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Decay of Groundplane Currents of Small Antenna Elements

J. C.-E. Sten and M. Hirvonen

Abstract—Expressions are derived for the surface current induced by vertical and horizontal dipoles on a nearby infinite groundplane. The expressions are obtained by means of the exact magnetic field. It is found that for a vertical dipole the surface current density decays roughly as the inverse of the distance from the source, while for a horizontal dipole it decays as the inverse distance squared. The result furnishes an explication for the empirical observation that antennas carrying mainly horizontal currents tend to be less sensitive to the dimensions of a finite groundplane than antennas carrying vertical currents. A formula is given by means of which the groundplane current of a compound source can be evaluated and controlled.

Index Terms—Antennas, ground plane, surface current.

I. INTRODUCTION

For some time, it has been recognized that the size and shape of the groundplane of an electrically small antenna have a significant effect not only on the radiation properties [1], [2] but also on the input impedance of the structure [3], [4]. Nevertheless, empirical studies aside no rigorous explanation has been given to the observation that different antennas are variously affected by the size and shape of the groundplane. In particular, antennas with an emphasis on the horizontal (or tangential) current component tend to be more independent—at least when the impedance is concerned—of the groundplane dimensions than antennas with an emphasis on the vertical (normal to the surface) current. It is thus conjectured that the distribution of surface currents is more widely spread, generally speaking, and the rate of decay of the current intensity slower for vertical sources than for horizontal ones.

In this paper we endeavour to show theoretically, by considering the near-fields of small antenna elements, that the surface currents induced by vertical and horizontal dipoles decay at a different rate according to the distance from the source region—for vertical dipoles at a rate of \( r \), while for horizontal dipoles at \( 1/r^2 \). The presented analysis also yields a simple means to evaluate groundplane currents for antennas with a prescribed current distribution. A numerical illustration of the method for an inverted L-structure (ILA) is presented.

II. DIPOLE FIELDS

The key to expressing the current distributions induced on an infinite groundplane is the magnetic field \( \mathbf{H} \). In spherical coordinates \((r, \theta, \varphi)\) a \( z \)-directed infinitesimal current element (an electric point dipole) located at the origin and having the moment \( \mathbf{u}_z[I] \), creates the field (implicitly assuming harmonic \( e^{j\omega t} \)-time dependence) [5]

\[
\mathbf{H}(r) = jk[I] \frac{e^{-jk\rho}}{4\pi r} \left(1 - \frac{j}{k^2 r^3} \right) \sin\theta \mathbf{u}_\varphi.
\]  

When a perfectly conducting plane is brought near the dipole, the plane creates a reflection which can be accounted for by means of an image source at the mirror image point. In the case of a vertical dipole (parallel to the surface normal) the image current is in phase with the original, while in the contrary case of horizontal (or tangential) current, the image is in the opposite phase [5].

A. Vertical Dipole

Let us introduce a system of two-dimensional primed coordinates \((x', y')\) on the plane perfectly conducting surface \( z = -z_0 \), with the origin at \((x, y, z) = (0, 0, -z_0)\). Writing the transformation

\[
r = \sqrt{z_0^2 + \rho'^2}, \quad \quad \theta = \frac{\pi}{2} + \arccos \frac{z_0}{r}, \quad \quad \rho' = \sqrt{x'^2 + y'^2},
\]

with \( \rho' = \sqrt{x'^2 + y'^2} \), we get

\[
\mathbf{H}(x', y') \approx -z_0 = jk[I] \frac{e^{-jk\rho'}}{4\pi r'} \left(1 - \frac{j}{k^2 r'^3} \right) \frac{\rho'}{r} \mathbf{u}_\varphi.
\]  

The total induced current is \( \mathbf{J}_s = 2n \times \mathbf{H} \), where \( \mathbf{n} \) is the surface normal \( \mathbf{u}_z \), the factor 2 being due to the reinforcing image source. Then

\[
\mathbf{J}_s = -j[I] \rho/(kr - j) \frac{e^{-jk\rho'}}{2\pi r'^3} \mathbf{u}_\varphi
\]  

where \( \mathbf{u}_{\varphi'} = (x' \mathbf{u}_x + y' \mathbf{u}_y)/\rho' \). At \( \rho' = 0 \), immediately below the original current, the surface current can be seen to vanish, while at a far distance, when \( \rho'/r \approx 1 \), the surface current decays roughly as \( 1/r \).

