

Jyri Pakarinen, 2005, Physical modeling of flageolet tones in string instruments, In: Proceedings of the 13th European Signal Processing Conference (EUSIPCO 2005), Antalya, Turkey, 4-8 September 2005, 4 pages.

© 2005 European Association for Signal Processing (EURASIP)

Reprinted with permission.

# PHYSICAL MODELING OF FLAGEOLET TONES IN STRING INSTRUMENTS

*Jyri Pakarinen*

Laboratory of Acoustics and Audio Signal Processing, Helsinki University of Technology  
 P.O. Box 3000, FI-02015 TKK, Espoo, Finland  
 phone: +358-9-451-6027, fax: +358-9-460-224, email: jyri.pakarinen@acoustics.hut.fi  
 web: www.acoustics.hut.fi/~jykk

## ABSTRACT

This paper introduces a physical modeling method for synthesizing flageolet tones played on string instruments. Flageolet tones are generated when a player gently damps the vibrating string at a certain location, so that only those modes that have a nodal point at the damping location remain ringing. A physical model of the spatially damped string is implemented using a wave digital resistor connected to a digital waveguide string. Also, a commuted waveguide version is derived and analyzed. The model is able to realistically synthesize flageolet tones with both static and time-varying damping parameters. Synthesis results are presented and analyzed.

## 1. INTRODUCTION

Flageolet tones, or harmonics, are a special kind of tone playable on many string instruments. The fundamental mechanism behind the generation of these sounds is that the player gently damps the vibrating string in a certain location, thus letting only those modes ring, which have a node at the damping location.

Flageolet tones are most often used as special effects, but a skillful player can also produce melodies and harmonies using this technique. Damping an open (i.e., not fretted) vibrating string with length  $L$  in a location  $x_d$  produces a flageolet tone consisting of components with frequencies

$$f_f = \frac{L}{x_d} n f_0, \quad \text{where } n = 1, 2, 3, 4, \dots \quad (1)$$

and  $f_0$  denotes the fundamental frequency of the open string, i.e., the first vibrational mode of the unfretted string. These flageolet tones are called natural harmonics in the guitar terminology.

On fretboarded instruments, the player can also produce flageolet tones by damping a fretted vibrating string in a desired location. The resulting tone will contain components with frequencies as evaluated in Eq. (1), except that then  $f_0$  will represent the fundamental frequency of the fretted string. In the guitar terminology, these tones are called artificial harmonics. Note that their only difference from natural harmonics is the fact that the string is fretted, rather than open.

On a guitar, there are also other possibilities of playing flageolet tones: the player can sweep his thumb or index finger on the string near the plucking location with the same movement as the plucking itself, thus creating a high-pitched flageolet tone (note that the node and antinode are now relatively close to each other). This is called playing pinch harmonics, an effect most emphasized with the usage of non-linear amplification (such as heavily distorted guitar ampli-

fiers). Flageolet tones can also be played on fretboarded instruments using a technique called “tap harmonics” where the player taps the string lightly with his finger in the location of a new “forced” antinode.

Probably the most straightforward way of creating flageolet tones from existing synthesis algorithms is to feed the output of the synthesizer into a comb filter which allows only frequencies  $f_f$  to ring. This is, however, not a physically justified process. This paper presents a physical modeling method for synthesizing flageolet tones on string instruments. We will concentrate on plucked string instruments here, although the same method could be used for struck and bowed strings as well. A somewhat similar damping technique for bowed strings has been outlined earlier in [1]. The major difference between the previous method and the one presented in this paper is that the earlier work effectively uses a waveguide string loaded with a mass, whereas this study uses a wave digital damper.

This paper is structured as follows: Section 2 presents a distributed waveguide model of a spatially damped string. Section 3 takes this idea further and presents a commuted single delay-loop (SDL) version with the same functionalities as the distributed one. The synthesis results are presented and analyzed in Section 4, and conclusions are drawn in Section 5.

