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Modal analysis of M-type-dielectric-profile optical fibers in the weakly guiding approximation

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We study the applicability of the weakly guiding approximation (WGA) to the modal analysis of an M-type optical fiber in which a ring-shaped core lies between two uniform cladding layers. Besides being dependent on the refractive indices, the accuracy of the approximation is shown to be substantially affected by the transverse dimensions of the core. The accuracy is characterized by calculating an overlap integral between the exact and WGA-approximated modal fields. Fibers that have an inner cladding similar to the outer cladding, or similar to vacuum, are considered in detail. The feasibility of the WGA in determining the fiber parameters for single-mode guidance is also discussed. © 2005 Optical Society of America

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1. INTRODUCTION

The guided modes of M-type-dielectric-profile optical fibers have attracted attention during the past years in several applications. If the inner cladding is missing, i.e., the volume inside the core is empty, the fiber can be used to guide atoms along the hollow core. In such a hollow optical fiber (HOF) (see Fig. 1) the atoms are confined in the dark near the fiber axis by repulsive dipole interaction between the atoms and the evanescent wave of the guided light, which is assumed to be detuned toward the blue from the atomic resonance. The output beam from the fiber can as well be used to guide and to trap atoms. Fiber lasers based on M-type fibers have attracted attention during the past years in several applications. If the inner cladding is missing, i.e., the volume inside the core is empty, the fiber can be used to guide atoms along the hollow core. The guided modes of M-type-dielectric-profile optical fibers are hence denoted by the value of $\Delta n_{1,2}$ where $\Delta n_{1,2} = n_1 - n_2$. For an ACF such an approximation would seem to be justified by the fact that the refractive-index differences at both of the core–cladding boundaries are then small, but for an HOF with a large index difference at the inner boundary, such an approach is not necessarily appropriate. Nevertheless, HOFs have been widely studied by use of the WGA, and the description has proven to work well when compared with rigorous vectorial calculations and with experimental observations of some guided modes. From these individual cases of agreement one cannot, however, deduce the range of fiber parameters for which the WGA will in general yield acceptable results.

In the first part of this paper, we show that the transverse dimensions of the ring-shaped core will have a major influence on the accuracy of the WGA in addition to the effect of the refractive-index differences. A significant reduction in the accuracy can be seen in a fiber with a core thickness of a few wavelengths and an inner-cladding radius much larger than the wavelength. In such a case, for example, the fundamental hybrid mode HE_{1,1} will no longer have purely linear polarization, which makes the description in terms of a strictly linearly polarized LP_{0,1} mode unsatisfactory. To establish the range of fiber parameters within which the WGA can successfully be applied to ACFs and HOFs, we compare some low-order LP_{m,p} modes with the corresponding superposition of the rigorous vector modes by calculating the overlap between the modal fields.

In the second part of the paper we discuss the feasibility of the WGA in finding the fiber parameters for single-mode propagation. The cutoff for the second lowest vector mode TE_{0,1} is degenerate with that of the LP_{1,1} mode in an M-type fiber, and thus it suffices to consider the cutoffs of the fundamental modes LP_{0,1} and HE_{1,1}. The paper is organized as follows. In Section 2 we outline the derivation of the characteristic equations from Helmholtz’s wave equation both rigorously and under the WGA. A simple measure is then presented to allow for...
Fig. 1. (a) Schematic cross section of an M-type fiber, as defined here, with an inner and outer radius \(a\) and \(b\) of the core, respectively. Below are shown the refractive-index profiles of an annular-core fiber (ACF) and a hollow optical fiber (HOF) for which the inner claddings have the refractive index of that of the outer cladding (of infinite extent) and of vacuum, respectively. The step-index profile of a conventional optical fiber can be obtained on taking the limit \(a \rightarrow 0\). (b) Transverse intensity profiles of the two lowest-order LP_{mn} modes calculated for \(a=2 \, \mu m, b=6 \, \mu m, n_1=1.46, n_2=1.45\), and wavelength \(\lambda=1.55 \, \mu m\).

