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MODELING OF NONLINEAR AND TIME-VARYING PHENOMENA IN THE GUITAR

Jyri Pakarinen

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Abstract

This dissertation studies some of the nonlinear and time-varying phenomena related to the guitar, and suggests new physics-based models for their realistic discrete-time simulation for sound synthesis purposes. More specifically, the tension modulation phenomenon is studied and three new algorithms are introduced for synthesizing it. Energy-related problems are discovered with conventional digital waveguide models when their pitch is varied, and two novel techniques are suggested as a remedy. A new wave digital filter based real-time model is presented for simulating a nonlinear vacuum-tube amplifier stage, found in typical high-quality guitar amplifiers. The first study of the handling noise on wound strings is presented. Using this information together with the time-varying digital waveguide energy compensation techniques mentioned above, a new real-time slide guitar synthesis algorithm is introduced. Also, the generation of flageolet tones on string instruments is discussed and a novel physics-based model for their simulation is presented. In general, the results presented in this dissertation can be used for improving the current physics-based string instrument synthesizers and vacuum-tube amplifier models.

Keywords digital signal processing, musical acoustics, model-based sound synthesis, computer music

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Tiivistelmä

Tämä väitöskirja käsittelee tiettyjä kitaraan liittyviä epälineaarisia ja aikamuuttuvia ilmiöitä, ja tarjoaa uusia, fysiikan lakeihin perustuvia, äänisynteesimalleja niiden todenmukaista simulointia varten. Kirjassa käsitellään kielen jännitysmodulaation aiheuttamaa epälineaarisuutta, ja ehdotetaan kolme uutta synteesialgoritmia sen mallinnukseen. Väitöskirjatyössä on havaittu soitinmallinnuksessa laajalti käytettyjen aaltojohtokielimallien vaimenemiseen liittyviä ongelmia kielen äänenkorkeuden muuttuessa, ja kaksi kompensaatiomenetelmää ehdotetaan ratkaisuksi tähän ongelmaan. Uusi aaltodigitaalisuodinpohjainen reaaliaikamalli esitetään kitaravahvistimissa yleisesti käytettyjen elektroniputkivahvistinasteiden simulointia varten. Kirjassa analysoidaan punottuja kieliä soitettaessa aiheutuvia hälyääniä sekä esitellään tätä analyysia ja edellä mainittuja kompensaatiomenetelmiä käyttämällä aikaansaatu slidekitaran äänisynteesimalli. Kirjassa selitetään myös kielisoittimiin liittyvien huiluäänten syntymekanismi, sekä tarjotaan uusi, kielen fysiikkaan pohjautuva algoritmi niiden syntetisoimiseksi. Tässä väitöskirjassa esitettyjä tuloksia voidaan käyttää hyödyksi nykyisten soitin- ja vahvistinmallien toiminnan parantamisessa.

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Preface

I consider myself a very lucky person. I have been given an opportunity to pursue two of my greatest passions, music and science, and to combine them into my doctoral work at the unit formerly known as Laboratory of Acoustics and Audio Signal Processing at Helsinki University of Technology. For this opportunity, I am most grateful to my supervisor and instructor, Prof. Vesa Välimäki. Vesa's guidance, ideas, and strive for efficiency have improved the quality of my thesis and helped me in getting this work done in a relatively short time. I wish to express my gratitude to Prof. Matti Karjalainen and Dr. Cumhur Erkut for support, inspiring discussions, and their endless patience for explaining and re-explaining many physics-based modeling issues to me. The contents of this thesis owe a lot to the co-writers of my publications: Dr. Stefan Bilbao, Dr. Balázs Bank, Dr. Henri Penttinen, and Mr. Tapio Puputti. I want to thank Mr. David Yeh for reading and commenting my manuscript, Mr. Luis Costa for his careful proofreading, and my opponent Prof. Julius Smith for finding some time in his busy schedule to travel from sunny California to not-so-sunny Finland for my doctoral defense. The effort of my pre-examiners, Prof. Federico Avanzini and Dr. Seppo Uosukainen, both geniuses in their respective fields, is also acknowledged.

The acoustics lab is an extraordinary working environment, filled with interesting and colourful personalities. Among the labsters, I especially want to thank our secretary Lea Söderman for taking care of the bureaucracy and numerous practicalities involved in my dissertation, my roommate Mairas for providing poor humor and five years worth of intensive tech support, Mara and Hynde for other technical and computer-related issues; Heidi-Maria Lehtonen, Laura Lehto, and Hannu Pulakka for second-lunch company; the Chechnyan rebels: Carlo Magi (In Memoriam 1980-2008), Toni Hirvonen, ex-member Petri Korhonen, Ville Pulkki, Toomas Altosaar, and Miikka Tikander. I am obliged to Prof. Gary Scavone, the Altosaar family, and all the great researchers at

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Also outside the lab, I feel privileged to be surrounded by such wonderful people. I wish to thank my parents, grandparents, brother, and friends (you know who you are) for supporting me. I greatly appreciate the rewarding counterbalance that our band Superjaded (Pekkeri, Anna, Jesse, Janne, and Mikko) has provided for my life. Finally, I wish to express my deepest gratitude and affection for my wife Annika and our lovely daughter Mimosa, for their unconditional love and support.

Espoo, 12th of February 2008,

Jyri Pakarinen

Contents

List of Publications

This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.

I J. Pakarinen, V. Välimäki, and M. Karjalainen, "Physics-based methods for modeling nonlinear vibrating strings." *Acta Acustica united with Acustica*, No. 2, pp. 312-325, March/April, 2005.

II J. Pakarinen, M. Karjalainen, V. Välimäki, and S. Bilbao, "Energy behavior in time-varying fractional delay filters for physical modeling of musical instruments." In *Proc. IEEE Intl. Conf. Acoust., Speech and Signal Proc.*, vol. 3, pp. 1-4, Philadelphia, PA, USA, March 19-23, 2005.

III V. Välimäki, J. Pakarinen, C. Erkut, and M. Karjalainen, "Discrete-time modelling of musical instruments." *Reports on Progress in Physics*, 69(1), pp. 1-78, Jan. 2006.

IV M. Karjalainen and J. Pakarinen, "Wave digital simulation of a vacuum-tube amplifier." In *Proc. IEEE Intl. Conf. Acoust., Speech and Signal Proc.*, vol. 5, Toulouse, France, May 14-19, 2006.

V J. Pakarinen, H. Penttinen, and B. Bank, "Analysis of the handling noises on wound strings." *Journal of the Acoustical Society of America*, 122(6), pp. EL197-EL202, Dec. 2007.

VI J. Pakarinen, T. Puputti, and V. Välimäki, "Virtual slide guitar." Accepted for publication in *Computer Music Journal*, 2008.

VII J. Pakarinen, "Physical modeling of flageolet tones in string instruments." In *Proc. 13th Eur. Signal Proc. Conf.*, Antalya, Turkey, September 4-8, 2005.

Author's Contribution

Publication I: "Physics-Based Methods for Modeling Nonlinear Vibrating Strings"

Previous digital waveguide models of tension modulated strings are lumped. This means that physically correct interaction with the simulated string is possible only at certain pre-defined locations. This article describes two novel spatially distributed string models with tension modulation nonlinearity, which allow interaction at any spatial location on the string. The author invented the novel finite difference algorithm, coded both algorithms, and wrote the article.

Publication II: "Energy Behavior in Time-Varying Fractional Delay Filters for Physical Modeling of Musical Instruments"

This publication discusses the unnatural damping of digital waveguide strings with time-varying length. Two novel algorithms are provided as a solution. The present author formulated and implemented one of these methods, the controllable wave digital filter delay line, and wrote Section 3, except for the second paragraph.

Publication III: "Discrete-Time Modelling of Musical Instruments."

This study is an attempt to summarize all current musical instrument modeling techniques into a thorough tutorial article. This is the first review article on the subject that covers different modeling methods at such a detailed level. The author wrote Sections 8, 11, and most of Section 4. Also, a novel technique for simulating nonlinear, energy-preserving strings in Sec. 11.2 was devised by the author.

Publication IV: "Wave digital simulation of a vacuum-tube amplifier"

In this article, a novel physical modeling technique for simulating vacuum-tube amplifier stages is presented. Unlike most previous techniques, this method simulates each circuit component in real time, resulting in a physically accurate but still real-time computable model. The author wrote Sections 3.1 and 3.4, and collaborated in writing Section 3.2. He also generated the related publication web page, and recorded the dry demo samples.

Publication V: ”Analysis of the handling noises on wound strings.”

The handling noise created by a moving finger on wound strings is studied. This is the first scientific report analyzing this common phenomenon. The presented work enables more realistic musical string synthesis in the future, since parametric models for the handling noise can be generated. Research results reveal that the noise consists of time-varying and static harmonic components. The former are due to the string windings, while the latter are caused by the excitation of longitudinal string modes. The present author is the main responsible for the underlying research. Discussions with co-authors supported the planning of the work, and the measurements were carried out with co-author #2. The author wrote the article, except for Sections IV and V.

Publication VI: ”Virtual Slide Guitar”

A new physics-based model for synthesizing slide guitar sounds is presented. This contains a new model for synthesizing the contact sounds between the slide tube and the strings. The synthesis model operates in real-time and it is controlled using a camera-based, gestural user interface. The author devised the model and wrote the article, except for some parts of the introduction and of the section discussing the real-time implementation. He also supervised the software implementation of the real-time virtual slide guitar.

Publication VII: ”Physical modeling of flageolet tones in string instruments”

A new synthesis model for simulating flageolet tones on string instruments is presented. This method allows a more physical interaction with the string than the previous comb-filter-based techniques. For example, the amount of spatial damping can be varied in time in a physically meaningful way. The present author is solely responsible for the research and writing of this publication.

List of Abbreviations

3D	Three-dimensional
BC	Before Christ
DC	Direct current
DSP	Digital signal processing
DWG	Digital waveguide
FIR	Finite impulse response
IIR	Infinite impulse response
K	Kirchhoff
LTI	Linear and time-invariant
MSW	McIntyre, Schumacher, and Woodhouse (algorithm)
PDE	Partial differential equation
W	Wave
WDF	Wave digital filter

List of Symbols

α	[rad]	Angle between the string and the soundboard at the termination
$\delta(\cdot)$		Dirac's delta function
κ	[m]	Radius of gyration
μ	[kg/m]	Linear mass density
τ_{air}	[s]	Time constant for decay caused by air damping
τ_{int}	[s]	Time constant for decay caused by internal string damping
τ_{sup}	[s]	Time constant for decay caused by energy loss through supports
τ_{tot}	[s]	Time constant for total decay
ϕ_n	[rad]	Initial phase of mode n
c_{t}	[m/s]	Transversal wave propagation velocity
c_{l}	[m/s]	Longitudinal wave propagation velocity
$\mathrm{d}s$	[m]	Elongated length of a string segment
e		Base for natural logarithms ($e \approx 2.7183$)
$f(x,t)$	[N/m]	Excitation force density
f_n	[Hz]	Frequency of mode $n = 1, 2, 3 \ldots$
f_0	[Hz]	Fundamental frequency
n		Mode index
t	[s]	Time variable
u	[m]	Transversal string displacement
u_0	[m]	Transversal string displacement at time $t = 0$
v	[m]	Longitudinal string displacement
w		Torsional component of the string vibration
x, y, z	[m]	Spatial coordinates of a string segment
z^{-N}		Integer delay of length N
z^{-d}		Fractional delay of length d
$(\cdot)_x$		Spatial derivative in the x-direction

$(\cdot)_t$		Temporal derivative
A	$[\mathrm{m}^2]$	Cross-sectional area of a string
B		Inharmonicity coefficient
$F_n(t)$	[N]	Excitation force of mode n
E	$[\mathrm{N/m}^2]$	Young's modulus
$H_n(t)$		Time-domain impulse response of mode n
$H_\mathrm{l}(z)$		Digital waveguide loss filter
I_gk	[A]	Grid-to-cathode current of a vacuum-tube
I_pk	[A]	Plate-to-cathode current of a vacuum-tube
L	[m]	String length
L'	[m]	Elongated string length
$R(f)$	[1/s]	Frictional resistance per unit mass
R_n	[1/s]	Frictional resistance per unit mass for mode n
T	[N]	String tension
T_0	[N]	Nominal string tension
T_u	[N]	Transversal component of string tension
V_g	[V]	Grid-to-ground voltage of a vacuum-tube
V_gk	[V]	Grid-to-cathode voltage of a vacuum-tube
V_k	[V]	Cathode-to-ground voltage of a vacuum-tube
V_p	[V]	Plate-to-ground voltage of a vacuum-tube
V_pk	[V]	Plate-to-cathode voltage of a vacuum-tube

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1 Introduction

1.1 History

Mankind has enjoyed the music of plucked string instruments for several millenia. The earliest archeological illustrations of plucked string instruments have been found in the Magdalenian cave in Ariége, France, where the rock art portrays a shaman holding an object believed to be a musical bow, drawn approximately 17500 years ago [Sieveking, 1998]. Eventually, the primitive musical bow evolved into various plucked string instruments, such as harps, psalteries, and lutes. For an extensive tutorial on the history of plucked instruments refer to [Jahnel, 1981]. One type of a gut-stringed lute, the 16th century Spanish vihuela, can be seen as the ancestor of the modern six-string guitar. The 19th century Spanish luthier Antonio de Torres (1817-1892) had a major impact on the construction of the guitar by enlarging the body and introducing the fan-shaped top-plate bracings [Fletcher and Rossing, 1988]. The concept of modern guitar was dramatically changed with the introduction of the solid-body electric guitar by musical instrument manufacturers Fender and Gibson in the 1950s.

