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BANDWIDTH LIMITATIONS OF DIPOLES MATCHED WITH NON-FOSTER IMPEDANCES

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Abstract

In this paper, the bandwidth limitations of dipoles matched with negative impedance components are discussed. In theory, infinite impedance bands are possible with non-Foster impedances, but in practice the fields stored inside the smallest sphere enclosing the antenna limit the band. Both theoretical and practical achievable bandwidths as well as non-Foster component tolerance issues are reported in this paper.

1 Introduction

Theoretical bandwidths of small antenna elements are traditionally limited by the size of the smallest sphere enclosing the current carrying region [1, 2]. In these formulations given for the radiation quality factor Q (which is taken to be inversely proportional to the fractional bandwidth), however, the limit takes only into account the non-radiating energy stored outside the smallest sphere. In principal, multi-stage matching can be used to achieve bandwidth enhancement. In the earlier work [3] the impedance matching properties of ideal dipoles achieved with infinite amount of passive matching components has been analysed. Promising results were shown, but still the actual size of the antenna limited the achievable bandwidth.

On the other hand, in theory, infinite bandwidths are achievable by cancelling out the antenna capacitance and inductance with corresponding negative reactances. Nevertheless, in practice limitations exist. In reality, also the energy stored inside the smallest sphere enclosing the antenna affects the radiation Q and decreases the bandwidth dramatically. In this paper, the bandwidth limitations of dipoles matched with one and two non-Foster impedance components are analysed both in the sense of theoretical limits based on energy outside the smallest sphere and in a practical case including the inner energy. The realization of the negative impedances has been left out scope of this paper, although implementations exist [4, 5].

2 Theory

In the model described by Chu [1], the antenna impedance linked to the fields outside the smallest enclosing sphere is represented by a ladder *RLC*-network. For the ideal electric dipole, TM_1, the equivalent circuit is presented in Fig. 1. The component values are $C = a\varepsilon_0$, $L = a\mu_0$, $R = \sqrt{\mu_0 / \varepsilon_0}$, where a is a radius of the smallest sphere enclosing the antenna. The wave impedance seen by the fields propagating outwards from the surface of the enclosing sphere is thus

$$Z_{Chu} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \left(\frac{1}{jka} + \frac{jka}{jka + 1} \right). \qquad (1)$$

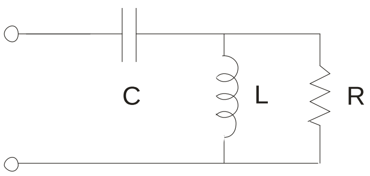

Fig. 1. Chu circuit model.

However, in practice strong fields are stored inside the enclosing sphere and thus, more circuit components are needed to model for the antenna impedance. According to Hansen [6] the measured input impedance of a short wire dipole can be described approximately as

$$Z_{Hansen} = 20(ka)^2 - j120 \frac{\ln(a/t) - 1}{\tan ka}, \qquad (2)$$

where $2a$ is the length and t thickness of the dipole. In Fig. 2 and 3 the real and imaginary parts of impedances calculated with above models are illustrated. The impedance described by Hansen predicts lower resistance value than Chu model. The imaginary part in Hansen model is greatly affected by the thickness of the wire. At thickness ratio $a/t = 50$, however, the imaginary part becomes similar to the one predicted by Chu model. By adding a network of two capacitances and an inductor to Chu circuit (see Fig. 4.), the impedance behaviour predicted by Hansen model may be achieved. Two capacitances act as voltage divider, which illustrates the ratio of the power transformed from the total power circulating in the antenna region to TM_1 mode. The inductor, together with shunt capacitance, represents a transmission line due to physical nature of the wire dipole. The input impedance of this extended Chu model is described as

$$Z_{Chu,extended} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \left(jka\frac{L_1}{L} + \frac{1}{jka\frac{C_2}{C}} + \frac{\frac{1}{jka} + \frac{jka}{jka+1}}{1+\frac{C_1}{C}\left(1-\frac{(ka)^2}{jka+1}\right)} \right)$$

(3)

With ratios C_1/C = 3.95, C_2/C = 1.48 and L_1/L = 0.32 the impedance match very well with Hansen model for thickness ratio $a/t = 50$ as illustrated in Fig. 2 and 3. Simulations of similar dipoles conducted with IE3D electromagnetic simulator based on MoM agree remarkably well with the results gained from the extended Chu and Hansen models for $ka << 1$.

