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Power and Q of a Horizontal Dipole Over a Metamaterial Coated Conducting Surface

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***Abstract*—The radiation properties of an electrically small antenna attached to an infinite conducting ground plane covered by a thin sheet of a metamaterial medium are considered. Using analytical field expressions, the radiation of an electric point dipole with arbitrary constitutive parameters of the material coating is studied. Attention is paid especially to permittivity values close to zero. Finite element simulations are employed to confirm the radiation enhancement and low quality factor Q characteristic of a zero-index coating as compared with regular "air-filling." The Q is evaluated partly through integration of the energy of the FEM-simulated fields within the antenna region and partly from the nonradiating component of the multipole fields outside the antenna region. Additionally, the influence of surface wave power is analyzed through simulations.**

***Index Terms*—Conducting surface, dipole antenna, Q factor, radiated power, zero-index materials.**

I. Introduction

ANTENNA miniaturization is a big engineering challenge because of the fundamental limitations that restrict the performance of electrically small antennas. Especially problematic are antennas designed to operate close to a conducting surface (ground plane) [1], since the radiation from impressed antenna currents flowing tangentially to the surface tend to be cancelled out by the surface currents induced on the plane [2]. Usually, this results in a high reactive field level, low radiation resistance as well as a narrow resonant bandwidth. In addition, the efficiency may be poor due to ohmic losses that are sometimes considerable compared to the radiation loss. An important approximate measure for the bandwidth capacity of a small antenna (an antenna whose largest dimension is a minor fraction of the free-space wavelength) is the reciprocal of the radiation quality factor Q. The quality factor is generally defined for ideal lossless antennas (see, e.g., [3] and references therein)

$$Q = \frac{\omega W}{P_r} \tag{1}$$

where ω is the angular frequency of the oscillation, W the total energy (both electric and magnetic) stored by the electromagnetic fields produced by the antenna and P_r, the total amount of radiated power (for lossy antennas, P_r is to be replaced by P_r plus the power lost within the antenna structure). In principle, multistage impedance matching can be used to achieve bandwidths somewhat larger than the $1/Q$-estimate (see, e.g., [4], [5]), but the required matching circuits tend to grow very complex and are, in practice, inherently lossy. In the present paper, matching issues are not considered, however, and therefore, the Q refers to the quality factor of the radiating element itself.

The theoretical limitations for the radiation Q for the most general multipole field [6] are based on the nonradiating energy (which, in turn, is proportional to the imaginary, reactive, power [7]) stored by the spherical multipole fields outside the source region, which is determined by the smallest sphere that encloses the current carrying region. These limitations are usually considered as unattainable because they neglect the energy inside the "source sphere," which, however, is often predominant in practice. Thus, in order to achieve the theoretical limit for the Q, the power radiated by the antenna should be maximized while, simultaneously, the electromagnetic energy stored by the antenna inside the "source sphere" should ideally vanish totally, or at least be minimized. To achieve this, according to an old maxim, one should "spread out" the source current to fill the spherical volume most efficiently, thereby avoiding the emergence of energetic field concentrations and singularities.

The rapid theoretical as well as experimental advances made recently in the field of artificially tailored media, so-called metamaterials (see, e.g., [8], [9] for recent reviews), characterized by unusual electromagnetic constitutive parameters, suggests yet another way of optimizing the power and Q of antennas [10]. In particular, the use of materials having a dielectric permittivity ε and a magnetic permeability μ near or equal to zero, also known as *zero-index* [8] or *nihility* medium [11], as a substrate for the antenna structure, could provide a natural means to minimize the radiation Q and maximize the power radiated into the far zone. In [12] it has actually been suggested that a certain kind of rod array exhibits a zero index of refraction on its plasma frequency, which property can be used to focus and enhance dipole radiation. In the present paper, we explore the consequences of the application of such a metamaterial substrate backed by an infinite conducting ground plane to the radiation performance and Q of a small dipole antenna attached to the substrate.