Comparison of the electric field patterns given by these two formalisms. In Section 3 we apply the measure to investigate the accuracy of the WGA for some low-order modes in HOFs and ACFs. The exact and the WGA cutoff equations are considered in Section 4. Summary and discussion are presented in Section 5.

2. CHARACTERISTIC EQUATIONS AND CONSTRUCTION OF LINEARLY POLARIZED MODES

The exact time-harmonic vector modes of an M-type fiber are found as solutions to Helmholtz’s wave equation, which in the geometry of Fig. 1 is most conveniently solved in cylindrical coordinates. The longitudinal component of the electric and magnetic field of a guided mode propagating in the positive \(z\) direction will then be of the general form \(F(r, \theta, z, t) = F(r, \theta) \exp[i(\omega t - \beta z)]\), where \(\beta\) is the propagation constant, \(\omega\) is the angular frequency of light, and \(t\) denotes time. The field amplitude \(F\) can be written as:

\[
F(r, \theta) = \begin{cases} 
C_1 I_l(r) \sin(\theta + \phi), & r < a \\
[C_2 J_l(r) + C_3 N_l(r)] \sin(\theta + \phi), & a < r < b \\
C_4 K_l(r) \sin(\theta + \phi), & b \leq r
\end{cases}
\]

(1)

Here the functions \(J_l\) and \(N_l\) are Bessel functions of the first and second kind of the order \(l\), respectively. Similarly, \(I_l\) and \(K_l\) denote modified Bessel functions of the first and second kind of the order \(l\), respectively. The parameters \(v, w,\) and \(u\) are given by \(v=(\beta^2-k^2n_0^2)^{1/2}\), \(w=(\beta^2-k^2n_2^2)^{1/2}\), and \(u=(k^2n_1^2-\beta^2)^{1/2}\), with \(k\) denoting the wave number in free space. The parameter \(\phi\) is an arbitrary phase angle and \(C_1, \ldots, C_4\) are constants.

The field in Eq. (1) is usually chosen to describe the longitudinal component \(E_z\) of the electric field. The corresponding magnetic field \(H_z\) is obtained from this expression by replacing “\(\sin\)” with “\(\cos\)” and introducing another set of coefficients \(C_5, \ldots, C_8\). The remaining transverse field components, denoted by the subscript \(t\), can then be derived from the equations:

\[
E_t(r, \theta) = \frac{i}{\beta^2-k^2n_j^2} \left[ \beta \nabla E_z(r, \theta) - \mu_j \omega \mathbf{u}_r \times \nabla H_z(r, \theta) \right],
\]

\[
H_t(r, \theta) = \frac{i}{\beta^2-k^2n_j^2} \left[ \beta \nabla H_z(r, \theta) + \epsilon_j \omega \mathbf{u}_r \times \nabla E_z(r, \theta) \right],
\]

(2)

where \(j=0, 1, 2\) denote the regions in the transverse dielectric profile of the fiber [see Fig. 1(a)], and where \(\mu_j = \mu_{vac}\) and \(\epsilon_j = \epsilon_{vac} n_j^2\) are, respectively, the permeability and permittivity of the region, with \(\mu_{vac}\) and \(\epsilon_{vac}\) being the corresponding values in vacuum. The explicit form of the differential operator is \(\nabla = \mathbf{u}_r \partial / \partial r + \mathbf{u}_\theta \partial / \partial \theta\), where \(\mathbf{u}_r\) and \(\mathbf{u}_\theta\) along with \(\mathbf{u}_z\) in Eq. (2), stand for the unit vectors of the coordinate system. The transverse components turn out to have the separable forms:

\[
E_r(r, \theta) = E_r(r) \sin(\theta + \phi),
\]

\[
E_\theta(r, \theta) = E_\theta(r) \cos(\theta + \phi),
\]

\[
H_r(r, \theta) = H_r(r) \cos(\theta + \phi),
\]

\[
H_\theta(r, \theta) = H_\theta(r) \sin(\theta + \phi).
\]

(3)