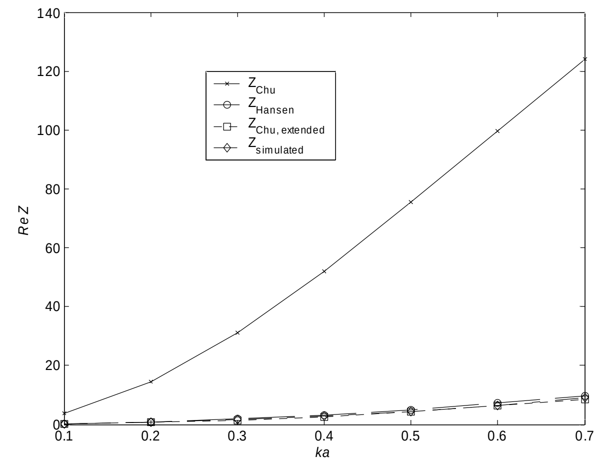

Fig. 2. Real part of impedance predicted with different models.

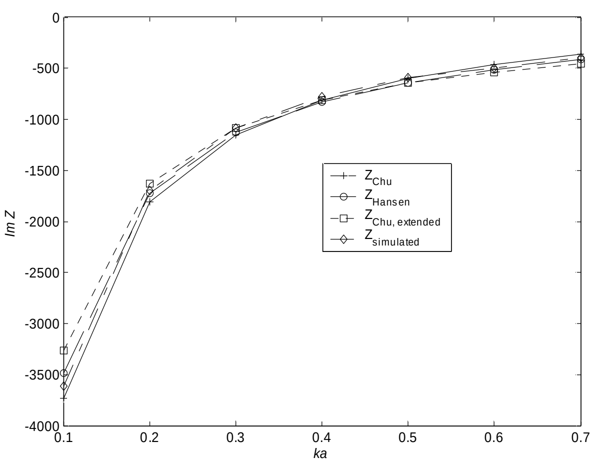

Fig. 3. Imaginary part of impedance predicted with different models.

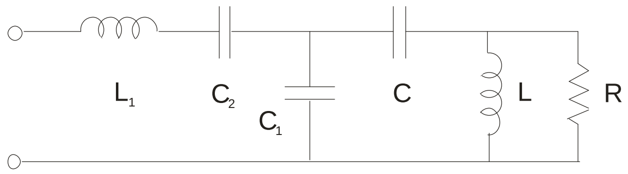

Fig. 4. Extended Chu circuit model describing approximately a short wire dipole.

3 Matching with a single non-Foster component

3.1 Ideal case

In the Chu model, the impedance of a small electric dipole is inherently capacitive. Cancelling out this capacitance by a single series negative capacitance leads to remarkable bandwidth enhancements. In the bandwidth calculation the antenna circuit including the negative C_x component is considered to be fed from an ideal impedance transformer, i.e.

$$\rho = \frac{Z_{in} - \mathrm{Re}\{Z_{in}\}}{Z_{in} + \mathrm{Re}\{Z_{in}\}}, \tag{4}$$

where Z_{in} in this case is Z_{chu} (see equation 1) in series with tuning capacitance C_x. From equation (4) the optimal value for C_x/C may be solved as

$$\frac{C_x}{C} = \frac{1}{\frac{(ka)^2}{(ka)^2+1} + \frac{2(ka)^3}{(ka)^2+1}\frac{|\rho|}{\sqrt{1-|\rho|^2}} - 1}. \tag{5}$$