We begin this paper by introducing expressions for the far field radiated by an elementary electric point dipole attached to a dielectric/magnetic slab with arbitrary combinations of μ_r and ε_r, involving particularly the cases $\mu_r = 1, 0$ or -1 and $\varepsilon_r = 1$, 0 or -1. We evaluate the radiated power both numerically and analytically and verify the results with Ansoft's HFSS finite element simulator for a short dipole. Radiation Q is analyzed by means of a spherical wave expansion of the approximate expressions for the fields radiated by the dipole and by integrating

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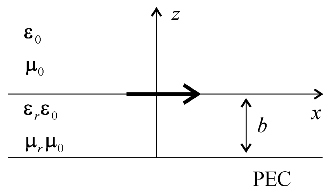

Fig. 1. Dipole on conductor backed slab.

the stored energy in the antenna structure. For comparison, Q is also derived from the antenna input impedance. Furthermore, we consider the amount of power propagating as surface waves, which is not accounted for by the expressions used for the radiated field. The pertinent question of realizability of such a special medium – albeit admittedly a very important one – is left out of the scope of the present study.

II. Fields Due to a Horizontal Dipole on Conductor Backed Slab

Consider a horizontal x-directed point dipole, or an electric current moment of magnitude IL, situated on the z-axis on a planar substrate layer backed by a perfectly conducting surface coinciding with the xy-plane (Fig. 1). The thickness of the layer is denoted b and its electric and magnetic material parameters relative to the ambient medium (say, vacuum) ε_r and μ_r, respectively. For the sake of simplicity it is assumed throughout that these parameters are real valued. All the field quantities are assumed to obey the harmonic $e^{j\omega t}$-time dependence, where $j = \sqrt{-1}$.

The electric field radiated into the far-zone (that is, $r \to \infty$, $\theta \in [0, \pi/2], \varphi \in [-\pi, \pi]$) is given by Jackson and Alexopoulos [13] as

$$\mathbf{E}(r,\theta,\varphi) = j\omega\mu_0 IL\left[\mathbf{u}_\varphi \sin\varphi\, F_\varphi(\theta) - \mathbf{u}_\theta \cos\varphi\, F_\theta(\theta)\right]\frac{e^{-jk_0 r}}{4\pi r} \tag{2}$$

where

$$F_\varphi(\theta) = \frac{2\tan(\beta b)}{\tan(\beta b) - \frac{jN(\theta)}{(\mu_r\cos\theta)}} \tag{3}$$

$$F_\theta(\theta) = \frac{2\tan(\beta b)\cos\theta}{\tan(\beta b) - \frac{j\varepsilon_r\cos\theta}{N(\theta)}} \tag{4}$$

and $\beta = k_0 N(\theta)$, $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$, $N(\theta) = \sqrt{n^2 - \sin^2\theta}$ and $n = \sqrt{\mu_r\varepsilon_r}$. In addition to space wave, a part of the power will be launched as a TM_0-surface wave if $\varepsilon_r \neq \mu_r^{-1}$. Surface wave power is discussed in Section III.

If the layer is electromagnetically thin, $\beta b \ll 1$, the expressions can be approximated as

$$F_\varphi(\theta) \approx \frac{2k_0 b}{k_0 b - \frac{j}{(\mu_r\cos\theta)}} \tag{5}$$

$$F_\theta(\theta) \approx \frac{2k_0(N(\theta))^2 b\cos\theta}{k_0(N(\theta))^2 b - j\varepsilon_r\cos\theta} \tag{6}$$

which are the basis for the subsequent developments. Below, we examine a few particular cases where these expressions can be expected to yield analytically tractable results.

1. When $kb\mu_r \ll 1$, a Taylor-expansion of $F_\varphi(\theta)$ gives

$$F_\varphi(\theta) \approx 2jk_0 b\mu_r\cos\theta(1 - jk_0 b\mu_r\cos\theta + \cdots) \tag{7}$$

   plus terms involving higher powers of $k_0 b\mu_r$ (presumed negligible).

2. When $\varepsilon_r = 1$ and $\mu_r = 1$ simultaneously, there is no substrate. Then

$$F_\varphi(\theta) \approx 2jk_0 b\cos\theta(1 - jk_0 b\cos\theta + \cdots) \tag{8}$$

   through substitution in (7) while $F_\theta(\theta)$ is

$$F_\theta(\theta) \approx 2jk_0 b\cos^2\theta(1 - jk_0 b\cos\theta + \cdots). \tag{9}$$

   This case of "air filling" has been examined in [1].

