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Propagation Parameter Tracking using Variable State Dimension Kalman Filter

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***Abstract*— The development of future wireless communication systems requires modeling of the radio propagation environment. These models need the estimation of the model parameters from channel sounding measurements. In this paper, we build a state-space model, and estimate the propagation parameters with the Extended Kalman Filter in order to capture the dynamics of the channel parameters in time. The model also includes the effect of distributed diffuse scattering in radio channels. The issue of varying state variable dimension, i.e., the number of propagation paths to track, is investigated. For this purpose, we rely also on maximum likelihood based estimation techniques. The proposed algorithm is investigated using both simulated and measured data.**

I. Introduction

The development of radio channel models for wireless MIMO communication systems requires multidimensional channel sounding measurements. The measurements are processed to estimate the radio channel parameters using double-directional channel models [1]. The double-directional modeling allows removal of the influence of the measurement antennas from the observation. This is necessary for using the measurement results for studying and comparing different MIMO transceiver structures.

The extraction of the channel model parameters from the measurement data is done using some parameter estimation algorithm, e.g., SAGE [2] or RIMAX [3]. A straight-forward approach for estimating the radio channel parameters (propagation path delays, angles of arrival and departure, polarized path weight components) is to do the estimation for each snapshot independently. However, it can be observed from measurements, that the specular component of the radio channel contains typically propagation paths which persist over a relatively large number of snapshots. The parameters of these paths change slowly in time. This observation can be exploited to track the path parameters over time, in order to reveal dynamic properties of the radio channel.

In this paper we use a state-space approach for tracking the radio propagation path parameters over time. This is done by deriving a state-space model based on the nonlinear data model presented in [3], and applying an Extended Kalman Filter (EKF) for parameter estimation. The approach for propagation path parameter estimation using Kalman filtering was introduced in [4]. It was pointed out that the use of recursive estimation algorithm is computationally less demanding compared to snapshot-by-snapshot estimation where the time correlation of the parameters is not exploited. One of the drawbacks of the proposed method in [4] is the fact that the state dimension, which is given by the number of paths, is kept fixed. This is not the case in practice, since paths may appear or disappear in time due to the dynamics in the propagation environment.

In this paper we propose a method which allows a dynamic adaptation of the state dimension. This makes the algorithm more reliable for the time characterization of the estimated parameters. In the following section we present the observation and the state-space models. In Section III we discuss the algorithm and its initialization. In Section IV we show some estimation results with both artificial, and measurement data. Section V concludes this paper.

II. System Model

The state-space model and Extended Kalman Filter were used for estimating a few propagation path parameters (Time Delay of Arrival (TDoA) τ, azimuth Direction of Arrival (DoA) φ_R, and the path weight γ for some fixed number of propagation paths in [4]. In this paper, the estimator is extended to take care of double directional channel model [3] with the propagation path parameters: TDoA τ, Direction of Departure (DoD) azimuth φ_T and elevation ϑ_T, DoA azimuth φ_R and elevation ϑ_R, and four polarimetric path weights γ_{HH}, γ_{HV}, γ_{VH}, and γ_{VV}. We address a problem related to the dynamics of the environment, i.e., how to choose the number of paths to track at a given time.

A. Model for a Radio Channel Observation

The employed radio channel model consists of two components: the specular (concentrated) propagation component and a multivariate circular complex normal distributed process to describe the dense multipath component (distributed diffuse scattering) of the radio channel [3]. In the following, the delay and angular parameters (for P paths) are referred to as structural parameters of the model

$$\boldsymbol{\mu} = \left[\boldsymbol{\tau}^{\mathrm{T}}\ \boldsymbol{\varphi}_T^{\mathrm{T}}\ \boldsymbol{\vartheta}_T^{\mathrm{T}}\ \boldsymbol{\varphi}_R^{\mathrm{T}}\ \boldsymbol{\vartheta}_R^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{R}^{5P\times 1}, \tag{1}$$

