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Adaptive Controller for the Avoidance of an Unknownly Guided Air Combat Missile

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***Abstract*— An adaptive controller for the avoidance of an air combat missile utilizing a guidance law unknown to the target is introduced. The controls of the aircraft are determined at discrete instants using a receding horizon control scheme providing near-optimal feedback controls of the aircraft against a closing missile. The controls are optimized with respect to the expected final distance between the vehicles. In the controller, the target's belief in the guidance law of the missile is represented as a discrete probability distribution over a set of guidance laws. As the missile closes on the aircraft, the probability distribution is updated on the basis of the realized vehicle trajectories using Bayesian reasoning. The controller is demonstrated with numerical examples.**

***Index Terms*— Missile avoidance, adaptive control, receding horizon control, Bayesian reasoning.**

I. Introduction

THE avoidance of guided missiles is crucial for the survival in air combat. Consequently, the problem has been researched extensively and a number of disciplines such as the theory of differential games [1], optimal control theory [2], nonlinear programming [3], and receding horizon control [4] have been applied in the solution. As a result, several models and methods providing optimal open-loop [5]–[8] and near-optimal feedback solutions in real-time [9], [10] have been introduced. However, the considered formulations are typically deterministic although the real setting is inherently stochastic. The uncertainty is manifested both in the observed states of the vehicles as well as in the target aircraft's information about the missile system.

We introduce a new adaptive controller [11] for the near-optimal solution of the missile avoidance problem under uncertainty. Specifically, we propose a novel way for taking into account the uncertainty regarding the target's information about the guidance law of the missile. This information is crucial considering optimal evasion, since the target must be aware of the missile dynamics for being able to solve the optimal evasion maneuver. To the authors' knowledge, this is one of the first studies that considers the identification of the missile's guidance law. In [12], the performance parameters and time-to-go of a closing surface-to-air missile guided by pure proportional navigation (PPN) are estimated with a maximum likelihood estimator by a nonmaneuvering target. Except for [12], the specific identification problem, much less the missile avoidance problem under uncertainty, has not been widely addressed in the open literature.

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Basically, a closing air combat missile may be avoided either by outrunning it or exploiting certain weakness of the missile system. Reasonable optimization criteria measuring the success of the avoidance include the capture time [5], closing velocity [13], miss distance, control effort [14], gimbal angle [8], and tracking rate [10] of the missile, see [9] for a brief review. Being the most relevant of the above criteria, only the miss distance maximization is considered in this paper.

Considering the variety of guidance laws in modern air combat missiles, proportional navigation and its variants are undoubtedly the prevailing alternatives. However, advances in beam-pointing technology have increased interest towards the command to line-of-sight (CLOS) guidance especially among surface-to-air missiles [15]. Combinations of various guidance laws are also common, since the missiles typically utilize different guidance schemes during the boost, mid-course, and terminal guidance phases [16]. In this paper, it is assumed that the missile is guided either by ideal proportional navigation (IPN), pure pursuit (PP), or CLOS that are suitable for demonstrating the aspects of the proposed controller.

In the missile avoidance problem at hand, the target aircraft seeks controls that maximize the miss distance of the closing air combat missile. The vehicles are modeled as three degree-of-freedom (3-DOF) point-masses [17]. For improved realism, the angular velocities and accelerations of the aircraft are limited [9]. It is also assumed that the guidance law of the missile is unknown to the target, but the state information received by the vehicles is accurate.

The above stated problem is solved with the adaptive controller introduced in this paper. The controller consists of the control and identification phases. In the first phase, the controls of the aircraft are solved using the receding horizon control scheme introduced in [9]. The controls of the target aircraft are determined at discrete instants. At each instant, the optimal open-loop controls with respect to the expected final distance between the vehicles are solved over a finite planning horizon. Subsequently, the state of the system is updated by implementing the optimal controls related to the current state and time. The computations are repeated at the propagated state, whereupon controls are obtained in a state feedback form. Since the controls are optimized over a limited planning horizon, the obtained solution is not necessarily globally optimal. Nevertheless, the scheme has proven to provide near-optimal solutions for various missile avoidance problems [9].

In the identification phase, the target's belief in the guidance law of the missile is updated using Bayesian reasoning. This refers to the updating of the subjective probabilities assigned to given hypotheses on the basis of the observed evidence using Bayes' theorem [11]. The target's belief in the guidance law of the missile is represented as a discrete probability distribution over a predetermined set of guidance laws. In the control phase, the expected final distance is computed over this distribution. The probability distribution is conditioned on certain feature variables derived from the vehicle states, where the variables obtain typically distinct values for the different guidance laws. As the missile closes on the aircraft, the probability distribution is updated using the measured features. The operational principles of the guidance laws are utilized in the construction of a suitable likelihood function of the guidance law required in the updating. The belief model resembles that applied in the assessment of the pilot's threat situation in the modeling of one-on-one air combat [18], [19]. As a whole, the identification approach is closely related to Multiple Model Adaptive Estimation [20].

The paper is structured as follows. In the following section, the vehicle models and the available guidance laws of the missile are reviewed. The adaptive controller is presented in Section III and demonstrated with numerical examples in Section IV. The aspects and possible extensions of the controller are discussed in Section V, followed by concluding remarks in Section VI.