3. In the case $\varepsilon_r \ll 1$ there are two possibilities

$$F_\theta(\theta) \approx \begin{cases} -\frac{2jk_0 b}{\varepsilon_r \sin^2\theta} & \text{if } |\varepsilon_r| \gg k_0 b \\ 2\cos\theta & \text{if } |\varepsilon_r| \ll k_0 b \end{cases}. \tag{10}$$

4. In the case of an "anti-vacuum", $\varepsilon_r = -1$ and $\mu_r = -1$

$$F_\varphi(\theta) \approx -2jk_0 b\cos\theta(1 + jk_0 b\cos\theta + \cdots) \tag{11}$$

$$F_\theta(\theta) \approx -2jk_0 b\cos^2\theta(1 + jk_0 b\cos\theta + \cdots). \tag{12}$$

   The fields are in this case the complex conjugates of those of the $\varepsilon_r = 1$ and $\mu_r = 1$-case mentioned above.

III. Radiated Power

The radiated power is generally given by the integral

$$P_r = \frac{1}{2}\Re\int_S (\mathbf{E}\times\mathbf{H}^*)\cdot\mathbf{n}\, dS \tag{13}$$

where S is an arbitrary surface enclosing the source, dS is the differential surface element and $\mathbf{n}$, the surface normal. It is often most convenient to evaluate the power in the far field at a certain distance r, whence (13) can be written in spherical coordinates as

$$P_r = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\int_0^{2\pi}\int_0^{\pi}(|E_\theta|^2 + |E_\varphi|^2)r^2\sin\theta\, d\theta\, d\varphi \tag{14}$$

which is easily computed numerically. However, in the special cases mentioned previously the radiated power can be obtained also analytically by direct integration. In particular, using (2) and the approximations (8) and (9) we get from (14)

$$P_r = (IL)^2\sqrt{\frac{\varepsilon_0}{\mu_0}}\frac{(\omega\mu_0)^2(k_0 b)^2}{15\pi} \tag{15}$$

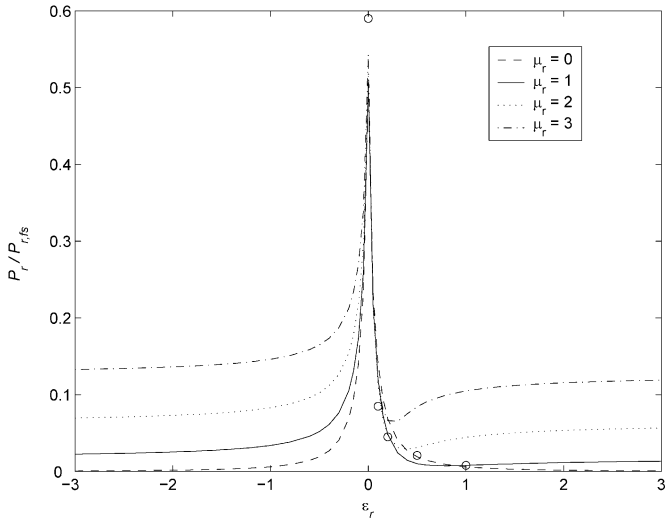

Fig. 2. Radiated power with positive μ_r values. Circles represent the analytical values gained from (15), (16) and (17) in the case $\mu_r = 1$.

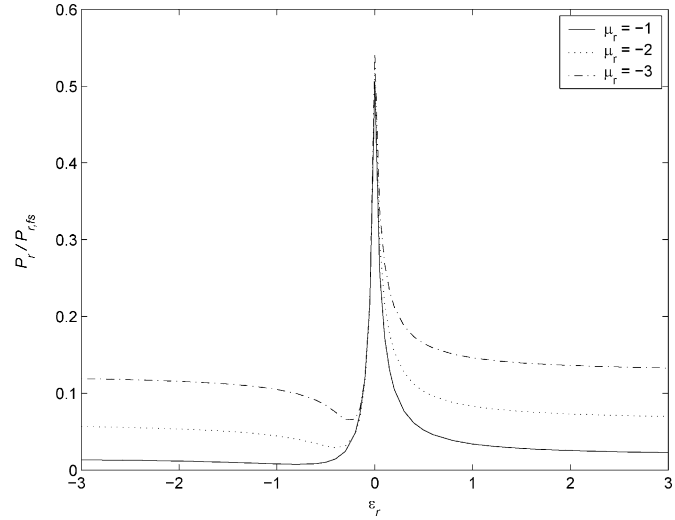

Fig. 3. Radiated power for negative μ_r values.