By demanding continuity of the tangential components \(E_r, E_\theta, H_r,\) and \(H_\theta\) over the core boundaries at \(r=a\) and \(r=b\), one can construct a matrix equation of the form \(\mathbf{Ax} = \mathbf{0}\), where the vector \(\mathbf{x}\) contains the coefficients \(C_1, C_2, \ldots, C_8\), and the matrix \(\mathbf{A}\) reads.
Here the prime denotes differentiation with respect to the radial coordinate. The propagation constants $\beta$ of the modes are given by the roots of the characteristic equation obtained by requiring that the determinant of this matrix vanish. The values of the coefficients $C_1, \ldots, C_8$ are then assigned by fixing the value of one of them, say $C_5$, and then applying Gaussian elimination to the original matrix equation $A\mathbf{x} = 0$. For the TM$_{0,p}$ modes, however, the coefficients $C_5, \ldots, C_8$ equal zero, and one proceeds by first fixing the value of $C_5$.

In the WGA the expression in Eq. (1) can be directly taken to represent a transverse electric field of a mode$^{18}$ $E = F$ as in the analysis of conventional weakly guiding fibers, with the tilde here referring to a quantity in the WGA. The WGA assumes that the refractive-index differences over the boundaries at $r = a$ and $r = b$ are so small that the transverse field components can be taken to pass continuously over these boundaries. Consequently, the first derivative of $E$, with respect to the radial coordinate will be continuous there as well. These boundary conditions can be collected by use of Eq. (1) as

$$
\frac{\omega}{v}I_m(\nu a) - J_m(\nu a) - N_m(\nu a) 0 0 0 0 0 0 0 0
$$

The TE$_{0,\phi}$(TM$_{0,p}$) mode$^{1,23}$ In such a construction, which is here generalized to an M-type fiber, the vector modes are blended to yield a vanishing transverse component for the electric field,$^{24}$ say, in the $x$ direction. The resulting field will then be polarized essentially in the $y$ direction, and we set $\phi = 0$ in Eq. (1) for both constitutive vector modes (for the TM$_{0,p}$ mode and the even HE$_{2,p}$ mode we choose $\phi = -\pi/2$). Neglecting the small longitudinal component will result in a field that is very close to the corresponding scalar-field LP$_{m,p}$ mode (taken to stand for the $y$ component of the electric field), for which the expression is, however, considerably easier to obtain.

The above approach is, on the other hand, equivalent to the requirement that the transverse components $E_x$ and $E_y$ of the EH$_{m-1,p}$ mode (or the nonvanishing component for the modes TE$_{0,p}$ and TM$_{0,p}$) and $E'_x$ and $E'_y$ of the HE$_{m+1,p}$ mode have similar radial dependencies. This can be seen, assuming $m > 1$, for example, by first resolving the Cartesian components of the superposed field by use of Eq. (3) as

$$
\tilde{E}_x(r, \theta) = -E'_x(r)\sin[(m-1)\theta]\cos \theta - E'_y(r)
\times \cos[(m-1)\theta]\sin \theta + E'_x(r)\sin[(m+1)\theta]
\times \cos \theta - E'_y(r)\cos[(m+1)\theta]\sin \theta,
$$

$$
\tilde{E}_y(r, \theta) = -E'_x(r)\sin[(m-1)\theta]\sin \theta + E'_y(r)
\times \cos[(m-1)\theta]\cos \theta + E'_x(r)\sin[(m+1)\theta]
\times \sin \theta + E'_y(r)\cos[(m+1)\theta]\cos \theta,
$$

where an explicit minus sign is added in front of $E'_x$ in both equations to account for a phase difference $\pi$ between $E'_x$ and $E'_y$. Then, by assuming a common form $E$ for the radial parts $E'_x$ and $E'_y$, the terms in Eq. (7) will add up destructively, yielding $\tilde{E}_x = 0$. In contrast the constructive addition of the $y$ components in Eq. (8) produces the expression
\[ \vec{E}_m(r, \theta) = 2\vec{\hat{E}}(r) \sin(m \theta + \pi/2), \]  

which is similar to Eq. (1) (with the phase angle \( \beta = \pi/2 \)) that was used above to describe the transverse field of an LP\(_{m,p}\) mode. In this construction both of the constituent vector modes carry the same amount of power, hence the factor 2 on the right-hand side of Eq. (9).