II. Vehicle models

In the controller and in the updating of the vehicles' states, the motions of the aircraft and the missile are described using 3-DOF vehicle models [17]. For the complete presentation of the vehicle models, see [9]. The state of the vehicle i is given by x_i, y_i, and h_i that refer to the horizontal coordinates and altitude, respectively, and flight path angle γ_i, heading angle χ_i, and velocity v_i. Here, $i = T, L, M$ denotes the target aircraft, launcher, and missile. Moreover, the missile model includes two additional state variables for the pitch and yaw accelerations denoted by a_p and a_y, respectively.

The aircraft is guided by the angle of attack α that controls the lift force normal to the velocity vector, bank angle μ that directs the lift force away from the vertical plane, and throttle setting η that controls the tangential acceleration. To maintain realism, rotational kinematics of the aircraft are taken into account by imposing limits on the angle of attack and roll rates and accelerations. The model also includes control and path constraints that prevent the violation of the minimum flight altitude, stalling, and exceeding of the maximum load factor and dynamic pressure levels.

The missile is guided by the pitch and yaw acceleration commands a_{pc} and a_{yc}. The dynamics of the guidance system are modeled as two first-order systems for each guidance channel that are assumed independent. The acceleration commands are given by the guidance law. The commanded accelerations are constrained by the stall and structural damage limits.

The commanded acceleration vectors $\mathbf{a}_c$ related to the guidance laws considered in this paper are presented in Table I. The pitch and yaw components of the commanded acceleration vector are obtained by projecting it on the pitch and yaw axes of the missile, see [9].

TABLE I
GUIDANCE LAWS OF THE MISSILE

Guidance law θ	Commanded acceleration vector
IPN	$\mathbf{a}_c = N\boldsymbol{\omega} \times \mathbf{v}_c$
PP	$\mathbf{a}_c = k_1 v_M \sin\delta \cdot \mathbf{e}_{(\mathbf{v}_M \times \mathbf{r}) \times \mathbf{v}_M}$
CLOS	$\mathbf{a}_c = k_1 \mathbf{d} + k_2 \dot{\mathbf{d}}$

In Table I, N, k_1, and k_2 are navigation constants. In IPN [21], the principle idea is to maintain the missile in the collision course to the target aircraft by driving the angular rate of the line-of-sight (LOS) vector $\boldsymbol{\omega} = \big(\mathbf{r} \times (-\mathbf{v}_c)\big)/\big(\mathbf{r} \cdot \mathbf{r}\big)$ towards zero. Above, $\mathbf{r} = \begin{bmatrix} x_T - x_M & y_T - y_M & h_T - h_M \end{bmatrix}^T$ is the LOS vector from the missile to the target (see Fig. 1) and $\mathbf{v}_c = -\dot{\mathbf{r}} = \begin{bmatrix} \dot{x}_M - \dot{x}_T & \dot{y}_M - \dot{y}_T & \dot{h}_M - \dot{h}_T \end{bmatrix}^T$ is the closing velocity vector.

In PP [14], the velocity vector of the missile is aligned with the LOS vector by guiding the missile towards the LOS vector in proportion to the angle δ between the velocity vector of the missile and the LOS vector (see Fig. 1). The angle δ, hereafter referred to as the bearing, is given by $\delta = \arccos\left(\mathbf{e}_{v_M} \cdot \mathbf{e}_r\right)$, where $\mathbf{e}_{(\cdot)}$ denotes the unit vector in the direction of the respective vector.

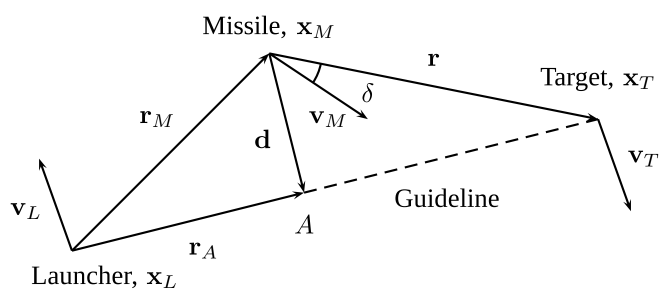


Fig. 1. Combat geometry.

In CLOS [22], the missile is guided towards the target by maintaining it along the guideline directed from the launcher to the target aircraft. The position vector of the missile relative to the guideline, whose magnitude d is hereafter referred to as the deviation, is given by $\mathbf{d} = \mathbf{r}_A - \mathbf{r}_M$, where $\mathbf{r}_A = \left(\mathbf{e}_G \cdot \mathbf{r}_M\right)\mathbf{e}_G$ is the LOS vector from the launcher to point A (see Fig. 1). Above, $\mathbf{e}_G$ is the unit vector in the direction of the guideline and $\mathbf{r}_M = \begin{bmatrix} x_M - x_L & y_M - y_L & h_M - h_L \end{bmatrix}^T$ is the LOS vector from the launcher to the missile. Overshoots are diminished by making the commanded acceleration vector proportional to the rate of change of the position vector $\dot{\mathbf{d}} = \mathbf{v}_{A_\perp} - \mathbf{v}_{M_\perp}$, where $\mathbf{v}_{A_\perp} = \boldsymbol{\omega}_G \times \mathbf{r}_A$ and $\mathbf{v}_{M_\perp} = \left(\mathbf{e}_G \times \mathbf{v}_M\right) \times \mathbf{e}_G$ are the components of the velocity of point A and the missile velocity vector perpendicular to the guideline, respectively.