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whereas the path weights

$$\boldsymbol{\gamma} = \left[\boldsymbol{\gamma}_{HH}^{\mathrm{T}}\ \boldsymbol{\gamma}_{HV}^{\mathrm{T}}\ \boldsymbol{\gamma}_{VH}^{\mathrm{T}}\ \boldsymbol{\gamma}_{VV}^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{4P\times 1} \tag{2}$$

are referred to as linear parameters. For notational convenience, we have dropped the time dependency, i.e., $\boldsymbol{\mu} = \boldsymbol{\mu}_k$ (k denotes discrete time), from all the parameters. Let us introduce the matrix valued function $\mathbf{B}(\boldsymbol{\mu}) \in \mathbb{C}^{M\times 4P}$ describing the nonlinear mapping of the structural parameters $\boldsymbol{\mu}$ as

$$\begin{aligned}\mathbf{B}(\boldsymbol{\mu}) = [\ & \mathbf{B}_{R_H}\Diamond\mathbf{B}_{T_H}\Diamond\mathbf{B}_f \quad \mathbf{B}_{R_V}\Diamond\mathbf{B}_{T_H}\Diamond\mathbf{B}_f \ldots \\ & \mathbf{B}_{R_H}\Diamond\mathbf{B}_{T_V}\Diamond\mathbf{B}_f \quad \mathbf{B}_{R_V}\Diamond\mathbf{B}_{T_V}\Diamond\mathbf{B}_f\ ],\end{aligned} \tag{3}$$

where $\Diamond$ denotes the Khatri-Rao product (column-wise Kronecker-product). For a complete description of the structure of the basis functions $\mathbf{B}_i$, see [3]. For future purposes we define also a parameter vector $\boldsymbol{\theta} \in \mathbb{R}^{LP\times 1}$ containing the L parameters for each of the P specular propagation paths as

$$\boldsymbol{\theta} = \left[\boldsymbol{\mu}^{\mathrm{T}}\ \Re\left\{\boldsymbol{\gamma}_{HH}^{\mathrm{T}}\right\}\ \Im\left\{\boldsymbol{\gamma}_{HH}^{\mathrm{T}}\right\}\cdots\Re\left\{\boldsymbol{\gamma}_{VV}^{\mathrm{T}}\right\}\ \Im\left\{\boldsymbol{\gamma}_{VV}^{\mathrm{T}}\right\}\right]^{\mathrm{T}}, \tag{4}$$

where $\Re\{\bullet\}$ and $\Im\{\bullet\}$ denote the real and imaginary parts. Using (3), the specular propagation path parameters $\boldsymbol{\theta}$ are mapped to an observation vector of length $M = M_f M_T M_R$ with the double-directional channel model as

$$\mathbf{s}(\boldsymbol{\theta}) = \mathbf{B}(\boldsymbol{\mu})\cdot\boldsymbol{\gamma} \in \mathbb{C}^{M\times 1}. \tag{5}$$

The second part of the observation model constitutes of the combination of the dense multipath components (DMC) and the measurement noise. The DMC is modeled with a multivariate circular Gaussian process $\mathbf{n}_d \sim \mathcal{N}_c(\mathbf{0}, \mathbf{R}_d) \in \mathbb{C}^{M\times 1}$. Furthermore the measurement noise is modeled by a zero-mean circular Gaussian process $\mathbf{n}_m \sim \mathcal{N}_c(\mathbf{0}, \sigma^2\mathbf{I}) \in \mathbb{C}^{M\times 1}$, where σ^2 denotes the measurement noise variance. We combine these random processes $\mathbf{n}_d$ and $\mathbf{n}_m$ into one process yielding

$$\mathbf{n}_y = \mathbf{n}_d + \mathbf{n}_m \sim \mathcal{N}_c(\mathbf{0}, \mathbf{R}_y), \tag{6}$$

with the covariance matrix $\mathbf{R}_y = \mathbf{R}_d + \sigma^2\mathbf{I}$.