First reported studies on vibrating strings were conducted by the Greek philosopher Pythagoras (about 580-500 BC). He discovered that two plucked strings in equal tension produced pitches in consonant, or pleasing, intervals when their lengths were in small integer ratios, such as $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$. Pythagoras' work was not continued until two thousand years later, when an Italian luthist and composer Vincenzo Galilei (1520-1591) found that contrary to the knowledge at that time, the string tension did not follow the Pythagorean consonance ratios [Caleon and Subramaniam, 2007]. Instead, he concluded that the tension ratio between two similar strings with equal lengths was 4:1 when they were tuned an octave apart [Foley, 2007].

Later, Vincenzo's son, Galileo Galilei (1564-1642) revealed that the perceived pitch of a vibrating string is determined by its frequency, i.e. the number of vibrations per time unit [Caleon and Subramaniam, 2007]. Therefore, he correctly reasoned that the pleasing consonance intervals were produced when the frequencies of the two strings, rather than other string properties, were in integer ratios. At the same time, a French philosopher and mathematician Marin Mersenne (1588-1648) discovered the relation between a string's length, tension, density, and the produced frequency [Foley, 2007].

The vibrational movement of an ideal string was studied in the early 18th century by English mathematician Brook Taylor (1685-1731), who first noted in 1715 that the acceleration of the string was proportional to its curvature [Mumford, 2006]. Using the theory of calculus, introduced by Sir Isaac Newton (1643-1727) and Gottfried Wilhelm Leibnitz (1646-1716) in the late 17th century, Daniel Bernoulli (1700-1782) was able to derive the partial differential equation for an ideal string. He also provided a solution in 1738 using separation of variables, which was later refined by Leonhard Euler (1707-1783) and Joseph Louis Lagrange (1736-1813) [Robinson, 1982].

In 1747 a new solution to the wave equation was presented by the French mathematician Jean le Rond d'Alembert (1717-1783). He stated [Lindsay, 1973] that the solution can be seen as a superposition of two arbitrary wave components, traveling in opposite directions on the string. After this, Bernoulli, Euler, Lagrange, and d'Alembert debated over the true general solution for the wave equation for several years [Shenitzer, 1998]. In 1759, starting from a finite set of linearly connected elementary masses, Lagrange showed that the harmonic vibration of a string is obtained as the number of the elements approaches infinity [Rayleigh, 1945]. The discussion was finally concluded in 1807, when the French mathematician and physicist Jean Baptiste Joseph Fourier (1768-1830) showed that any function, e.g. the initial displacement of a string, can be given as an infite sum of sinusoids.

As musicians have known for long, the vibrating string has to be connected to an instrument body for sufficient sound radiation. Thus, the vibrational properties of the musical instrument itself have a major impact on the produced sound. There are many studies on the vibration of a guitar body in the literature (e.g. [Fletcher and Rossing, 1988], see [Richardson, 2003] for an overview), but since it is essentially a linear and time-invariant phenomenon[1], it will not be discussed further in this thesis. For electric guitars, the magnetic pickup and amplifier properties also contribute a great deal to the sound. Electric guitar body vibrations have been studied in [Esposito, 2003], and a report on magnetic pickups can be found in [Jungmann, 1994]. The effect of a guitar amplifier will be discussed in Chapter 4.2.

The birth of computers in the mid-twentieth century gave new, powerful tools for musical instrument research. The first discrete-time string models were presented by Hiller and Ruiz in the early 1970s [Hiller and Ruiz, 1971a,b]. Article III provides a more thorough discussion on the simulation of a vibrating string. Several discrete-time guitar body models have been presented during the last two decades, starting from simple filters [Karjalainen and Laine, 1991; Karjalainen et al., 1991, 1993b], to more sophisticated admittance-based [Cuzzucoli and Lombardo, 1999; Woodhouse, 2004] and finite element [Elejabarrieta et al., 2001; Derveaux et al., 2003; Bader, 2003, 2005; Bécache et al., 2005] models. Hybrid body models containing warped filters, resonators, and reverb algorithms have been presented in [Karjalainen et al., 1999; Penttinen et al., 2000, 2001b]. A comparison between synthesized and measured guitar tones has been published in [Woodhouse, 2004].

[1]An opposing view has been presented in [Besnainou et al., 2001], but it is generally considered invalid [Erkut, 2002; Penttinen, 2006].

1.2 Nonlinearity and time-variance

The definition of nonlinearity is probably best made through negation. A system with input x and output $f(x)$ is considered linear, if it is both

- additive, i.e. if $f(x_1 + x_2) = f(x_1) + f(x_2)$ and
- homogeneous, i.e. if $f(\alpha x) = \alpha f(x)$, for all α.

An intuitive way of illustrating linearity is to plot the output of a system as a function of the input. For memoryless linear systems, the result is always a straight line through the origin, hence the term linear. On the other hand, if a system fails to fulfill either of the requirements above, it is nonlinear. The input-output relation of a nonlinear system is a curve, which is not a straight line.

A system is considered time-variant, if its response properties depend explicitly on time. In other words, an input signal $x(t)$ produces the output

- $y(t) = f(x(t), t)$ at a given time instant t.

Thus, due to the time-variance, the system output can change even if $x(t)$ remains constant. In conclusion, nonlinearity and time-variance are two distinct properties. Thus, a system might be either nonlinear or time-variant, both, or neither. Systems, which are linear and do not depend explicitly on time are called linear and time-invariant (LTI) systems.

So, what is the practical relevance of whether a system is LTI or non-LTI, one might ask. The answer is that for LTI systems, a special set of analysis tools, called LTI system theory, can be used. Probably the most fundamental property of an LTI system is that its behavior is explicitly defined by its impulse response. The impulse response is, as its name implies, the output of the system when a single impulse is given as the input. When the impulse response of a lumped, i.e. dimensionless, LTI system is known, its response for any input signal can be obtained by convolving the input signal in the time domain with the impulse response. This is a remarkable simplification since it reduces all the functionality of the system into one signal. For spatially distributed systems, convolving the impulse response in the space domain with an arbitrary excitation gives meaningful results only if the system is space invariant, as in the hypothetical case of an infinitely long string, for example. Since real strings are not space invariant, simple spatial convolution does not suffice. This will be discussed further in Sec. 2.

For non-LTI systems, this reduction is not possible. The impulse response of a nonlinear system tells only how the system reacts to an impulse input, but in general it does not tell anything about the system's response to any other signal. Therefore, if the behavior of a nonlinear system is to be defined only by its input-output relation (the so-called black-box approach), one would have to measure the system's output for every imaginable input signal. For time-varying systems, the case is even more complicated, since the response for a given input depends also on when the input was fed to the system.

Obviously, LTI systems are a lot easier to analyze or simulate than non-LTI systems from the engineering point of view. Thus, it is not surprising that various systems are often considered LTI, although, in the strict sense, they are not. In many cases, the parameter ranges are chosen so that the inaccuracy due to this erroneous assumption is negligible, i.e. the system is nearly-enough LTI. However, as nature does not follow simple mathematical restrictions, the LTI assumption does not hold for real systems in general. Especially in the case of musical instruments, there are various phenomena in which the LTI assumption does not yield realistic results.

1.3 Aim and contents of this thesis

The scope of this thesis is physics-based sound synthesis of string instruments. The purpose of this branch of research is to use the laws of physics to artificially create the sounds of real sounding objects, such as musical instruments. These synthesis techniques can then be implemented for example in electronic keyboards, computer programs, and mobile devices. Traditionally, most plucked string instruments have been considered LTI in the synthesis point of view. Although usually faster to compute, LTI string instrument models often sound too artificial or static to be interesting for a human listener. The present thesis aims to address this problem by adding the effect of many important non-LTI properties involved in real string instruments, guitars in particular. The inclusion of these phenomena results in more realistic sound synthesizers that respond to the user's action more intuitively than previous synthesis models.

In addition to the physical accuracy of these methods, their ability to produce real-time sound synthesis is of paramount importance. Usually, there is a trade-off between these requirements, so a choice has to be made between physical accuracy and system simplicity. Since the author's main motive for physical modeling of musical instruments is real-time sound synthesis, the choice of generating models which are sufficiently accurate for human listeners but can still produce synthesized sound in real time has been made.

This thesis consists of a summary and seven articles. The articles are published in international peer-reviewed journals and conferences, and the summary aims to give a concise description of the results obtained as well as to provide more thorough background information. The summary consists of five sections. Section 1 discusses the history of string instrument research, introduces the concept of nonlinearity and time-variance, and clarifies the aim and purpose of this work. Section 2 studies the vibrational behavior of a linear string, and section 3 gives a brief overview of the simulation methods used in this thesis. Section 4 discusses the guitar-related nonlinear and time-varying phenomena that are studied and modeled in articles I-VII. Section 5 gives a conclusion of the scientific results presented in this thesis. The rest of the thesis consists of the articles.

Articles I and III introduce new spatially distributed sound synthesis algorithms for tension modulated strings. These models simulate the initial pitch descent and mode coupling effects of real string instruments. Unlike previous nonlinear string models, the new algorithms allow the user to correctly interact with the string at any location along its length, thus increasing the flexibility of string synthesizers.

Article II presents two new methods for compensating for artificial energy losses encountered in current varying-pitch string models. Using the presented techniques, more realistic models for strings with rapidly varying pitch can be generated. The nonlinear string model introduced in article III, and the virtual slide guitar introduced in article VI use these methods.

Article IV introduces a new model for real-time simulation of a guitar tube amplifier stage. In contrast to most previous algorithms, this technique allows a realistic component-level simulation of the amplifier circuit. More importantly, the proposed model is modular, which means that the circuit topology can be easily varied. This simulation scheme is a first step towards a new type of a virtual guitar amplifier, where the user could easily alter the device's component values and circuit connections and immediately hear the resulting change in amplifier's tone. This future guitar amplifier simulator could also be used a prototyping tool for conventional tube amplifier designers.

Handling noises on wound strings is analyzed in article V, which is the first scientific study of this nonlinear and time-varying phenomenon to the author's best knowledge. These squeaky contact sounds are created by a moving finger on a wound string, and they can frequently be heard in most guitar recordings, even though musicians often try to avoid creating them. The inclusion of these effects is crucially important if realistic guitar synthesis is desired, since a total lack of handling sounds make a synthetic string instrument sound too machine-like and dull. Using the results presented in article V, more realistic synthesis algorithms for wound string instruments can be generated.