Above, $\boldsymbol{\omega}_G$ denotes the angular rate vector of the guideline. Note that with CLOS, also the position of the launcher must be given to be able to determine the guideline.

III. Adaptive controller

The adaptive controller is separated into the control and identification phases. The controls of the target aircraft maximizing the expected miss distance are solved using the receding horizon control scheme introduced in [9] and the target's belief in the guidance law is updated using Bayesian reasoning, see, e.g., [11].

The controls are determined at discrete instants $t_k = k\Delta t$, where k denotes the stage and Δt is a constant interval. It is assumed that the vehicles are equipped with sensors capable of measuring the accurate state of the system at each instant. After computing the optimal controls, the state of the system is updated by integrating the state equations over the interval Δt using the obtained controls and the true guidance law of the missile. The same 3-DOF vehicle models are utilized in the controller as well as when updating the states of the vehicles. In principle, also more delicate vehicle models could be used in the updating.

A. Control phase

At state $\mathbf{x}(t_k)$ and time t_k, the optimal open-loop controls of the aircraft $\mathbf{u}^*\big(\mathbf{x}(t_k), t\big)$ over the interval $t \in [t_k, t_k + T]$ that maximize the expected distance between the vehicles at the end of the planning horizon T are solved at first. The controls of the aircraft at instant t_k to be utilized over the interval Δt are then obtained from $\mathbf{u}^*\big(\mathbf{x}(t_k), t_k\big) \equiv \mathbf{u}_k^*$. Repetition of the computation at each instant provides a sequence of controls in feedback form.

In the optimal control problem at instant t_k, the expected distance given by

$$\begin{aligned} \tilde{J}_k(\mathbf{u}) &= E\left\{r(t_k + T)\right\} \\ &= \sum_{\theta \in G} P_k(\theta \mid Z_k) r(t_k + T; \theta) \end{aligned} \tag{1}$$

is maximized subject to

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t; \theta), \quad \mathbf{x}(t_k) = \mathbf{x}_k, \tag{2}$$

$$\mathbf{g}(\mathbf{x}, \mathbf{u}) \leq \mathbf{0}. \tag{3}$$

Above, the state vector $\mathbf{x}$ contains the states of the vehicles, $\mathbf{u}$ is the control vector of the aircraft, and $\mathbf{x}_k$ denotes the state of the system at t_k. In (1), $P_k\left(\theta \mid Z_k\right)$ denoting the probability that the guidance law equals $\theta \in G = \{\text{IPN}, \text{PP}, \text{CLOS}\}$ represents the target's belief in the guidance law of the missile at t_k. These probabilities are conditioned on a set of past state measurements denoted by $Z_k = \{\mathbf{z}_0, \mathbf{z}_1, \ldots, \mathbf{z}_k\}$, where the measurements $\mathbf{z}_k$ are described in the following subsection. $r(t_k + T; \theta)$ denotes the distance between the vehicles at $t_k + T$ when the missile uses the guidance law θ. The performance measure (1) results in maneuvers where at first, the aircraft tries to outrun the missile due to the ongoing maximization of the distance between the vehicles. In the end, the aircraft performs a high-g maneuver that increases the miss distance due to the dynamic delay of the missile guidance system. For the feasibility of the above performance measure in miss distance maximization, see [9]. The differential equations (2) correspond to the state equations of the aircraft and the missile where θ determines the guidance law of the missile. The constraints (3) limit the controls and states of the aircraft, see [9].

The miss distance equals the distance between the vehicles at the moment of the closest approach, that is, when the closing velocity of the missile is zero. At each instant, an estimate of the vehicle states at the end of the planning horizon is first computed by using the optimal controls computed at the previous instant. If the missile is estimated to reach the aircraft at the end of the planning horizon, the terminal constraint

$$v_c(t_k + T) = \mathbf{r}(t_k + T) \cdot \mathbf{v}_c(t_k + T)/r(t_k + T) = 0 \tag{4}$$

that fixes the moment of the closest approach is included in (1)–(3), and T is set free. Also, the computation is stopped, if the estimated time-to-go calculated as $\tau = r(t_k)/v_c(t_k)$ exceeds a missile specific threshold which indicates that the missile cannot reach the target.

B. Identification phase

The guidance laws presented in Table I produce typically mutually distinct bearing and deviation histories. For example, with PP, the bearing is likely to be near zero for the duration of the encounter since the missile is continuously directed at the target. On the other hand, the deviation presumably obtains arbitrary values since it is not taken into account in the PP guidance law. We therefore base the identification of the guidance law on the measurement of the bearing and deviation. At instant t_k, the measured bearing δ_k and deviation d_k (see Fig. 1) are collected into a feature vector $\mathbf{z}_k = \begin{bmatrix}\delta_k & d_k\end{bmatrix}^T$.