when $\varepsilon_r = 1$, $\mu_r = 1$ (note: the same power is obtained in the case $\varepsilon_r = -1, \mu_r = -1$) and from (2), (7), and (10)

$$P_r = (IL)^2 \sqrt{\frac{\varepsilon_0}{\mu_0}} \frac{(\omega\mu_0)^2(\frac{8}{5\varepsilon_r} + \mu_r)(k_0 b)^2}{24\pi} \tag{16}$$

when $1 \gg |\varepsilon_r| \gg k_0 b$ and

$$P_r = (IL)^2 \sqrt{\frac{\varepsilon_0}{\mu_0}} \frac{(\omega\mu_0)^2(1 + \mu_r k_0 b)^2}{24\pi} \tag{17}$$

when $1 \gg |\varepsilon_r| \ll k_0 b$. In these cases (where a ground plane is involved) the integrals are of course evaluated over the upper hemisphere only ($\theta \in [0, \pi/2]$). In comparison, a dipole in free space radiates the power

$$P_{r,\text{fs}} = (IL)^2 \sqrt{\frac{\varepsilon_0}{\mu_0}} \frac{(\omega\mu_0)^2}{12\pi} \tag{18}$$

over the entire sphere ($\theta \in [0, \pi]$).

Figs. 2 and 3 depict the radiated power for different combinations of ε_r and μ_r. The distance of the dipole to the conducting plane in all these examples is $k_0 b = 0.1$ and the feed current is considered constant. The values of (14) are normalized to the power radiated by the same dipole source in free space (18).

For $\varepsilon_r, \mu_r > 1$ a growing permittivity or permeability leads to an enhanced radiated power, i.e., the radiation resistance increases, although for μ_r the enhancement is notably stronger, as has been noticed in [14]. Similarly in the case of negative parameters, $\varepsilon_r, \mu_r < -1$, the smaller ε_r and μ_r, the more power is radiated. However, when ε_r approaches zero, the power increases and reaches its peak at $\varepsilon_r = 0$, in accordance with (17). The radiated power with permittivity zero is (in this particular case of $k_0 b = 0.1$) roughly 75 times larger than in the case $\varepsilon_r = 1$, corresponding to $(1 + \mu_r k_0 b)^2/2$ times (i.e., more than a half of) the power radiated by the same dipole source in free space. This is a remarkable improvement.

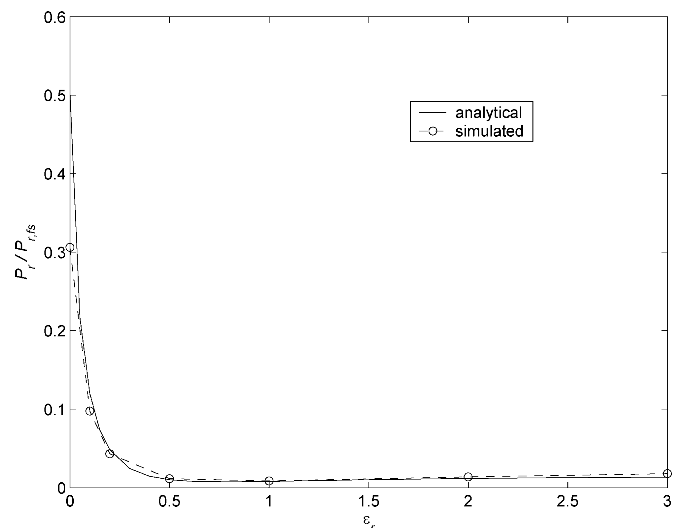

Fig. 4. Simulated radiated power versus the power given by the theoretical field expressions.

In Fig. 4 the radiated power of the quarter wave dipole simulated with HFSS (Ansoft Corp.) is compared with the theoretical result of Fig. 2 in the case of $\mu_r = 1$. In the simulations the dipole was a center fed infinitesimally thin PEC strip of width 0.0033 λ. Again, the height of the dipole over the conducting plane $k_0 b$ is 0.1 and the values are normalized to the radiated power of the corresponding dipole source in free space. As can be seen, the simulated results resemble very closely the numerical ones, a part of the difference being due to surface-wave power. Only positive ε_r- values are considered here, because HFSS (Version 10.1.2) can only handle material parameters having a positive real part with a reasonable accuracy [15].

