approx \varepsilon_2 \approx 0$ and $\Delta_1 \approx \Delta_2$. As shown above, $|\Delta_2 - \Delta_1|$ should be compared to $J = MI_p^1 I_p^2$ where M is the mutual (kinetic) inductance between the qubit loops. A realistic sample[11,24] may have quite similar tunneling energies and a large coupling so as an example we assume $(\Delta_2 - \Delta_1)/h = 200$ MHz, $\Delta_1/h = 6$ GHz, and $J/h = 1$ GHz. Choosing the bias of one of the qubits, say qubit 2, to be $\varepsilon_2 = 0$ is easy using a global magnetic field and a typical e-beam patterned sample with nominally the same area may then have $\varepsilon_1/h = 200$ MHz. This last assumption further modifies the eigenstates $|\tilde{d}\rangle$, $|\tilde{\phi}_s\rangle$, $|\tilde{\phi}_a\rangle$, and $|\tilde{u}\rangle$. These are reasonable and quite conservative assumptions as the suggested sample geometry has perfect symmetry about the center conductor and e-beam patterning is very accurate. A numerical calculation then gives for a symmetric coupling energy g

$$\Gamma_{\tilde{\phi}_s \to \tilde{d}} = 1.7 \times \frac{g^2}{\hbar^2} S_x(2\pi \times 7.2\text{ GHz}), \tag{10}$$

$$\Gamma_{\tilde{\phi}_a \to \tilde{d}} = 4.0 \times 10^{-3} \times \frac{g^2}{\hbar^2} S_x(2\pi \times 5.2\text{ GHz}). \tag{11}$$

Assuming that the noise spectrum $S_x(\omega)$ does not have too strong frequency dependence we then expect two orders of magnitude different relaxation times for the subradiant and super-radiant states even with very typical parameters. As shown in the beginning of the paper, the factor $g^2/\hbar^2 S_x(\omega)$ appearing in the above formulas is the characteristic relaxation rate for individual qubits. This could be typically, say, 1 μs. This translates into a 250 μs lifetime of the antisymmetric state under global noise while the symmetric state decays in about 0.6 μs. Considering that presently energy relaxation is limiting coherence in our flux qubits,[4] very long overall coherence can then be expected if a significant amount of the high-frequency noise is global. The large coupling energy J not only protects from parameter scatter but also provides a gap of about $2J$ between $|\tilde{\phi}_s\rangle$ and $|\tilde{\phi}_a\rangle$. Although this transition is suppressed for single-qubit noise (flipping both qubits required), it is better to have the difference as large as possible to avoid stimulated emission.

The apparent contradiction in the present setting is, on the one hand, the stability of $|\phi_a\rangle$ under any kind of global high-frequency field and, on the other hand, the desire to excite the transition. It is clear that a symmetric drive cannot achieve this, as demonstrated in Ref. 11. As shown schematically in Fig. 1 we therefore assume that the qubits are coupled antisymmetrically (due to the left- and right-handed configurations of the qubits) to the center conductor such that a resonant drive via the transmission line can excite the $|d\rangle \leftrightarrow |\phi_a\rangle$ transition and ideally only that. That is, the microwave Hamiltonian can be estimated as $H_{\mathrm{mw}} = \alpha(t)(\sigma_x^{(2)} - \sigma_x^{(1)})$ (if the drive and cavity are far detuned from the cavity angular frequency ω) for which clearly the excitation of $|\phi_a\rangle$ is possible since $\langle d|(\sigma_x^{(2)} - \sigma_x^{(1)})|\phi_a\rangle \neq 0$, but transitions between the symmetric states are forbidden. The antisymmetric microwave drive amplitude $\alpha(t)$ obeys $\alpha(t) = \delta\Phi(t)I_p$, where I_p is the persistent current of the qubit and $\delta\Phi(t)$ is the ac flux drive.

The coupling to the transmission line cavity must be weak enough such that the antisymmetric coupling does not allow for significant relaxation to the 50 Ω environment due to the finite quality factor Q of the cavity. In the case of a transmission measurement and coupling via current it is most natural to use a half-wavelength resonance since this mode has an antinode of voltage and a node of current in the middle. Also all other modes are guaranteed to have a higher resonant frequency. The relaxation via this route can be estimated for a given detuning $\delta = \hbar\omega - (\sqrt{\Delta^2 + J^2} - J)$ between the cavity and the qubit singlet similar to the Purcell effect discussed in Ref. 25. The presence of the cavity modifies the Hamiltonian by two terms, $H_{\mathrm{cav}} = (\hbar\omega + 1/2)\hat{a}^\dagger\hat{a}$ and $H_{\mathrm{cav}\text{-}q} = \gamma(\sigma_x^{(2)} - \sigma_x^{(1)})(\hat{a} + \hat{a}^\dagger)$. The first excited state corresponding to $|\tilde{\phi}_a\rangle$ has a photonic nature with approximately $p = 2(b-a)^2\gamma^2/\delta^2$ probability. Here, the coupling energy $\gamma = M_{\mathrm{cav}\text{-}q} I_p I_{\mathrm{rms}}$ between the cavity mode and the qubits depends on the mutual inductance $M_{\mathrm{cav}\text{-}q}$ between each qubit loop and the center conductor (sign difference is built in the antisymmetric coupling) and the rms current I_{rms} in the ground state of the cavity. The relaxtion rate of the antisymmetric singlet limited by the cavity quality factor Q is thus simply $\Gamma_Q = p\omega/Q$. If, e.g., $\Delta/h = 6$ GHz, $J/h = 1$ GHz, $g/h = 0.08$ GHz, $\omega/2\pi = 10$ GHz, and $Q = 10^4$, we get $1/\Gamma_Q = 260$ μs. This is long enough to detect the difference between the lifetimes of the subradiant and the super-radiant states. Furthermore, a numerical calculation for these values shows that the resonant frequency of the cavity will be shifted down by about 1 MHz when the singlet is excited compared to when the qubit is in the ground state. This shift revealing the qubits' state is well detectable in a microwave transmission measurement using a low-noise cold amplifier in the same way as in Ref. 23 since the width of the resonator transmission peak is comparable, i.e., $\omega/(2\pi Q) = 1$ MHz.

Owing to symmetry the effect of any global fluctuation is minimized in the present system. Testing whether a significant part of the relaxation is due to global fluctuations amounts to measuring the lifetime of the state $|\phi_a\rangle$. Whether the result will be positive or negative is not known but in any case this should give valuable information about the origin of the noise.

We studied relaxation in an interacting two-qubit system exposed to a global relaxation mechanism and showed how interference effects lead to a dramatic deviation from the independent relaxation picture. The small detuning of bath couplings leads to a slow relaxation of the subradiant state while the super-radiant state decays much faster than individual excitation. Experimental realization of phenomena was discussed in detail in the context of superconducting flux qubits, where the phenomenon can be utilized to extract information of an incompletely understood relaxation process and possibly construct long-lived quantum states.

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