The target's belief in the guidance law of the missile is updated as follows. At each instant, the prior probabilities $P_{k+1}\left(\theta \mid Z_k\right)$, $\theta \in G$ are assumed equal to the previous posterior probabilities $P_k\left(\theta \mid Z_k\right)$. Given these probabilities and the measured features, the posterior beliefs at t_{k+1} denoted by $P_{k+1}\left(\theta \mid Z_{k+1}\right)$ are computed using Bayes' theorem (see, e.g., [11]) as

$$\begin{aligned} P_{k+1}(\theta \mid Z_{k+1}) &= \frac{P_{k+1}(\theta \mid Z_k) f(\mathbf{z}_{k+1} \mid \theta)}{\sum_{\xi \in G} P_{k+1}(\xi \mid Z_k) f(\mathbf{z}_{k+1} \mid \xi)} \\ &= \frac{P_k(\theta \mid Z_k) f(\mathbf{z}_{k+1} \mid \theta)}{\sum_{\xi \in G} P_k(\xi \mid Z_k) f(\mathbf{z}_{k+1} \mid \xi)}. \end{aligned} \tag{5}$$

The likelihood function of the parameter θ is calculated by

$$f(\mathbf{z}_k \mid \theta) = f^\delta(\delta_k \mid \theta) f^d(d_k \mid \theta) \tag{6}$$

where $f^\delta(\cdot)$ and $f^d(\cdot)$ are probability density functions of the bearing and deviation given the guidance law θ.

The likelihood function gives the likelihoods of the different guidance laws for the measured features. It is an essential part of the belief model because it determines the evolution of the probabilities of the guidance laws over time.

C. Probability density functions

Feasible probability density functions of the features that determine the likelihood function (6) are given in Table II. The functions are characterized by design parameters δ_0, λ_1, λ_2, λ_3, and D that scale the functions appropriately.

TABLE II
PROBABILITY DENSITY FUNCTIONS FOR THE FEATURES

Probability density function	Support
$f^\delta(\delta \mid \text{IPN}) = \delta_0^{-2}\exp(-\delta/\delta_0)\delta$	$\delta \geq 0$
$f^\delta(\delta \mid \text{PP}) = \lambda_1\exp(-\lambda_1\delta)$	$\delta \geq 0$
$f^\delta(\delta \mid \text{CLOS}) = \lambda_2\exp(-\lambda_2\delta)$	$\delta \geq 0$
$f^d(d \mid \text{IPN}) = 1/D$	$d \in [0, D]$
$f^d(d \mid \text{PP}) = 1/D$	$d \in [0, D]$
$f^d(d \mid \text{CLOS}) = \lambda_3\exp(-\lambda_3 d)$	$d \geq 0$

Since an IPN guided missile tends to maintain a constant bearing, a suitable probability density function of the bearing is a Gamma distribution with the parameters $\alpha = 2$ and δ_0 illustrated in Fig. 2, where δ_0 equals, e.g., the theoretical lead angle. This angle can be obtained from the collision triangle in which the vehicles are assumed to fly straight ahead at constant velocities till the moment of interception, see [22]. With PP, the bearing is likely to be near zero, whereupon an exponential distribution with a relatively large parameter λ_1 is a possible choice. The effect of the parameter on the shape of the distribution is illustrated in Fig. 2. With CLOS, the missile tends to be directed more or less towards the target, and thus an exponential distribution with a smaller parameter λ_2 compared to PP is a suitable choice.

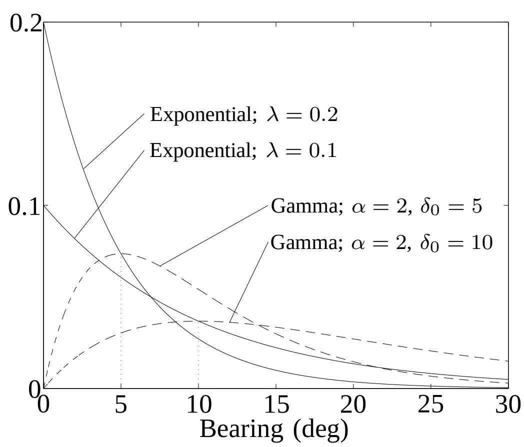


Fig. 2. Illustrations of probability density functions.

Since the IPN and PP guidance laws do not consider the deviation at all, a uniform distribution of the deviation is suitable for these guidance laws. With CLOS, the deviation is likely to be small, so an exponential distribution with a relatively small parameter λ_3 is a viable choice here.

D. Implementation aspects

The optimal control problem (1)–(4) is solved by the direct shooting method [23], which is robust and fast when the length of the planning horizon remains within a few seconds. Here, the problem is transcribed into a nonlinear programming problem (NLP) by discretizing the time and evaluating the controls of the aircraft at the discrete instants. The state equations are then integrated explicitly by using the controls evaluated at the instants. In order to achieve a longer planning horizon T with a smaller number of NLP variables, the intervals between the instants are increased towards the end of the planning horizon, see [9].

Since the missile is positioned near the guideline at the launch time and in the endgame, the deviation is close to zero during these periods. In addition, as the missile closes on the aircraft, the bearing increases rapidly in the end due to the increasing misalignment between the missile's velocity vector and the LOS vector. Consequently, the identification of the guidance law is difficult if not impossible during these periods. For example, the likelihood of CLOS prevails the other guidance laws at the launch time, whereupon the guidance law would be identified always as CLOS irrespective of the true guidance law.

