J. Rauhala and V. Välimäki, Tunable dispersion filter design for piano synthesis, IEEE Signal Processing Letters, vol. 13, no. 5, pp. 253-256, 2006.

Tunable Dispersion Filter Design for Piano Synthesis

Jukka Rauhala and Vesa Välimäki, *Senior Member, IEEE*

***Abstract*—The tunable dispersion filter is a new design approach presented in this letter to provide dispersion modeling for digital waveguide synthesis of musical instruments, which do not produce extremely inharmonic sounds, such as the piano. We propose to use a cascade of second-order allpass filters for modeling dispersion. The filter coefficients can be calculated by using simple formulae based on the Thiran allpass filter design, which is usually used for fractional delay approximation. Unlike the previous allpass filter approximations, this filter design is easily scalable to produce various inharmonicity values for a wide range of fundamental frequencies.**

***Index Terms*—Acoustic signal processing, delay filters, discrete time filters, music, tunable filters.**

I. Introduction

DISPERSION is a phenomenon occurring in string instruments caused by the stiffness of the string. It stretches the partial frequencies higher compared to harmonic frequencies. This makes the spectrum inharmonic, which affects the timbre of the instrument. For some musical instruments, such as the piano, this effect is characteristic, and it is important that it is implemented in the sound synthesis [1].

Dispersion can be modeled in different ways, depending on the synthesis method in use. From the filter design point of view, dispersion can be considered as frequency-dependent phase delay. In digital waveguide synthesis [2], one of the most widely used physical modeling techniques, dispersion can be implemented by inserting an allpass filter into the feedback loop [3]–[6]. The goal is to find an allpass filter with a phase delay best matching the inharmonicity.

The idea of using an allpass filter to model the dispersion was introduced by Jaffe and Smith [3]. In order to create a computationally efficient dispersion filter, Van Duyne and Smith proposed the use of multiple first-order allpass filters in cascade [4]. Rocchesso and Scalcon suggested that it is possible to create a dispersion filter of satisfactory quality by using a single allpass filter of order less than 20 [5]. In addition, there have been a number of implementations with good results using a high-order allpass filter [5], or a low-order filter [6], designed with optimization algorithms. A common characteristic between the solutions mentioned above is that they use much computation for designing the filter, and hence, these filters can be considered static. Moreover, none of the solutions offer a simple relation between the filter parameters and the desired dispersion coefficient values.

The goal in this letter is to design a tunable dispersion filter in a way that the coefficients can be determined by a lightweight algorithm taking the fundamental frequency and the dispersion coefficient as input parameters. Moreover, the performance of the filter should be good enough for high-quality piano synthesis. The major advantage of the proposed filter is that it enables real-time tuning of the dispersion characteristics by using closed-form minimal parameterization. This is important in sound synthesis, as, in the end, the synthesis model must be fine-tuned by listening. Additionally, it is desirable that the user can adjust the synthesis parameters during playing [7].

In this letter, the tunable dispersion filter is designed by using Thiran allpass filters [8]–[10]. This letter is structured as follows. In Section II, the design process of the tunable dispersion filter is presented, followed by the results measuring the performance and validating the model in piano synthesis in Section III. Finally, the conclusions of this letter are presented in Section IV.

II. Design of Tunable Dispersion Filter

A. Thiran Allpass Filter as Dispersion Filter

The key idea of this letter is to use the Thiran allpass filter in a new way for approximating the dispersion. The original Thiran allpole filter design [8] can be modified, resulting in a maximally flat fractional delay allpass filter [9], [10]. The filter coefficients can be calculated as

$$a_k = (-1)^k \binom{N}{k} \prod_{n=0}^{N} \frac{D-N+n}{D-N+k+n} \text{ for } k = 1, 2, \ldots, N \tag{1}$$

where a_k is the kth filter coefficient, N is the order of the filter, and D is the desired delay. Thus, the filter can be parameterized by using a single parameter D.

