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PID CONTROLLER TUNING FOR INTEGRATING PROCESSES: ANALYSIS AND NEW DESIGN APPROACH

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ABSTRACT

This paper discusses PID controller tuning for integrating processes with time-delay and first order lag. Most of the existing tuning rules for this kind of processes have the same general structure, and the properties of these rules are discussed especially in connection with varying time-delay systems. The paper proposes a novel tuning method that optimizes the closed-loop performance with respect to certain robustness constraint while considering the delay variance via jitter margin maximization. Further, we develop new PID controller tuning rules based on the tuning method. The paper discusses the new tuning rules in detail and compares them with some of the recently published results. The work was originally motivated by the need for robust but simultaneously well performing PID controller tuning parameters in an agricultural machine case process. We also demonstrate the superiority of the proposed tuning rules with this case process.

1. INTRODUCTION

The tuning of the PID controller has been discussed in numerous articles and books, and there exists a variety of tuning methods. Maybe the best known tuning rules are those proposed by Ziegler and Nichols already in 1942. Still today the Z-N methods are popular in process control. It is obvious that the Z-N tuning methods do not meet the requirements of all the processes in today's industry. An example of this is a networked control system where varying time-delays might endanger the stability.

PID tuning is not a completely solved problem despite of the decades of research. On the contrary its research seems to grow [1]. Some of the recent tuning methods are presented in [2]. The research described in this paper was motivated by the need for PID tuning rules for integrating processes where variable transport delays and gain parameters affect the system stability and performance. The present tuning rules are investigated in this framework and a new tuning method and rules are also proposed.

The need for tuning rule development was experienced when prototyping some of the most recent tuning rules [3] in a case process that is presented in Section 2. The section also presents the preliminaries required to understand the proposed tuning approach and reviews the current tuning rules for the FOLIPD (first order lag plus integral plus delay) process model that is considered throughout the paper. Section 3 analyzes the properties of PID controlled FOLIPD systems. Section 4 presents the new tuning approach and rules that are compared with other tuning rules in Section 5. Section 6 states the conclusions.

2. CONTROL SYSTEM

The general layout of the control system and its components are discussed in this section. In addition, the case process is described. The PID controller tuning rules currently found in the literature are also reviewed. The control system goodness measures used in this paper such as the distance from the "robustness circle" and the jitter margin are presented.

2.1. Process model

The general layout of the control system is seen in Figure 1. We consider an integrating process in connection with a low-pass measurement filter. Alternatively, the low-pass filter can be part of the process (integrator + first order lag). In both cases the process model is given as

$$P(s) = \frac{K_v}{s\left(1+sT_F\right)} e^{-sL} . \tag{1}$$

This is also known as the FOLIPD model [1], where K_v is the velocity gain, T_F the filter or lag time constant, L the time-delay, and s the Laplace variable. The tuning rules are later developed by assuming that T_F is fixed by the process measurement setup such that adequate noise compensation is achieved.

2.2. Case process

In the experiments a real integrating process is used. The process is a part of an agricultural tractor, and it consists of a hydraulic system, an electronically controllable hydraulic valve, a hydraulic cylinder actuator connected to a weight and a position sensor. The position of the weight is controlled. The mass of the weight varies and also occasional counterforces by ground contact are evident. The hydraulic valve is controlled via CAN bus, and this limits the control cycle to 100 ms. The delay of the complete system is identified to vary between 200 and 300 ms, consisting of communication bus delay, valve dynamics, oil pressure and flow in the hydraulic pipes and position measurement delays. The integrating case process has a variable transport delay and -gain, and with a measurement filter it can be modeled as (1).