The problem is overcome by starting the updating of the target's belief after a certain warm-up period, and stopping the updating when the estimated time-to-go falls below a given limit. Thereafter, the prevailing probabilities are applied. Suitable values for the duration of the warm-up period and the time-to-go limit are obtained from computational experiments.

IV. NUMERICAL EXAMPLES

We next demonstrate the introduced controller with five examples. The vehicle models correspond to a generic fighter aircraft and a medium range air-to-air missile. The minimum altitude, maximum dynamic pressure, and maximum load factor of the aircraft are initialized to $h_{a,\min} = 100$ m, $q_{\max} = 80$ kPa, and $n_{a,\max} = 9$, respectively. The aircraft employs an afterburner. The navigation constants appearing in Table I and the maximum load factor of the missile are set to $N = 4$, $k_1 = 50$, $k_2 = 30$, and $n_{m,\max} = 40$, respectively. The durations of the missile's boost and sustain phases are 3 and 5 seconds, respectively.

In the controller, the discretization interval is set to $\Delta t = 0.25$ s. The planning horizon equals about $T = 3$ s. We assume that the target has no prior knowledge about the guidance law of the missile, hence the probabilities representing the target's belief are initialized to $P_0(\theta \mid \mathbf{z}_0) = 1/3$ for all $\theta \in G$. Following the reasoning given in Section III-C, the parameters of the probability density functions are set to $\delta_0 = 5$ deg, $\lambda_1 = 1.5$, $\lambda_2 = 0.01$, $\lambda_3 = 0.0075$, and $D = 1000$. Based on the computational experience, the duration of the warm-up period and the time-to-go limit are set to 0.5 s and 3.0 s, respectively. All the parameters are chosen by the authors and are suitable for demonstrative purposes.

In the examples, the missile is launched towards the aircraft with the lead angle of 5 deg at the range of 14000 m whereas the aircraft is flying towards the launch point with the aspect angle of 20 deg. The initial states of the aircraft and the missile are summarized in Table III. The initial values of the angle of attack and bank angle are set to $\alpha_0 = 0$ and $\mu_0 = 0$, respectively.

TABLE III
INITIAL STATES OF THE VEHICLES

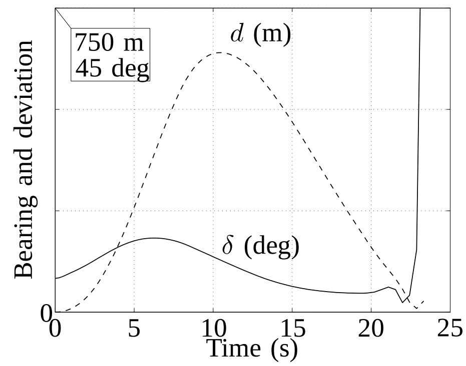
	x (m)	y (m)	h (m)	γ (deg)
Aircraft	0	0	5000	0
Missile	14000	0	5000	0
	χ (deg)	v (m/s)	a_p (m/s^2)	a_y (m/s^2)
Aircraft	20	250	—	—
Missile	175	250	0	0

1) Ideal Proportional Navigation: In this example, the missile is guided by IPN. We begin by analyzing the histories of the features and target's belief distribution presented in Figs. 3a and 3b, respectively. Now, the probability of PP decreases rapidly to zero since the bearing remains clearly positive over the duration of the encounter. Also, the target cannot at first separate between IPN and CLOS due to the small deviation. However, while the bearing and deviation increase, the probabilities of IPN and CLOS increase and decrease towards zero and one, respectively. After about 4 seconds, the probability of IPN saturates to one, whereupon the guidance law is identified correctly. Note that in the end, the bearing increases rapidly. The optimal endgame evasion is a vertical S-maneuver [8] induced by a rapid increase of the angle of attack and bank angle, see Fig. 3c. The resulting miss distance is 27.7 m.

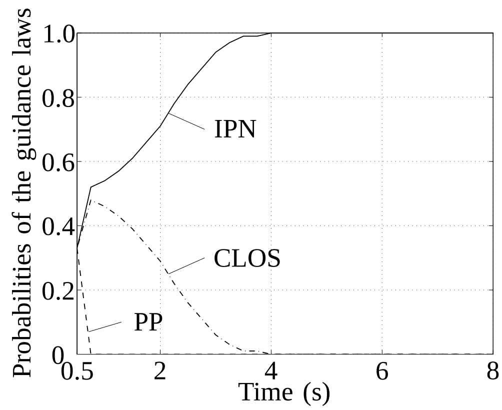
2) Pure Pursuit: Here, the missile is guided by PP, which drives the bearing rapidly to zero, see Fig. 4a. On the other hand, the deviation begins to increase. Consequently, the target identifies the guidance law as PP within one second while the probabilities of IPN and CLOS decrease rapidly to zero. In the end, the aircraft induces the missile to attain large lateral acceleration by curving strongly, see Fig. 4c. The resulting miss distance is 59.3 m. Comparison to Fig. 3c indicates that the trajectory of the missile and the endgame maneuvers of the aircraft differ considerably from those of the previous example. In both cases, the dynamic delay of the missile guidance system is exploited.

