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Determining the irradiance signal from an asymmetric source with directional detectors: application to calibrations of radiometers with diffusers

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The energy transfer integral between radiating rectangular and detecting circular parallel plates having nonideal angular characteristics is solved for modeling the distance dependence of the irradiance signal. The equation derived for the irradiance signal, which is called the modified inverse-square law, depends on the position, shape, size, and angular characteristics of the light source and the detector. We apply the new model equation to the calibration of a spectroradiometer to determine accurately the distance offsets, which fix the positions of the effective receiving apertures of diffusers used in the entrance optics of spectroradiometers. Earlier measurement results, e.g., for solar UV irradiance, may include uncorrected effects and can be corrected reliably as diffuser offsets and other correction factors are determined with the modified inverse-square law. Simplifications of the modified inverse-square law for analyzing the distance offsets and the correction factors are studied. Simplified equations for the diffuser offset analysis may be used without losing the accuracy when the cosine response of the diffuser is reasonably good. However, for diffusers whose angular responsivities deviate much from the cosinusoidal angular responsivity, large approximation errors in the diffuser offset values may appear if the angular effects are not properly taken into account.

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1. Introduction

In optical radiometry, radiation is often measured with a detector having a small circular entrance aperture while radiation sources are more or less rectangular. Source-to-detector distances are usually so large that the conventional inverse-square law of the point source and point detector is accurate enough. However, the calibration of a spectroradiometer is usually performed at relatively short distances from the signal source due to the noise at the short wavelengths. On the other hand, at short distances from the signal source, the transverse dimensions of the source and detector and the nonidealities in their angular distribution and responsivity characteristics may play a significant role in modeling the distance dependence of the irradiance.

Recently, the distance dependences of the signals of spectroradiometers [1] and photometers [2] equipped with diffusers have been studied for determining the positions of their effective receiving planes. In most cases, these planes were located inside the diffusers and, in some cases, several millimeters away from the outermost surfaces of the diffusing plates that are usually used as the reference planes in the distance setting. It was noted that pure geometric consideration of the diffuser does not give the real receiving plane position for the diffuser but that plane should be experimentally determined. Without taking this effect into account, the irradiance levels measured are overestimated.

Similar findings based on solar ultraviolet (UV) irradiance measurements were reported for a

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dome-shaped diffuser of type J1002 commonly used in solar UV spectroradiometers [3]. It is of great importance, particularly for the international quality assurance projects of monitoring solar UV irradiance [4–7], to improve the accuracy of the distance determination to be able to calibrate the spectroradiometers as reliably as possible.

In the earlier analyses of the diffuser reference planes, the angular responsivities were assumed to be cosinusoidal and the sources and detectors were assumed to be spatially uniform [1]. The point-source and point-detector approximation was used in [2,3]. However, the radiation sources used were approximately rectangular and some of the diffusers were noted to have significantly noncosinusoidal angular responsivities [1]. The diffusers used in broadband UV radiometers [8] and Brewer spectroradiometers [9] have also been observed to collect the incoming radiation nonuniformly over the receiving surface. It is of importance to study how large effects these nonidealities have in the simplified model.

The energy transfer between two parallel plates with various geometries has been studied in several papers [10–14]. However, these studies do not cover the cases where both plates have noncosinusoidal angular characteristics. In this paper, the integral of the energy transfer between radiating rectangular and detecting circular parallel plates with noncosinusoidal angular characteristics is solved to model the irradiance signal at varying distances. The accuracy of the model equation is evaluated in terms of the transverse dimensions of the measurement system. The irradiance signal equation derived is based on the position, shape, size, and directivity of the radiation source and detector. The equation is applied to the earlier measurement data used in diffuser offset analysis [1] to determine the positions of the effective receiving planes of diffusers more accurately. The angular characteristics of the radiation source and detector are modeled by $\cos^m\theta$ functions. The sizes of the effective receiving apertures of the diffusers are experimentally determined. Finally, it is investigated whether the experimental effort required for the use of the model equation can be reduced by doing different approximations and assumptions without sacrificing the required accuracy.

2. Theory

A. Derivation of the Model Equation

Consider such a measurement geometry where a rectangular emitting surface with width x_0 and height y_0 and a receiving aperture of a circular detector with radius r_0 are parallel and their centers are situated on the z axis of the coordinate system at distance D from each other (Fig. 1). The differential signal dS of a detector surface element dA_D at (r,φ,D) produced by a small surface element dA_S at $(x,y,0)$ is given by

$$dS = R(r,\varphi;\theta)L_e(x,y;\theta)dA_\perp d\Omega, \tag{1}$$

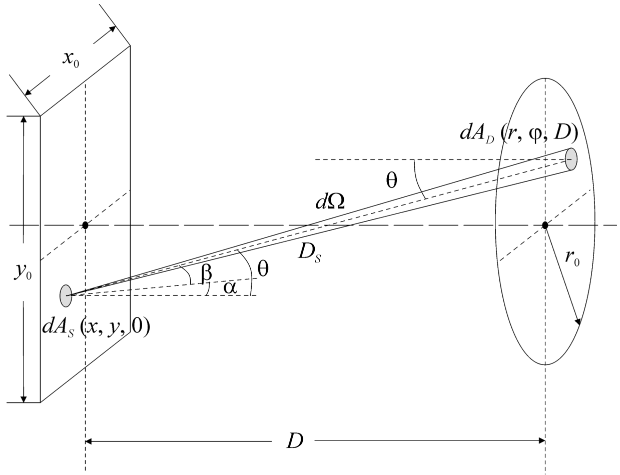

Fig. 1. Geometric parameters between rectangular and circular parallel plates.