The foregoing notion that an LP\(_{m,p}\) mode corresponds to a sum of equal-power vector modes can be taken as the basis for a procedure of evaluating the accuracy of the WGA. By normalizing the power in both of the vector modes according to the relation \( \int \int |\vec{E}|^2 r dr d\theta = 1/2 \), with the electric field vector \( \vec{E} = \vec{E}_s \vec{u}_s + \vec{E}_p \vec{u}_p + \vec{E}_r \vec{u}_r \) given by Eqs. (1)–(3), the superposition field will carry unit power (the HE\(_{1,1}\) mode can be normalized directly to unity). Similarly the normalization of the corresponding LP\(_{m,p}\) mode can be done by use of the relation \( \int \int |\vec{E}|^2 r dr d\theta = 1 \), where the electric field is expressed as \( \vec{E} = \vec{F} \vec{u}_y \) through Eq. (1), with \( \vec{u}_y \) denoting a unit vector in the \( y \) direction. A mismatch between an exact modal field and the corresponding scalar-field approximation can then be revealed by taking an inner product between them and subsequently integrating the result over a transverse plane of the fiber. By denoting the electric fields of the vector modes by \( \vec{E}_s \) and \( \vec{E}_p \), one can formulate this procedure as

\[ W = \int_0^{2\pi} \int_0^r [|\vec{E}_s(r, \theta) + \vec{E}_p(r, \theta)|^2 - |\vec{E}_s(r, \theta)|^2 + |\vec{E}_p(r, \theta)|^2] \cdot |\vec{E}(r, \theta)|^2 r dr d\theta, \]  

which defines a quantity \( W \) to characterize the accuracy of the WGA. An upper limit to the squared modulus \( |W|^2 \) is unity as a result of the chosen normalization of the fields. In such a case an LP\(_{m,p}\) mode would exactly describe a modal field in the fiber, both in polarization and amplitude. Any deviation from this situation, either in polarization or amplitude (and implicitly also in the propagation constants), will be reflected in the value of \( |W|^2 \) by its reduction below unity.

### 3. Modal Description of Light Fields in Annular-Core and Hollow Optical Fibers

In this section we apply the above formalism to study modal fields in ACFs and HOFs with their transverse dimensions selected according to applications in telecommunications and in atom guiding, respectively. For ACFs we select the wavelength of light to be the telecommunications wavelength \( \lambda = 1.55 \mu m \), whereas for HOFs we choose \( \lambda = 780 \) nm, which would correspond to the guiding of Rb atoms. To begin with we give in Fig. 2 an illustration of the effect of the core dimension of an HOF on the accuracy of describing the fundamental HE\(_{1,1}\) mode as an LP\(_{0,1}\) mode. In Fig. 2(a) the inner-cladding radius \( a \) is first taken to be zero, which corresponds to the case of a conventional step-index optical fiber. The HE\(_{1,1}\) mode then has perfectly linear polarization, and the corresponding plot for the LP\(_{0,1}\) mode coincides with the one presented, i.e., the accuracy parameter has the value of \( |W|^2 = 1.00 \).

If a hole of radius \( a = 8 \mu m \) is then introduced in the center of the structure as in Fig. 2(b), an HOF with a core thickness of a few wavelengths is formed. The polarization of the fundamental HE\(_{1,1}\) mode is seen clearly to deviate from linear in such an HOF, and the amplitude distribution is far from being rotationally symmetric, both of which are features of the corresponding LP\(_{0,1}\) mode shown in Fig. 2(c) for comparison. The number describing the accuracy of the WGA is now \( |W|^2 = 0.84 \).