The complete model for a radio channel observation is thus approximated as a superposition of P specular propagation paths $\mathbf{s}(\boldsymbol{\theta})$ and the random process $\mathbf{n}_y$ as

$$\mathbf{y}_k = \mathbf{s}(\boldsymbol{\theta}_k) + \mathbf{n}_{y,k}. \tag{7}$$

The vector $\mathbf{y}_k$ models the output of the channel sounder at time k.

B. State-Space Model

We assume that the specular propagation path parameters of the radio channel can be described using a Gauss-Markov model [5]. This allows us to formulate the problem as a state-space model:

$$\boldsymbol{\theta}_k = \boldsymbol{\Phi}\boldsymbol{\theta}_{k-1} + \mathbf{v}_k \in \mathbb{R}^{LP\times 1} \tag{8}$$

$$\mathbf{y}_k = \mathbf{s}(\boldsymbol{\theta}_k) + \mathbf{n}_{y,k} \in \mathbb{C}^{M\times 1}, \tag{9}$$

where (8) is the linear state equation and (9) is the nonlinear measurement equation at time k. The parameters in the state vector $\boldsymbol{\theta}_k$ are assumed to be uncorrelated, which leads to a diagonal structure of the state transition matrix $\boldsymbol{\Phi}$. The spectral radius of $\boldsymbol{\Phi}$ is assumed to be less than unity to ensure stability. The state noise $\mathbf{v}_k$ is a real valued white Gaussian process, and it is assumed to be uncorrelated with the state. The covariance matrix of the state noise $\mathbf{Q}$ is a $LP \times LP$ diagonal matrix containing the noise variance of each parameter on the diagonal. The observation noise $\mathbf{n}_{y,k}$ (6) is assumed to be uncorrelated with the state and it has the covariance matrix $\mathbf{R}_{y,k}$.

III. Path Parameter Estimation

The proposed parameter estimation procedure consists of multiple estimators. The core of the algorithm, tracking the propagation path parameters over time, is the EKF. For searching new paths for each observation a Maximum Likelihood (ML) based (RIMAX [3]) estimator is applied. Also, an independent ML estimator is used for estimating the parameters of the DMC component, i.e., to estimate $\mathbf{R}_{y,k}$. The general principle of the estimation procedure is presented in Figure 1, whereas Figure 2 reveals the algorithm in closer detail.

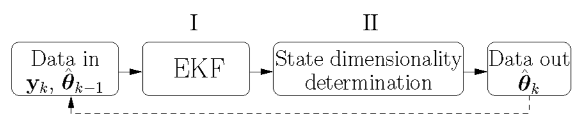


Fig. 1. Estimation procedure principle. The state vector $\hat{\boldsymbol{\theta}}_{k-1}$ (as well as other EKF system matrices) of previous time instant may have different dimensions than the current one $\hat{\boldsymbol{\theta}}_k$.

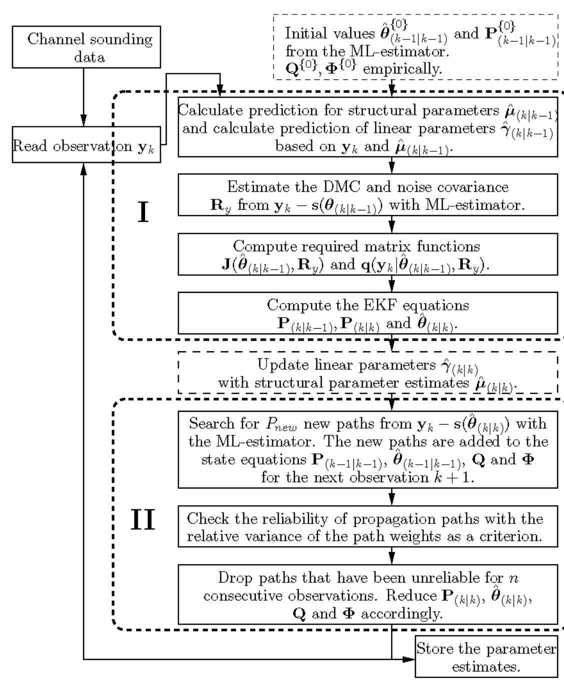


Fig. 2. Complete estimation procedure. The upper level concepts in Figure 1 are marked with areas I and II.