On the other hand solving equation (4) as a function k leads to bandwidth information. Equation (4) may be presented as a third order polynomial for k as

$$k^3 - k^2\frac{a^2C}{yC_x} - \frac{\frac{C}{C_x}+1}{y} = 0, \tag{6}$$

where

$$y = \frac{2|\rho|a^3}{\sqrt{1-|\rho|^2}}. \tag{7}$$

Third order polynomial (6) has three real solutions at $C_x/C(k = 1)$. The roots represent three points where the matching curve passes the matching limit $|\rho|$. One of the solutions is a trivial solution, $k = 1$, representing the lower limit of the band. Second solution represents the upper band limit and the third solution the third passing point, which, however, is irrelevant in the bandwidth calculation.

The maximum relative bandwidth achieved with an ideal negative C_x matching component is presented in Fig. 5 as a function of antenna size ka. Above certain size of the antenna ka, the second root gives imaginary values indicating infinite impedance bandwidth. The numerical circuit simulator calculations conducted with Microwave Office agree very well with the analytical results. The corresponding relative bandwidth values achieved with infinite amount of passive components [3] is presented in Fig. 6. In theory, larger bandwidths are possible with smaller size antennas with one negative C_x component that with infinite amount of passive components.

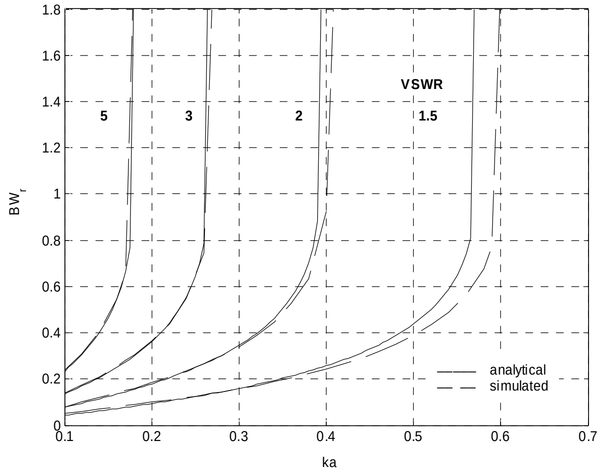

Fig. 5. Maximum bandwidth achieved with negative C_x component.

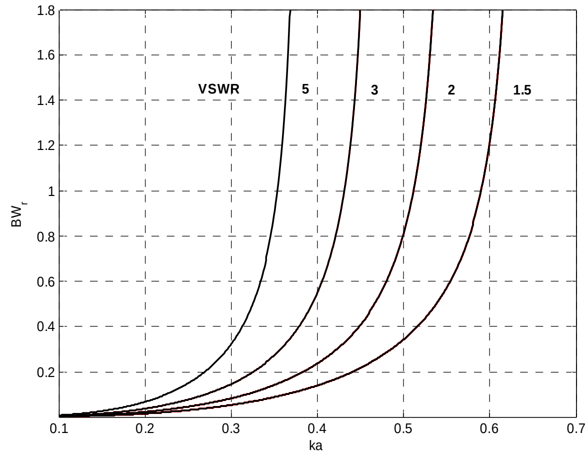

Fig. 6. Maximum bandwidth achieved with infinite amount of passive components.

3.2 Practical case

In a case of a real antenna, the achievable bandwidths with negative C_x element tuning are considerably smaller due to the fields stored inside the smallest sphere. The analytical solution for C_x/C from equation (4), where in this case Z_{in} is $Z_{Chu,extended}$ (see equation 3) in series with negative capacitance C_x, leads to complicated formula and k is a function of sixth order. The function is, however, analytically solvable with symbolic computation solvers like Mathematica. k has two real solutions at $C_x/C(k = 1)$. First real solution is a trivial solution, $k = 1$, representing the lower limit of the band and the second real solution the upper band limit. The functions are, however, extremely sensitive to C_1/C, C_2/C, L_1/L and ρ parameter values.