3) Command to Line-of-Sight: In this example, the missile is guided by CLOS and the launcher is assumed to fly straight ahead with a constant velocity and heading. Here, the deviation stays near zero for the duration of the encounter, see Fig. 5a. Also, the bearing remains near zero but is still clearly positive. In the beginning, the bearing decreases rapidly to zero which causes the target to believe that the missile is guided by PP, see Fig. 5b. However, as the bearing increases and the deviation stays near zero, the probabilities of PP and CLOS decrease and increase, respectively. Finally, the probability of CLOS saturates to one within 6 s. The attained miss distance is 37.6 m. Due to the assumed launcher maneuvering, the trajectory of the missile resembles that produced by PP, see Figs. 5c and 4c.

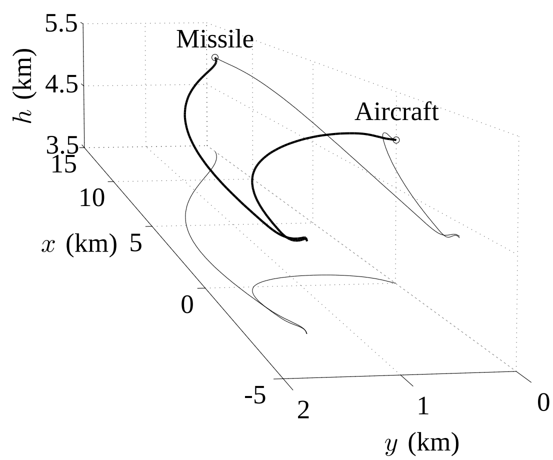
4) Pure Pursuit and Ideal Proportional Navigation: We next study three cases where the guidance law of the missile is changed in the course of flight which is typical for modern air combat missiles. Here, the missile is initially guided by PP and the guidance law is switched from PP to IPN after 6, 8, or 10 s.



(a) Feature histories



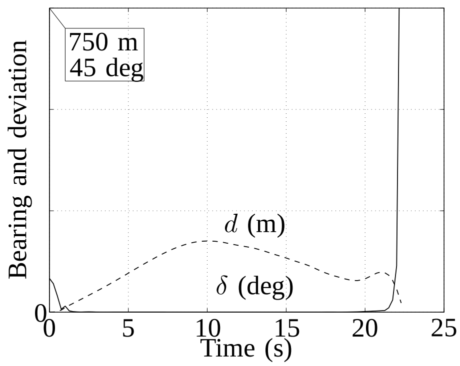
(b) Target's belief distribution histories



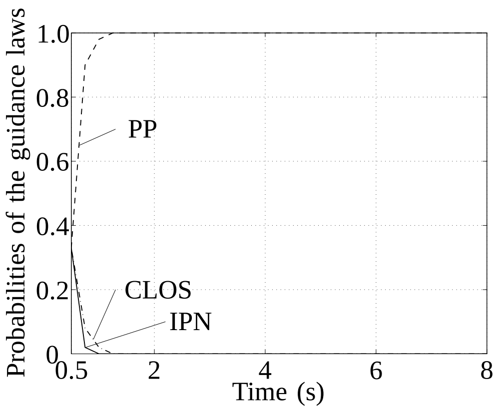
(c) Trajectories of the vehicles

Fig. 3. Example 1; ideal proportional navigation

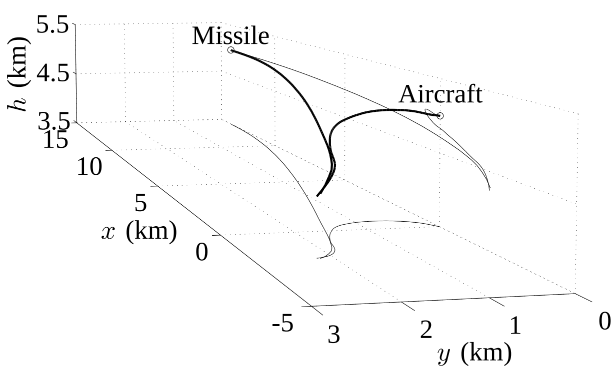
The bearing remains near zero while the missile is guided by PP, see Fig. 6a. After the switch to IPN, the bearing stabilizes to a constant positive level until the rapid increase in the end. The guidance laws utilized by the missile are identified correctly, see Fig. 6b. Since the bearing stabilizes quite slowly after the guidance law is changed, the probabilities of IPN and PP are not instantaneously swapped after