## 2. SPATIALLY DISTRIBUTED STRING MODEL

For modeling the string, a digital waveguide (DWG) approach [2] was chosen due to its computational efficiency. The string damper representing the player’s finger is implemented as a wave digital (WD) resistor. For an introduction to digital waveguides and wave digital filters, see [3] and [4], respectively. The WD resistor is connected to the DWG string via a 3-port series connection [4]. Note that since both techniques are based on the traveling-wave decomposition, the DWG and the WD structures are fully compatible, provided that the wave decomposition of the WD approach is chosen appropriately [5].

The location of the damper, and thus the harmonic structure of the flageolet tone, can be altered by moving the WD resistor’s connection point. The amount of damping, in turn, can be controlled by altering the value of the resistor. Figure 1 illustrates the DWG string connected to a damping WD resistor  $R_S(z)$  via a 3-port scattering junction. In the figure,  $a_1$ ,  $a_2$ , and  $a_3$  denote the wave components traveling to the junction from the left and right parts of the string and from the damper, respectively. The symbols  $b_1$ ,  $b_2$ , and  $b_3$  stand for the corresponding wave components leaving the junction, while  $Z_1$ ,  $Z_2$ , and  $Z_3$  denote the corresponding port impedances. The transfer functions  $R_f(z)$  and  $R_b(z)$  repre-

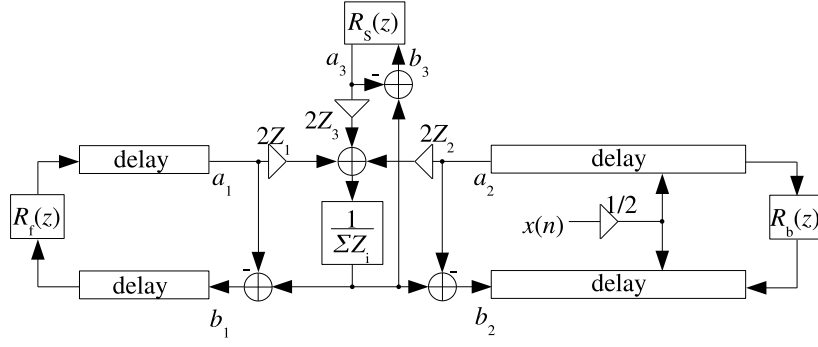


Figure 1: A dual delay-line waveguide model of a plucked string with a wave digital damper connected via a 3-port scattering junction. Signal  $x(n)$  denotes the excitation for the string.

sent the losses taking place at the string terminations which, in the case of the guitar, would be the nut or the fret and the bridge. The excitation signal  $x(n)$  is summed to both delay lines, as usual.

Since the WD resistor is essentially a broken connection [4], we can discard the wave components entering and leaving the damper ( $b_3$  and  $a_3$ , respectively) and reformulate the scattering junction as

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} \rho_1 & \rho_2 \\ \rho_3 & \rho_4 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad (2)$$

where

$$\begin{aligned} \rho_1 &= \frac{Z_1 - Z_2 - Z_3}{Z_1 + Z_2 + Z_3} \\ \rho_2 &= \frac{2Z_2}{Z_1 + Z_2 + Z_3} \\ \rho_3 &= \frac{2Z_1}{Z_1 + Z_2 + Z_3} \\ \rho_4 &= \frac{Z_2 - Z_1 - Z_3}{Z_1 + Z_2 + Z_3}. \end{aligned} \quad (3)$$

This spatially damped waveguide string model is illustrated in Fig. 2. Here, acceleration waves are used as wave components, and the string output is taken as a force signal  $F(s)$  at the bridge. We have used a Laplace-domain formulation here to ease the commutation process taking place in the following section. The symbols  $d_1$ ,  $d_2$ , and  $d_3$  denote the length of the delay lines between the nut and the damper, damper and the plucking location, and the delay between the plucking and the bridge, respectively. The symbol  $X(s)$  represents the excitation acceleration signal to the string (plucking), while  $X_1(s)$  and  $X_2(s)$  denote the excitation components for the upper and lower delay lines, respectively. Note that an ideal pluck would correspond to an impulse excitation [6], [7].