B. Horizontal Dipole

A plane conducting surface is now introduced at \( y = y_0 \) along with an associated system of primed coordinates \((x', z')\) originating at \((x, y, z) = (0, y_0, 0)\). The transformation then reads

\[
r = \sqrt{x'^2 + y_0^2 + z'^2},
\]

\[
\theta = \arccos \frac{x'}{r},
\]

\[
\varphi = \arccos \frac{y'}{\sqrt{x'^2 + y_0^2}}
\]

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where
\[ \mathbf{H}(x', z')|_{y=y_0} = jk[H] \frac{e^{-jkr}}{4\pi r} \left(1 - \frac{j}{kr}\right) \sqrt{x'^2 + y_0^2} \mathbf{u}_\phi \]  \hspace{1cm} (6)

where
\[ \mathbf{u}_\phi = \frac{(-y_0 \mathbf{u}_x + x' \mathbf{u}_y)}{\sqrt{x'^2 + y_0^2}}. \]

Using \( \mathbf{J}_s = 2\mathbf{n} \times \mathbf{H} \), where \( \mathbf{n} = -\mathbf{u}_y \), the current density induced on the surface becomes
\[ \mathbf{J}_s = -j[H] y_0 (kr - j) \frac{e^{-jkr}}{2\pi r^3} \mathbf{u}_z \]  \hspace{1cm} (7)

which is seen to decay roughly as \( 1/r^2 \) at large distances \( r \). In addition, when \( y_0 \ll x' \) and \( z' \), the current density is directly proportional to \( y_0 \), the height of the dipole above the surface (in the limit \( y_0 \to 0 \) radiation ceases and \( \mathbf{J}_s \to 0 \)).

C. General Case

Consider now a general point current element
\[ \mathbf{J}(\mathbf{r}) = [(H)_{x'} \mathbf{u}_x + (H)_{y'} \mathbf{u}_y + (H)_{z'} \mathbf{u}_z] \delta(\mathbf{r} - \mathbf{r}_0) \]  \hspace{1cm} (8)

residing at \( \mathbf{r}_0 = x_0 \mathbf{u}_x + y_0 \mathbf{u}_y + z_0 \mathbf{u}_z \) in front of a perfectly conducting surface, \( z = 0 \). The current distribution arising in the \( x'y' \)-plane may be written by transforming the expressions (4) and (7) as
\[
\begin{align*}
J_{s,x} &= -j(kr - j) \frac{e^{-jkr}}{2\pi r^3} [(H)_{x'}(x - x_0) + (H)_{y'}y_0) \]  \hspace{1cm} (9) \\
J_{s,y} &= -j(kr - j) \frac{e^{-jkr}}{2\pi r^3} [(H)_{x'}(y - y_0) + (H)_{y'}z_0) \]  \hspace{1cm} (10)
\end{align*}
\]

with \( r = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} \).

III. APPLICATION

By exploiting the general case formulas (9) and (10), the surface currents of actual antennas may now be studied. As an example, the surface currents induced by the inverted L wire antenna are presented. Two variants of the geometry are selected. Variant one, depicted in Fig. 1, is a very low-profile structure with a height of only \( \lambda/28 \) and a total wire length of \( \lambda/4 \). For variant two, appearing in Fig. 2, the vertical and horizontal components are of the same length, \( \lambda/8 \). For both variants, the current is assumed to be a quarter sinusoid, reaching maximum at the feed point and tending to zero at the open end of the wire. This assumption, despite being an approximation, gives an adequate picture of the surface current distribution.

In the evaluation of the currents, the ILA-structure was modeled as several vertical and horizontal point currents with amplitudes proportional to the integral of sine current. The surface currents of the two variants are presented in Figs. 1 and 2, respectively. The scale of the contour lines is the same for both cases, representing half-values. As can be seen by comparing the two figures, the surface current induced by the higher profile variant with a dominating vertical current extends in a larger area than the surface current induced by the lower profile structure with dominating horizontal current.

IV. CONCLUSION

Antennas backed with a groundplane can be divided into a set of horizontal and vertical current elements. It has been demonstrated in this paper that such elements generate different kinds of surface current patterns, and that the rates of decay of these current densities are disparate. A formula was derived by means of which the surface current pattern induced by an antenna in front of a groundplane can be constructed. Knowledge of the surface current pattern is essential when one tries to reduce...
the size of the groundplane without compromising the antenna performance.

REFERENCES