(a) Feature histories



(b) Target's belief distribution histories



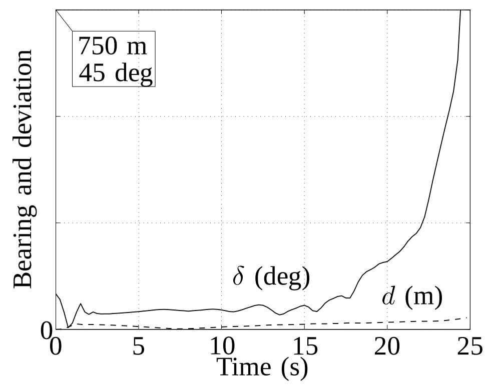
(c) Trajectories of the vehicles

Fig. 4. Example 2; pure pursuit

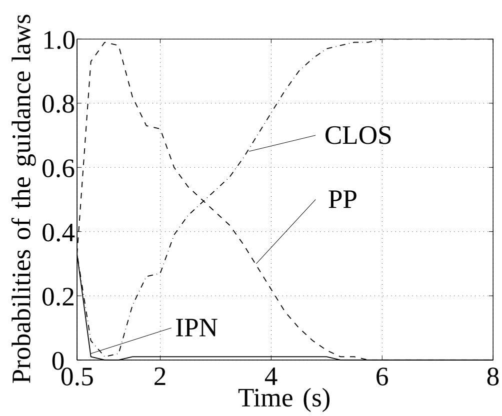
the switch. The achieved miss distances with the switches at 6, 8, and 10 s are 22.6, 22.6, and 22.3 m, respectively. The trajectory of the missile with the switch at 8 s presented in Fig. 6c corresponds to a combination of those presented in Figs. 4c and 3c.

5) Sensitivity analysis: We finally study how the variations in the initial bearing affect the identification of the guidance law and the miss distances with IPN. The initial states of the vehicles are those given in Table III except for the heading angle of the missile. We compare three cases where the inital heading is set to $\chi_m = 180, 175, 170$ deg, whereupon the initial bearing equals $\delta(t_0) = 0, 5, 10$ deg, respectively. Note that the second case corresponds to Example 1.

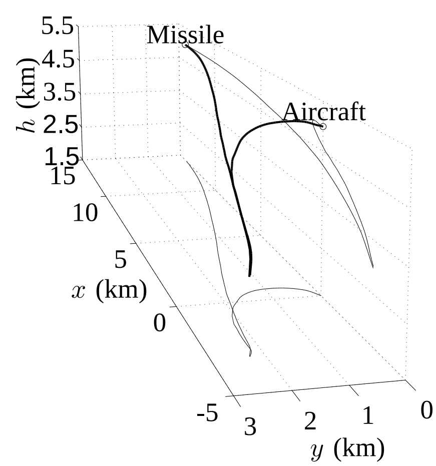
With $\delta(t_0) = 0$, the bearing stays near zero during the first two seconds, whereupon the guidance law is identified incorrectly as PP at first, see Figs. 7a and 7b. As the bearing increases, the probability of PP decreases rapidly to zero. However, since the deviation remains under a few hundred meters within the first 4 s, the guidance law is misidentified as CLOS. Eventually the bearing and deviation reach the levels that result in the correct identification. The probability of IPN saturates to one in 4.5 s. The achieved miss distance



(a) Feature histories



(b) Target's belief distribution histories



(c) Trajectories of the vehicles

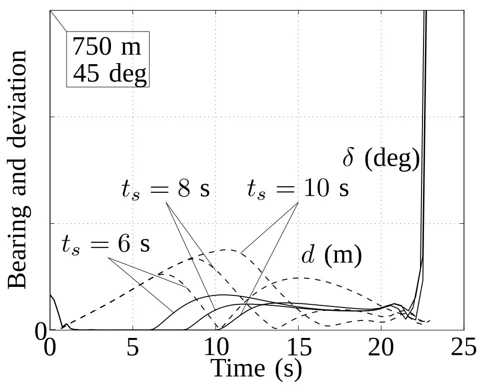
Fig. 5. Example 3; command to line-of-sight

is 36.0 m.

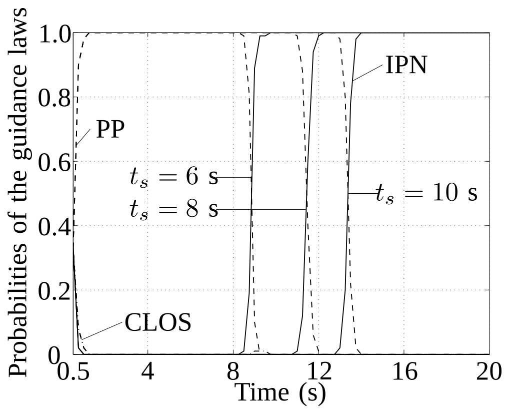
With $\delta(t_0) = 5$ deg, the guidance law is identified as IPN within 4 seconds. For the detailed analysis of this case, see Example 1 on page 5.

With $\delta(t_0) = 10$ deg, the small deviation and large bearing in the beginning result in the misidentification of the guidance law at first. During the first 2.5 s, the probability of CLOS prevails over IPN and PP, of which PP decreases rapidly to zero due to the large bearing, see again Figs. 7a and 7b. However, as the deviation increases, the probability of IPN saturates to one in 4 s. The resulting miss distance is now 22.2 m.