A. Extended Kalman Filter

The Extended Kalman Filter uses Taylor series expansion to linearize a nonlinear state-space model about the current estimates. To apply the EKF one needs to compute the first order partial derivatives to the parameters $\boldsymbol{\theta}$ of the data model $\mathbf{s}(\boldsymbol{\theta})$, i.e., the Jacobian (see [3])

$$\mathbf{D}(\boldsymbol{\theta}) = \frac{\partial}{\partial \boldsymbol{\theta}^{\mathrm{T}}}\mathbf{s}(\boldsymbol{\theta}) = \left[\frac{\partial}{\partial \theta_1}\mathbf{s}(\boldsymbol{\theta}) \cdots \frac{\partial}{\partial \theta_{LP}}\mathbf{s}(\boldsymbol{\theta})\right]. \quad (10)$$

The expressions for the computation of the EKF can be summarized as [4]:

$$\hat{\boldsymbol{\theta}}_{(k|k-1)} = \boldsymbol{\Phi}\hat{\boldsymbol{\theta}}_{(k-1|k-1)} \quad (11)$$

$$\mathbf{P}_{(k|k-1)} = \boldsymbol{\Phi}\mathbf{P}_{(k-1|k-1)}\boldsymbol{\Phi}^{\mathrm{T}} + \mathbf{Q}_k \quad (12)$$

$$\mathbf{J}(\hat{\boldsymbol{\theta}}_{(k|k-1)}, \mathbf{R}_y) = 2 \cdot \Re\left\{\mathbf{D}_k^{\mathrm{H}}(\boldsymbol{\theta})\mathbf{R}_y\mathbf{D}_k(\boldsymbol{\theta})\right\} \quad (13)$$

$$\mathbf{P}_{(k|k)} = \left(\mathbf{P}_{(k|k-1)}^{-1} + \mathbf{J}(\hat{\boldsymbol{\theta}}_{(k|k-1)}, \mathbf{R}_y)\right)^{-1} \quad (14)$$

$$\mathbf{q}(\mathbf{y}_k|\boldsymbol{\theta}, \mathbf{R}_y) = 2 \cdot \Re\left\{\mathbf{D}_k^{\mathrm{H}}\mathbf{R}_y^{-1}\left(\mathbf{y}_k - \mathbf{s}\left(\hat{\boldsymbol{\theta}}_{(k|k-1)}\right)\right)\right\} \quad (15)$$

$$\Delta\hat{\boldsymbol{\theta}}_k = \mathbf{P}_{(k|k-1)}\left(\mathbf{I} - \mathbf{J}(\hat{\boldsymbol{\theta}}, \mathbf{R}_y)\mathbf{P}_{(k|k)}\right)\mathbf{q}(\mathbf{y}_k|\boldsymbol{\theta}, \mathbf{R}_y) \quad (16)$$

$$\hat{\boldsymbol{\theta}}_{(k|k)} = \hat{\boldsymbol{\theta}}_{(k|k-1)} + \Delta\hat{\boldsymbol{\theta}}_k, \quad (17)$$

where

$$\mathbf{q}(\mathbf{y}|\boldsymbol{\theta}, \mathbf{R}_y) = \frac{\partial}{\partial \boldsymbol{\theta}}\mathcal{L}(\mathbf{y}|\boldsymbol{\theta}, \mathbf{R}_y)$$