In the previous formulas the surface wave power is not taken into account. The amount of surface wave power P_{sw} carried by

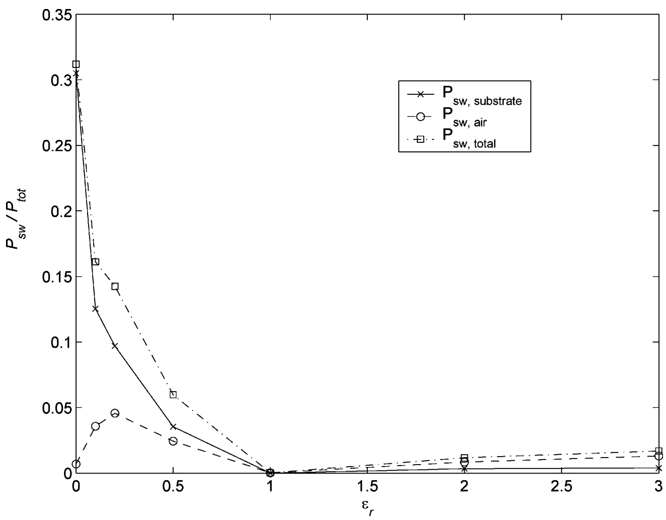

Fig. 5. Surface wave power of the quarter wave dipole extracted from HFSS simulations.

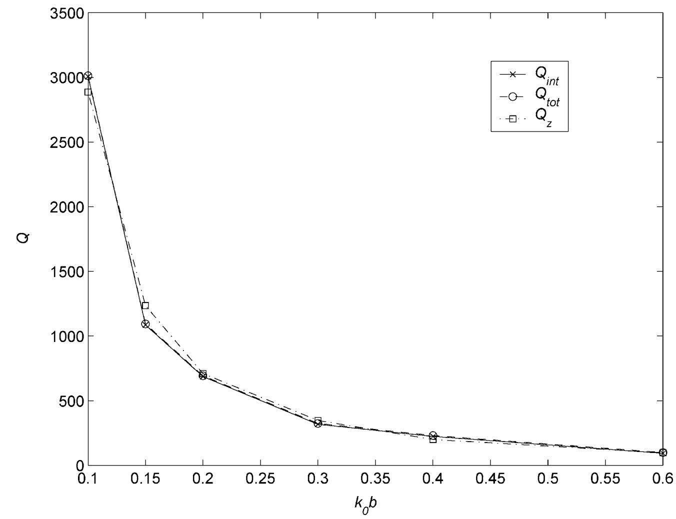

Fig. 7. Internal and total Q factors as a function of the layer thickness.

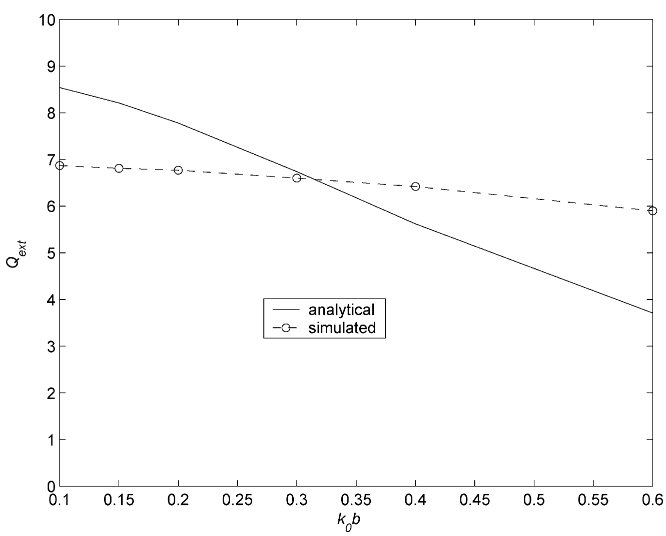

Fig. 6. External Q factor as a function of the layer thickness.

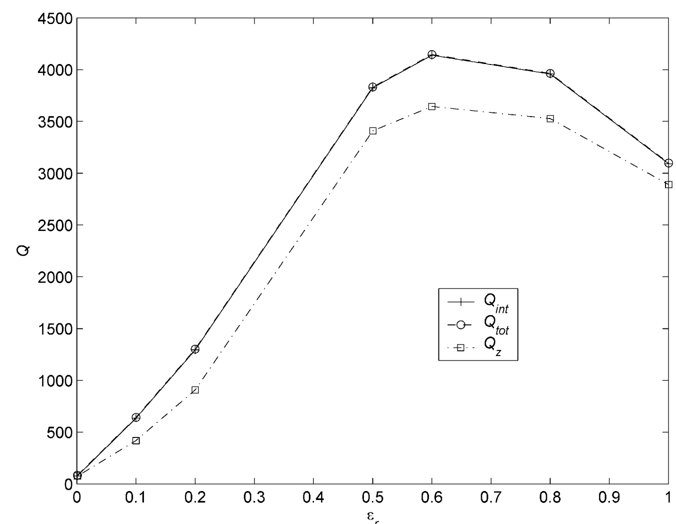

Fig. 8. Internal and total Q factors when ε_r varies and $k_0 b = 0.1$ and $\mu_r = 1$.