where $R(r,\varphi;\theta)$ is the flux responsivity of the detector in incidence direction θ at surface element dA_D, $L_e(x,y;\theta)$ is the radiance of the source at surface element dA_S in direction θ, $dA_\perp = dA_S\cos\theta$ is the projected area of dA_S, and $d\Omega = dA_D\cos\theta/D_S^2$ is the solid angle from the point $(x,y,0)$ to the surface element dA_D. The distance between the area elements dA_S and dA_D is $D_S = [D^2 + (x - r\cos\varphi)^2 + (y - r\sin\varphi)^2]^{\frac{1}{2}}$.

For an asymmetric source, angle θ can be divided into horizontal and vertical parts α and β (see Fig. 1) with $\tan^2\theta = \tan^2\alpha + \tan^2\beta$. On the basis of the law of cosines, $\cos\theta = \cos\alpha\,\cos\beta$ is valid and $dA_\perp = dA_S\cos\alpha\,\cos\beta$. The angular dependence of the radiance of a homogeneous asymmetric source is modeled by

$$L_e(x,y;\alpha,\beta) = \frac{I_0}{x_0y_0}\frac{I(\alpha,\beta)}{\cos\alpha\,\cos\beta}, \tag{2}$$

where $I(\alpha,\beta)$ is the normalized angular distribution of the source $[I(0,0) = 1]$ and I_0 is the radiant intensity, which does not depend on x or y. Function $I(\alpha,\beta)$ can be modeled by two parabolic curves in the horizontal and vertical directions within a few degrees around the optical axis as

$$I(\alpha,\beta) \approx \left(1 - \frac{1}{2}p\alpha^2\right)\left(1 - \frac{1}{2}q\beta^2\right) \approx \cos^p\alpha\cos^q\beta, \tag{3}$$

where p and q are the directivity parameters of the source in the horizontal and vertical directions, respectively. When $p = q = n$, Eq. (3) is reduced to the form $\cos^n\theta$, where $n = 1$ for a Lambertian source.

The flux responsivity $R(r,\varphi;\theta)$ of a homogeneous detector can be expressed as

$$R(r,\varphi;\theta) = \frac{R_0}{\pi r_0^2}\frac{K(\theta)}{\cos\theta}, \tag{4}$$

where R_0 is the irradiance responsivity of the detector, which does not depend on r or φ, and $K(\theta)$ is the normalized angular responsivity of the detector. Function $K(\theta)$ can be approximated by a parabolic curve within a few degrees from the normal of surface element dA_D:

$$K(\theta) \approx 1 - \frac{1}{2} m\theta^2 \approx \cos^m\theta, \tag{5}$$

where m is the directivity of the detector. The flux responsivity of the detector with ideal, cosinusoidal angular responsivity ($m = 1$) does not depend on θ. The differential irradiance signal of Eq. (1) can be expressed in the form

$$dS = \frac{R_0 I_0}{x_0 y_0 \pi r_0^2} \frac{\cos^m\theta \cos^p\alpha \cos^q\beta}{D_S^2} dA_D dA_S. \tag{6}$$

The total irradiance signal $S(D)$ as a function of distance is obtained by integrating Eq. (6) over the emitting and receiving surface elements. As $dA_D = rd\varphi dr$, $dA_S = dx\,dy$, $D_S^2 = D^2 + r^2 + x^2 + y^2 - 2r(x\cos\varphi + y\sin\varphi)$, $\cos\theta = D/D_S$, $\cos\alpha = D/D_\alpha$, $\cos\beta = D/D_\beta$, $D_\alpha = [D^2 + (x - r\cos\varphi)^2]^{1/2}$, and $D_\beta = [D^2 + (y - r\sin\varphi)^2]^{1/2}$, the integration yields

$$S(D) = \frac{R_0 I_0 D^{m+p+q}}{x_0 y_0 \pi r_0^2} \int_{-\frac{y_0}{2}}^{\frac{y_0}{2}} \int_{-\frac{x_0}{2}}^{\frac{x_0}{2}} \int_0^{r_0} \int_0^{2\pi} \frac{1}{[D^2 + r^2 + x^2 + y^2 - 2r(x\cos\varphi + y\sin\varphi)]^{(m+2)/2}} \times \frac{rd\varphi dr dx dy}{[D^2 + x^2 - 2rx\cos\varphi + r^2\cos^2\varphi]^{p/2}[D^2 + y^2 - 2ry\sin\varphi + r^2\sin^2\varphi]^{q/2}}. \tag{7}$$

The denominator factor D_S^{m+2} of the integrand of Eq. (7) can be approximated with the assumption that D is much larger than r, x, and y:

$$\left[1 + \left(\frac{r}{D}\right)^2 + \left(\frac{x}{D}\right)^2 + \left(\frac{y}{D}\right)^2 - 2\frac{r}{D^2}(x\cos\varphi + y\sin\varphi)\right]^{-(m+2)/2} \cong 1 - \frac{m+2}{2D^2}[r^2 + x^2 + y^2 - 2r(x\cos\varphi + y\sin\varphi)], \tag{8}$$

where terms of order x^4/D^4, y^4/D^4, r^4/D^4, etc., were omitted. The corresponding approximations apply to D_α^p and D_β^q. After the integration, the equation for the distance dependence of the irradiance signal can be presented in the form

$$S(D) = \frac{R_0 I_0}{D^2}\left(1 - \frac{r_S^2 + r_D^2}{D^2}\right), \tag{9}$$

where the new modified radius parameters are defined as

$$\begin{bmatrix} r_S \\ r_D \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \sqrt{\frac{m+p+2}{6}x_0^2 + \frac{m+q+2}{6}y_0^2} \\ r_0\sqrt{m + \frac{p+q}{2} + 2} \end{bmatrix}. \tag{10}$$