is the score function, i.e., the partial derivative of the log-likelihood function with respect to the parameters $\boldsymbol{\theta}$, and

$$\mathbf{J}(\boldsymbol{\theta}, \mathbf{R}_y) = -\mathrm{E}\left\{\frac{\partial}{\partial \boldsymbol{\theta}}\mathcal{L}(\mathbf{y}|\boldsymbol{\theta}, \mathbf{R}_y)\left(\frac{\partial}{\partial \boldsymbol{\theta}}\mathcal{L}(\mathbf{y}|\boldsymbol{\theta}, \mathbf{R}_y)\right)^{\mathrm{T}}\right\}$$

is the Fisher information matrix [6]. Equations (11-17) are in the information form of the EKF, which is derived from the covariance form using the matrix inversion lemma. It should be noted that the computation of the Kalman gain is embedded in equations (15) and (16). These expressions have computational advantages over a separate expression for the Kalman gain (see [3] for a discussion).

B. Initialization of the Proposed Algorithm

We propose a dynamic initialization procedure for the EKF algorithm. The number of propagation paths to be tracked should not be restricted, due to the time-varying nature of the mobile radio channel. This, in terms of state-space modeling, means that the dimension of the state vector $\boldsymbol{\theta}$ may change in time. As can be observed from the flow chart in Figure 2, we use a maximum likelihood estimator for searching parameter estimates of new paths. This is done for a larger number of paths at the very beginning of the estimation, and for, e.g., $P_{new} = 2$, paths from the residual $\mathbf{y}_k - \mathbf{s}(\hat{\boldsymbol{\theta}}_{(k|k)})$ of each observation. In addition to the parameter estimates for the state vector, the ML-estimator also provides information on the estimation error variance in form of the inverse of the Fisher information matrix [3]. These estimation error variance estimates are used already inside the ML-estimator to determine if new paths are valid or not. If new paths are found, they are taken into account in the state equations for the subsequent observations trough

$$\hat{\boldsymbol{\theta}}_{(k-1|k-1)}^{\{new\}} = \hat{\boldsymbol{\theta}}_{ML} \quad (18)$$

and

$$\left\{\mathbf{P}_{(k-1|k-1)}^{\{new\}}\right\}_{ij} = \begin{cases} \left\{\mathbf{J}(\hat{\boldsymbol{\theta}}_{ML}, \mathbf{R}_y)^{-1}\right\}_{ij} & , i = j \\ 0 & , i \neq j. \end{cases} \quad (19)$$

Changing the state vector dimension results in modifying (the dimensions of) other system matrices ($\mathbf{Q}$ and $\boldsymbol{\Phi}$) also. The state noise covariance $\mathbf{Q}$ is, for now, chosen through trial and error. Too low values result in the estimator losing its track, whereas too high values prevent the Kalman filter from fully utilizing its filtering capabilities.

C. Reducing the Number of Paths

As new path parameter estimates are added to the state, also paths which have faded out or whose tracking is lost must be dropped out. This is done based on the estimated relative variance of the path weight estimates as it was proposed in [3] for a ML-estimator. The relative variance is used as a path reliability criterion and it can be formulated as

$$\sum_{n=1}^{N_{pol}} \frac{\gamma_{r_{n,p}}^2\sigma_{rr_{n,p}}^2 + 2\,\gamma_{r_{n,p}}\gamma_{i_{n,p}}\sigma_{ri_{n,p}}^2 + \gamma_{i_{n,p}}^2\sigma_{ii_{n,p}}^2}{|\hat{\gamma}|_{n,p}^4} < \varepsilon_{|\gamma|}^2 < 1, \quad (20)$$

where $\gamma_{r_{n,p}}$ denotes the real part of polarization n ($n \in \{HH, HV, VH, VV\}$) of the p^{th} path. The values $\sigma_{rr_{n,p}}$, $\sigma_{ri_{n,p}}$, and $\sigma_{ii_{n,p}}$ denote the path weight real ($_r$) and imaginary ($_i$) part estimation error variances and covariances extracted from the filtering error covariance matrix $\mathbf{P}_{(k|k)}$ (14). If a path fails this criterion then it can be dropped from the state. This can be done either immediately, or after some consecutive fails during multiple observations.