the E_z and H_φ components of the fields of the dominant TM_0 surface wave can be evaluated from [13]

$$P_{\mathrm{sw}} = -\frac{1}{2}\Re \int_0^\infty \int_0^{2\pi} E_z H_\varphi^* \rho \, d\varphi \, dz \tag{19}$$

using the values of E_z, H_φ given by the simulator program at the interface along the boundary surface of the simulation volume. The resulting surface wave power of a quarter wave long dipole as a function of ε_r is illustrated in Fig. 5. The distance to the conducting plane $k_0 b$ is 0.1 and the power values are normalized to the amount of total power (i.e., the power radiated into free space plus the surface wave power). For all the cases $\mu_r = 1$.

In Fig. 5 the surface wave power is drawn separately for the substrate and in the "air region" above the interface. It turns out that for $\varepsilon_r < 1$, the largest part of the surface wave power flows inside the substrate while for $\varepsilon_r > 1$ above it. The total surface wave power can be seen to grow as a function of index of refraction at the interface, thus reaching its maximum at $\varepsilon_r = 0$ and minimum at $\varepsilon_r = 1$.

IV. Quality Factor

The quality factor Q, being a ratio between energy and power (1), is not as such directly measurable. The theoretical limitations for the radiation Q are based on the nonradiating energy stored by the spherical multipole fields outside the smallest sphere enclosing the current carrying region. The formulas for determining this external energy W_{ext} and hence the external quality factor Q_{ext} are based on [6] and summarized in the Appendix .

However, in real antenna structures also the energy stored inside the smallest enclosing sphere, W_{int}, needs to be considered. Although expressions for internal energy are generally impossible to describe analytically, finite element simulation programs (such as Ansoft's HFSS) allow the electromagnetic en-

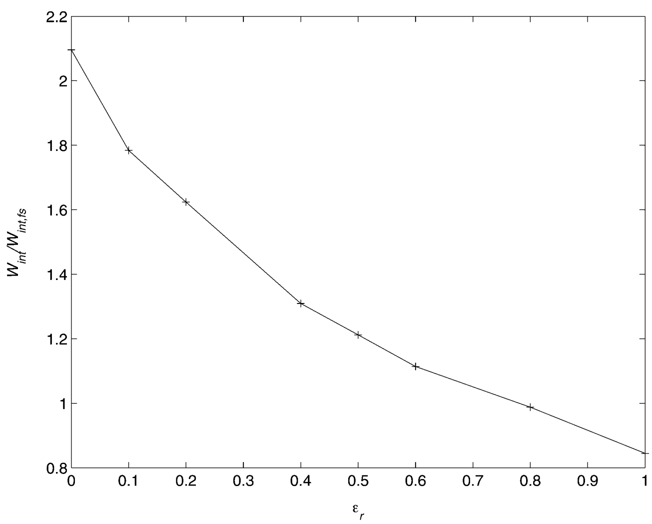

Fig. 9. Internal stored energy $W_{\rm int}$ when ε_r varies and $k_0b = 0.1$ and $\mu_r = 1$.

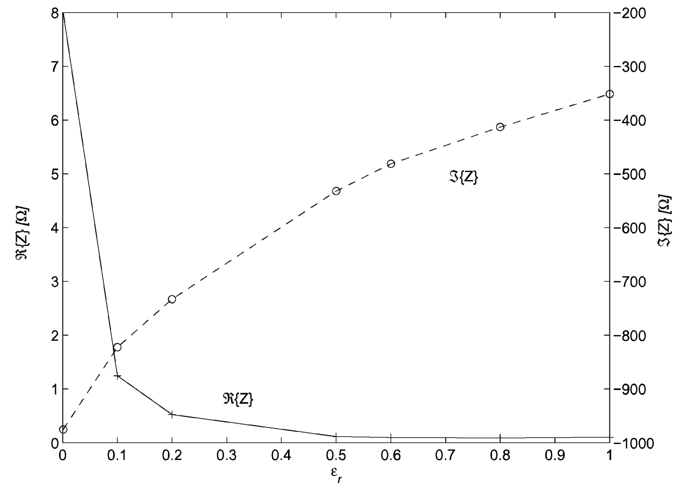

Fig. 10. Input impedance Z when ε_r varies and $k_0b = 0.1$ and $\mu_r = 1$.