Article VI introduces a new musical instrument, the virtual slide guitar. The physics-based synthesis engine of this instrument uses the research results presented in articles II and V, and it is capable of creating realistic slide guitar sounds. The virtual slide guitar has a gestural camera-based user interface, so that the user can play this virtual instrument simply by making slide guitar playing movements in front of a camera, similarly as presented in Karjalainen et al. [2006].

Finally, a physics-based synthesis model for flageolet tones on string instruments is presented in article VII. Flageolet tones (a.k.a. harmonics) can be generated nearly with all string instruments by damping the string at some specific locations. The advantage of the proposed technique over previous flageolet tone modeling mechanisms is that it allows a more realistic simulation also when the damping is varying in time, which often happens in real playing situations.

2 Linear string acoustics

2.1 Ideal string vibration

The vibration of a string can be decomposed into four orthogonal components. This means that each infinitely short string segment can move in four independent directions, called polarizations. The first two polarizations, horizontal and vertical, are transversal to the string, i.e. they move perpendicular to the direction of the string. The third polarization, longitudinal, expresses the compressional waves within the string medium. Finally, the fourth polarization, torsional, describes the string's movement around its longitudinal axis.

Although the torsional polarization plays an important role in characterizing the vibration of a bowed string, it has a negligible effect in the case of plucked strings. Thus, only transversal and longitudinal vibrations are discussed in what follows. Also, since the physical laws dictating the behavior of the horizontal and vertical polarizations are the same, mainly the transverse string displacement u is discussed in what follows. The mathematics in the remainder of this section follow the derivations presented in several textbooks, e.g. [Fletcher and Rossing, 1988], except that here the focus is on obtaining the string's impulse response.

Consider an ideal string – a perfectly flexible, lossless cord – which is tightly fixed at both ends. The transversal vibration of this theoretical string is governed by the linearized inhomogeneous wave equation

$$u_{tt} - c_{\mathrm{t}}^2 u_{xx} = \frac{f(x,t)}{\mu}, \tag{2.1}$$

where u_t and u_x denote the temporal and spatial partial derivatives, respectively. Variable

$$c_{\mathrm{t}} = \sqrt{\frac{T}{\mu}} \tag{2.2}$$

denotes the transversal wave velocity, where T is the tension of the string and μ is its linear mass density, i.e. mass per unit length. Variable f in Eq. (2.1) denotes an external excitation force density, which depends on the longitudinal coordinate x and time, t. In other words, Eq. (2.1) relates the string's acceleration to its curvature and an external excitation. The longitudinal vibration of the string obeys a similar rule as in Eq. (2.1), except that for the longitudinal wave velocity

$$c_{\mathrm{l}} = \sqrt{\frac{EA}{\mu}}, \tag{2.3}$$

where E is Young's modulus and A is the cross-sectional area of the string.

As discussed in Sec. 1.1, d'Alembert solved Eq. (2.1) using the traveling-wave solution

$$u(x,t) = u_0(x + c_{\mathrm{t}}t) + u_0(x - c_{\mathrm{t}}t), \tag{2.4}$$

where $u_0(x) = \frac{1}{2}u(x,0)$ denotes the initial string displacement. Although the synthesis algorithms discussed later in this thesis are mainly based on d'Alembert's solution, it is educational to take a closer look at the closed-form solution of Eq. (2.1), presented by Bernoulli:

$$u(x,t) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi x}{L}\right)\left[a_n \cos\left(\frac{n\pi c_{\mathrm{t}} t}{L}\right) + b_n \sin\left(\frac{n\pi c_{\mathrm{t}} t}{L}\right)\right]. \tag{2.5}$$

Here, L is the length of the string, so that $0 \leq x \leq L$. Symbols a_n and b_n are constants defining the modal amplitudes. By setting $t = 0$ in Eq. (2.5), the initial displacement of the string can be given as

$$u(x,0) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right). \tag{2.6}$$

For the initial velocity, taking the time derivative of Eq. (2.5) and setting $t = 0$ yields

$$u_t(x,0) = \sum_{n=1}^{\infty} \frac{n\pi c_{\mathrm{t}}}{L} b_n \sin\left(\frac{n\pi x}{L}\right). \tag{2.7}$$

In order to solve a_n, Euler and Lagrange suggested multiplying each side of Eq. (2.6) by $\sin(k\pi x/L)$ and integrating from 0 to L [Robinson, 1982] to get[2]

$$a_n = \frac{2}{L}\int_0^L u(x,0)\sin\left(\frac{n\pi x}{L}\right)\mathrm{d}x. \tag{2.8}$$

Similarly, Eq. (2.7) gives

$$b_n = \frac{2}{n\pi c_\mathrm{t}}\int_0^L u_t(x,0)\sin\left(\frac{n\pi x}{L}\right)\mathrm{d}x. \tag{2.9}$$

In other words, if the initial string displacement and velocity are known, the vibration of an ideal string is explicitly given by Eqs. (2.5), (2.8), and (2.9). It must be noted that Eqs. (2.5) through (2.9) give the solution for the homogeneous wave equation, corresponding to the case where $f(x,t) = 0$ in Eq. (2.1).

Since the ideal string is an LTI system, the closed-form solution to the inhomogeneous wave equation can be obtained by using the impulse response. This would mean setting $f(x,t) = \delta(x-x_0)\delta(t)$ in Eq. (2.1), i.e. using a (spatial and temporal) force density impulse as an excitation. If the initial string displacement is zero, applying a force impulse at $x = x_0$ at time $t = 0$ corresponds to setting the initial acceleration to

$$u_{tt} = \frac{\delta(x-x_0)\delta(t)}{\mu}, \tag{2.10}$$

due to Newton's second law. Hence, the initial velocity becomes

$$u_t = \int \frac{\delta(x-x_0)\delta(t)}{\mu}\mathrm{d}t = \frac{\delta(x-x_0)}{\mu}. \tag{2.11}$$

Substituting Eq. (2.11) into (2.9) and reformulating Eq. (2.5) (note that for zero initial displacement $a_n = 0$) gives

$$u(x,t) = \sum_{n=1}^{\infty}\sin\left(\frac{n\pi x}{L}\right)\frac{2}{n\pi c_\mathrm{t}}\int_0^L \frac{\delta(x-x_0)}{\mu}\sin\left(\frac{n\pi x}{L}\right)\mathrm{d}x\,\sin\left(\frac{n\pi c_\mathrm{t} t}{L}\right). \tag{2.12}$$

[2]More rigorously, it can be stated that the functions $\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$ are an orthonormal system of the space $\mathcal{L}^{(2)}[0,L]$.

Additionally, noting that since the frequency of mode n is $f_n = nc_\mathrm{t}/(2L)$, the transversal wave velocity can be written as $c_\mathrm{t} = 2Lf_n/n$. Also, since it is easy to see that the integral in Eq. (2.12) equals $\sin\left(n\pi x_0/L\right)/\mu$, the string vibration resulting from the force impulse becomes

$$u(x,t) = \frac{1}{\pi L\mu}\sum_{n=1}^{\infty}\sin\left(\frac{n\pi x}{L}\right)\frac{1}{f_n}\sin\left(2\pi f_n t\right)\sin\left(\frac{n\pi x_0}{L}\right). \tag{2.13}$$

Thus, Eq. (2.13) represents the impulse response of an ideal string, fixed at both ends. In order to get the response for any point-like excitation $f(x_0,t)$, time-domain convolution must be applied.

For obtaining the string response to any spatially distributed excitation $f(x,t)$, the case is different. It must be noted that although the system under discussion is LTI, it is not spatially invariant. This means that the vibration of a point on the string depends on its longitudinal coordinate, and simply convolving $f(x,t)$ with Eq. (2.13) in space does not yield the correct solution. Instead, one must spatially integrate $f(x,t)$ with the modal shapes in order to get the temporal excitation for each mode separately. In other words, since Eq. (2.13) gives the string response for any point-like excitation, the string response for a spatially distributed excitation can be obtained using superposition, i.e. summing over the spatial points. Formally, the string response for arbitrary force excitation is thus given as

$$u(x,t) = \frac{1}{\pi L\mu}\sum_{n=1}^{\infty}\sin\left(\frac{n\pi x}{L}\right)H_n(t) * F_n(t), \tag{2.14}$$

where $*$ denotes convolution and

$$H_n(t) = \frac{1}{f_n}\sin\left(2\pi f_n t\right) \tag{2.15}$$

is the impulse response of mode n and

$$F_n(t) = \int_0^L \sin\left(\frac{n\pi x}{L}\right) f(x,t)dx \tag{2.16}$$

is the excitation force of that mode.

2.2 Losses and stiffness

In contrast to the ideal string, real strings lose their energy and their vibration gradually decays. Three main reasons for the decay are [Fletcher and Rossing, 1988] air damping, internal damping, and transfer of mechanical energy through the supports. The first two of these are dissipative mechanisms due to viscoelastic and thermodynamic losses. They convert the kinetic energy of the string into heat and are therefore irreversible processes. Mechanical energy transfer is a process in which the kinetic energy flows from the string to the instrument body. A small portion of this energy can also flow back to the string, thus making the energy transfer bidirectional in nature. These loss types are briefly discussed in what follows. For a more detailed study of loss mechanisms in musical strings, refer to [Valette, 1995].

Air damping is caused by the viscous flow that retards the movement of the string. There is also a small energy transfer from the string directly into the surrounding air, but since the string itself is a poor radiator (its diameter is small compared to the wavelength) this loss is often considered negligible. For a given frequency, air damping causes an exponential decay of vibration. This decay is expressed using the time constant τ_{air}, which is a function of the properties of the string and air and the vibration frequency [Fletcher and Rossing, 1988]. A physical interpretation for τ_{air} is the time it takes for the string vibration to decay to $1/e$ part due to air damping only.

Internal damping consists of all thermo-[3] and viscoelastic forces that resist movement within the string structure. The decay caused by internal damping is characterized with time constant τ_{int}, which has a similar physical interpretation as τ_{air}, except that instead of air damping, only internal damping is considered. String stiffness introduces additional hysteretic damping, which can also be included in τ_{int}, if the frequency-

[3] Although thermoelastic losses generally play a minor role in metallic strings, they can have a noticeable effect in a certain frequency region, as stated in [Zener, 1937; Valette, 1995].

dependent behavior of the internal losses are suitably selected. For wound strings, dry friction, i.e. the friction between contiguous turns of the wound wire, also damps the string movement. The detailed effect of this phenomenon is, however, not well understood [Valette, 1995].

Damping through the end supports is often characterized by the coupling impedance, or its reciprocal, admittance (mobility), between the string and the instrument body. The impedance gives the ratio between force and the resulting velocity, and it depends on the string and body properties, contact type, and vibration frequency. The damping effect of the end supports is again of exponential type and thus it can be given as a time constant τ_{sup}, which essentially is a function of impedance [Fletcher and Rossing, 1988].

The net effect of the major losses can now be expressed using a resistance term [Morse, 1948; Bank, 2006]

$$R(f) = \frac{1}{\tau_{\mathrm{tot}}} = \frac{1}{\tau_{\mathrm{air}}} + \frac{1}{\tau_{\mathrm{int}}} + \frac{1}{\tau_{\mathrm{sup}}}, \tag{2.17}$$

which is the effective frictional resistance per unit mass. Naturally, R depends not only on the vibration frequency, but also on the physical properties of the string and air discussed above. In practice, the correct value of R cannot be defined analytically, but must be measured from the decay times of real string instruments over some frequency range. For this reason, it is most practical to denote R as a function of frequency only.

As opposed to ideal strings, real strings are not perfectly flexible, but possess some bending stiffness. This means that in addition to the string tension T, there is another restoring force that tends to keep the string in its equilibrium position. In practice, the stiffness causes dispersion, i.e. harmonic frequencies become stretched so that the upper partials end up higher in frequency when compared to the perfectly harmonic case. Stiffness plays a major role in the vibration of thick strings, such as those used in the piano, but it is not important for guitar strings due to their smaller diameter [Järveläinen and Karjalainen, 2006].