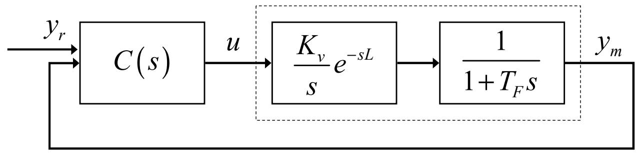

Figure 1. *The general layout of the control system.*

2.3. PID controller

We consider the continuous-time PID controller of the form [4]

$$u(t) = k\left(by_r(t) - y_m(t)\right) + k_i\int_0^t \left(e(\tau)\right)d\tau + k_d\left(c\frac{dy_r(t)}{dt} - \frac{dy_m(t)}{dt}\right), \tag{2}$$

where $e(\tau) = y_r(\tau) - y_m(\tau)$ is the error signal between reference signal and measured (filtered) output. The parameters k, k_i and k_d are proportional-, integration- and derivative gains, respectively. The set-point weighting parameters b and c are fixed in advance, here $b = 1$ and $c = 0$. The transfer function of (2) is

$$C(s) = k + k_i\frac{1}{s} + k_d s\ . \tag{3}$$

2.4. AMIGO tuning

The AMIGO tuning rules (see [3], [5]) were recently developed both for non-integrating and integrating processes. The good experiences with these tuning rules for non-integrating processes encouraged the authors to prototype the AMIGO approach for integrating processes in the case process. Nevertheless, the results from the case process indicated that the performance of the control system could be improved by replacing the AMIGO tuning with some other method.

The tuning method proposed in this paper takes the similar approach as the AMIGO tuning, where the process is modeled with a simple first order linear model or the integrator model. The model is developed in the spirit of Ziegler-Nichols via step response experiments. The tuning rules are then derived based on the few process parameters (gain, time constant, delay).

The AMIGO tuning for integrating processes is based on characterizing the process using the IPD (integral plus delay) model structure [1]

$$P(s) = \frac{K_v}{s}e^{-sL}\ . \tag{4}$$

After numerous analyses of parameter relationships and extensive studies of robustness and performance, the following tuning rules are proposed in [3] and [5] for integrating processes.

$$\begin{aligned} k &= 0.45 / K_v, \\ k_i &= 0.05625 / \left(K_v L\right), \\ k_d &= 0.225 L / K_v. \end{aligned} \tag{5}$$

This tuning is called AMIGO tuning (approximate M-constrained integral gain optimization).

2.5. PID tuning rules for FOLIPD

As the AMIGO tuning rules were originally tested with the case process with unsatisfactory results, other PID tuning rules for integrating processes were investigated. For the pure integrator process there are several tuning methods, but for the FOLIPD there are not so many.

Numerous tuning rules are collected into Handbook of PI and PID Controller Tuning rules [1], also for FOLIPD process model. Most of the tuning rules for FOLIPD are given in the form

$$k = \frac{a}{K_v L},\ k_i = 0,\ k_d = \frac{aT_F}{K_v L} \tag{6}$$

where a is tuned with various methods. Vitečkova *et al.* [6] have tuned a based on overshoot criterion and O'Dwyer [1] has derived a from gain and phase margins. Numerical values of a vary roughly from 0.3 to 1.0. By writing out the open-loop equation

$$\begin{aligned} C(s)P(s) &= \left(\frac{a}{K_v L} + \frac{aT_F}{K_v L}s\right)\frac{K_v}{s\left(1+sT_F\right)}e^{-sL} \\ &= \frac{a}{Ls}e^{-sL} \end{aligned} \tag{7}$$

it can be seen that the open-loop, and thus also the closed-loop, is independent of the values of K_v and T_F. In other words the controller structure eliminates those known process parameters.

Other tuning rules for FOLIPD are presented by Rivera and Jun [7], and these rules are converted in [1] into form

$$k = \frac{L + T_F + 2\lambda}{K_v\left(L+\lambda\right)^2},\ k_i = \frac{1}{K_v\left(L+\lambda\right)^2},\ k_d = \frac{T_F\left(L+2\lambda\right)}{K_v\left(L+\lambda\right)^2}, \tag{8}$$

where λ is an adjustable parameter, value that should correspond approximately to the closed-loop response speed.

If the lag T_F in (1) is rather small compared to the delay L, the tuning rules for IPD can also be used. Åström and Hägglund [4] propose the following rules based on Ziegler-Nichols ultimate cycle equivalent method

$$k = \frac{0.94}{K_v L},\ k_i = \frac{0.94}{2K_v L^2},\ k_d = \frac{0.47}{K_v}\ . \tag{9}$$

All the tuning rules collected in [1] for IPD and FOLIPD were tested with the case process using simulation. Some rules seemed to work only for a certain range of process parameters L and T_F, and some rules gave unsatisfactory performance versus robustness ratio that is an important factor in the case process.