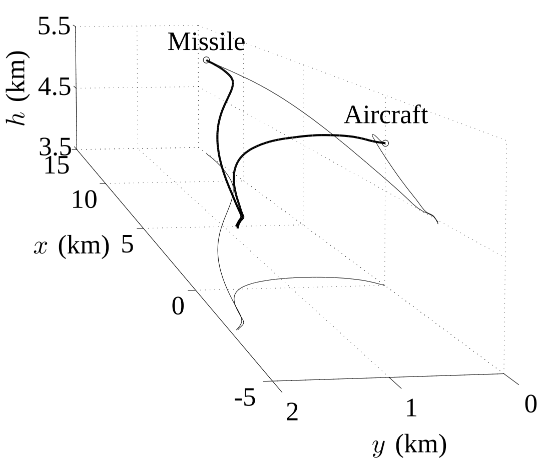
In summary, the uncertainty regarding the guidance law is larger during the first few seconds when the initial bearing is different from the design parameter $\delta_0 = 5$ deg. Consequently, the identification of the guidance law is somewhat delayed. However, the differences between the identification times are small, whereby the controller as a whole appears to be insensitive to variations of the initial bearing from δ_0.



(a) Feature histories. t_s denotes the switching time.

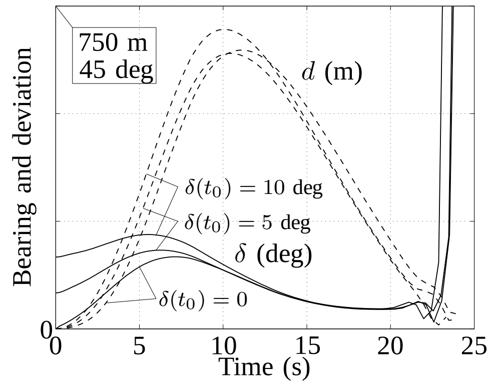


(b) Target's belief distribution histories. t_s denotes the switching time.

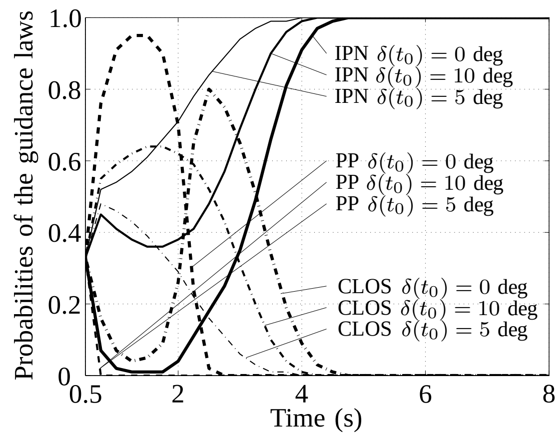


(c) Trajectories of the vehicles

Fig. 6. Example 4; pure pursuit and ideal proportional navigation



(a) Feature histories



(b) Target's belief distribution histories

Fig. 7. Example 5; sensitivity analysis

V. Discussion

According to the numerical examples, the introduced adaptive controller identifies the guidance law rapidly and reliably, whereupon efficient endgame evasion maneuvers can be performed. The bearing and deviation are suitable choices for the features, on the basis of which the guidance law of the missile can be identified. Likewise, the utilized receding horizon control scheme appears to provide competent feedback controls in the maximization of the expected miss distance. Finally, the utilization of a suitable warm-up period and time-to-go limit in the updating of the target's belief distribution prevents the potential misidentification of the guidance law, which would result from disinformative values of the features at the launch time and during the endgame, see, e.g., Fig. 3a.

The repertoire of the guidance laws available in the adaptive controller can be extended in a straightforward manner. At first, the considered feedback guidance law must be implemented in the controller. The more demanding part is the determination of suitable features and probability density functions characterizing the particular guidance law. However, simulations and the operational principle of the guidance law can be used to provide an insight into this task. At first, a set of features should be extracted by studying the operational principle of the respective guidance law. Then, suitable probability density functions of the features could be constructed on the basis of feature histories obtained from the simulations. Finally, the feasibility of the choices should be validated computationally. For example, guidance laws based on optimal control theory and the theory of differential games [14], [24] as well as various loft schemes could possibly be appended in the controller via the procedure described above.

In the control phase, the optimization is performed with respect to the expected distance between the vehicles at the end of the planning horizon. This renders the computational complexity of the optimization problem proportional to the number of available guidance laws. Hence, the larger the arsenal of the guidance laws, the less tractable the problem is. One way to circumvent the increase in the computational load is to assume that the guidance law associated with the highest probability is the true one, and optimize the controls of the aircraft against the missile guided by the particular guidance law.

The applied belief model can be extended to cover also other elements of the problem including uncertainty. For example, different missile types could be categorized by another parameter which could be identified similarly than the missile's guidance law. Again, feasible features and probability density functions of them should be derived on the basis of the simulations and properties of the missile types. In addition, the state of the system at each instant could be estimated from the state measurements by using, e.g., the extended Kalman filter that could be incorporated into the Bayesian framework as well [25].

VI. Conclusion

Considering the avoidance of a guided missile, it is essential that different properties of the missile are known or identified correctly by the target. We introduced an adaptive controller for computing near-optimal controls of the aircraft avoiding an air combat missile using a guidance law unknown to the target. The numerical examples presented in the paper suggest that the introduced controller identifies the guidance law by the endgame, whereupon efficient last-ditch evasion maneuvers can be performed. In addition to the guidance law identification, the controller can be extended to take into account also other sources of uncertainty in the missile avoidance problem.

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