In Fig. 7 the maximum achievable bandwidths with an ideal negative C_x component in a case including the inner field model are illustrated as a function of size of the antenna ka. Also the maximum bandwidths of corresponding simulated electric dipoles tuned with negative C_x component are presented. The analytical and simulated bandwidths correspond very well. As can be seen from Fig. 7 in reality only limited bands are possible with negative capacitance matching. For comparison, the maximum bandwidths of dipoles tuned with infinite amount of passive components are presented in Fig. 8. Infinite amount of passive components was modelled here as a cascade of 14 LC resonators generated by Genesys circuit simulator as in [3]. Tuning with negative capacitance shows considerably larger bands than tuning with passive components.

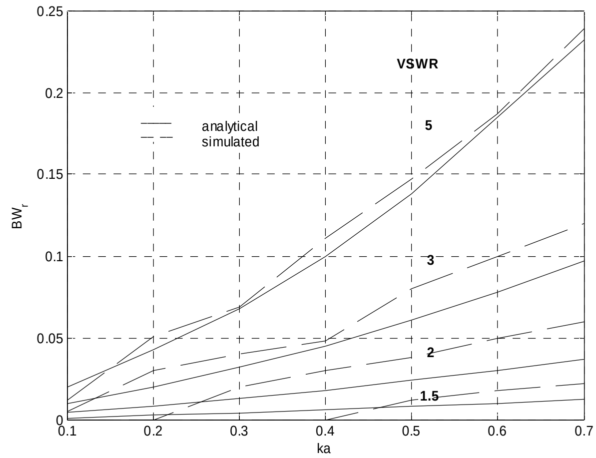

Fig. 7. Maximum bandwidth achieved with negative C_x component

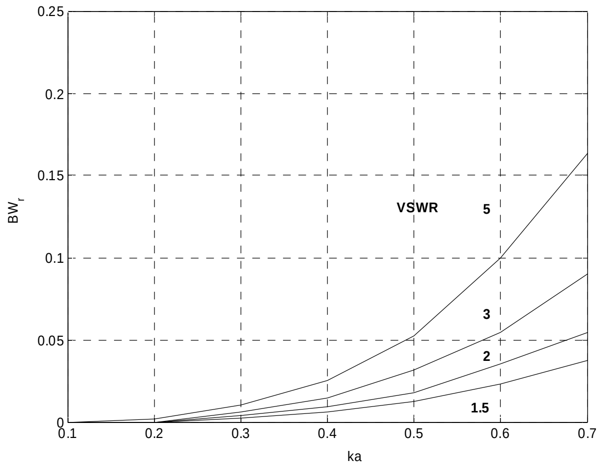

Fig. 8. Maximum bandwidth achieved with infinite amount of passive components.

The variation tolerance of the negative C_x component is illustrated in Fig. 9. The tolerance is defined in a margin, where frequency shifts maximum of 1.0 per cents from the base frequency f_0. The tolerance is really low and increases as a function of antenna size ka.

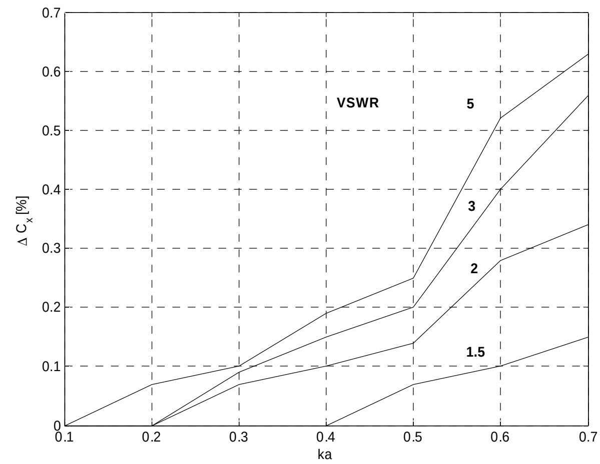

Fig. 9. Variation tolerance of negative C_x component.