Equation (9) may be approximated by the expression

$$S(D) \approx \frac{R_0 I_0}{D^2 + r_S^2 + r_D^2}, \tag{11}$$

because, in Eq. (9), there are two lowest-order terms of the series expansion of Eq. (11). The validity of the approximations leading to Eqs. (9) and (11) is evaluated in Appendix A.

B. Principles of the Diffuser Offset Determination

Equation (11) is applied to determining the positions of the effective receiving apertures of four spectroradiometer diffusers [1]. Distance D in Eq. (11) is decomposed as $D = d + \Delta d_S + \Delta d_D$, where d is the measurement distance between the auxiliary reference planes of the lamp and the diffuser under study and Δd_S and Δd_D are the distance offsets of the lamp and the diffuser, respectively. These offsets fix the effective positions of the lamp filament and the receiving aperture of the diffuser relative to their auxiliary reference planes. Offset Δd_S is determined from the distance dependence of the signal with a reference detector having a well-known aperture plane ($\Delta d_D = 0$ for the reference detector) and, when Δd_S is known, Δd_D is determined from the distance dependence of the signal with a detector having the diffuser under study. In [1], the distance dependences of the irradiance signals with different diffusers were investigated in four wavelength bands, effective wavelengths being 350, 570, 700, and 860 nm. Here, the corresponding measurement results for determining the sizes of the effective receiving apertures of the diffusers are presented in Subsection 3.B.

3. Accurate Diffuser Offset for Reliable Spectroradiometer Calibration

A. Estimation of Parameters of Lamp and Reference Detector

A 1 kW FEL lamp of type BN-9101 from Giga Hertz Optik was used as the light source in the

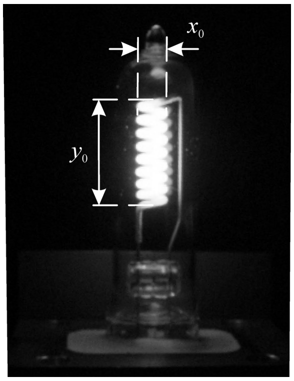

Fig. 2. Illustration of the rectangular approximation of the FEL lamp. The parameter values are $x_0 = 5\,\text{mm}$ and $y_0 = 18\,\text{mm}$.

measurements [1]. The directivities p and q of the lamp are here determined on the basis of the intensity distribution graph presented in [15] for a lamp of the same type. The lamp can be considered as an isotropic radiator in the horizontal plane and as a directional radiator in the vertical plane because the projected area of the filament changes with the vertical direction but does not change with the horizontal direction. The directivity values of the FEL lamp fitted in the horizontal and vertical directions were $p = 0.0$ and $q = 5.0$, respectively. Figure 2 shows the FEL lamp, which can be modeled quite well with a rectangular, homogeneous source whose dimensions are $x_0 = 5\,\text{mm}$ and $y_0 = 18\,\text{mm}$.

The reference detector used has a knife-edge entrance aperture with 8 mm diameter [1]. It was evaluated to work as a homogeneous detecting element whose angular responsivity was cosinusoidal within small incident angles of the order of a few degrees. Thus, the values $r_0 = 4\,\text{mm}$ and $m = 1$ for the reference detector were used.

B. Estimation of Diffuser Parameters

The spectroradiometer measuring heads studied included planar Bentham diffusers of types D3, D5, and D7 and a dome-shaped Schreder diffuser of type J1002-01 having a quartz dome [1]. The values of parameters r_0 and m for these diffusers were estimated from their spatial and angular responsivity data as follows. Parameter m describing the degree of the detector directivity was determined by fitting the $\cos^m(\theta - \theta_0)$ function to the measured angular responsivity data, normalized to unity at 0° angle (see Section 2), by using m and angular offset θ_0 as free-

Table 1. Values of Parameter *m* Fitted to the Angular Responsivity Data of the Diffusers at Four Wavelengths[a]

Band\Diffuser	Parameter m			
(nm)	D3	D5	D7	J1002
350	3.2	1.8	1.5	1.7
570	3.9	1.9	0.8	-
700	3.1	10	0.7	3.2
860	2.9	28	-	-

[a] See the insets of Figs. 3–6.

fitting parameters. Results from the angular responsivity measurements and analyses are shown in Table 1 and in Figs. 3–6. In most cases, modeling the angular responsivity of the diffuser by the ideal cosine curve ($m = 1$) does not match with the measurement results even close to the normal incidence angles. The values of m were experimentally determined by analyzing the angular dependences of the diffusers more accurately, as shown in the insets of Figs. 3–6.