Figure 3(a) shows the behavior of the quantity \( |W|^2 \) for some ACFs of different outer radii \( b \) of the core. Figure 3(b) shows the corresponding curves for HOFs. Four curves are plotted for each \( b \) as a function of the inner-cladding radius \( a \) by varying the refractive-index difference \( \Delta n_{1,2} \) between the core and the outer cladding. In Fig. 3(a) the parameter \( \Delta n_{1,2} \) ranges from 0.50% to 2.00%. For the smallest value \( b = 10 \mu m \), the LP\(_{0,1}\) mode is very close to the HE\(_{1,1}\) mode, causing the value of \( |W|^2 \) to be near unity for all the considered values of \( \Delta n_{1,2} \). However, when the value of \( b \) is increased, a dip emerges in the curves for values of \( a \) a few wavelengths smaller than \( b \). The depth of the dip scales with the values of \( \Delta n_{1,2} \) and \( b \), and near the bottom of the dip the HE\(_{1,1}\) and LP\(_{0,1}\) modes differ qualitatively as in Figs. 2(b) and 2(c). When \( a \) is very close to \( b \), the accuracy is again recovered. In general, for a small \( \Delta n_{1,2} \) (much smaller than would be allowed in a conventional fiber), the fundamental mode is essentially an LP\(_{0,1}\) mode irrespective of the core dimensions, but for values on the order of \( \Delta n_{1,2} \approx 2\% \) or higher, the core has to be much thicker (or thinner) than the-

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**Fig. 2.** Effect of the inner radius of the core on the polarization and amplitude of the fundamental mode. The fiber parameters are \( \Delta n_{1,2} = 1.00\% \) and \( b = 10 \mu m \) and the wavelength is \( \lambda = 780 \) nm. The core boundaries are shown by dashed circles. (a) HE\(_{1,1}\) or LP\(_{0,1}\) mode in a conventional step-index fiber. (b) HE\(_{1,1}\) mode in an HOF with an inner radius of \( a = 8 \mu m \). (c) LP\(_{0,1}\) mode of the fiber in Fig. 2(b) calculated by use of the WGA.
wavelength, or altogether relatively small, for this to be true.

In HOFs intended for use as atom guides, the refractive-index difference and the core dimensions are typically smaller than in the ACF examples considered above, because the light field needs to be very smooth and intense on the fiber’s inner surface. In Fig. 3(b) the highest value of \( b \) is thus chosen to be 9 \( \mu m \) with the parameter \( \Delta n_{1,2} \) ranging from 0.25% to 1.00%. Qualitatively, the curves begin to behave similarly to those for the ACFs for small values of \( a \). As \( a \) approaches \( b \), the HE\(_{1,1}\) mode reaches its cutoff before the LP\(_{0,1}\) mode (see Section 4), causing the value of \( |W|^2 \) to drop sharply near the cutoff. Before this, the curve exhibits a dip similar to those with the ACFs of Fig. 3(a). Again the dip is more clearly visible in the larger fibers. As a rule one might say that for the values of \( \Delta n_{1,2} \) considered here, the thickness of the core should not be less than \( b/2 \) for the fundamental mode to be well described by an LP\(_{0,1}\) mode, unless \( \Delta n_{1,2} \) is very small, in which case being far from the cutoff is sufficient.

Figure 4 shows the corresponding plots for the second-order modal fields in the fibers of Fig. 3. The curves corresponding to the superposition of the odd HE\(_{2,1}\) mode and TE\(_{0,1}\) mode are solid, and those for the superposition of the even HE\(_{2,1}\) mode and the TM\(_{0,1}\) mode are dashed. In Fig. 4 the values of \( |W|^2 \) are in general much closer to unity than in Fig. 3, although the qualitative behavior is very similar. However, all the curves in Fig. 4(a) eventually bend down for very thin fiber cores because of the cutoffs for the TE\(_{0,1}\) and LP\(_{1,1}\) modes, which are degenerate.\(^{20,21}\) In Fig. 4(b) the declines due to these cutoffs are more visible and one can see that the dashed curves fall off monotonically, whereas the solid curves rise slightly for some of the largest fibers, as in Fig. 3(b). In brief a second-order modal field can effectively be described as an LP\(_{1,1}\) mode below modal cutoffs, if similar guidelines are applied in choosing the fiber parameters as with the fundamental modes above. Note that the analysis for an HOF with the other wavelength \( \lambda = 1.55 \mu m \) can be obtained by scaling \( a \) and \( b \) with the ratio of the wavelengths (=2) in Figs. 3(b) and 4(b).