2.6. Robustness for disturbances: The *M*-circle

The development of the AMIGO rules was based on the following robustness criterion: if the Nyquist curve of the loop transfer function does not intersect a circle with center c_R and radius r_R defined as

$$c_R = -\frac{2M^2 - 2M + 1}{2M(M-1)},\ r_R = \frac{2M-1}{2M(M-1)}, \tag{10}$$

the sensitivity function and the complementary sensitivity functions are less than M for all frequencies [8]. The robustness is thus captured by one parameter only, M. The value $M = 1.4$ was used in the AMIGO rule development, although finally the rules did not quite satisfy the constraint. For the test process batch a 15 % increase of M was reported, resulting in $M \approx 1.6$.

2.7. Robustness for delay variance: The jitter margin

Whereas the robustness parameter M concerns the disturbances such as measurement noise, in the case process also other type of robustness is required. The process suffers from varying time-delays as mentioned in Section 2.2. Often such time-delays are known to be bounded, and it might be tempting to design a worst-case controller using the maximum delay. Unfortunately, a controller designed for the maximum delay does not guarantee that the closed-loop system would be stable as the delay varies in the range from the minimum to the maximum value. [9]

Recently proposed stability criteria for systems with varying time-delays [10] are suitable for our usage, since they can be formulated as objective functions in the optimization of PID controller parameters. The *jitter margin* is an upper bound for additional delay that can be added into a closed-loop control system while maintaining stability. The delay can be of any type (constant, time-dependent, random), but the jitter margin determines the bound for the maximum value of the delay. A continuous-time SISO system is stable for any time-varying delays defined by

$$\Delta(v) = v\left(t - \delta(t)\right), \qquad 0 \le \delta(t) \le \delta_{\max} \tag{11}$$

if

$$\left|\frac{P(j\omega)C(j\omega)}{1+P(j\omega)C(j\omega)}\right| < \frac{1}{\delta_{\max}\omega}, \qquad \forall \omega \in [0, \infty[\,, \tag{12}$$

where $\delta_{\max}$ is the maximum additional delay (the jitter margin). This criterion has been successfully applied in the derivation of PID tuning rules for non-integrating processes in varying time-delay systems [11].

3. ANALYSIS OF FOLIPD TUNING RULES

For the FOLIPD process model most of the PID tuning rules have the same general structure (6), i.e. PD controller. According to (7), when these tuning rules are applied, the open-loop system becomes independent of T_F and K_v. The Nyquist curve of the open-loop transfer function is

$$G_{ol}(j\omega) = \frac{a}{Ls}e^{-sL}\bigg|_{s=j\omega} = -\frac{a}{\omega L}\left(\sin(\omega L) + j\cos(\omega L)\right). \tag{13}$$

This indicates that with high frequencies the Nyquist curve converges to the origin. The parameters a and L determine the distance from (-1,0) such that with higher values of a the gain margin decreases. The jitter margin of the system becomes

$$\delta_{\max} < \left|\frac{1+G_{ol}(j\omega)}{j\omega G_{ol}(j\omega)}\right| = \sqrt{\left(\frac{L}{a}\right)^2 - \frac{2L}{a\omega}\sin(\omega L) + \frac{1}{\omega^2}}\,. \tag{14}$$

In order to calculate the analytical expression for the jitter margin in the FOLIPD case with the controller tuning (6), the expression (14) should be minimized with respect to frequency ω. This turns out to be a hard problem analytically, but rather easy using numerical methods. For the analysis of the jitter margin we concentrate on parameter ranges $0.368 \le a \le 1.008$ and $0.01 \le L \le 100$. The range for a is chosen similarly as in [6] where a determines the overshoot of the closed-loop response (0 – 50 %). The range for delay L is simply chosen to be very wide. The jitter margin for the FOLIPD process model with the PD controller (6) is shown in Figure 2 with respect to parameters a and L. A closer look at the jitter margin surface reveals that for a fixed a the jitter margin is nearly linear function of delay L. For practical use of this analysis, it would be convenient to have an expression for a (the closed-loop "performance" parameter) as a function of L and $\delta_{\max}$. Often the minimum delay (L) and the possible additional delay ($\delta_{\max}$) of the system are known, but the problem is how to select between robustness and performance (a). Thus we calculate an approximation for the jitter margin and solve it for a. The jitter margin surface can be approximated by