The spatial responsivity measurements were conducted by scanning the diffusing plates at two perpendicular directions using a linear translator and lasers emitting at wavelengths of 325, 543, and 633 nm. The results from the spatial responsivity measurements are presented in Fig. 7. The spatial responsivities of the studied diffusers did not depend on wavelength and the diffusers were rotationally symmetric. Effective radii r_0 of the receiving apertures were calculated on the basis of the full width at half-maximum (FWHM) of the spatial responsivity curves. The values of r_0 for diffusers D3, D5, D7, and J1002 were estimated to be 2.1, 2.0, 2.6, and 7.0 mm, respectively. When those values are compared with their physical radii 12.5, 11.5, 5.0, and 9.0 mm, one can see that the earlier assumption in [1] on the spatially uniform response over the entire surface of the diffuser is not valid, but much smaller values for r_0 in the offset analysis should be used. Table 2 shows the values of the modified radius parameters r_S and r_D for the studied diffusers as calculated by Eq. (10). The values of r_S and r_D for the reference detector ($m = 1$) were 10.5 and 4.7 mm.

In the spectroradiometer used, an optical fiber behind the diffuser was used to couple the light either to the monochromator or a photodiode detector. The spatial responsivity results may be explained by the small numerical aperture of the optical fiber (acceptance angle of ±5°), the short distance between the rear surface of the diffusing plate and the entrance port of the fiber (3, 0, and 10 mm for flat diffusers

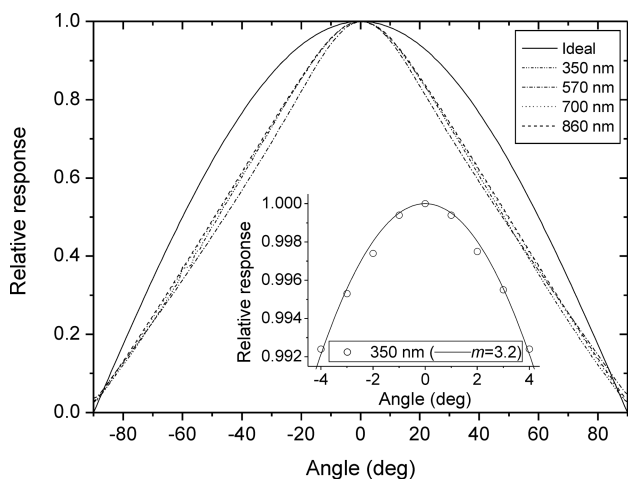

Fig. 3. Angular responsivities of the Bentham D3 diffuser at four wavelength bands. The inset shows the fit of the angular responsivity curve at the 350 nm band.

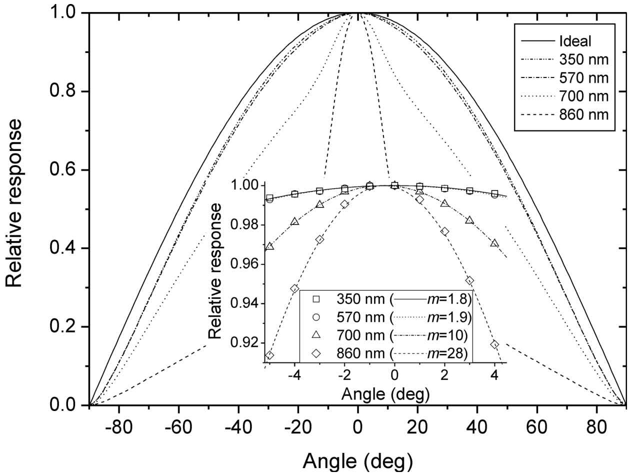

Fig. 4. Angular responsivities of the Bentham D5 diffuser at four wavelength bands. The inset shows the fits of the angular responsivity curves.

D3, D5, and D7), and the large size of the diffuser relative to that of the fiber entrance port (4 mm in diameter). Thus, the results of Fig. 7 do not mean that the studied diffusers would be poor measuring heads. As seen in Fig. 7 and considering only the flat diffusers, the optical fiber collects light from the widest area for diffuser D7 due to the longest distance between the rear surface of the diffusing plate and the fiber end. On the other hand, light from the rear surface of the diffusing plate D3 is coupled into the fiber a bit more on the edges of its diffusing plate than in cases of the other planar diffusers (see Fig. 7). Figure 7 indicates very different spatial responsivity for J1002 as for the planar diffusers above. The shapes of its spatial responsivity curves can be explained by its dome-shaped diffusing plate and specific structure behind the dome (see [16]). As a conclusion, the spatial responsivities should be measured for each diffuser separately when different kinds of fibers, detectors, and adapters are used.

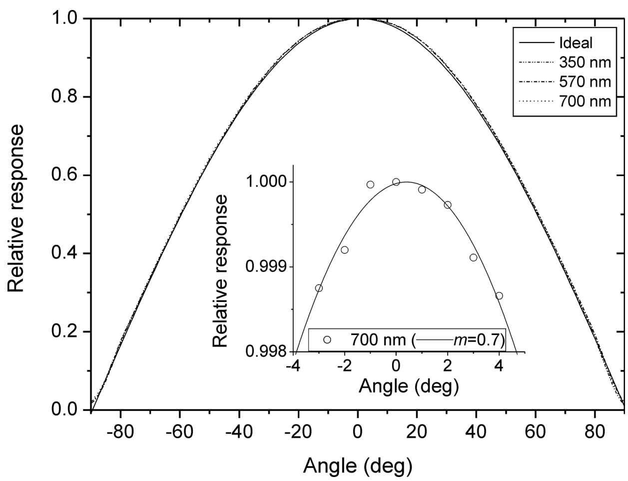

Fig. 5. Angular responsivities of the Bentham D7 diffuser at three wavelength bands. The inset shows the fit of the angular responsivity curve at the 700 nm band.