Finally, the partial differential equation (PDE) governing the motion of a lossy string with nonzero stiffness is given as

$$u_{tt} - c_{\mathrm{t}}^2 u_{xx} + 2R(f)u_t + \frac{EA\kappa^2}{\mu} u_{xxxx} = f(x,t), \tag{2.18}$$

where κ is the radius of gyration. The solution to Eq. (2.18) is the same as that given in Eq. (2.14), except that the losses cause the mode impulse response presented in Eq. (2.15) to have an additional multiplicative decay term [Morse, 1948]:

$$H_{n,\mathrm{lossy}}(t) = \frac{e^{-R_n t}}{f_n} \sin\left(2\pi f_n t\right), \tag{2.19}$$

where R_n is the frictional resistance for mode n. In principle, a finite damping through the supports lowers the modal frequencies since the vibrational length of the string increases. However, if the damping is small, this shift in frequencies can be considered negligible, i.e.

$$f_n = \frac{1}{2\pi}\left[\left(\frac{\pi n c_{\mathrm{t}}}{l}\right)^2 - R(f)^2\right]^{\frac{1}{2}} \approx \frac{n c_{\mathrm{t}}}{2l}. \tag{2.20}$$

The stiffness causes the spreading of the modal frequencies so that

$$f_n = f_0 n\sqrt{1 + Bn^2}, \tag{2.21}$$

where f_0 is the fundamental frequency and

$$B = \frac{EA}{T}\left(\frac{\pi\kappa}{L}\right)^2 \tag{2.22}$$

is called the inharmonicity coefficient [Fletcher et al., 1962].

3 Physics-based sound synthesis

The purpose of physics-based sound synthesis is to create artificial instrument sounds using the laws of physics. In most cases, it is desirable that the synthesized sounds resemble the sound of real instruments as much as possible. In some cases, however, the synthesized sounds should represent the tones played on an imaginary instrument, a musical tool for which a real-life implementation would be impractical or even impossible to build (consider, e.g. the 24-string virtual super guitar, introduced in [Laurson et al., 2002]).

At the time of the writing of this thesis, the largest drawback of physics-based sound synthesis is the poor sound quality compared to sample-based synthesis methods. This is caused by the fact that in order to be able to run in real-time, the physical models often have to be simplified greatly. Also, insufficient control data causes the models to sound unrealistic; high-quality guitar synthesis, for example, is hard to implement with a different user interface than the one in a real guitar. Sample-based synthesis methods can obviously produce very realistic instrument sounds, but they are greatly limited in flexibility, since they are essentially just modified record-and-playback machines. Although sample-based approach is suitable for synthesizing instruments with a relatively small expressive range, such as the organ, it fails to reproduce the nuances of a more expressive instrument, such as the violin. An excellent article on evaluation of different sound synthesis techniques can be found in [Jaffe, 1995].

The major advantage in all model-based sound synthesis is the flexibility; the same model can produce a myriad of different sounds. For example, a general bowed-string instrument model could produce all the tones of the whole bowed-string instrument family. Preferably, the synthesizer would conceptually act like a real instrument, so that the user's action would produce an intuitive change in the produced sound. This way, a musician could play the virtual model just like a real instrument, without first having to undergo special training for getting used to the model parameters. It must be noted that such a system does not necessarily have to be implemented using physics; sample-based synthesizers and spectral synthesis models can aim to do this as well, although it usually is much more complicated.

Here lies, however, probably the greatest advantage of physics-based sound synthesis: the mapping between the model parameters and the aspects of the resulting sound is objective, i.e. defined by the universal laws of nature. In other synthesis methods, this mapping is subjective. This means that in non-physics-based synthesis techniques the system designer gets to choose the representation of the model parameters. Thus, he might call a certain parameter “distance” just because changing that parameter seems to change the distance between the listener and the virtual instrument in his own opinion. In physics-based models, mother nature has made these decisions already, and there is no need for the error-prone subjective part. It must be noted that in practice, it is advisable to use also perceptual information in deciding which physical phenomena are to be simulated and to which extent. If real-time sound synthesis is to be obtained, there is little use of simulating processes that do not yield audible results. Perceptual studies of synthesized string instrument sounds have been reported e.g. in [Järveläinen, 2003].

An exhaustive tutorial on physics-based sound synthesis methods is provided in article III. For this reason, only those two modeling techniques, digital waveguides (DWGs) and wave digital filters (WDFs), that are essential for understanding the research results presented in this thesis are discussed in sections 3.2 and 3.3, respectively. Furthermore, the discussion is focused on guitar-related modeling, although these paradigms can be used for simulating various other systems as well. Both digital waveguides and wave digital filters are based on the wave-formalism, which is elaborated in the following.

3.1 Kirchhoff vs. wave models

As discussed in article III, physics-based sound synthesis techniques can be divided into two categories: those that operate with Kirchhoff (K for short) variables, and those that operate with wave (W) variables. Operating with K variables means that in addition to the system states, also the digital signals inside the model represent physical quantities directly. Often these quantities are arranged in pairs (called Kirchhoff pairs, hence the name) such as voltage and current or pressure and volume flow. The operation of a system is then usually defined using immittances, i.e. impedances or admittances, which give the relation between the Kirchhoff pair.

Operating with W variables means that instead of using physical quantities directly, the model uses d'Alembert's approach (see Sec. 2) and represents its variables as wave decompositions of the physical quantities. Thus, in a W-based model, the signals inside the synthesis engine represent waves, e.g. pressure waves, traveling in opposite directions, and the actual physical quantities are obtained by summing the waves together. In W-based models, the operation of a system is defined by its reflectance instead of its immittance. Reflectance is defined as the relation between the wave going into the system and the wave coming out (i.e., reflecting) from the system. From the signal processing point of view the system reflectance can be seen as equivalent to the system's transfer function, representing the ratio between the input and output signals in the frequency domain.

A major conceptual difference between K- and W-models is that in W-based systems the direction of causality is fixed, as discussed in article III. In other words, a decision has been made that the input causes the output. In K-models, the direction of causality is left open. For example, in a mass-spring oscillator one might think that the restoring force of the spring causes the mass to gain velocity, or alternatively that the velocity of the mass results in spring displacement and therefore causes a restoring force. If the system is LTI, the direction of causality is not important from the modeling point of view, since both interpretations yield the same model. However, with non-LTI systems a different interpretation leads to a different model. W-based modeling techniques avoid this ambiguity by forcing the system designer to choose explicitly the input and output variables.

It must be noted that although immittances and reflectances are frequency-domain quantities, the actual modeling is most often carried out in the time domain. In practice, this is obtained by approximating them with digital filters.

3.2 Digital waveguides

The term digital waveguide modeling was introduced by Julius Smith in the 1980s [Smith, 1985, 1987]. The same principle of using wave variables and scattering junctions had been used already earlier in the Kelly-Lochbaum speech synthesis model [Kelly and Lochbaum, 1962; Strube, 2000]. Also the Karplus-Strong string model [Karplus and Strong, 1983] can be seen as a simplified case of a DWG system, although its relation to physics-based modeling was not apparent at the time of its introduction. For an excellent tutorial on DWG modeling, see [Smith, 1992].

In practice, DWG systems can be efficiently constructed using delay lines which contain the traveling wave components. In DWG strings, the time delay of the delay loop equals the inverse of the string's fundamental frequency. For correct tuning, fractional delay filters [Laakso et al., 1996] are used. Figure 3.1 illustrates a simple DWG string. In the most straightforward implementation, only a pointer update per time step is required for modeling wave propagation. Simulation of losses, inharmonicities, etc. can

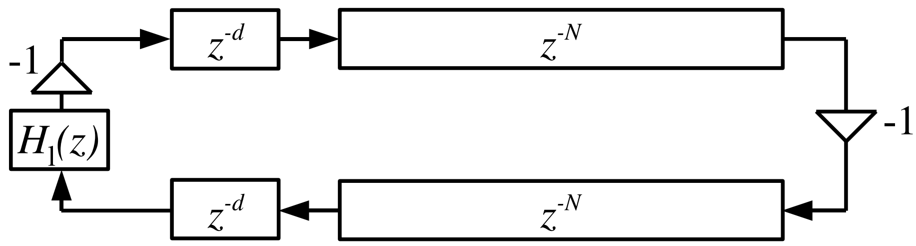

Figure 3.1: A simple DWG string implementation consists of two delay lines, a fractional delay filter for applying the fractional part of the delay, and a loop filter for simulating losses (after [Smith, 1992]).

be implemented with FIR and IIR filters. Formally, DWG blocks can be seen as reflectance functions, discretized using the impulse invariant transform. It must be noted that the DWG system itself does not have an anti-aliasing mechanism, thus the user has to ensure that the input signals are properly band-limited to half of the Nyquist limit to avoid aliasing.

As shown in Fig. 3.1, the simulation of a simple vibrating string can be very efficiently carried out using DWGs. Concatenating the two delay lines in Fig. 3.1 leads to an even simpler single-delay-loop [Karjalainen et al., 1998] DWG model. However, since string instruments do not consist only of the vibrating string, the sound coloration due to the instrument body is an important auditory aspect and must be simulated also. The most straightforward implementation would be to apply a digital filter simulating the instrument body's transfer function at the string's output. The shortcoming of this approach is that for realistic sound synthesis, a large filter would have to be used, which would slow down the total operation of the instrument model. A solution for this problem was introduced in 1993 independently in articles [Smith, 1993] and [Karjalainen et al., 1993b], which suggested that the FIR body filter be commuted with the DWG input. In other words, the body's impulse response would be used as the string excitation. This approach simplifies greatly the synthesis algorithm and is perfectly valid as long as the string can be considered LTI. For nonlinear or time-varying strings, e.g. strings that change their pitch during vibration, commuted DWG synthesis cannot directly be used, since changing the total delay length would vary the instrument body resonances.

3.3 Wave digital filters

Wave digital filters are a special class of digital filters with physically meaningful parameters. The WDF technique was formulated by Alfred Fettweis in the late 1960s [Fettweis, 1971] for discrete-time modeling of analog electric circuits. For a tutorial on WDF modeling, see [Fettweis, 1986]. Unlike DWGs, WDFs are designed for simulating lumped, i.e. point-like, systems, although they can be extended for simulating multidimensional systems in some cases [Bilbao, 2001].

Another difference between DWGs and WDFs is the type of discretization: WDFs discretize the system reflectance using the bilinear transform, which maps the analog frequency axis in the s-domain inside the unit circle in the z-domain. This avoids aliasing of the system response, but introduces warping of the high frequencies since the infinite analog frequency is mapped onto the Nyquist frequency in the digital domain. It must be noted that this warping takes place only in the system response and not in the wave variables themselves. Thus, also with WDFs, the input signal must be band-limited to half of the Nyquist limit to avoid signal aliasing. In practice, DWG and WDF systems can be interconnected (through a scaling coefficient in some cases), as the wave variables are essentially the same. Wave digital filters are especially well-suited for modeling electric circuits. This is convenient for simulating the electric guitar, since the circuitry involved in the magnetic pickups and the amplifier forms an essential part of the instrument's sound.

A basic WDF model consists of elementary blocks called one-ports, which are interconnected using N-port adaptors. The one-ports simulate the basic circuit elements, such as resistors, capacitors, and inductors. The adaptors come in two types: series and parallel, and they represent series and parallel connections, respectively. The purpose of an adaptor is to implement wave scattering between the one-ports so that Kirchhoff's continuity rules are preserved. A straightforward implementation method is to connect the one-ports using three-port adaptors so that the WDF circuit becomes a binary tree [De Sanctis et al., 2003]. Since the operation of the WDF network is based on Kirchhoff's continuity rules, energetic stability is guaranteed automatically[4] in time-invariant structures. In the time-varying case, if the system response does not change rapidly, the system usually remains stable even though stability can no longer be guaranteed theoretically. Figure 3.2 illustrates a simple electrical circuit and its WDF representation.