ergy in a given region of space (for a specific excitation) to be integrated numerically. Thus, one may estimate the quantity

$$W_{\rm int} = \frac{1}{4}\Re \int_{\rm sphere} dV\,(\varepsilon|\mathbf{E}|^2 + \mu|\mathbf{H}|^2) \tag{20}$$

where the integration volume "sphere" signifies the smallest spherical volume that surrounds the current carrying region. In the following dispersionless, positive material parameters are considered. For a discussion on the exact definitions of the electric, magnetic and magnetoelectric energies in a general, dispersive metamaterial medium, we refer to [3], [10], [16].

Moreover, as shown in [3] a fair estimate of the Q-factor can be achieved by means of the input impedance $Z(\omega)$ through the formula

$$Q_Z \approx \frac{\omega}{2\Re\{Z\}}\left|\frac{dZ}{d\omega}\right|. \tag{21}$$

In Fig. 6 the external quality factor $Q_{\rm ext}$ of the simulated quarter wave dipole on top of an air-filled conductor backed slab as a function of thickness k_0b of the slab is illustrated. Again, the feed current is considered constant. Although the analytical formula given in the Appendix predicts a deeper slope compared to the simulated one, the order of magnitude of the results is the same. For comparison, the external quality factor of the same dipole in free space is approximately 3.5. In Fig. 7 the internal quality factor $Q_{\rm int}$ and total quality factor $Q_{\rm tot}$ (the external plus the internal Q) are presented in the same case. For reference, the quality factor Q_Z evaluated from the simulated input impedance Z is also shown. Clearly, the Q's given by the different methods are seen to increase exponentially as the thickness of the slab decreases. The internal quality factor $Q_{\rm int}$, being proportional to the energy $W_{\rm int}$ stored inside the smallest sphere, is in the order of total Q and is thus dominant.

Fig. 8 displays the simulated internal and total quality factors of the quarter wave dipole for different values of permittivity. The distance to the conducting plane k_0b is 0.1 and the permeability value one. The external quality factor in this case is small compared to the internal one, near 7 for all values of permittivity. The internal Q decreases from 3000 to 120 as the permittivity goes from one to zero, which is also in tune with the simultaneously increasing radiated power. As a reference, the internal Q of a similar dipole in free space would be around 30. On the other hand, the internal stored energy presented in Fig. 9 slightly increases as ε_r approaches zero. The values in Fig. 9 are normalized to the stored energy of a similar dipole in free space. The stored energy is of the same order of magnitude as in the free space case indicating the dominant role of the radiated power in the Q value. To understand this, we remind that in these examples the feed current is held constant. If the radiated power or input power in matched case would be held constant instead, one would encounter notably larger levels of internal energy compared to the free space case and a steeply decreasing trend as permittivity approaches zero.

In Fig. 10 the input impedance of the simulated dipole is presented. The real part of the impedance increases as the permittivity approaches zero indicating an increasing radiation resistance. The result parallels the increasing radiated power illustrated in the Figs. 2–4. On the other hand, the imaginary part of the input impedance decreases somewhat as the permittivity becomes smaller. This phenomenon can be understood by the growth of the wavelength in a low epsilon substrate, which makes the dipole appear electromagnetically smaller than in the air.

V. Conclusion

In this paper we have considered both theoretically and computationally the effect on the quality factor Q and radiated power of letting the permittivity and permeability of an antenna substrate be less than the free space values ε_0 and μ_0, respectively. Attention was paid especially to the case of zero or near-zero permittivity, that can potentially be achieved using special microstructured substrate materials exhibiting a "plasma resonance" frequency. It was shown that a remarkable increase of the radiated power (due to a simultaneously growing radiation resistance) compared to the stored internal energy would occur for such a near-zero permittivity substrate medium, leading to exceptionally low quality factor in dispersionless case. These properties are highly desirable, because antennas attached to a ground plane suffer from both low radiation resistance and narrow bandwidth.

For comparison, different methods to evaluate the quality factor were provided. One of the methods is based on a division of the stored electromagnetic energy into two respective volumes – the antenna region (where the energy is computed using simulated fields) and the free-space surrounding the antenna (where the stored energy is associated with the reactive near fields of the antenna) while the other method employs the frequency derivative of the input impedance. To our satisfaction, both methods were found to yield similar results.