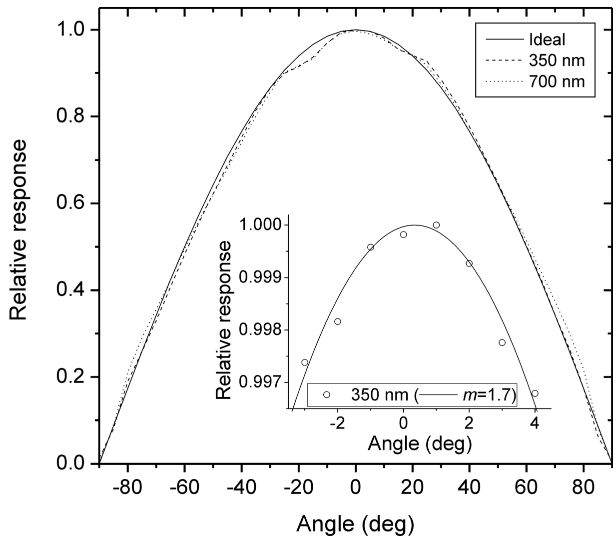

Fig. 6. Angular responsivities of the Schreder J1002-01 diffuser at two wavelength bands. The inset shows the fit of the angular responsivity curve at the 350 nm band.

C. Diffuser Reference Plane Offsets

Irradiance signals S of the reference detector and the detectors under study were measured in a distance range of 300–2000 mm [1]. Equation (11) was fitted to the measured irradiance values to determine the diffuser distance offset values. Results from the diffuser offset analysis are presented in Fig. 8.

If the calibration of a spectroradiometer equipped with a diffuser having nonzero offset was made with respect to the outermost point of the diffuser, systematically erroneous measurement results would be obtained at all other measurement distances except that used in the calibration. The distance offsets of the diffusers that are independent of wavelength, e.g., D3 and D7, can straightforwardly be taken into account in the calibration of the spectroradiometer by setting the distances from the lamp with respect

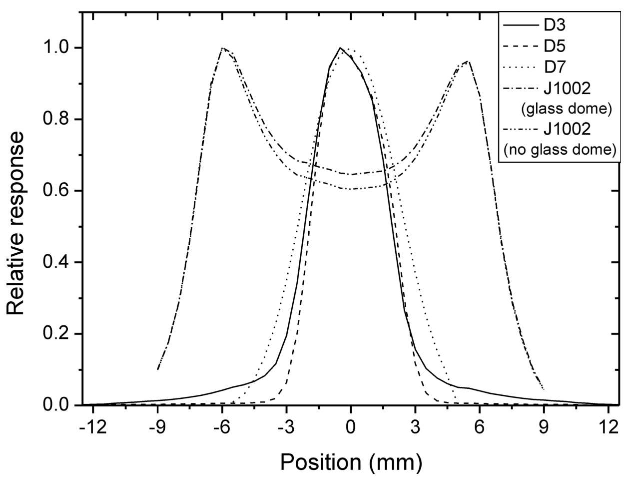

Fig. 7. Spatial responsivities of the spectroradiometer with the diffusers studied. The effective radii were estimated from the FWHM of the responses. The physical radii of the D3, D5, D7, and J1002 diffusers are 12.5, 11.5, 5, and 9 mm, respectively.

Table 2. Modified Radius Parameters r_S and r_D for the Diffusers

Band\Diffuser (nm)	Modified Radius Parameters r_S/r_D (mm) D3	D5	D7	J1002
350	12.0/2.9	11.1/2.5	10.9/3.2	11.0/8.7
570	12.4/3.0	11.1/2.5	10.4/3.0	-/-
700	11.9/2.9	15.6/3.8	10.3/3.0	12.0/9.7
860	11.8/2.9	22.4/5.7	-/-	-/-

to the effective receiving planes of the diffusers. On the other hand, if the diffuser offset was dependent of wavelength, as in the case of D5, the calibration could be made in such a way that the signal of the spectroradiometer is first recorded over the whole wavelength region of interest with respect to one distance reference plane, e.g., the outermost surface of the diffuser and then a spectrally varying correction for the calibration is calculated with the known positions of the effective receiving aperture. This applies especially to measurements where the wavelength range is of the order of hundreds of nanometers. If narrow measuring bands, where the receiving plane position does not change significantly, were needed, it might be convenient to use one suitably selected reference plane in the calibration. In the next section, a correction factor is presented which can be used in the calibration of a spectroradiometer with a spectrally varying diffuser offset.

D. Correction Factors for Calibration Data and Earlier Measurement Data

Spectroradiometers are usually calibrated at a certain distance from an irradiance standard lamp where the spectral irradiance is known. If the distance reference plane of the diffuser in the spectroradiometer calibration was inconsistent with its receiving plane, a distance correction taking into account the reference plane displacement should be made to the calibration data or to the earlier irradiance measurement results. A reliable correction may be evaluated when parameters $\Delta d_S, x_0, y_0, p$, and q of the calibration lamp and parameters Δd_D, m, and r_0 of the spectroradiometer diffuser are known.