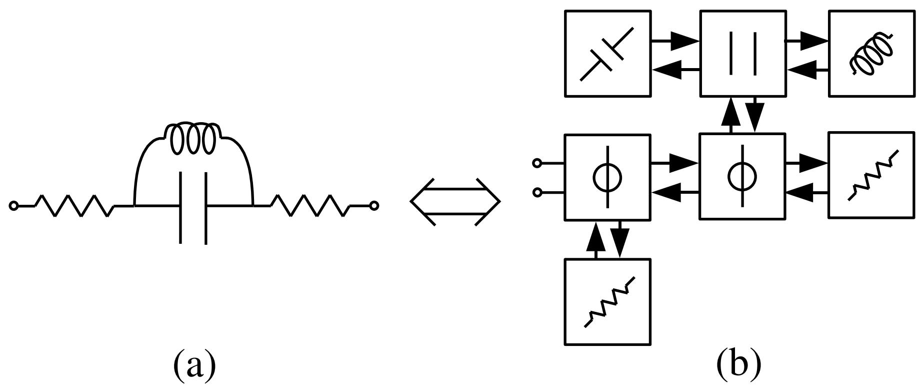

Figure 3.2: A simple electrical circuit, the Kilroy bandstop (a) and its WDF equivalent (b). In (b), resistor, capacitor, and inductor elements are connected using three-port adaptor blocks. The lower adaptors (marked with ϕ) denote series connections, while the upper adaptor (marked with $||$) stands for parallel connection.

[4]With nonlinear WDFs, power-normalized waves [Bilbao, 2001] or variable turns-ratio transformers [Meerkötter and Felderhoff, 1992] should be used for ensuring energy conservation.

Each port in a WDF network holds a computational parameter, the port impedance, which is used in calculating the wave scattering. Effectively, the port impedance values of all circuit elements are interdependent. By choosing the individual port impedances correctly, the one-port elements become extremely simple DSP blocks, where the instantaneous dependency between input and output is removed. For a list of some typical circuit components and their WDF realizations, see article III, p. 41. Since the signal flow between each element is bidirectional, special scheduling is needed in order for the WDF network to be realizable. The binary-tree approach [De Sanctis et al., 2003] uses reflection-free ports for implementing the scheduling. A detailed description of the binary-tree method is provided in Sec. 8 of article III.

If the modeled circuit is LTI, the port impedances remain constant throughout the simulation. Unfortunately, this is not the case with nonlinear WDF elements. Consider, for example, that the leftmost resistor in Fig. 3.2(a) would be nonlinear. This would mean that its resistance value, and thus the port impedance, would vary as a function of the incoming signal. Since the port impedances are interconnected through adaptor elements, changing the port impedance of any element would require a recalculation of every port impedance within the circuit. The binary-tree approach [De Sanctis et al., 2003] can handle one nonlinear element in the WDF network using its special scheduling technique. Other nonlinearities can be connected through delay blocks, if desired. For memoryless nonlinearities, i.e. nonlinear resistors, the reflectance can be implemented as a simple lookup table, as done in article IV. For nonlinearities with memory (nonlinear reactances), special mutator elements can be used [Sarti and De Poli, 1999].

It must be noted that, with nonlinearities, aliasing cannot always be avoided even with properly bandlimited input signals. The reason for this is that the nonlinear distortion creates high-frequency signal components that will alias back to the baseband. This aliasing is audible if the nonlinearity is too strong. In article IV, aliased components are suppressed by using temporal oversampling.

4 Modeling of nonlinear and time-varying phenomena

This chapter focuses on explaining a set of non-LTI phenomena related to the guitar, namely tension modulation, vacuum-tube nonlinearity, time-varying pitch and damping, and handling noise. Previous simulation attempts are recapitulated, and new synthesis models introduced in articles I-IV and VI-VII are discussed. For a general overview of musical instrument nonlinearities, see [Fletcher, 1998]. Modeling of various non-LTI effects in musical instruments are discussed in article III and [Bilbao, 2007].

4.1 Geometric string nonlinearities

The term "nonlinear strings", widely used in the literature, usually refers to a special vibrational aspect, where the spatial structure of the string causes nonlinear behavior. Thus, the nonlinearity is caused by the geometry of the string rather than its material properties, for example. This type of nonlinearity will be studied more thoroughly in what follows.

4.1.1 Previous work

Many publications considering geometric string nonlinearities can be found in the literature; see overviews in [Narasimha, 1968; Tolonen, 2000; Erkut, 2002; Bank, 2006]. First studied by Kirchhoff in the late 19th century and later revised by Carrier [Carrier, 1945], the geometric nonlinearities in strings are responsible for various phenomena.

One of the most audible effects is the initial pitch glide phenomenon [Carrier, 1945; Valette, 1995], where a heavily plucked string experiences a pitch descent as its vibration decays due to tension modulation. Another interesting effect is the generation of missing modes due to nonlinear coupling, where new vibrational modes are generated after the plucking event [Miles, 1965; Legge and Fletcher, 1984; Feng, 1995; Valette, 1995; Conklin, 1999]. Transversal polarizations are also coupled due to the nonlinearity, causing the vibration to have a whirling motion [Murthy and Ramakrishna, 1964; Miles, 1965; Anand, 1969; Elliott, 1980; Gough, 1984; Miles, 1984]. The coupling between transversal and longitudinal modes [Morse and Ingard, 1968; Giordano and Korty, 1996] in turn leads to generation of another set of harmonics [Nakamura and Naganuma, 1993], called phantom partials [Conklin, 1999]. The generation mechanism of phantom partials is explained in detail in [Bank and Sujbert, 2005].

In thin strings with relatively high tension, such as those used in a guitar, the longitudinal and transverse vibrations can be considered separable [Oplinger, 1960; Narasimha, 1968; Anand, 1969], as will be discussed later in this thesis. Coupling between the transverse modes through torsional vibration has been discussed in [Watzky, 1992]. The nonlinearity also causes the string to experience amplitude jumps under forced oscillation [Murthy and Ramakrishna, 1964; Tufillaro, 1989; Hanson et al., 1994]. An excellent classification of geometric string nonlinearities can be found in [Bank, 2006].

4.1.2 Tension modulation

A more thorough explanation for the pitch glide and generation of missing harmonics due to tension modulation is given in the following. The derivation follows the one presented in [Legge and Fletcher, 1984]. A similar approach has also been used in [Bank, 2006].

Consider the short string segment illustrated in Fig. 4.1. The horizontal bold line denotes the segment of length $\mathrm{d}x$ in its equilibrium position. Next, the string is displaced so that the end originally at $(x_1, 0)$ is moved by v in the longitudinal, and by u in the transversal direction. The other end, originally at $(x_2, 0)$ experiences a displacement of $v + v_x\mathrm{d}x$ in the longitudinal and $u + u_x\mathrm{d}x$ in the transversal direction. The bold arc in Fig. 4.1 is the segment after the displacement. Since the new length $\mathrm{d}s$ of the segment differs from the original length $\mathrm{d}x$, the local tension of the string can be expressed as [Oplinger, 1960]

$$T(x) = T_0 + EA(\mathrm{d}s - \mathrm{d}x)/\mathrm{d}x \tag{4.1}$$

assuming that the relative strain $(\mathrm{d}s - \mathrm{d}x)/\mathrm{d}x$ is sufficiently small that Hooke's law still holds. Using linear approximation in Fig. 4.1, the tension can be written in terms of u and v [Carrier, 1945]:

$$T(x) = T_0 + EA\left(\sqrt{(1 + v_x)^2 + u_x^2} - 1\right). \tag{4.2}$$

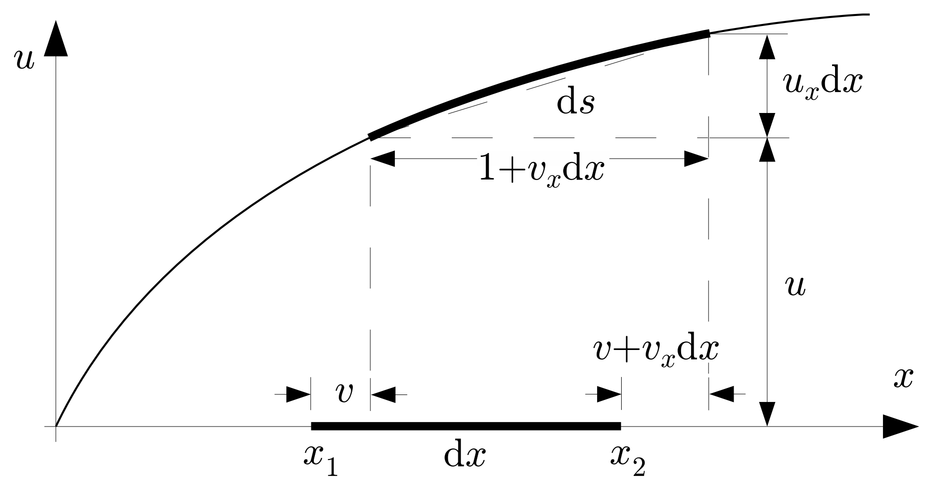

Figure 4.1: Displaced string segment (after [Carrier, 1945] and [Murthy and Ramakrishna, 1964]).

In the case of guitar strings, the inequality $T/EA \ll 1$ usually holds (typical values for a low-e string made of steel are $T \approx 76N$, $E \approx 20 \times 10^{10} \frac{\mathrm{N}}{\mathrm{m}^2}$, and $A \approx 1.04\mathrm{mm}^2$). Thus, looking at Eqs. (2.2) and (2.3) it can be seen that the longitudinal wave velocity is considerably larger than the transversal one. A simplifying assumption can now be made; if the longitudinal tension propagation is considered instantaneous[5], each point on the string will experience the same tension at a given time instant, and the tension becomes uniform [Oplinger, 1960][6]:

$$T = T_0 + EA(L' - L)/L, \tag{4.3}$$

where

$$L' = \int_0^L \sqrt{1 + u_x^2}\mathrm{d}x \approx L + \frac{1}{2}\int_0^L u_x^2 \mathrm{d}x \tag{4.4}$$

is the elongated length, i.e. the total length of the displaced string. Substituting Eq. (4.4) in Eq. (4.3) gives an approximation for the spatially uniform tension

$$T = T_0 + \frac{1}{2}\frac{EA}{L}\int_0^L u_x^2 \mathrm{d}x. \tag{4.5}$$

In the lossy case, the impulse response of the string can be written using Eqs. (2.14) and (2.19) as

$$u(x,t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{L}\right) \sin(2\pi f_n t + \phi_n) e^{-R_n t}, \tag{4.6}$$

where

$$a_n = \frac{1}{\pi L \mu f_n}. \tag{4.7}$$

The ϕ_n term in Eq. (4.6) simply denotes the initial phase of mode n. Substituting Eq. (4.6) into Eq. (4.5) yields the time-varying tension as a function of the string vibration:

$$T(t) = T_0 + \frac{1}{2}\frac{EA}{L}\int_0^L \left(\sum_{n=1}^{\infty} a_n \frac{n\pi}{L}\cos\left(\frac{n\pi x}{L}\right)\sin(2\pi f_n t + \phi_n)e^{-R_n t}\right)^2 \mathrm{d}x. \tag{4.8}$$

[5]More precisely, the excitation bandwidth must be small enough so that the longitudinal modes are excited well below their resonances [Bank, 2006].

[6]This simplification is sometimes referred to as Anand's argument due to a more detailed three-dimensional study presented in [Anand, 1969]. However, it first appears in [Oplinger, 1960].

Carrying out the integration and neglecting some higher-order terms leads to the expression [Legge and Fletcher, 1984]:

$$T(t) \approx T_0 + \frac{\pi^2 EA}{8L^2} \sum_{n=1}^{\infty} n^2 a_n^2 \left(1 - \cos(4\pi f_n t + 2\phi_n)\right) e^{-2R_n t}. \tag{4.9}$$

Two remarkable observations can now be made by studying Eq. (4.9). Firstly, the exponential term reveals that, due to the frictional resistance, the modulating tension component decays in time. This explains the initial pitch glide effect, i.e. the fact that with heavily plucked strings the pitch has initially a higher value that decreases as the vibration decays. Secondly, the cosine term tells that the tension oscillates with a double frequency compared to the transversal vibration.