4. SINGLE-MODE M-TYPE FIBERS

In this section we consider the cutoffs of the modes considered in the previous section to determine how well the WGA works in finding the parameters for single-mode guidance in M-type fibers. The exact cutoff equation follows by requiring that the determinant of the matrix \( A \) in Eq. (4) vanish in the limit of \( w \to 0 \). The relevant cutoffs are then found from the equation

\[
\begin{align*}
\text{Fig. 3.} & \quad \text{Accuracy } |W|^2 \text{ of the WGA in describing the HE}_{1,1} \text{ mode (a) in an ACF at } \lambda = 1.55 \mu m \text{ and (b) in an HOF at } \lambda = 780 \text{ nm with } n_2 = 1.45 \text{ as a function of the inner radius } a \text{ of the core. Three different outer radii } b \text{ of the core, denoted by vertical dotted lines, are considered in both plots. The curves correspond to refractive-index differences increasing from top to down, as indicated.}
\end{align*}
\]

\[
\begin{align*}
\text{Fig. 4.} & \quad \text{Correspondence between LP}_{1,1} \text{ mode with the superposition of odd HE}_{2,1} \text{ and TE}_{0,1} \text{ (solid curves) and even HE}_{2,1} \text{ and TM}_{0,1} \text{ (dashed curves) modes in terms of the quantity } |W|^2 \text{ as a function of the inner radius } a \text{ of the core. The different outer radii } b \text{ of the core, the wavelengths, and the refractive indices are as in Fig. 3. Second-order modes do not exist for } b = 3 \mu m \text{ with } \Delta n_{1,2} = 0.25\%; \text{ thus only three pairs of curves are given.}
\end{align*}
\]
\[
\left\{ \frac{1}{u} N_l^{(u)}(ua) + \frac{1}{v} \frac{I'_l(ua)}{I_l(ua)} N_l^{(u)}(va) \right\} J_l(ub)
\]
\[
- \left\{ \frac{1}{u} N_l^{(u)}(ua) + \frac{1}{v} \frac{I'_l(ua)}{I_l(ua)} N_l^{(u)}(va) \right\} J_l(ub)
\]
\[
\times \left\{ \frac{n_l^2}{u} N_l^{(u)}(ua) + \frac{n_l^2}{v} \frac{I'_l(ua)}{I_l(ua)} N_l^{(u)}(va) \right\} J_l(ub)
\]
\[
- \left\{ \frac{n_l^2}{u} N_l^{(u)}(ua) + \frac{n_l^2}{v} \frac{I'_l(ua)}{I_l(ua)} N_l^{(u)}(va) \right\} J_l(ub)
\]
\[
- \left( \frac{n_l}{a} \right)^2 \left( \frac{1}{u^2} + \frac{1}{v^2} \right)^2
\]
\[\times (J_l(ua) N_l^{(u)}(ub) - J_l(ub) N_l^{(u)}(ua))^2 = 0. \] (11)

This equation is similar to the equation displayed in Ref. 1 with the difference that here none of the expressions in the square brackets explicitly depend on \(b\). The cutoffs for the TE_{0,p} (TM_{0,p}) modes are obtained by setting the first (second) term in the curly brackets equal to zero with \(l = 0\). For the cutoffs of the HE_{1,p} (and the EH_{1,p}) modes one sets \(l = 1\) throughout the equation.

In the WGA the cutoff equation follows from Eq. (6) on taking the limit \(\tilde{w} \to 0\). On substituting the coefficient \(\tilde{C}_2\) from Eq. (5) and using the recurrence relations of the regular and modified Bessel functions,\(^{25}\) one can cast the resulting equation in the form

\[
\left\{ \frac{1}{\tilde{u}} N_m^{(\tilde{u})}(\tilde{u}a) + \frac{1}{\tilde{v}} \frac{I'_m(\tilde{u}a)}{I_m(\tilde{u}a)} N_{m-1}^{(\tilde{u})}(\tilde{u}a) \right\} J_{m-1}(\tilde{u}b)
\]
\[
- \left\{ \frac{1}{\tilde{u}} J_m(\tilde{u}a) + \frac{1}{\tilde{v}} \frac{I'_m(\tilde{u}a)}{I_{m-1}(\tilde{u}a)} J_{m-1}(\tilde{u}a) \right\} N_{m-1}(\tilde{u}b) = 0.
\] (12)

In this equation, one sets \(m = 0\) and \(m = 1\) for the LP_{0,p} and LP_{1,p} modes, respectively.