Points E1 and E2 represent the excitation locations in the upper and lower delay lines, respectively, while  $A_1(s)$  and  $A_2(s)$  stand for the respective acceleration waves entering and leaving the bridge at locations R1 and R2. The symbols  $a_1$  and  $a_2$ , as before, stand for the respective waves entering the damping location from left and right, while  $b_1$  and  $b_2$  denote the waves leaving the damper from left and right, respectively.

### 3. COMMUTED STRING MODEL

If we assume the string to be linear and time-invariant (LTI), we can construct a commuted version of the string, presented

as a cascade of filters. In a real-time implementation, this single delay-loop model might be computationally more efficient than the dual delay-line version, as is often the case with commuted waveguide models. In addition, it is interesting from a theoretical point of view to see if the different phenomena in string vibration can be formulated as separate filter structures. The following commuting process follows the guidelines presented in [8], the difference being that the string is no longer a simple delay loop due to the scattering junction caused by the spatial damping. The commutation will be done in the Laplace domain because of its simple form of presenting derivations and integrations. In the following, we will refer to  $H_{A,B}(s)$  as the transfer function from point A to B, as done in [8].

We start simplifying the model by defining a single excitation at point E1 that represents the superimposed effect of the two excitation components  $X_1(s)$  and  $X_2(s)$ . The equivalent single excitation at E1 can be stated as

$$\begin{aligned} X_{E1,eq}(s) &= H_{E1,R1}(s)X_1(s) + [\alpha(s) + \beta(s) + \gamma(s)]X_2(s) \\ &= H_E(s)X(s), \end{aligned} \quad (4)$$

where

$$H_E(s) = \frac{1}{2} [H_{E1,R1}(s) + \alpha(s) + \beta(s) + \gamma(s)] \quad (5)$$

and

$$\begin{aligned} \alpha(s) &= H_{E2,a2}(s)\rho_4 H_{b2,R1}(s), \\ \beta(s) &= H_{E2,a2}(s)\rho_2 H_{b1,a1}(s)\rho_3 H_{b2,R1}(s), \\ \gamma(s) &= H_{E2,a2}(s)\rho_2 H_{b1,a1}(s)\rho_1 H_{b1,a1}(s)\rho_3 H_{b2,R1}(s), \end{aligned} \quad (6)$$

where

$$\begin{aligned} H_{b1,a1}(s) &= H_{b1,Rf}(s)R_f(s)H_{Rf,a1}(s), \\ H_{b2,R1}(s) &= H_{b2,E1}(s)H_{E1,R1}(s). \end{aligned} \quad (7)$$

Here,  $\alpha(s)$  is due to the signal route from E2, via  $\rho_4$  to R1,  $\beta(s)$  is due to the signal from E2 via  $R_f(s)$  to R1, and  $\gamma(s)$  is caused by the route  $\beta(s)$  added with the loop via  $\rho_1$ . The force signal at the bridge is defined as [8]

$$\begin{aligned} F(s) &= Z(s)[A_1(s) - A_2(s)]/s \\ &= Z(s)[A_1(s) - R_b(s)A_1(s)]/s \\ &= H_B(s)A_1(s), \end{aligned} \quad (8)$$

where

$$H_B(s) = Z(s)[1 - R_b(s)]/s \quad (9)$$

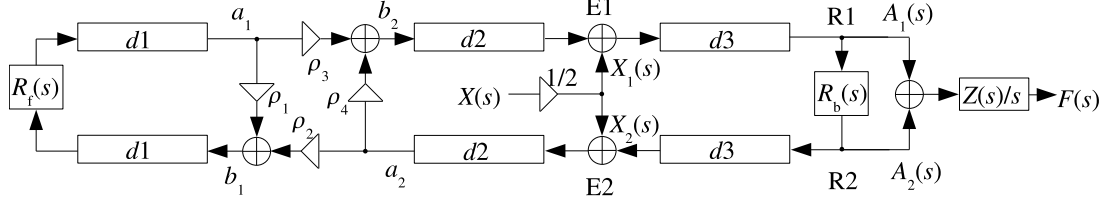


Figure 2: A dual delay-line waveguide model of a plucked string with a simplified spatial damper. The output of the string is taken as the force signal at the string termination.

denotes the acceleration-to-force transfer function at the bridge.