From the 1D wave equation (Eq. (2.1)), one would now assume that the varying tension would modulate the string curvature and excite vibrational modes near the corresponding sum and difference frequencies. A closer look, however, proves this assumption wrong. Consider a vibrating string fixed at both ends and characterized by Eqs. (2.1) or (2.18). For each transversal mode n, the tension varies with frequency $2f_n$. If the transversal vibration also carries a mode m, the driving force Tu_{xx} has components at frequencies $2f_n \pm f_m$. Note that even though the tension is the same for all points on the string, the driving force is not. For efficient mode excitation, two criteria must be filled: (1) the spatial shape of the exciting force must match the shape of the mode under excitation, and (2) their frequencies must be relatively close. In other words, both their spatial and temporal frequencies must match [Legge and Fletcher, 1984].

Listing out different mode numbers for n and m reveals that the two criteria above are met only when $n = m$. This means that if the string ends are fixed, the modes can only act back on themselves, and tension modulation cannot excite modes that are not initially present in the transversal vibration. In reality, the string ends are not perfectly fixed, but have a finite impedance. Figure 4.2 shows a typical termination case for a guitar.

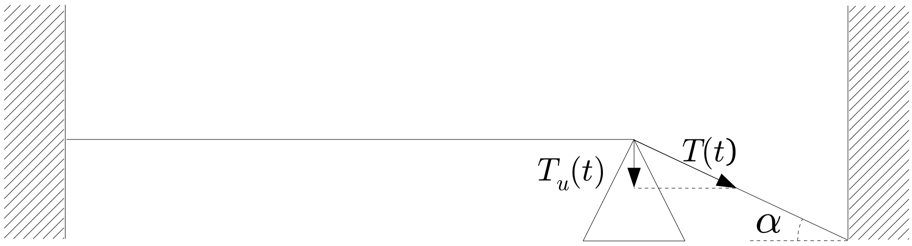

Figure 4.2: Typical string termination in a guitar. The tension variation $T(t)$ has a transversal component $T_u(t) = \sin(\alpha)T(t)$ (after [Legge and Fletcher, 1984]).

Figure 4.2 shows that due to the angled termination, the tension variation is directly coupled to the transversal vibration, providing excitation for the double-frequency modes. Thus, for a middle-plucked string, although the even modes are initially missing from the spectrum (since they have a node at the excitation location), they experience a gradual onset as the vibration continues. In reality, the coupling from tension modulation to transversal vibration is not unidirectional, i.e. also the transversal vibration is coupled to the tension modulation due to the nonrigid bridge. However, since this phenomenon is likely to be less significant in practice, it is not discussed further here.

It must be noted that although the tension modulation was considered only in the two-dimensional case above, the results are similar for the whole three-dimensional system. Equations for the motion of a nonlinear string in 3D are provided in [Morse and Ingard, 1968] and [Bank, 2006][7].

[7]Note that the PDEs in [Morse and Ingard, 1968] differ slightly from the ones in [Bank, 2006]. The latter ones seem to be physically more valid, as explained in [Bank, 2006].

4.1.3 Modeling of tension modulation

Numerical simulation of the tension modulation nonlinearity has been carried out by several techniques. Energy-preserving finite-difference models have been introduced in [Furihata, 2001; Bilbao, 2004a,b] (see an overview in [Bilbao, 2005]). The advantage of these models is that their numerical stability can be guaranteed. On the other hand, their computational requirements are often too demanding for real-time sound synthesis. A modal-based approach for nonlinear string simulation has been taken in [Trautmann and Rabenstein, 2000] and [Bilbao, 2004b]. Also, hybrid models using finite-difference strings with resonators for simulating longitudinal modes have been presented [Bank and Sujbert, 2004; Bank, 2006], as well as models which use separate DWGs for simulating different polarizations [Bank and Sujbert, 2003]. Stabilization issues related to undamped nonlinear strings have been addressed in [Shahruz, 1999; Kobayashi and Sakamoto, 2007].

One popular modeling method has been to use DWGs, where the length of the delay line has been varied in order to simulate the initial pitch glide phenomenon [Karjalainen et al., 1993a; Välimäki et al., 1998; Välimäki et al., 1999; Tolonen et al., 1999, 2000; Erkut et al., 2002]. A similar approach had been taken already earlier by Pierce and Van Duyne for modeling a vibrating string terminated in a nonlinear spring [Pierce and Van Duyne, 1997]. However, since these algorithms use a lumped fractional delay at the termination of the waveguide, the whole string essentially becomes a dimensionless black-box model, where physically meaningful interaction is restricted to the string ends only. Article I tackles this deficiency by presenting a spatially distributed nonlinear digital waveguide string model, which allows interaction with the entire length of the string. This new waveguide model uses time-varying first-order allpass filters evenly distributed along the string. The desired delay change is evaluated from the tension variation, and the allpass filter coefficients are updated each time sample. It must be noted that although the model is presented in a spatially distributed form, the string tension is considered uniform.

Article I also introduces an experimental finite-difference model of a spatially distributed nonlinear string. The nonlinearity is simulated by using fractional delay filters to estimate the string state between sampling instants and thus varying the temporal sampling interval during run-time. Although the digital waveguide model discussed above performs better than the finite difference one, the derivation of the latter nicely shows that tension modulation can be simulated in a finite difference scheme by simply varying the time step of the algorithm during run-time.

4.1.4 Energy compensation

Another shortcoming of the previous nonlinear DWG models (and time-varying waveguides in general), is the fact that changing the waveguide length at run time introduces artificial energy variation. Consider, for example, a DWG string which is rapidly shortened to half of its original length. If no precautions are taken, half of the signal samples in the delay lines are discarded, and the string loses its energy. It must be noted that even though the energetic behavior of a real string under pitch change has not been clarified, this type of artificial energy loss caused by the delay-line structure of the DWG is clearly not physical and must be compensated for.

Article II presents two novel energy-compensation methods to resolve this problem. The first method uses the string length variation to evaluate the artificial energy change each time sample, and adjusts a scaling parameter inside the waveguide accordingly. The second method uses power-normalized WDF one-ports in series connection as delay lines in a DWG string. Due to the energy-preserving properties of power-normalized WDF elements, the resulting varying-tension DWG does not encounter the artificial energy change. Using the power-normalized DWG string presented in II, a new energy-preserving nonlinear DWG string model is proposed in article III. Basically, this nonlinear DWG model can be seen as an extension of the spatially distributed DWG string in article I, with inclusion of the WDF energy-conservation technique described in article II. The energy-scaling method, also introduced in II, is used in the virtual slide guitar, presented in article VI.

4.2 Vacuum-tube nonlinearity

Probably the single most important nonlinear phenomenon related to the electric guitar is the distortion effect caused by nonlinear amplification, vacuum-tube amplifiers in particular. From the engineering point of view, vacuum-tube technology seems outdated in comparison to the modern solid-state (i.e. transistor-based) technology, due to the large size, high power consumption, poor durability, and expensive price of the tube elements. Therefore, it might be surprising that throughout the history of the electric guitar, vacuum-tube amplifiers have been favored by most professional guitarists. A major reason for this is that the distinctive tone of tube amplifiers was popularized in the 1950s and 60s by the early rock'n roll bands. At that time, before the transistor revolution, vacuum-tube technology was the only viable solution for electric audio amplification. Since then, the perceptually favorable warm distortion of tube amplifiers has become the de-facto standard of the rock'n roll guitar sound. From the auditory point of view, vacuum tubes and transistors behave very similarly when operated in their linear range. However, major audible differences emerge when the signal starts to saturate [Hamm, 1973], as happens nearly always with the electric guitar.

A typical guitar amplifier unit consists of a preamplifier, equalization circuit, and a power amplifier. The purpose of the preamplifier is to magnify the relatively weak signal from the magnetic guitar pickups and to provide buffering so that the pickup response is not changed by the amplifier circuitry. The equalization unit (a.k.a. tone-stack) provides a typical V-shaped equalization for compensating the pickup's high resonance at mid-frequencies, and gives the user additional control over the tone. The power amplifier boosts the signal so that it is strong enough to drive a loudspeaker. In the so-called all-tube guitar amplifiers, both the pre- and power amplifier circuits are implemented with tube technology. Typically, they consist of one or more tube stages, i.e. circuits that contain a vacuum-tube and other electric components. The power amplifier also contains an output transformer for generating a suitable voltage for the loudspeaker and often a phase-splitter circuit for feeding two equal but opposite signals to the power tubes.

The nonlinear behavior of a triode tube-stage, encountered in preamps, is studied further in the following. The same principles can be applied for tetrode and pentode stages found in power amplifiers, although their operational details are different. An overview of vacuum-tubes used in audio applications can be found in [Barbour, 1998], while a more detailed tutorial on different vacuum-tube circuits is provided in [Langford-Smith, 1954]. The detailed vacuum-tube physics is discussed in [Spangenberg, 1948].

4.2.1 Operation of a triode stage

The circuit diagram of a typical triode tube stage is shown in Fig. 4.3, where the oval symbol denotes the triode tube. Physically, the tube consists of three electrodes: plate, cathode, and grid, thus the name triode. The electrodes are marked with black dots in Fig. 4.3, and the symbols V_{p}, V_{k}, and V_{g} denote the voltages between these respective points and the ground. The electrodes are enclosed in a glass shell, and a vacuum is generated inside this casing. The operational idea of the triode tube is that the current flowing from the plate to the cathode is controlled by the grid, which is located in

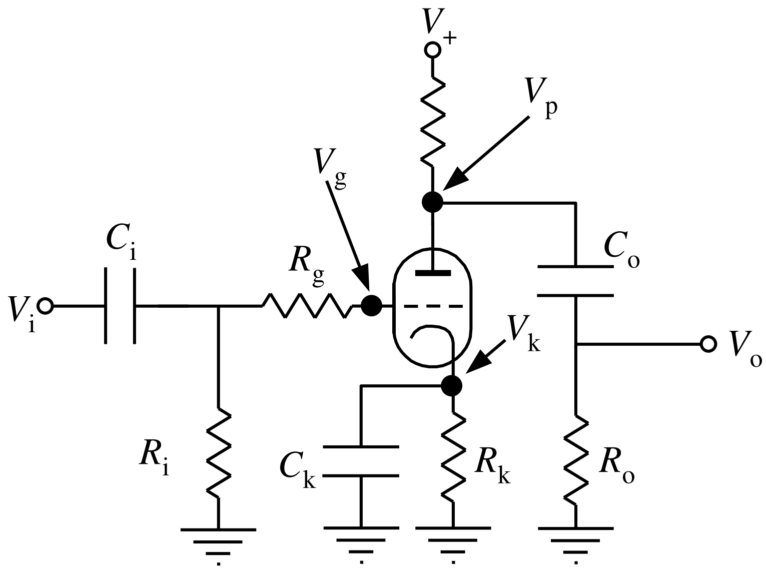

Figure 4.3: Circuit diagram of a typical triode tube stage (adapted from article IV).

between them.

More specifically, when the plate-to-cathode voltage $V_{\mathrm{pk}} = V_{\mathrm{p}} - V_{\mathrm{k}}$ is positive, current tends to flow from the plate to the cathode. When a negative grid-to-cathode voltage $V_{\mathrm{gk}} = V_{\mathrm{g}} - V_{\mathrm{k}}$ is applied, the plate-to-cathode current is disturbed, since less electrons leaving the cathode reach the plate due to the repelling negative charge of the grid in between. If the grid-to-cathode voltage reaches a large negative value, the plate-to-cathode current ceases altogether.

The input voltage V_{i} in Fig. 4.3 represents the tube stage input signal, while the output voltage V_{o} represents its output signal. The supply voltage V_{+} is typically in the range of one hundred volts or more. The capacitance C_{i} and resistance R_{i} provide high-pass filtering so that any DC component of the input signal is omitted. The resistance R_{g} serves for limiting the current in the grid circuit and to avoid instabilities. The cathode resistor R_{k} is used for setting the correct operation range for the tube, i.e. for making a negative bias for $V_{\rm gk}$. The cathode capacitance $C_{\rm k}$ bypasses the cathode resistor, increasing gain at audio frequencies. The output capacitance $C_{\rm o}$ and resistance $R_{\rm o}$ separate the DC component from the plate voltage $V_{\rm p}$ and yield the output voltage $V_{\rm o}$.