When the spectroradiometer with the reference plane offset Δd_D has been calibrated at distance $d + \Delta d_S$ from the lamp filament to the outermost surface of the diffuser (i.e., $D = d + \Delta d_S + \Delta d_D$), the calibration signal of the spectroradiometer is proportional to factor $[(d + \Delta d_S + \Delta d_D)^2 + r_S^2 + r_D^2)]^{-1}$ [see Eqs. (10) and (11) and Subsection 2.B]. If the calibration distance $d + \Delta d_S$ had been correctly measured with respect to the reference plane of the diffuser (i.e., $D = d + \Delta d_S$), the calibration signal would be proportional to factor $[(d + \Delta d_S)^2 + r_S^2 + r_D^2)]^{-1}$. The multiplicative correction factor for the spectroradiometer measurement signal can then be expressed as

$$C = \frac{(d + \Delta d_S)^2 + r_S^2 + r_D^2}{(d + \Delta d_S + \Delta d_D)^2 + r_S^2 + r_D^2}, \tag{12}$$

where the correction factor for the irradiance responsivity of the spectroradiometer is C^{-1}.

Modified radius parameters r_S and r_D of the diffuser in Eq. (10) might be estimated by the tabulated values of Table 2 if the equipment were similar as herein (FEL lamp, Bentham monochromator, and optical fiber). For the best accuracy, parameter Δd_S should be experimentally determined during the same lamp burn when the calibration signal of the spectroradiometer is recorded [2]. The lamp offset can be precisely measured with a well-known detector by using Eq. (11) as the basis of the analysis. Because the effective optical filament position of the FEL lamp is approximately the same as the physical position, a reasonable estimate for Δd_S may be obtained using visual observation with a telescope perpendicular to the optical axis.

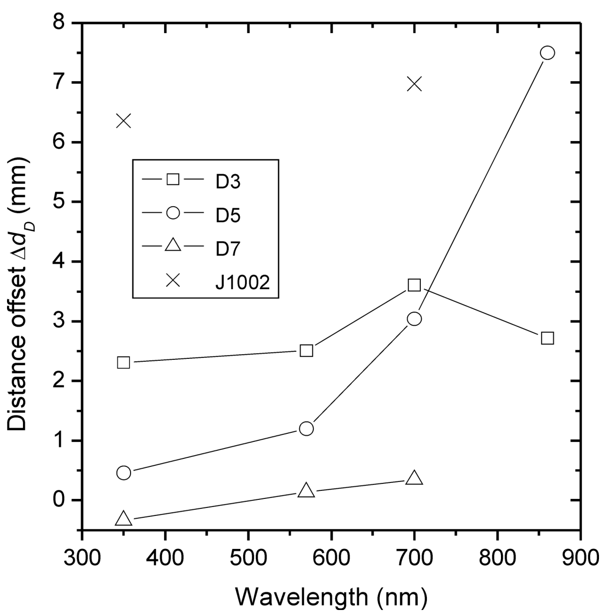

Fig. 8. Distance offsets of diffusers as a function of wavelength determined by the method in Subsection 2.B. The offsets fix the position of the effective receiving surface of diffusers relative to the outermost surface of the diffuser.

E. On Simplification of the Modified Radius Parameters

It is of interest to investigate how large errors are made when experimental efforts are reduced by making additional assumptions. Approximation errors for the diffuser offset values and correction factor values in three cases are determined.

The alternative approximations of the modified radius parameters and their characterization requirements are listed in Table 3. Case 1, where the inverse-square law of the point source and point detector is applied, is the most straightforward way to make analyses and measurements. In Case 2, the data analysis is made as in [1], where the rectangular source is approximated by a circular source of the same area and the physical radius r_{ph} of the diffusing plate is used as r_D. This circular approximation assumes a Lambertian source and cosinusoidal angular responsivity of the detector. When the spatial

Table 3. Approximations of the Modified Radius Parameters r_S and r_D in Three Cases and the Corresponding Characterization Requirements[a]

Approximation	Modified Radius Parameters	Characterization Requirement	Comment
Case 1	$\begin{bmatrix} r_S \\ r_D \end{bmatrix} = 0$	-	Inverse-square law (see [2,3])
Case 2	$\begin{bmatrix} r_S \\ r_D \end{bmatrix} = \begin{bmatrix} \sqrt{x_0 y_0/\pi} \\ r_{\mathrm{ph}} \end{bmatrix}$	Source dimensions	Circular approximation (see [1])
Case 3	$\begin{bmatrix} r_S \\ r_D \end{bmatrix} = \begin{bmatrix} \sqrt{(x_0^2+y_0^2)/6} \\ r_0 \end{bmatrix}$	Spatial responsivity	Equation (9) as $m = p = q = 1$
Case 4	$\begin{bmatrix} r_S \\ r_D \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \sqrt{\frac{m+p+2}{6}x_0^2 + \frac{m+q+2}{6}y_0^2} \\ r_0\sqrt{m+(p+q)/2+2} \end{bmatrix}$	Angular and spatial characteristics of source and diffuser	Equation (9)

[a]Parameter r_{ph} is the physical radius of the diffuser.

responsivity data of the detector are available, cosinusoidal angular characteristics for the source and detector are assumed in Case 3, which corresponds to Eq. (10) with $m = p = q = 1$. In Case 4, both the spatial and the angular characteristics of the source and detector are used in Eqs. (10) and (11) with the most complete set of parameter values.

The diffuser offset values were determined using Eq. (10) and three approximations of Cases 1–3. The results on the applicability of the approximations for diffuser D5 are shown in Fig. 9. The offset error caused by each approximation is significant in the 700 and 860 nm bands. The use of Case 1 for the J1002 diffuser offset analysis gave offset errors of 0.09 and 0.15 mm in the 350 and 700 nm bands. The uses of Cases 2–3 for the J1002 in the 700 nm band gave 0.11 mm errors for the offset values. Other cases for each diffuser gave smaller errors.