Next, we can formulate the expression for  $A_1(s)$

$$\begin{aligned} A_1(s) &= H_{E1,R1}(s)X_{E1,eq}(s) + H_{loop}(s)A_1(s) \\ &= \frac{H_{E1,R1}(s)X_{E1,eq}(s)}{1 - H_{loop}(s)} \\ &= H_{E1,R1}(s)S(s)X_{E1,eq}(s), \end{aligned} \quad (10)$$

where

$$H_{loop}(s) = \delta(s) + \varepsilon(s) + \zeta(s) \quad (11)$$

denotes the transfer function when the signal propagates through the string loop once and

$$S(s) = \frac{1}{1 - H_{loop}(s)} \quad (12)$$

is the transfer function of the whole string that represents the recursion around the string loop. For the terms in Eq. (11),

$$\begin{aligned} \delta(s) &= R_b(s)H_{R2,a2}(s)\rho_4H_{b2,E1}(s)H_{E1,R1}(s), \\ \varepsilon(s) &= R_b(s)H_{R2,a2}\rho_2H_{b1,a1}(s)\rho_3H_{b2,R1}(s), \\ \zeta(s) &= R_b(s)H_{R2,a2}(s)\rho_2H_{b1,a1}(s)\rho_1H_{b1,a1}(s)\rho_3H_{b2,R1}(s), \end{aligned} \quad (13)$$

where

$$\begin{aligned} H_{b2,R1}(s) &= H_{b2,E1}(s)H_{E1,R1}(s), \\ H_{R2,a2}(s) &= H_{R2,E2}(s)H_{E2,a2}(s). \end{aligned} \quad (14)$$

Here,  $\delta(s)$  results from the signal route from R1 via  $\rho_4$  back to R1,  $\varepsilon(s)$  is due to the loop from R1 via  $R_f(s)$ , and  $\zeta(s)$  is caused by the route  $\varepsilon(s)$  added with the loop through  $\rho_1$ .

Putting this all together, we can formulate the transfer function from excitation to bridge output as

$$\begin{aligned} H_{E,B}(s) &= \frac{F(s)}{X(s)} \\ &= H_B(s)H_{E1,R1}(s)S(s)H_E(s). \end{aligned} \quad (15)$$

Next, we will approximate this continuous-time model with a discrete  $z$ -domain representation. Simplifying Eqs. (6) and (13), we have

$$\begin{aligned} \alpha(z) &= \rho_4 z^{-(2d2+d3)}, \\ \beta(z) &= \rho_2 \rho_3 R_f(z) z^{-(2d1+2d2+d3)}, \\ \gamma(z) &= \rho_1 \rho_2 \rho_3 R_f(z)^2 z^{-(4d1+2d2+d3)}, \end{aligned} \quad (16)$$

and

$$\begin{aligned} \delta(z) &= \rho_4 R_b(z) z^{-(2d2+2d3)}, \\ \varepsilon(z) &= \rho_2 \rho_3 R_f(z) R_b(z) z^{-(2d1+2d2+2d3)}, \\ \zeta(z) &= \rho_1 \rho_2 \rho_3 R_f(z)^2 R_b(z) z^{-(4d1+2d2+2d3)}. \end{aligned} \quad (17)$$

Thus, the transfer function from the excitation acceleration to the bridge force becomes

$$H_{E,B}(z) = H_{E1,R1}(z)H_B(z)H_E(z)S(z), \quad (18)$$

where

$$H_B(z) = Z(z)I(z)[1 - R_b(z)], \quad (19)$$

and  $I(z)$  represents the discrete-time approximation of the time-domain integration. For the rest of the terms in Eq. (18)

$$H_E(z) = \frac{1}{2} [H_{E1,R1}(z) + \alpha(z) + \beta(z) + \gamma(z)] \quad (20)$$

and

$$S(z) = \frac{1}{1 - H_{loop}(z)} = \frac{1}{1 - [\delta(z) + \varepsilon(z) + \zeta(z)]}. \quad (21)$$

Since  $H_{E1,R1}(z) = z^{-d3}$  is a common delay in the nominator of Eq. (18), it can be neglected in a practical sound synthesis implementation.