The plate-to-cathode current $I_{\rm pk}$ (which causes the output voltage $V_{\rm o}$) is a nonlinear function of the plate-to-cathode and grid-to-cathode voltages:

$$I_{\rm pk} = f(V_{\rm pk}, V_{\rm gk}). \tag{4.10}$$

Figure 4.4 illustrates the function in Eq. (4.10) for a typical triode tube 12AX7. It must be noted that besides being nonlinear, this relation also is essentially implicit, since $V_{\rm pk}$ depends on $I_{\rm pk}$. If a high-amplitude signal is fed to the input of the tube

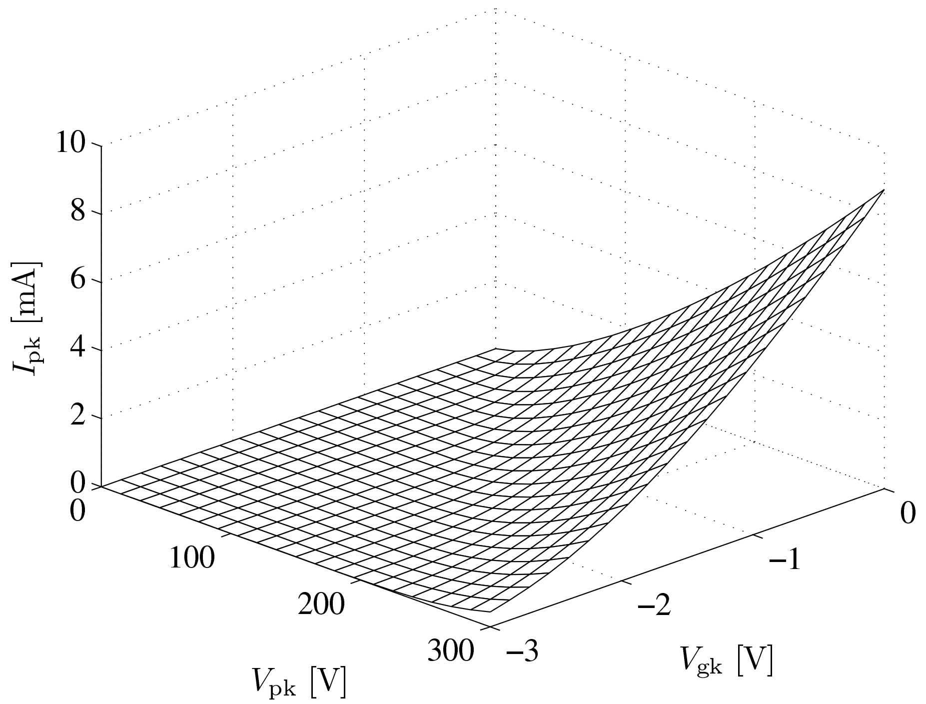

Figure 4.4: A simulated characteristic plane of a typical triode tube (12AX7). The current $I_{\rm pk}$ is a nonlinear function of both the plate-to-cathode and grid-to-cathode voltages $V_{\rm pk}$ and $V_{\rm gk}$.

stage, it might happen that the grid-to-cathode voltage becomes positive. In this case, a current I_{gk} starts to flow from the grid to the cathode, thus producing a voltage drop $I_{\mathrm{gk}}R_{\mathrm{g}}$ across R_{g}. This reduces the grid voltage and causes an additional nonlinearity.

4.2.2 Modeling of vacuum-tube amplifiers

Despite their acclaimed tone, tube amplifiers have some impractical properties: they are heavy, expensive, and relatively fragile. On the other hand, different classes of tube amplifiers are known to have quite different tonal qualities. Thus, for obtaining a variety of high-quality tones, guitarists have traditionally had to resort to an expensive arsenal of different tube amplifiers. For these reasons, there has been a demand for modeling amplifiers in recent decades. Several companies (e.g. Roland, Line6, Vox, Fender, Yamaha) have pursued making products that realistically simulate a collection of tube amplifiers.

Although solid-state amplifiers also produce nonlinear distortion, it is essentially different from the vacuum-tube distortion (see, e.g. [Santo, 1994]). Thus, for realistic solid-state modeling of tube amplifiers, special transistor-based emulation circuits have been suggested: [Todokoro, 1976; Sondermeyer, 1981, 1984; Pritchard, 1989, 1991; Butler, 1991; Tiers and Kieffer, 1991; Pritchard, 1992, 1995; Sondermeyer, 1996; Pritchard, 1997, 1998a,b,c]. Also, transistor-based models for simulating the guitar amplifier's loudspeaker [Pittman and Buck, 1990], and the loudspeaker-microphone-setup [Kelsey, 1998] have been reported.

Since the 1990s, the trend of tube amplifier modeling has shifted towards simulation via digital signal processing (DSP). The advantage of digital algorithms over analog electric circuits is their versatility: new model parameters are simply loaded into the system memory when the user switches between amplifier models. Also, the whole virtual amplifier can be implemented as a computer program, and model updates can be made available through the Internet. The term virtual analog modeling describes this general approach for digitally simulating analog electric circuits. Some digital amplifier modeling patents can be found in the literature [Kuroki and Ito, 1998; Doidic et al., 1998; Suruga et al., 2002; Gustafsson et al., 2004] while some others incorporate also the simulation of special audio effects [Pennock et al., 2003], and multi-channel amplifier modeling [Limacher et al., 2005, 2006]. Tube amplifiers and other nonlinear signal processing systems can also be approximated by varying the coefficients of a linear filter according to the input signal level, as suggested in [Kemp, 2006].

It is interesting to note that although there are various patents in solid-state tube modeling, surprisingly few can be found on tube simulation using DSP. One explanation for this could be that in the quickly evolving music technology market, it is more advisable for the manufacturers to implement the software-based technology as quickly as possible than to patent it. Solid-state tube amplifier modeling products are more easily reverse-engineered by rival companies, and thus they require the additional protection by the patent. Despite the marketing claims, current digital tube amplifier modeling products in general do not simulate the tube circuitry using physics-based modeling. Instead, most commercial DSP amplifier emulators use very simple signal-based models matched to the processing power of the hardware [Zölzer, 2007]. In recent years, also DSP modeling of guitars has been introduced [Celi et al., 2004].

There are some academic studies on simulation of nonlinear audio circuits available in the literature; a short overview is provided in [Schattschneider and Zölzer, 1999]. Although nonlinear system theory offers tools such as the Volterra series [Rugh, 1981] for approximating mild nonlinearities [Schattschneider and Zölzer, 1999; Abel and Berners, 2006], they are in general not well-suited for efficient modeling of strongly saturating nonlinearities, such as those encountered in guitar amplifiers. Static nonlinearities can also be approximated using broken-line approximation, as suggested in [Schimmel and Misurec, 2007].

Non-real-time vacuum-tube simulation has been implemented using the SPICE [Quarles et al., 2007; Leach, 1995; Möller et al., 2002] and Matlab [Matlab; Zölzer, 2002] softwares, while simplified real-time tube-amplifier models have been reported in [Sapp et al., 1999; Möller, 2004; Schimmel, 2003; Karjalainen et al., 2006; Santagata et al., 2007]. An analytic model of a triode tube stage has been presented in [Serafini], and computational issues of nonlinear electric circuits have been addressed in [Serafini and Zamboni]. The equalization circuitry of guitar amplifiers have also been simulated [Curtis et al., 2001; Yeh and Smith, 2006]. In addition to vacuum-tube amplifiers, other nonlinear electric audio circuits, such as the Moog ladder filter [Moog, 1965; Huovilainen, 2004; Välimäki and Huovilainen, 2006; Hèlie, 2006], chorus and flanger [Huovilainen, 2005], and distortion and overdrive pedals [Yeh et al., 2007a,b] have been modeled.

The WDF modeling approach, reviewed in Sec. 3.3, is a promising approach for modeling nonlinearities in real-time [Sarti and De Poli, 1999]. Although several studies on nonlinear WDF electric circuit simulation have been conducted [Meerkötter and Scholtz, 1989; Meerkötter and Felderhoff, 1992; Fränken et al., 2001; WDInt], WDF models for vacuum-tubes have not been previously introduced. Article IV of this thesis presents WDF model for a triode vacuum-tube amplifier stage. A real-time implementation of this model is carried out using the BlockCompiler software [Karjalainen, 2003]. This new algorithm correctly mimicks the basic nonlinear operation of a single tube stage and realistically simulates the distorted vacuum-tube sound. Although the WDF implementation of a vacuum-tube stage is computationally less efficient than a direct signal-based filtering approach, this new algorithm provides a more accurate, physics-based model suitable, e.g. for rapid prototyping of novel tube amplifier circuits. Sound examples of the WDF tube stage are available at `http://www.acoustics.hut.fi/publications/papers/icassp-wdftube/`.

4.3 Nonlinearities in other string instruments

As one might have guessed, the set of guitar-related phenomena discussed so far is by no means a complete listing of all nonlinear string behavior. In fact, many other nonlinear aspects affect the sound of various string instruments. One of the most important ones (and one of the most studied ones, too) is the nonlinear friction between the string and the bow in violin-type instruments. The nonlinearity causes the bow-string contact to undergo a stick-slip movement, where the string repeatedly attaches to and detaches from the constantly moving bow. This periodic excitation is the origin of bowed string vibration, and thus the source of violin-type instrument sounds.

Many literary works study the bow-string nonlinearity in detail: [McIntyre and Woodhouse, 1979; Schumacher, 1979; Cremer, 1983; Woodhouse, 1993; Schumacher and Woodhouse, 1995b; Pitteroff and Woodhouse, 1998; Guettler, 2002; Woodhouse and Galluzzo, 2004]. Also, various modeling approaches have been suggested for simulating this phenomenon (see an overview in [Serafin, 2004]), for example DWGs [Smith, 1986, 1997; Takala et al., 2000; Holm and Toiviainen, 2004][8], finite differences [Palumbi and Seno, 1999], mass-spring models [Cadoz et al., 2001], and modal synthesis [Antunes et al., 2000]. Other modeling approaches have been presented in [Schumacher and Woodhouse, 1995a,b; Woodhouse, 2003; Woodhouse and Galluzzo, 2004].

When a piano key is pressed and the hammer hits a string, the layer of felt covering the hammer is compressed in a nonlinear manner. This causes the hammer to appear harder when played with a greater intensity, and softer when played lightly. Thus, a heavily pressed key results in a sound that has a lot of energy in the high frequencies of the initial transient, while a softly pressed key produces a mellower tone. This hammer-

[8]Also the MSW algorithm, presented in [McIntyre et al., 1983] can be seen as a waveguide-based model, although it was presented before the waveguide formalism had been developed.

felt nonlinearity has been studied, e.g. in [Hall, 1986, 1987a,b, 1992] and simulated in [Suzuki, 1987; Boutillon, 1988; Chaigne and Askenfelt, 1994a]. For a performance evaluation of the synthesis model presented by Chaigne and Askenfelt, see [Chaigne and Askenfelt, 1994b]. The phantom partials, mentioned already in section 4.1.1, are also an important nonlinear effect in the piano.

Another interesting nonlinearity in plucked string instruments is the displacement constraint that the instrument body or fretboard poses to the string. This effect is best illustrated by slap bass players, who vigorously pop the string so that it hits the fretboard and produces a percussive sound. Naturally, this technique can be applied to the guitar as well, although it is less frequently encountered. This effect is discussed and modeled in the following publications: [Rank and Kubin, 1997; Karjalainen, 2002; Krishnaswamy and Smith, 2003]. In some string instruments, such as the indian sitar or tanpura, the displacement constraint takes place at the curved bridge. This phenomenon, introduced in [Raman, 1921], has been thoroughly studied in [Burridge et al., 1982] and [Valette, 1995].

4.4 Time-varying phenomena in guitars

In guitars, almost all musically interesting events are caused by time-varying string properties. This section summarizes three important time-varying phenomena: the varying-length string, fret noise, and time-varying spatial damping.