Usually the Thiran allpass filter is used for producing a fractional delay. The value of parameter D is typically chosen to be between $N-1$ and N. In such cases, the phase delay of the Thiran filter is approximately constant at low frequencies. However, when much larger values of D are used, the phase delay decreases monotonically with frequency from D samples at dc to N samples at the Nyquist frequency. With proper choice of D and N, and by cascading several identical filters, the phase delay behavior is similar to the phase delay required in a dispersive string model. The phase delay P_k, corresponding to the inharmonicity value B [11] at the frequency of partial k, can be calculated as

$$P_k = \frac{f_s}{f_0\sqrt{1+Bk^2}} \tag{2}$$

Manuscript received October 17, 2005; revised December 15, 2005. This work was supported by the Academy of Finland under Project 53537 and Project 104934. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Patrick A. Naylor.

The authors are with the Laboratory of Acoustics and Audio Signal Processing, Helsinki University of Technology, FI-02015 TKK Espoo, Finland (e-mail: jukka.rauhala@acoustics.hut.fi; vesa.valimaki@tkk.fi).

Digital Object Identifier 10.1109/LSP.2006.870376

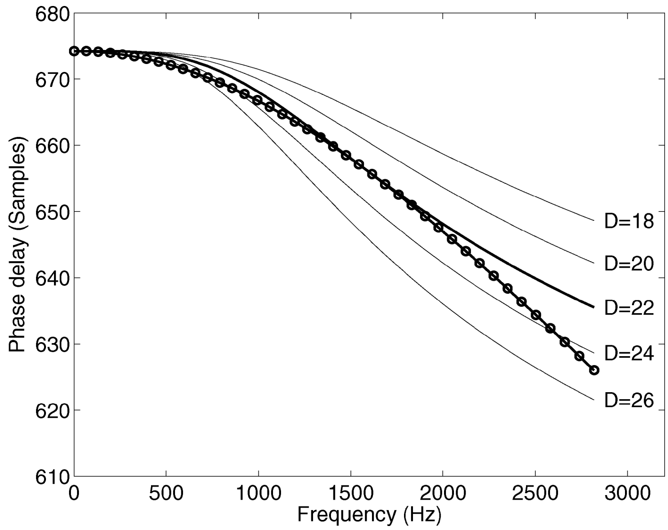

Fig. 1. Thiran allpass filter (cascade of four second-order filters) phase delay response when $B = 0.000\,10$, $f_0 = 65.406$ Hz (key C_2), sample rate is 44.1 kHz, and D gets values 18, 20, 22, 24, and 26 (the estimated delay line length and the tuning filter phase delay are taken into account). The phase delay that corresponds to the target dispersion is marked with a line with circles, and the closest approximation ($D = 22$) is marked with a thick line.

where f_s is the sampling frequency, and f_0 is the nominal fundamental frequency. Fig. 1 shows the phase delay of a cascade of four Thiran allpass filters and a long delay line with various D values as well as the accurate dispersion phase delay when $B = 0.000\,10$ and $f_0 = 65.406$ Hz. The figure suggests that the cascade of Thiran allpass filters is well suited for modeling the dispersion, because it approximates well the desired phase delays with large D values.

The Thiran allpass filter adds an extra delay, which can be quite large, to the delay loop. This can be taken into account by modifying the length of delay line and the delay of the tuning filter. The modified delay line length L_1 can be calculated from the phase delay $d_{\mathrm{disp},1}$ of the dispersion filter at the fundamental frequency as $L_1 = \mathrm{floor}(f_s/f_1 - d_{\mathrm{disp},1}M) - 1$, where f_1 is the frequency of the first partial, and M is the number of filters in cascade. Similarly, the required tuning filter delay d_t can be calculated as $d_t = f_s/f_1 - d_{\mathrm{disp},1}M - L_1$.

B. Filter Order and Number of Filters in Cascade

The first problem in designing a Thiran allpass filter is to decide the order N. Additionally, the option of using M cascaded filters with the same coefficients is considered. The design criteria for evaluating the different options is defined as the number of consecutive partials within the error limit and is set to 0.5% of the frequency of the partial, which is a conservative limit (0.5% is the difference just noticeable for the frequency of a single sine tone [12]). Since the interest is in piano synthesis, the focus in the B domain is on the expected range of piano inharmonicity at a certain f_0. The optimal D was searched for all cases with B values $r10^k$ for all $r = 1, 2, \ldots, 9$ and $k = -5, -4, -3$ in all the frequencies of piano range (27.5 Hz – 4186 Hz) with a resolution of 0.1 for D.