To conclude, the number of paths, i.e., the dimensionality of the state, is determined for each snapshot based on the reliability of the path weight estimates. This criterion is also used in the ML estimator, while searching for new paths from the data after removing the contribution of the known paths.

IV. Estimation Examples

A. Estimation Performance in Simulations

The performance of the proposed estimation algorithm is first demonstrated using simulated data. The artificial sounder output is created using the data model (7) with generated time evolving parameters. A Power Delay Profile (PDP) of the simulated observation is shown in Figure 3. Figure 4 shows the parameters used for azimuth DoA over 100 snapshots, as well as the EKF estimates. Also the estimates using ML based RIMAX [3] algorithm are shown. It should be kept in mind that, although we present here only estimates of one parameter for demonstration purposes, the actual algorithm estimates all the $L \cdot P$ parameters jointly. Knowing the original parameter

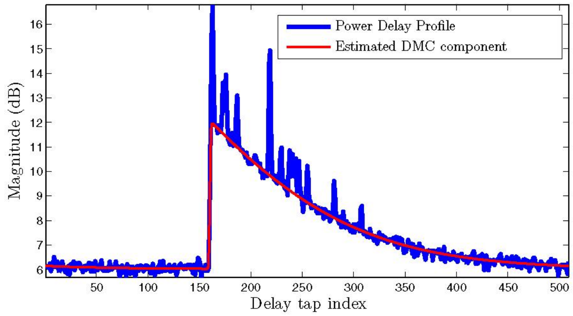


Fig. 3. Power Delay Profile (PDP) of a simulated observation.

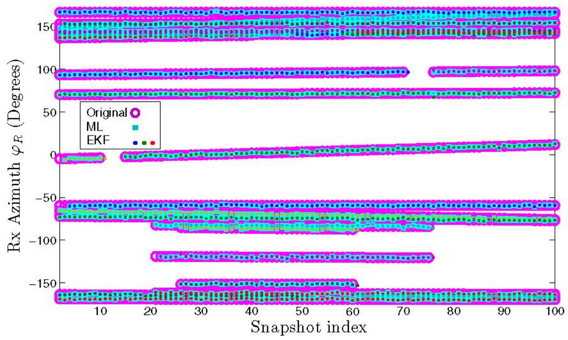


Fig. 4. Rx azimuth angle estimation over 100 snapshots of simulated observations. Both the EKF and the ML estimates follow the original parameters with the correct number of paths.

values enables us to compute the estimation error for the ML-ss (snapshot-by-snapshot) and the EKF estimates. After pairing the path estimates of different estimators, we can compute the standard deviations (STD) of the estimation error for the angular parameters as

$$\sigma_\mu = \sqrt{\frac{1}{KP}\sum_{p=1}^{P}\sum_{k=1}^{K}(\mu_{k,p} - \hat{\mu}_{k,p})^2}, \tag{21}$$

where P stands for the number of paths and K is the number of snapshots. The STD errors of the angular parameters computed from 20 paths over 100 observations are shown in Figure 5. From this figure it can be observed that the performance of the EKF algorithm is better compared to the ML-ss estimator.

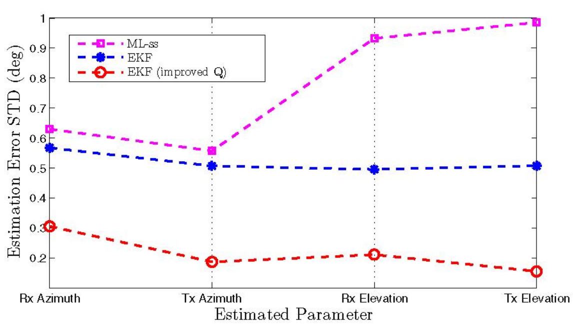


Fig. 5. Standard deviation for estimation error of simulated parameters for 20 paths over 100 observations. The smallest error is obtained by using $\mathbf{Q}$ which is computed from the known original parameters.