4.4.1 Varying-length string

An important time-varying phenomenon encountered in many string instruments is the change in the playing length of the string during its vibration. This takes place in every string instrument where multiple pitches are to be played on a single string. As with other non-LTI phenomena, physically correct simulation of this aspect might turn out to be problematic, depending on the modeling technique. In DWG string models, varying the length (or pitch) of the string corresponds to changing the delay line length during the running of the algorithm. As discussed in Sec. 4.1.4, this causes an artificial energy change in the DWG string. Also, when the delay line length is varied, commuted DWG synthesis, discussed in Sec. 3.2, cannot be used, since changing the total delay length would also shift the simulated instrument body resonances. Physically, this would correspond to dynamically varying the size of instrument body during playing. Although this non-physical behavior might be modeled as a special effect (as done in [Penttinen et al., 2001a]), it is generally not desired. Varying-length digital waveguides form the core of the synthesis algorithms presented in articles I, II, III, and VI.

4.4.2 Fret noise

The guitar-related term fret noise refers to the usually unintentional handling noises generated by the guitarist's moving fingers on wound strings. These squeaky sounds are especially audible in acoustic guitar music, and their imitation is essential for realistic guitar synthesis. It must be noted that despite the name, these noises have nothing to do with the actual frets, and they are encountered also in fretless string instruments, such as violins. Since the fret noise is caused by the friction between a moving finger and the strings, it is essentially nonlinear [Urbakh et al., 2004]. Also, it can be seen as a time-varying phenomenon, since the characteristics of the excitation and the resulting noise vary considerably in time. Tutorials on friction-based sounds are provided in [Serafin, 2004] and [Rocchesso and Fontana, 2003].

Due to the complicated nature of friction, general simulation models are considered impossible to formulate [Olsson et al., 1998]. Thus, there are many different friction models available in the literature. These models can be divided into static (a.k.a. kinetic) and dynamic models [Rocchesso and Fontana, 2003]. For static models, the friction force is a function of the relative instantaneous velocity between the moving bodies. For dynamic models, the friction force depends on the velocity via a differential equation. Generally, transient simulation is more accurate with dynamic models [Altpeter, 1999]. Under constant- or low velocities, both model types behave similarly.

The first simple static friction model was already proposed by Leonardo da Vinci [20-sim] in the 15th century. As later improved by Coulomb (1785), this model is usually referred to as the Coulomb friction model. Basically, it states that the frictional force is opposite to the direction of the movement, but it does not take the magnitude of the velocity into account. The concept of static friction was introduced by A. J. Morin in 1833, and a viscous friction model was developed by O. Reynolds in 1866. In 1902, Stribeck suggested that the friction force is a non-monotonic function of the sliding velocity.

The first dynamic friction model was introduced by Dahl in the late 1960s [Dahl, 1968] (reviewed in [Olsson et al., 1998]). This was the first model that could simulate the dynamic pre-sliding displacement. Almost three decades later, a more advanced modeling scheme, the LuGre friction model [Canudas de Wit et al., 1995], which also simulates the frictional lag and the varying break-away force between the moving bodies, was introduced. The LuGre model has later been extended to include hysteresis in the friction force [Swevers et al., 2000]. Also, a class of elasto-plastic models that allow a purely elastic regime for small displacements [Dupont et al., 2002] have been formulated.

Article V of this thesis is the first scientific study analyzing the handling noise on wound strings. Based on this analysis, a contact sound synthesizer is proposed for simulating the slide guitar in article VI. As presented in article V, the fret noise is generated by an exciter (the moving finger-string contact) whose output is modified by a resonator (the vibrating string). The resulting noise has a time-varying harmonic structure with also a few static harmonics. The frequencies and root-mean-square amplitudes of the time-varying harmonic components were found to be linearly dependent on the sliding velocity. Longitudinal wave components were discovered to be responsible for static harmonics in the noise. It must be noted that although most friction-related studies are interested in finding the relationship between the sliding velocity and the resulting force, article V studies how the sliding velocity directly affects the resulting sound. In a way, the handling noise model in V can be seen as a static friction model, since the excitation force depends only on the sliding velocity. Recorded sound examples of the contact noise discussed in article V can be found at `http://www.acoustics.hut.fi/publications/papers/jasael-handling-noise/`.

There is a close resemblance between the sliding contact sound model presented in article VI and the rolling sound synthesis model in [Rocchesso and Fontana, 2003]. However, the algorithm in [Rocchesso and Fontana, 2003] uses a fractal approach for surface modeling, and implements the friction noise as a special type of pink noise, called fractal noise. This approach is based on the assumption that the surface texture is more or less random, and has a high self-similarity. In the case of wound strings, however, the surface of the string consists of windings, and can therefore be considered periodic from the sliding point of view. Rolling sound synthesis has been developed further in publications [Rath, 2003; Rath and Rocchesso, 2004, 2005].

In conclusion, the handling noise model proposed in article VI falls somewhere between a friction model and a periodic impact model. A video of the virtual slide guitar described in article VI can be found at `http://www.acoustics.hut.fi/publications/papers/vsg/`. Impact models for sound synthesis have been implemented using modal techniques [Adrien, 1991; Pai et al., 2001; Rocchesso and Fontana, 2003; Aramaki et al., 2006] and granular synthesis [Cook, 2002]. Resynthesis of natural impact sounds is discussed in [Aramaki and Kronland-Martinet, 2006]. Also, perceptional aspects of impact sounds have been studied [Gaver, 1988; Freed, 1990; Lutfi and Oh, 1997; Kunkler-Peck and Turvey, 2000; Klatzky et al., 2000; Avanzini and Rocchesso, 2001b,a; Rocchesso and Fontana, 2003; Avanzini et al., 2005].

4.4.3 Time-varying spatial damping

Spatial damping of a vibrating string refers here to the effect where a musician gently touches the vibrating string at some location with a finger, setting the string displacement to zero at that point. If performed at a correct location, this will force a vibrational node at the touched point, but will leave the string vibrating. From the resulting flute-like tone, called a flageolet tone (or a harmonic in the guitar terminology), all modes which do not have a vibrational node at the touch point will be missing. In a way, the plucking event (spatial excitation) can be seen as an opposite operation to the spatial damping, since the plucking excites those modes that do not have a node at the plucking point, whereas the spatial damping attenuates all modes that do not have a node at the damping point.

By damping the string at different locations, the musician can produce flageolet tones with different pitches. In reality, the spatial damping is carried out in a time-varying manner; the damping is released quickly right after it has been applied, so that the string vibration does not attenuate too much. It must be noted that since the human finger is not a point-like object, also the flageolet components will attenuate gradually if the damping is not released.

Article VII presents a novel physics-based method for synthesizing flageolet tones. Unlike previous filter-based implementations (e.g. [Penttinen et al., 2006]), this new technique is able to correctly simulate the spatial damping also in the time-varying case. The model is based on a DWG string, where a WDF damper (i.e. mechanical resistor) is connected at the desired damping location. This damping location can be changed during the running of the algorithm. By varying the amount of damping, realistic flageolet tones can be generated. Sound examples are available on the article's companion web-page: `http://www.acoustics.hut.fi/publications/papers/eusipco-flageolets/`. Article VII also introduces a commuted version of the string model, where the LTI parts have been lumped as separate filters.

5 Conclusions and future development

This thesis studies some of the nonlinear and time-varying aspects related to the guitar and presents physics-based methods for their realistic discrete-time modeling. Nonlinearities caused by tension modulation were studied and three new synthesis algorithms were proposed for simulating them. The first algorithm, introduced in article I, uses time-varying fractional delay filters, evenly distributed inside the delay lines of a DWG string. The second algorithm, also introduced in article I, uses time-varying fractional delay filters in varying the temporal sampling instants in a finite-difference string. The third algorithm, introduced in article III, uses a series connection of time-varying power-normalized WDFs as a delay line inside a DWG string. Unlike most previous nonlinear string models, all of these novel algorithms are spatially distributed, so that they allow interaction with various locations on the string.

Energy-related problems were found to arise with time-varying DWG models, and two new techniques presented in article II were offered as a remedy. The first technique evaluates the undesired energy variation from the string elongation each time instant and tunes a scaling coefficient inside the DWG loop for compensation. The second technique uses power-normalized WDFs, a.k.a. generalized allpass filters, as delay elements inside a DWG string. Due to the energy-preserving properties of power-normalized WDFs, the string energy remains unaltered even though the string tension is varied. This second technique allows the DWG string to remain spatially distributed, as shown in the case study of a nonlinear string in article III. The nonlinear operation of a vacuum-tube amplifier stage was analyzed and a new model for simulating a guitar tube amplifier stage was proposed in article IV. This novel real-time algorithm simulates the entire amplifier circuit as a WDF network. Unlike previous guitar amplifier models, this new design is modular, so that more flexible editing of the circuit structure is enabled.

A first study of the handling noise created by a moving finger–string contact was presented in article V. The handling noise was found to consist of a lowpass-type noise with both static and dynamic harmonic components. The lowpass cutoff frequency, frequencies of the dynamic harmonics, and the total amplitude of the contact noise were found to be dependent on the sliding velocity of the finger–string contact point. Also, the longitudinal string vibration modes were discovered to generate the static harmonics. Using the information presented in article V together with the energy-compensation method in article II, a new real-time synthetic instrument, the virtual slide guitar, was introduced in article VI. The synthesis model is controlled using a camera-based gestural user interface, similar to what presented in [Karjalainen et al., 2006]. Also, the virtual slide guitar includes a novel physics-based synthesis block for generating the contact noise between the slide tube and the strings.

The generation of flageolet tones was studied and a novel physics-based synthesis algorithm for their synthesis was presented in article VII. This DWG model uses a WDF damper for simulating the effect of a finger, gently attenuating the string in the desired location. Due to the physics-based realization of the damper, correct simulation of time-varying damping is obtained.

In conclusion, the new results obtained in this thesis provide new understanding and modeling tools for simulating the guitar-related non-LTI phenomena and can be used for making the current guitar synthesis sound more realistic.

An obvious future task for physics-based sound synthesis research is to find new algorithms for realistic simulation of those musical instruments that have not been previously modeled. Also, the improvement of current synthesis models in both sound quality and computational efficiency remains to be done. As discussed in article III, there is no silver bullet for physics-based instrument modeling: different vibrational systems often require different modeling schemes. Thus, an important future challenge in physics-based sound synthesis is to create a unifying simulation strategy that efficiently combines the different modeling techniques. The idea of interconnecting different modeling paradigms has been studied in [Rabenstein et al., 2007], and methods for combining state-space and wave digital filter models [Petrausch and Rabenstein, 2005], as well as waveguide and finite difference models [Karjalainen and Erkut, 2004; Smith, 2004] have been presented.

For creating realistic synthetic instruments, extraction of control data for the models must be further developed. This can be obtained using sensors (such as pressure or acceleration) or optics-based gesture recognition, for example. Haptic interactivity with the synthesizers can be implemented using e.g. vibrotactile actuators, as described in [Marshall and Wanderley, 2006]. In many cases the synthesis model parameters are not directly measurable, so physical parameter estimation must be carried out from instrument recordings. Some studies applying this for plucked strings [Traube and Smith, 2000; Liang and Su, 2000; Nackaerts et al., 2001; Erkut, 2002; Riionheimo and Välimäki, 2003; Penttinen, 2006], bowed strings [Serafin et al., 2001], piano strings [Aramaki et al., 2001], and the trumpet [D´haes and Rodet, 2003] have already been conducted. Synthesis parameter estimation from recordings is closely related to automatic music transcription, where playing gestures are extracted from recorded music; see, e.g. [Klapuri and Davy, 2006] for a tutorial. By using automatic music transcription together with high-quality physics-based sound synthesis, extremely efficient music compression algorithms could be developed, since only the synthesis parameters and control gesture data would have to be stored or transmitted.

More interestingly, with a unified modeling approach, the user could create the instrument using a virtual lutherie system, which would allow the compilation of a musical tool from physical subsystems, such as strings, plates, horns, etc., and the resulting instrument would behave similar to a real, physical instrument. This system could be implemented on a computer or a hand-held device, and it could be used by real instrument builders for rapid prototyping of new designs, or by music enthusiasts just for entertainment. When combined with realistic control, such systems could, e.g., enable live music performances through the Internet, where each musician of the orchestra could participate by playing one's self-made virtual instrument anywhere in the world.

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