The best values for both M and N seem to be between 1 and 4. It is impossible to find a single filter that would perform well for the whole frequency range. Hence, the frequency range was split by using one filter (A_{disp1}) for low and another filter (A_{disp2}) for high fundamental frequencies. A filter A_{disp1} consisting of a cascade of four second-order allpass filters was chosen for key numbers 1–44, and a single second-order allpass filter A_{disp2} was chosen for key numbers 45–88.

Larger filter orders do not necessarily produce better results because of the Thiran filter design method. The Thiran filter design produces maximally flat group delay responses, which means that also the phase delay is approximately constant at low frequencies. Larger filter orders produce a wider "constant" region, which does not imitate well the desired phase response.

C. Filter Parameterization

The next step is to find how to compute the delay parameter D from the dispersion coefficient B and the fundamental frequency. The parameterization process can be simplified by dividing it into two phases. First, B is considered as constant, and the best value for parameter D is determined at different B values by varying f_0. When D is replaced by $\ln(D)$ and f_0 is replaced by the key number I_{key} obtained from the formula

$$I_{\mathrm{key}}(f) = \log_{\sqrt[12]{2}} \frac{f\,\sqrt[12]{2}}{27.5} \quad (3)$$

$\ln(D)$ as a function of I_{key} is approximately a straight line in the low-frequency range. The values of $\ln(D)$ saturate at high frequencies toward two, which corresponds to the order $N = 2$ of the allpass filter. Since the focus is on the lower frequency range, this is not taken into account.

Denoting

$$\ln(D(I_{\mathrm{key}}, B)) = C_{\mathrm{d}}(B) - k_{\mathrm{d}}(B) I_{\mathrm{key}} \quad (4)$$

where k_{d} and C_{d} are coefficients of a polynomial approximation, the values k_{d} and C_{d} can be calculated with different B values. The values k_{d} and C_{d} are obtained by fitting a straight line to the data in the least-squares sense. A good solution to parameterize coefficients k_{d} and C_{d} seems to be to use logarithmic scale and to approximate k_{d} with a second-order polynomial

$$k_{\mathrm{d}}(B) = e^{\left(k_1 (\ln B)^2 + k_2 \ln B + k_3\right)} \quad (5)$$

where k_1, k_2, and k_3 are polynomial coefficients. Similarly, C_{d} can be modeled with a straight line in the $\ln(B)$ domain

$$C_{\mathrm{d}}(B) = e^{(C_1 \ln B + C_2)} \quad (6)$$

where C_1 and C_2 are straight line coefficients.

As a final result, the following formula for D is obtained

$$D(I_{\mathrm{key}}, B) = e^{(C_{\mathrm{d}}(B) - I_{\mathrm{key}} k_{\mathrm{d}}(B))} \quad (7)$$

where I_{key} is the key number when $A_4 = 440$ Hz (which corresponds to $I_{\mathrm{key}} = 49$) and the other keys are in tune with it, and the parameters have the values shown in Table I. The exact key number I_{key} should be calculated from the fundamental frequency by using (3) rather than using the key number directly.

TABLE I
OPTIMAL DESIGN PARAMETERS FOR THE PROPOSED FILTER

Parameter	A_{disp1} (N=2, M=4)	A_{disp2} (N=2, M=1)
k_1	-0.00050469	-0.0026580
k_2	-0.0064264	-0.014811
k_3	-2.8743	-2.9018
C_1	0.069618	0.071089
C_2	2.0427	2.1074

D. Steps for Inserting Filter Into a String Model

The following steps are simple guidelines for inserting the tunable filter into a waveguide string model.

1) Filter Parameter Calculation: The first step is to calculate the parameters for the dispersion filter. By using the determined I_{key} [computed from f_0 by using (3)] and B values, the D values for each key can be obtained from (7). The actual filter parameters a_1 and a_2 for a second-order allpass filter can be calculated from the D value by deriving formulae from (1) by setting $N = 2$.

The filter parameters can be stored in a table, or they can be calculated on the fly, if there is a sufficient amount of computing power available. Moreover, it is possible to provide real-time control over the inharmonicity coefficient B by repeating this step and the next step every time the B value is changed.