$$\delta_{\max} = \left(\frac{0.9485}{a} - 0.6356\right)L\,, \tag{15}$$

which gives

$$a = \frac{0.9485L}{\delta_{\max} + 0.6356L}\,. \tag{16}$$

This is the maximum value for gain a that can be used with certain jitter margin requirement. For example, if the process minimum time-delay is $L = 0.5$ and the required jitter margin is 50 % of L, it is possible to use gain $a = 0.8352$ which gives approximately 30 % overshoot for the closed-loop system with good performance. In order to have less overshoot, the gain a can be decreased without endangering the stability, since the smaller values of a increase the jitter and gain margins. In the chosen range of parameters the estimate (15) gives a maximum error of ±2.5 % of the true jitter margin (14).

4. NEW TUNING APPROACH

In this section we propose a new PID controller tuning approach that explicitly takes into account the robustness criteria presented in Section 2. We introduce two objective functions to be optimized simultaneously, and use simulation based constrained optimization to solve the optimal parameters for the PID controller in connection with the FOLIPD process. The tuning rules are then derived based on the parameter surfaces produced in the optimization phase. Multi-objective optimization is used for solving the problem, since the optimal controller parameters should minimize more than one conflicting objectives simultaneously.

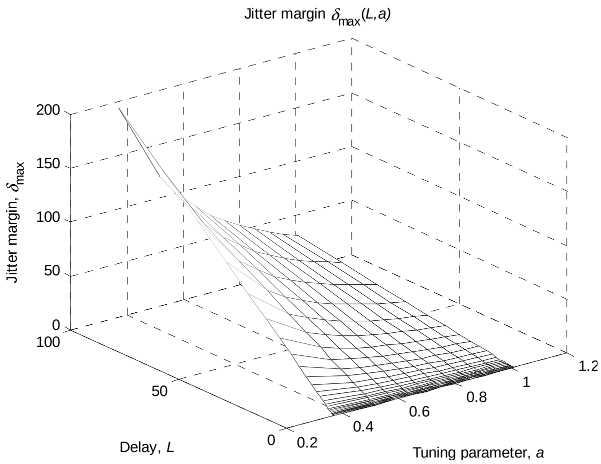

Figure 2. *The jitter margin (FOLIPD + PD controller).*

4.1. Multi-objective optimization

In order to solve the controller tuning problem we use multi-objective constrained optimization. A general multi-objective optimization (here minimization) problem is given as

$$\begin{aligned} &Min && F(x) = \{f_1(x) \quad \cdots \quad f_k(x)\} \\ &s.t. && x \in \Omega \\ & && \Omega = \left\{ x \left| \begin{matrix} g_i(x) \le 0 & i = 1,...,m_1 \\ h_j(x) = 0 & j = 1,...,m_2 \\ x = [x_1 \cdots x_n]^T & \end{matrix} \right. \right\}, \end{aligned} \tag{17}$$

where $f_l(x)$, $l = 1\ldots k$, are nonlinear objective functions that are to be minimized simultaneously, x_i are the decision variables and $g_i(x)$ and $h_j(x)$ are the nonlinear inequality and equality constraints, respectively (see e.g. [12]). There are numerous algorithms for solving the above problem, of which the *goal attainment method* will later be used for deriving the tuning rules. The goal attainment problem is defined as

$$\begin{aligned} &Min && \gamma \\ &\text{s.t.} && F(x) - \alpha \cdot \gamma \le F_g \\ & && x \in \Omega, \end{aligned} \tag{18}$$

where γ is an auxiliary variable, α is a vector of weights and F_g is a vector of goals, i.e. the objective function values that should be attained.

4.2. Problem formulation

In order to apply multi-objective optimization the objective functions must be set. We use the ITAE cost criterion to measure the performance of the closed-loop system. The other criterion is the jitter margin that should be maximized. The robustness with respect to disturbances is also taken into account by introducing an optimization constraint that keeps the open-loop Nyquist curve outside the robustness circle (10) similarly as in the AMIGO rules. Here $M = 1.5$ is used as the robustness parameter, which approximately corresponds to the obtained value that was reported with AMIGO tuning rules. Note that we use this value as a *hard* constraint, whereas in AMIGO rules $M = 1.4$ was rather a *soft* constraint or an objective. As mentioned before the AMIGO rules finally had to relax (increase) this value up to 15 %.