4 Matching with two non-Foster components

4.1 Ideal case

In the Chu model, cancelling out the dominating capacitance led to remarkable bandwidth enhancements. However, by adding a parallel negative inductor to the model, the input impedance of the circuit may be written as

$$Z_{in} = \frac{j\omega L_x \left(\frac{1}{j\omega C} + \frac{1}{j\omega C_x} + \frac{j\omega LR}{j\omega L + R} \right)}{j\omega L_x + \left(\frac{1}{j\omega C} + \frac{1}{j\omega C_x} + \frac{j\omega LR}{j\omega L + R} \right)}. \tag{8}$$

If $-L_x$ is equal to L and simultaneously $-C_x$ is equal to C, equation (8) becomes

$$Z_{in} = R, \tag{9}$$

indicating frequency independent, infinite band.

4.2 Practical case

In practice, as already mentioned, the inner field limits the bandwidth behaviour. In this case, the function for impedance ($Z_{Chu,extended}$ in series with negative C_x and L_x components) has two variables and analytical formulation gets difficult. However, according to the numerical optimization done with simulated dipoles, relative bandwidths more than 2 are possible with two negative series tuning components. Unfortunately, the tuning is very sensitive to matching component values. In Fig. 10 and 11 the component tolerances at 1 per cent margin are presented.

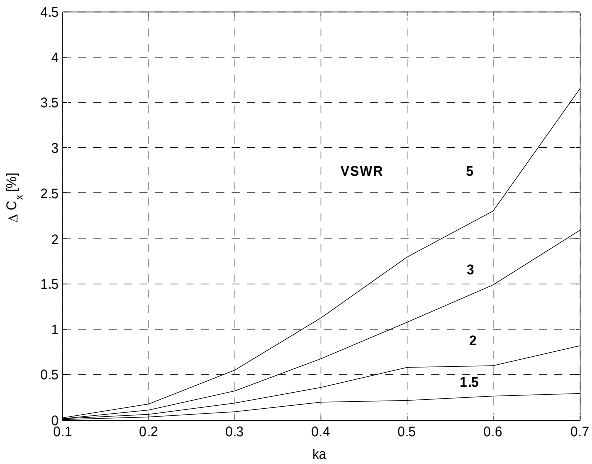

Fig. 10. Variation tolerance of negative C_x component.

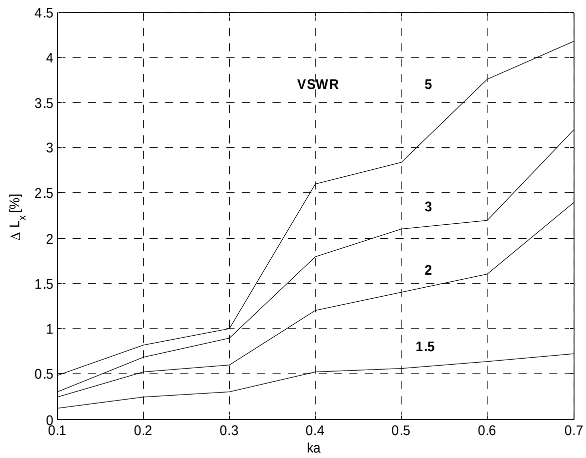

Fig. 11. Variation tolerance of negative L_x component.

5 Conclusions

In this paper, the active element tuning for small antennas is analysed. In theory, infinite bandwidths are achievable with very small antennas using only one negative tuning element. However, in practice strong near fields limit the antenna Q and only limited bands are achievable with one active element. On the other hand, by using two negative tuning elements bandwidths larger than 200 % are achievable even with very small antennas. Still, the variation tolerances are very small and limit the component implementation possibilities in practice.

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