2) Modifications in the Delay Line and in the Tuning Filter: Since the dispersion filter adds a large amount of delay, the delay line and the tuning filter need to be updated as presented in Section II-A. The phase delay $d_{\text{disp},1}$ of the dispersion filter at the frequency f_0 can be calculated. It is found that the filter parameter D, which corresponds to the phase delay at dc, is a satisfactory approximation, since it adds an extra error smaller than the just-noticeable difference of 0.5% in the frequency [12] for practical fundamental frequencies and B values. Only when the key number is greater than 84 and the B value is larger than 0.018 is the error larger than 0.5%. However, the inharmonicity itself affects the perceived fundamental frequency, as shown by Järveläinen *et al.* [13]. Hence, it is necessary to set the tuning by hand for the highest keys.

3) Filter Implementation: The next step is to build the actual dispersion filter block. The transfer function of the dispersion filter can be written as

$$A(z) = \left(\frac{a_2 + a_1 z^{-1} + z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \right)^4. \quad (8)$$

4) Excitation Signal Modification: In commuted synthesis, the excitation signal may have to be modified because of added dispersion. In other words, the spectral peaks of the partials have to be shifted according to the inharmonicity coefficient B. If the excitation signal is extracted by using an inverse filter of the string model, it has to be recomputed with the inverse filtering of the dispersive string model when B is changed.

III. RESULTS

The main goal for the filter design was to create a dispersion filter model, which is controlled with a simple algorithm usable for real-time synthesis. The goal was met, as the resulting equation (7) has five static parameters and takes two input parameters. In order to evaluate the performance, the filter was tested by estimating the filter response (it would not be valid to compare the proposed algorithm to existing ones, because they require a lot of computation, whereas the proposed algorithm is the first closed-form design, which can be used in real-time synthesis).

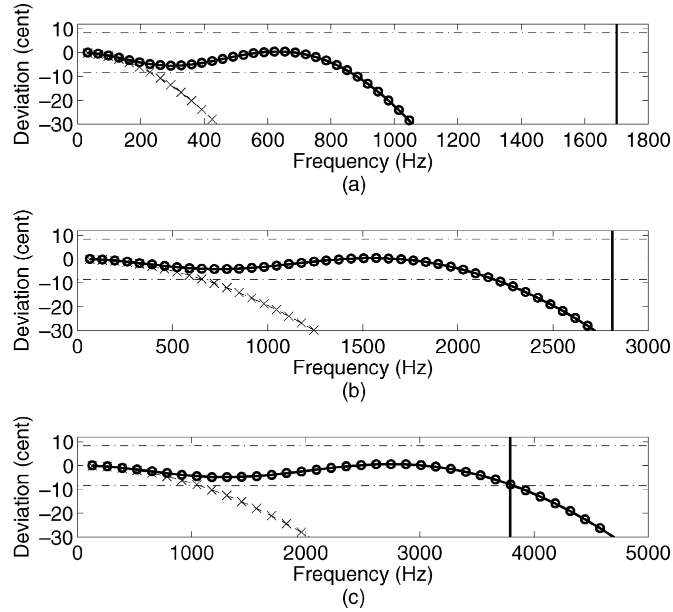

Fig. 2. Partial deviation as a function of frequency for the proposed filter (solid line with circles at the partials), when (a) $B = 0.000\,20$, $D = 54.99$, $L_1 = 1127$, $d_t = 1.38$, $f_0 = 32.703$ Hz (key C_1), (b) $B = 0.000\,10$, $D = 22.05$, $L_1 = 585$, $d_t = 1.00$, $f_0 = 65.406$ Hz (key C_2), and (c) $B = 0.000\,15$, $D = 12.63$, $L_1 = 285$, $d_t = 1.57$, $f_0 = 130.81$ Hz (key C_3). The dashed line with crosses is the calculated deviation for a harmonic tone, the vertical line is the maximum number of perceived inharmonic partials according to Rochesso and Scalcon [14], and the dash-dotted lines are the 0.5% error limits, which correspond to $+/-8.39$ cents.

Fig. 2 shows the phase delay of the example filter proposed in this letter in three different cases: piano keys C_1, C_2, and C_3 with reasonable inharmonicity coefficient values. The performance of the dispersion model can be evaluated in two ways: the smoothness of the phase curve [5] and the number of partials within the error limit compared to the maximum number of perceived partials [14]. As seen in Fig. 2, the phase curves seem to be smooth, but the maximum number of perceived partials is reached only for key C_3. However, good error limits or the number of partials to be modeled accurately in order to produce perceptually good inharmonicity are currently unknown. Hence, not enough is known in order to analyze the quality of the design examples. The proposed filter was implemented in a waveguide piano model. Sound examples are available at the laboratory website (http://www.acoustics.hut.fi/demos/tunable-disp/). The filter produced natural-sounding tones in informal listening tests. Formal subjective testing of the sound quality is left for future work.