APPENDIX

In this Appendix, expressions for the radiated power and Q in terms of multipole moments are summarized [6]. We write a spherical harmonics expansion of outward propagating electric and magnetic fields as

$$\mathbf{E}(\mathbf{r}) = \sum_{n=1}^{\infty}\sum_{m=-n}^{n}\left[a_{mn}\mathbf{M}_{mn} + b_{mn}\mathbf{N}_{mn}\right] \tag{22}$$

$$\mathbf{H}(\mathbf{r}) = \frac{jk}{\omega\mu_0}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\left[a_{mn}\mathbf{N}_{mn} + b_{mn}\mathbf{M}_{mn}\right] \tag{23}$$

where the spherical vector wavefunctions $\mathbf{M}_{mn}$ and $\mathbf{N}_{mn}$ are defined

$$\mathbf{M}_{mn}(r,\theta,\varphi) = \nabla\times\left\{\mathbf{r}h_n^{(2)}(kr)\,P_n^{|m|}(\cos\theta)\,e^{-jm\varphi}\right\} \tag{24}$$

and $\mathbf{N}_{mn}(r,\theta,\varphi) = (1/k)\nabla\times\mathbf{M}_{mn}$, $\mathbf{r}$ being the position vector, $h_n^{(2)}$ the spherical Hankel function and $P_n^{|m|}$ the associated Legendre function.

By defining the power density in the far field as

$$p(r,\theta,\varphi) = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\mathbf{E}\cdot\mathbf{E}^\star = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}(|E_\theta|^2 + |E_\varphi|^2) \tag{25}$$

the power radiated into the upper half-space can be obtained by integration over the hemisphere as

$$P = \int_0^{2\pi}\int_0^{\pi/2} p\,\sin\theta d\theta d\varphi = \frac{\pi}{k^2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\sum_{n=1}^{\infty}(a_n^2 + b_n^2) \tag{26}$$

where

$$a_n^2 = \sum_{m=-n}^{n}\Lambda_{mn}|a_{mn}|^2 \qquad b_n^2 = \sum_{m=-n}^{n}\Lambda_{mn}|b_{mn}|^2 \tag{27}$$

$$\Lambda_{mn} = \frac{n(n+1)(n+|m|)!}{(2n+1)(n-|m|)!}. \tag{28}$$

To compute the Q from (1) we need an expression for the total energy stored by the multipole moments outside the sphere, defined as the smallest sphere of radius r_0 that can be drawn outside the source. Such an expression has been derived in [6], but since only one half-space is of concern, only one half of this expression is taken viz.

$$W = \frac{\pi\varepsilon_0}{4k^3}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)w_n \tag{29}$$

where

$$w_n = 2x - x^3(|h_n^{(2)}|^2 - j_{n-1}j_{n+1} - y_{n-1}y_{n+1}) - x^2(j_n j_n' + y_n y_n') - x|h_n^{(2)}|^2. \tag{30}$$

For brevity, the argument $x = kr_0$ of the spherical Hankel functions $h_n^{(2)}$, the spherical Bessel j_n and Neumann functions y_n is omitted. The lowest five w_n's are given by

$$w_1 = (1 + 2x^2)x^{-3} \tag{31}$$
$$w_2 = (18 + 9x^2 + 6x^4)x^{-5} \tag{32}$$
$$w_3 = 3(225 + 60x^2 + 12x^4 + 4x^6)x^{-7} \tag{33}$$
$$w_4 = 5(8820 + 1575x^2 + 180x^4 + 20x^6 + 4x^8)x^{-9} \tag{34}$$
$$w_5 = 15(297675 + 39690x^2 + 3150x^4 + 210x^6 + 15x^8 + 2x^{10})x^{-11}. \tag{35}$$

The case $\varepsilon_r = \mu_r = 1$ can be handled explicitly. In effect, the field (2) including the approximations (8) and (9) can be matched with a spherical wave expansion (22) by means of the following pair of dominant multipole moments

$$a_{\pm1,1} = \pm jILb\frac{k^3}{8\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}} \qquad \text{and} \qquad b_{\pm1,2} = ILb\frac{k^3}{24\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}. \tag{36}$$

Hence, from (1) and (29) the Q-factor is

$$Q = \frac{27 + 16x^2 + 14x^4}{16x^5} \tag{37}$$

while the radiated power is given by (17).

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