The corresponding approximations of the radius parameters shown in Table 3 were used for the calculation of the correction factors in Eq. (12), as well. Parameters Δd_S and Δd_D were analyzed with Eq. (11) using the radius approximations of Table 3. The correction factors at three calibration distances

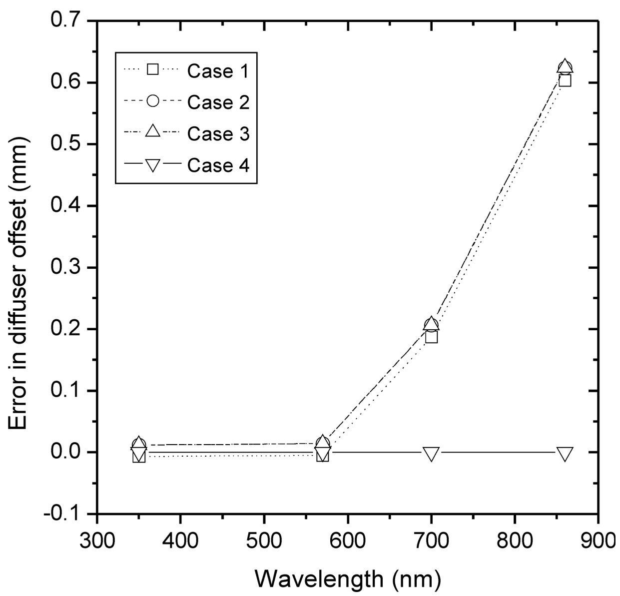

Fig. 9. Deviations between Eq. (11) and approximation Cases 1–3 for diffuser D5 distance offsets (see text for details).

are presented in Fig. 10, where the approximation cases producing the largest deviations were selected.

The relative approximation error for the correction factor is less than 0.1% for most diffusers, but Cases 1–3 for diffuser D5 in the 860 nm band gave errors ranging from 0.3% to 0.1% depending on the calibration distance. In conclusion, the receiving plane distance offset of a diffuser with small m may be quite accurately determined using any of the approximation Cases 1–3. When m is larger than 10, the approximation error is generally more than 0.1%. In that case, the use of Eq. (10) is recommended, if accurate calibrations are needed.

If the spatial responsivity data of the diffuser were missing, the estimate of r_0 could be assumed to be radius r_{ph} of the diffusing plate. As Fig. 7 shows,

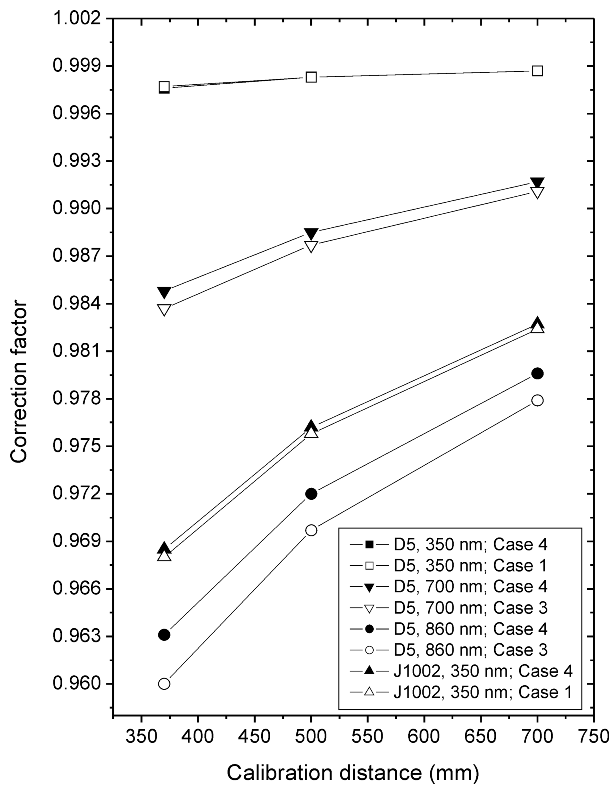

Fig. 10. Correction factors for a spectroradiometer with different diffusers at three calibration distances. The solid symbols refer to Eq. (12) and the open symbols to the corresponding approximation case.

the physical area of the diffusing plate may deviate quite a lot from the active area of the diffuser. If the distance offsets and correction factors had been calculated by using r_{ph} in Eqs. (11) and (12), instead of r_0, the approximation errors would have been larger than 0.1% in the cases of diffusers D3 and D5, which have large physical dimensions. The determination of the effective radius of the diffuser by measuring its spatial responsivity is highly important in the offset analysis of such diffusers, whose physical diameters are larger than 20 mm.

4. Conclusions

Many aspects of the theory needed for determining the position of the effective receiving aperture of diffusers have been presented. An approximation of the complex integral of the energy transfer produces a new model equation which can be used for analyzing the distance dependences of irradiance signals in cases where the radiation source is studied at varying distances on its optical axis. The new model equation is based on the position, shape, size, and angular characteristics of the source and detector. The analysis method has been applied to a wide variety of spectroradiometer diffusers for determining their effective receiving planes. A spectroradiometer should be calibrated relative to the effective reference plane to get rid of potential systematic measurement errors. The use of the new model equation necessitates several characterizations of the source and detector. To find out whether the experimental effort can be reduced, the applicability of different approximation schemes was tested. Some approximation schemes produced good results for some diffusers depending strongly on their angular and spatial responsivities. If a planar diffuser was smaller than 20 mm in diameter and its angular responsivity was close to the cosine curve at small incidence angles, the analyses for the receiving plane distance offset of the diffuser might be simplified without losing the accuracy.