The optimization problem is formulated in (19)-(22), where $f_{1,2}(x)$ are the objective functions to be minimized and $g(x)$ is the constraint function that the decision variables x must satisfy.

$$f_1(x) = \int_0^\infty t\left|e(t)\right| d\tau = \int_0^\infty t\left|y_r(t) - y(t)\right| d\tau \tag{19}$$

$$f_2(x) = \frac{1}{\delta_{max}} = \frac{1}{\min\limits_{\omega \in [0,\infty[} \left| \frac{1 + H(j\omega)}{j\omega H(j\omega)} \right|} \tag{20}$$

$$\begin{aligned} g(x) &= -d \\ &= -\min_{\omega} \left(\sqrt{\left(\mathrm{Re}(H) - c_R\right)^2 + \left(\mathrm{Im}(H)\right)^2} - r_R \right) \le 0 \end{aligned} \tag{21}$$

where

$$x = \begin{bmatrix} k & k_i & k_d \end{bmatrix}^T \text{ and } H(j\omega) = C(j\omega)P(j\omega)\,. \tag{22}$$

4.3. Tuning results

The controller tuning problem was solved using MATLAB's Optimization Toolbox and *fgoalattain* function. The weights of the goal attainment method were chosen such that both performance (19) and jitter margin (20) were equally weighted. The goal for ITAE criterion was set equal to the ITAE criterion for the Vitečkova *et al.* [6] tuning (6) with $a = 0.4$ corresponding to overshoot of 10 % for the closed-loop system. The goal for the jitter margin was set to $T_F + L$ corresponding to the effective dead-time of the process. This margin would allow the delay to increase by 100 % from the effective dead-time while guaranteeing the stability. Note that this is in some cases quite a high objective, but as it is handled as an objective rather than a constraint, it is reasonable. The initial values of the controller parameters for the optimization were chosen according to the Vitečkova *et al.* [6] tuning. The range for parameters T_F and L was chosen from 0.01 to 100, but only values for which the ratio T_F / L remains in the range [0.1, 10] were considered. This restriction was motivated on one hand by simulation accuracy, since the system tends to become stiff for values outside of this range, and on the other hand by reasoning. One of the parameters easily becomes negligible outside this range.

The optimization results are presented in Figure 3, where controller gains (k, k_i and k_d) and the ITAE cost are presented with respect to the process nominal delay L and process time constant T_F. The controller gain k increases as L and T_F decrease. The integral gain k_i remains in zero. The derivative gain k_d decreases as L and T_F decrease. The ITAE cost increases naturally as the delay increases, but also as T_F increases. Figure 4 shows the open-loop Nyquist curves when applying the optimal tuning to the FOLIPD process (1).

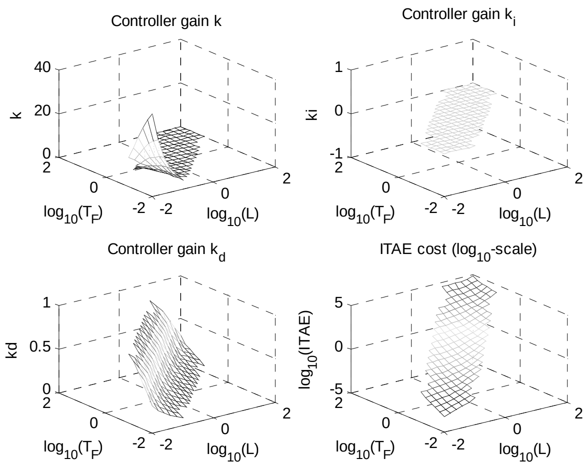

Figure 3. *Optimized PID controller parameters.*

4.4. Tuning rules

Based on the controller parameter surfaces the tuning rules were developed. This phase included both determining the rule structure and estimating the coefficients. The rule identification was done similarly as in [11]. As with the other FOLIPD tuning rules the integral gain is zero, and the proportional and the derivative gains are inversely proportional to the process velocity gain K_v. The proposed tuning rules for the FOLIPD processes are