By comparing Eq. (12) with \(m = 1\) to the first two rows of Eq. (11) with \(l = 0\), one can see that the expressions become identical. In particular the cutoffs of the second lowest modes LP_{1,1} and TE_{0,1} are degenerate in any M-type fiber. For the fundamental modes with \(m = 0\) and \(l = 1\), however, Eqs. (12) and (11) will yield differing results. Figure 5 shows an example of the ratio of the cutoff wavelengths \(\tilde{\lambda}_c\) and \(\lambda_c\) of the LP_{0,1} and HE_{1,1} modes, respectively, obtained from the two equations as a function of the refractive index \(n_0\). A few values of the ratio \(a/b\) between the inner and outer radii of the core are considered.

As the value of \(n_0\) decreases from the value of \(n_1\), the HE_{1,1} mode will attain a finite cutoff wavelength slightly earlier than the LP_{0,1} mode, yielding a high value for the ratio \(\tilde{\lambda}_c/\lambda_c\). These cutoffs take place for the values\(^{21}\)

\[
n_{0,c} = [n_1^2 - (b/a)^2(n_1^2 - n_2^2)]^{1/2},
\] (13)

which are marked in Fig. 5 as vertical dotted lines. Below this critical value the ratio of the cutoff wavelengths reaches a local minimum that is closer to unity in value and occurs nearer to \(n_{0,c}\) for fibers with a high value of the ratio \(a/b\). On the whole the WGA is seen to determine the cutoff wavelength more accurately for such fibers than for fibers with a small value of \(a/b\).

5. SUMMARY AND DISCUSSION

We have studied the effect of the transverse dimensions of an M-type fiber on the accuracy of modal analysis by use of the weakly guiding approximation (WGA). When the inner and outer claddings of the fiber have the same index of refraction, the LP_{0,1} and LP_{1,1} modes were shown to describe the modal field well for core thicknesses much greater than the optical wavelength. On the other hand, a refractive index of unity of the inner cladding requires the core to be thicker than roughly half the outer radius of the core for the modal description obtained by means of the WGA to be accurate. For a fixed wavelength in general, the discrepancies are more significant in larger fibers, and the approximation is more accurate for the second-order modal fields than for the fundamental. In addition, it was found that in the WGA, the cutoff wavelength for the fundamental mode can be most accurately determined if the ratio between the inner and outer radii of the core is high, i.e., if the core is thin.

When applying a hollow single-mode fiber to guiding of cold atoms, the efficient transfer of the atoms into the fiber will strongly depend on the intensity distribution just outside the fiber. We note that in cases where the LP_{0,1} mode well describes the rigorous HE_{1,1} mode, the scalar diffraction calculation will bring out the true output field.\(^{4,26}\) On the other hand when an incident light beam is coupled into an M-type fiber, the LP-mode approximation will predict inaccurate modal coupling efficiencies when \(|W|^2\) differs from unity. This can be important, for instance, in calculations of the optimum pump-power coupling in an M-type-fiber laser,\(^{6,7}\) since a thin-core fiber is required for single-radial mode propagation. A similar size requirement is found also in self-imaging applications of ACFs.\(^{15,17}\) As regards optical fibers in general, the justification of the WGA often relies on the smallness of the refractive-index differences. Rigorous investigations, such as the work reported here, will then be valuable for justifying the approach selected for the modal analysis.

![Fig. 5. Ratio of cutoff wavelengths \(\tilde{\lambda}_c\) (WGA) and \(\lambda_c\) (exact) of the fundamental mode as a function of the refractive index \(n_0\) of the inner cladding for different ratios between the inner and outer radii, \(a\) and \(b\), respectively, of the core. The fixed refractive indices are \(n_1 = 1.4525\) and \(n_2 = 1.45\).](image-url)
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