Appendix A: Accuracy of the Model Equation

The accuracy of the series expansion in Eq. (7) was evaluated by calculating the approximation error matrix in terms of the integrating parameters within the integration limits of Eq. (6). The matrix was calculated by determining the relative deviation between the approximation and the exact result, as r, x, y, and φ were varied by 50 equidistant steps within their integration limits. Earlier shown values for lamp parameters p, q, x_0, and y_0 were used in the calculation. The mean value of the four-dimensional error matrix in the case of diffuser D5 at the 860 nm band ($D = d + \Delta d_S + \Delta d_D = 330\,\mathrm{mm}$, $m = 28$, and $r_0 = 2\,\mathrm{mm}$), which gives the largest approximation error in the case of this paper, was 0.0021%. The approximation error in Eq. (7) reduces when decreasing the transverse dimensions and the values of m, p, and q.

The accuracy of the inverse series expansion to Eq. (9) was evaluated by calculating the relative deviation between Eqs. (9) and (11) with the above values of parameters D, m, p, q, r_0, x_0, and y_0. The use of the reverse series expansion for Eq. (9) caused a maximum error of 0.0024% with parameter values in this paper. This error is of the same order as the approximation error of the integrand but opposite in sign and, thus, the total approximation error of Eq. (10) is very small. It can be concluded that the use of the series expansion as the integrand causes negligible deviation between the approximation and the exact integral result in the cases of the diffusers studied here.

The use of Eq. (10) for the reference plane offset of the J1002 diffuser necessitates that its spatial responsivity is uniform within the circle with radius r_0. As one can see in Fig. 7, the spatial responsivity of the spectroradiometer with the J1002 diffuser is the largest in the middle area between the center and the edge of the diffuser. However, it is quite convenient to use Eq. (10) because the value of parameter m of the J1002 is small. If m were larger than 10, the spatial responsivities should be mathematically modeled for deriving a more accurate irradiance signal equation.

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References

1. P. Manninen, J. Hovila, L. Seppälä, L. Ylianttila, P. Kärhä, and E. Ikonen, "Determination of distance offsets of diffusers for accurate radiometric measurements," Metrologia **43**, S120–S124 (2006).
2. J. Hovila, M. Mustonen, P. Kärhä, and E. Ikonen, "Determination of the diffuser reference plane for accurate illuminance responsivity calibrations," Appl. Opt. **44**, 5894–5898 (2005).
3. J. Gröbner and M. Blumthaler, "Experimental determination of the reference plane of shaped diffusers by solar ultraviolet measurements," Opt. Lett. **32**, 80–82 (2007).
4. J. Metzdorf, K. D. Stock, P. Sperfeld, A. Sperling, S. Winter, and T. Wittchen, "Aspects of quality assurance in monitoring solar UV irradiance," Metrologia **40**, S66–S69 (2003).
5. J. Gröbner, M. Blumthaler, S. Kazadzis, A. Bais, A. Webb, J. Schreder, G. Seckmeyer, and D. Rembges, "Quality assurance of spectral solar UV measurements: results from 25 UV monitoring sites in Europe, 2002 to 2004," Metrologia **43**, S66–S71 (2006).
6. G. Bernhard, R. L. McKenzie, M. Kotkamp, S. Wood, C. R. Booth, J. C. Ehramjian, P. Johnston, and S. E. Nichol, "Comparison of ultraviolet spectroradiometers in Antarctica," J. Geophys. Res. **113**, D14310 (2008).
7. R. R. Cordero, G. Seckmeyer, D. Pissulla, L. DaSilva, and F. Labbe, "Uncertainty evaluation of spectral UV irradiance measurements," Meas. Sci. Technol. **19**, 045104 (2008).
8. G. Xu and X. Huang, "Characterization and calibration of broadband ultraviolet radiometers," Metrologia **37**, 235–242 (2000).
9. K. Lakkala, "A decade of spectral UV measurements at Sodankylä," Ph.D. dissertation (University of Helsinki, 2001).
10. D. I. Lawson and D. Hird, "A photometric method of determining configuration factors," Br. J. Appl. Phys. **5**, 72–74 (1954).
11. J. W. T. Walsh, *Photometry* (Dover, 1965).

12. D. S. Goodman, "Linear systems methods in some problems of radiative transfer between parallel plates," Appl. Opt. **24**, 3240–3248 (1985).
13. S. Tryka, "The flux of radiation incident on a circular planar detector from a coaxial circular plane extended source," J. Mod. Opt. **53**, 365–380 (2006).
14. E. Ikonen, P. Manninen, and P. Kärhä, "Modeling distance dependence of LED illuminance," Light Eng. **15**, 57–61 (2007).
15. Y. Ohno, "Photometric standards," in *OSA/AIP Handbook of Applied Photometry*, C. DeCusatis, ed. (Optical Society of America, 1997), pp. 55–90.
16. J. G. Schreder, M. Blumthaler and M. Huber, "Design of an input optic for solar UV-measurements" (Internet Photochemistry and Photobiology, 1998), http://www.photobiology.com/UVR98/schreder/index.htm.