Note that the effect of the spatial damping cannot be isolated as a separate filter like the effect of excitation or bridge coupling in [8], since the damping is a part of the filters  $H_E(z)$  and  $S(z)$  themselves. Also, damping due to the losses in the string itself (i.e., not caused by the player) cannot be commuted into just one end of the string without giving up the physical validity, since otherwise a loss-free loop would be formed between the other end and the damping location.

## 4. RESULTS

The distributed string model and the commuted version are capable of simulating flageolet tones with both constant and time-varying damping. Applying spatial damping in runtime requires changing the  $Z_3$  parameter of Eq. (3) during the filtering operation of function  $H_{E,B}(z)$  and thus making the system time-variant. While in theory the commuting process done in Section 3 is not valid for time-variant systems, we did not encounter any problems with the time-varying commuted version. Figure 3 illustrates a spectrogram of a guitar tone, synthesized by the time-varying commuted string model.

Figure 4 illustrates the magnitude responses of filters  $H_{E,B}(z)$ ,  $S(z)$ ,  $H_E(z)$ , and  $H_B(z)$  when the string is plucked at 1/4 of its length and damped at the middle. The transfer function of the string itself,  $S(z)$  (magnitude response drawn with a dashed line), causes the amplification of the normal modes as well as the attenuation of the odd modes due to the damping at the middle.

The excitation filter  $H_E(z)$  (magnitude response drawn with a dash-dotted line) is responsible for the comb-filtering effect attenuating every 4th harmonic due to the plucking position. This effect is discussed more thoroughly in an earlier

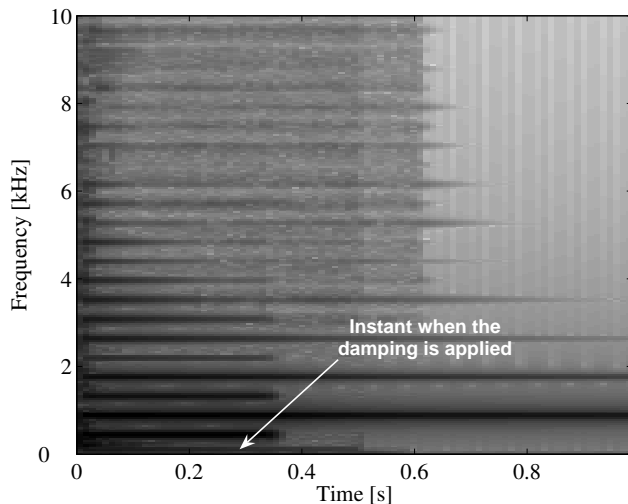


Figure 3: Spectrogram image of a synthesized guitar tone, damped in the middle after 0.3 seconds. The termination filters are set to  $R_f(z) = R_b(z) = -0.8 - 0.2z^{-1}$ , the integrator has the value  $I(z) = \frac{0.5}{1-0.5z^{-1}}$ , and the bridge impedance is  $Z(z) = 1$ . The WDF resistance is varied within  $[0..0.51]$ , while the string segment impedances are  $Z_1 = Z_2 = 0.5$ . The string has the total length of 50 samples. As can be seen, the odd harmonics die out when damping is applied.

study [9]. The bridge filter  $H_B(z)$  (magnitude response drawn with a dotted line) effectively becomes a simple lowpass filter with a relatively flat magnitude response if the bridge impedance  $Z(z)$  is assumed to be a real constant.

## 5. CONCLUSIONS

A physical model of a spatially damped vibrating string was presented. The string contains a wave digital resistor that is connected to a digital waveguide. A commuted equivalent structure of the model was also derived, consisting of relatively simple cascaded filters. Simulation results reveal that both models are able to synthesize flageolet tones when the spatial damping is either static or varied in time.