$$k = \frac{10^{f(L,T_F)}}{K_v L},\ k_i = 0,\ k_d = \frac{T_F^{g(L,T_F)}}{K_v} 10^{h(L)}, \qquad (23)$$

where

$$\begin{aligned} f(L,T_F) &= 0.0027(T_F/L)^2 - 0.0794 T_F/L - 0.34, \\ g(L,T_F) &= 0.02 + \left(0.51 - 0.076\log_{10}(T_F)\right)L^{0.15}, \qquad (24) \\ h(L) &= 0.97 - 1.48 L^{0.15}. \end{aligned}$$

5. EXPERIMENTS

The developed tuning rules were utilized in the case process. The step response of the process was recorded in varying cases. Identification of the process parameters was automated with a MATLAB script. It was found out that the process gain K_v varies between 1.3 and 2.2 and the transport delay L between 0.25 and 0.35. Reasonable values for the measurement filter time constant T_F are 0.10-0.20. Based on the set of identification results, the process was fixed with parameters $K_v = 1.8$, $L = 0.25$, $T_F = 0.15$.

5.1. Comparison of tuning rules by simulation

All the tuning rules collected in [1] for IPD and FOLIPD were tested with the case process model using simulation. The new tuning rules were compared to these. Figure 6 presents the robustness properties and the simulated step responses of the case process when using different tuning rules. On the left, there are the Nyquist curves and on the right, the unit step responses.

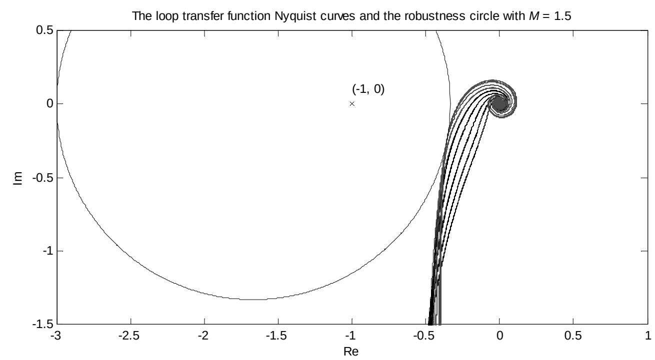

Figure 4. *Optimized open-loop systems' Nyquist curves.*

The circles in Figure 6 represent the robustness circle (10) with $M = 1.5$. Only the proposed tuning and Vitečkova *et al.* [6] tuning give good performance with adequate robustness properties that fulfill the servo control requirements presented above. Nevertheless, the proposed tuning gives a better settling time and robustness properties simultaneously, and there is no overshoot. The lowest plots of Figure 6 compare the step responses and robustness properties between the proposed rules and the Vitečkova *et al.* rules with overshoot of 10 %. It can be concluded that the new rules are superior to the other rules in many respects.

5.2. Experiments with the real process

In the case process the response of the hydraulic valve is not linear to the control signal in the whole range, but in the experiments only a part of the full range was used (0-40 %), where the valve is approximately linear to the control signal. This control signal limitation was also taken into account in the simulations. The comparison of the simulated and real process step responses using the new PID tuning rules is presented in the Figure 5.

6. CONCLUSIONS

In this paper the PID controller tuning for integrating processes was considered. The few existing tuning rules for FOLIPD process model were analyzed with respect to robustness for disturbances and for varying time-delay. The common structure of these tuning rules was analyzed and the dependency of tuning rule performance parameter and time-delay robustness criterion was shown. In addition, a novel tuning method for the PID controller was proposed and based on the design concept, new tuning rules were developed. The new tuning rules were tested by simulation and in a case process. The new tuning rules were shown to outperform the other known tuning rules for FOLIPD process.