The computational load of the filter is estimated to be 16 multiplications and 16 additions per sample for keys 1–44 and four multiplications and additions for keys 45–88. Moreover, when the dispersion value is changed, it requires additionally 12 multiplications and 16 additions, and the algorithm also calls the exponential function three times, the rounding function once, and performs one division to compute the new filter coefficients.

IV. Conclusion

In this letter, a new dispersion filter design was introduced for digital waveguide synthesis of the piano and other stringed instruments. The proposed filter system includes a cascade of four second-order filters for the low fundamental frequencies and a single second-order filter for the high frequencies. The filter coefficients, approximating the desired inharmonicity coefficient at a desired fundamental frequency, can be computed by using a simple formula. Because of this, the filter design method enables fine-tuning of the dispersion characteristics of the model.

Acknowledgment

The authors would like to thank Mr. J. Pakarinen, Mr. B. Bank, and Dr. H. Järveläinen for their comments related to this work.

References

[1] H. Järveläinen, V. Välimäki, and M. Karjalainen, "Audibility of the timbral effects of inharmonicity in stringed instrument tones," *Acoust. Res. Lett. Online*, vol. 2, no. 3, pp. 79–84, 2001.

[2] J. O. Smith, III, "Physical modeling using digital waveguides," *Comput. Music J.*, vol. 16, no. 4, pp. 74–91, 1992.

[3] D. A. Jaffe and J. O. Smith, III, "Extensions of the Karplus-Strong plucked-string algorithm," *Comput. Music J.*, vol. 7, no. 2, pp. 56–69, 1983.

[4] S. A. Van Duyne and J. O. Smith, III, "A simplified approach to modeling dispersion caused by stiffness in strings and plates," in *Proc. Int. Computer Music Conf.*, Århus, Denmark, Sep. 1994, pp. 407–410.

[5] D. Rocchesso and F. Scalcon, "Accurate dispersion simulation for piano strings," in *Proc. Nordic Acoustical Meeting*, Helsinki, Finland, Jun. 1996, pp. 407–414.

[6] J. Bensa, S. Bilbao, R. Kronland-Martinet, and J. O. Smith, III, "The simulation of piano string vibration: From physical models to finite difference schemes and digital waveguides," *J. Acoust. Soc. Amer.*, vol. 114, no. 2, pp. 1095–1107, Aug. 2003.

[7] P. R. Cook, *Real Sound Synthesis for Interactive Applications*. Natick, MA: A. K. Peters, 2002.

[8] J.-P. Thiran, "Recursive digital filters with maximally flat group delay," *IEEE Trans. Circuit Theory*, vol. CT-18, no. 6, pp. 659–664, Nov. 1971.

[9] A. Fettweis, "A simple design of maximally flat delay digital filters," *IEEE Trans. Audio Electroacoust.*, vol. AU-20, no. 2, pp. 112–114, Jun. 1972.

[10] T. I. Laakso, V. Välimäki, M. Karjalainen, and U. K. Laine, "Splitting the unit delay—tools for fractional delay filter design," *IEEE Signal Process. Maga.*, vol. 13, no. 1, pp. 30–60, Jan. 1996.

[11] H. Fletcher, E. D. Blackham, and R. Stratton, "Quality of piano tones," *J. Acoust. Soc. Amer.*, vol. 34, no. 6, pp. 749–761, 1962.

[12] J. G. Roederer, *The Physics and Psychophysics of Music*, 3rd ed. New York: Springer-Verlag, 1995, pp. 26–27.

[13] H. Järveläinen, T. Verma, and V. Välimäki, "Perception and adjustment of pitch in inharmonic string instrument tones," *J. New Music Res.*, vol. 31, no. 3, pp. 311–319, 2002.

[14] D. Rocchesso and F. Scalcon, "Bandwidth of perceived inharmonicity for physical modeling of dispersive strings," *IEEE Trans. Speech Audio Process.*, vol. 7, no. 5, pp. 597–601, Sep. 1999.