## 6. ACKNOWLEDGEMENTS

This work was supported by the ALMA project (IST-2001-33059), the Academy of Finland project SA 104934, Tekniikan edistämissäätiö, and the Helsinki Graduate School of Electrical and Communications Engineering. The author is grateful for Prof. Vesa Välimäki, Prof. Matti Karjalainen, Dr. Cumhur Erkut, and the members of the Pythagoras Graduate School for comments and helpful discussions.

## REFERENCES

- [1] T. Takala, J. Hiipakka, M. Laurson, and V. Välimäki. An expressive synthesis model for bowed string instruments. In *Proc. ICMC*, pages 70–73, Berlin, Germany, Sept. 2000.
- [2] J. O. Smith. Physical modeling using digital waveguides. *Computer Music Journal*, 16(4):74–87, Winter 1992.

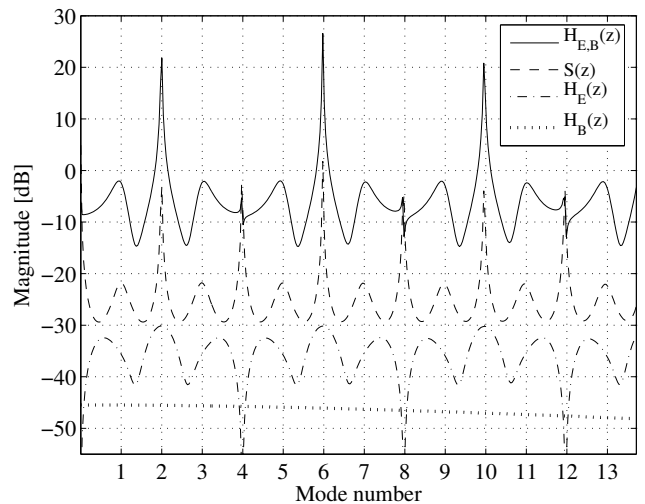


Figure 4: Magnitude responses of a spatially damped commuted DWG filter  $H_{E,B}(z)$  (solid line) and its components  $S(z)$  (dashed line),  $H_E(z)$  (dash-dotted line), and  $H_B(z)$  (dotted line) when the string is plucked at  $1/4$  of its length and damped at the middle. The magnitude response curves are drawn to different levels for illustrational purposes. The horizontal axis denotes the vibrational mode number and the first mode vibrates at a frequency of 153 Hz.

- [3] J. O. Smith. *Applications of Digital Signal Processing to Audio and Acoustics*, chapter Principles of Digital Waveguide Models of Musical Instruments, pages 417–466. Kluwer Academic Publishers, Feb 1998.
- [4] A. Fettweis. Wave digital filters: Theory and practice. *Proc. IEEE*, 74(2):270–327, Feb. 1986.
- [5] M. Karjalainen and C. Erkut. Digital waveguides vs. finite difference schemes: Equivalence and mixed modeling. *EURASIP Journal on Applied Signal Processing*, (7):978–989, June 2004.
- [6] J. O. Smith. *Techniques for Digital Filter Design and System Identification with Application to the Violin*. Ph.D. dissertation. Stanford, California, Stanford University, Dept. of Music, CCRMA Tech. Report no. STAN-M-14, 1983.
- [7] M. Karjalainen and U. K. Laine. A model for real-time sound synthesis of guitar on a floating-point signal processor. In *Proc. ICASSP*, volume 5, pages 3653–3656, Toronto, Canada, May 14–17, 1991.
- [8] M. Karjalainen, V. Välimäki, and T. Tolonen. Plucked-string models: From the Karplus-Strong algorithm to digital waveguides and beyond. *Computer Music Journal*, 22(3):17–32, 1998.
- [9] H. Penttinen and V. Välimäki. A time-domain approach to estimating the plucking point of guitar tones obtained with an under-saddle pickup. *Applied Acoustics*, 65(12):1207–1220, Dec. 2004.