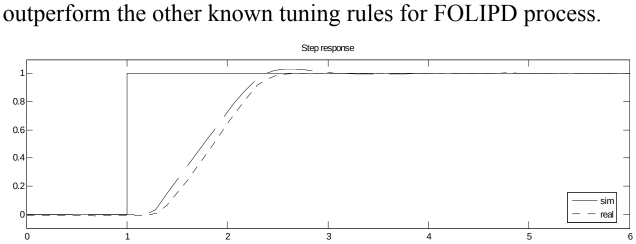

Figure 5. *Step response with simulator and real process using the new rules.*

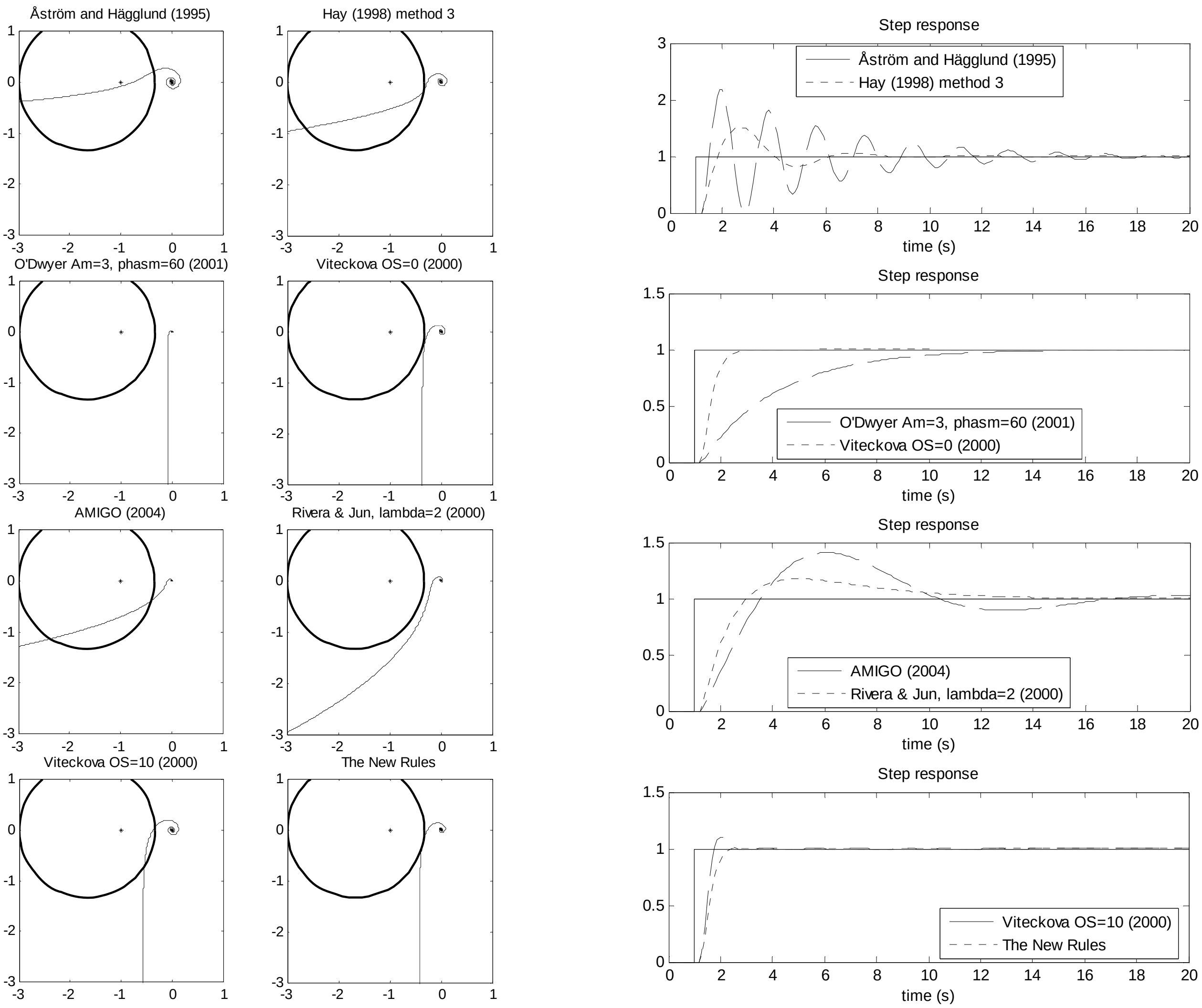

Figure 6. *Comparison of loop transfer function Nyquist curves (robustness circle with M = 1.5) and unit step responses.*

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