The simulation in Figure 4 (STD error labeled *EKF* in Figure 5) was conducted with manually selected, fixed values for the state transition matrix $\mathbf{Q}$. Our simulations show that using a better estimate for $\mathbf{Q}$, results in lower estimation error. This can be seen in Figure 6, showing estimation result from simulations, where the variance for the state covariance matrix $\mathbf{Q}$ were computed from the original parameters. In practice, the optimal values for $\mathbf{Q}$ are difficult to obtain. Thus, for running the algorithm with measurement data in the next section, we use some manually fixed values.

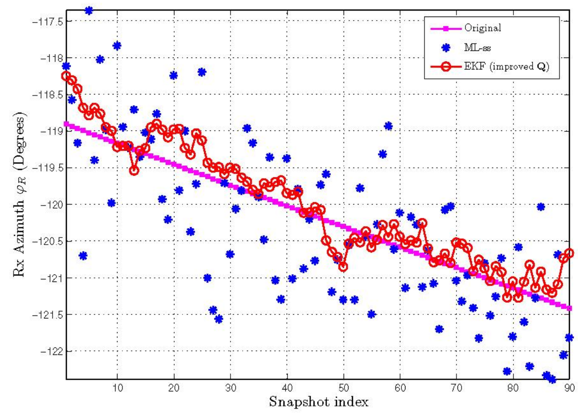


Fig. 6. DoA azimuth angle estimates of a single path in simulations. The elements of the matrix $\mathbf{Q}$ for the EKF were computed from the variance of the known original parameters.

Figure 7 shows the time taken by the estimation of the artificial data comparing the EKF with the ML-ss (RIMAX) algorithm. The recursive computation of the EKF has lower complexity than the iterative ML algorithm, resulting in better performance in terms of computation time (Matlab implementation).

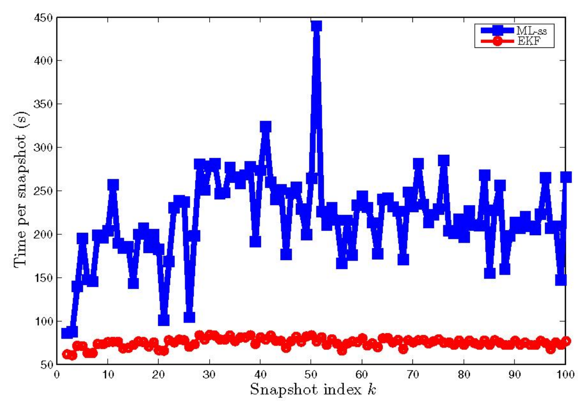
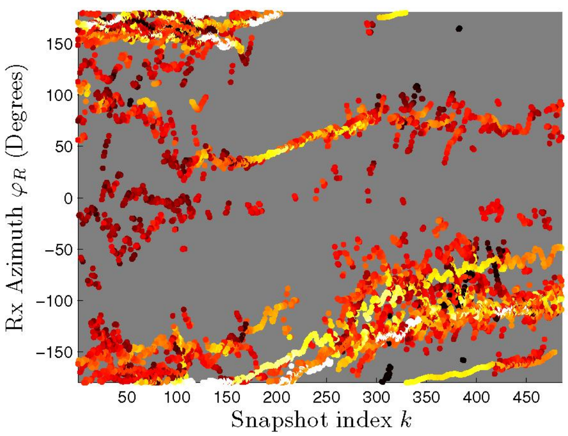


Fig. 7. Processing time taken per snapshot for ML vs. EKF implementation with simulated data (Matlab implementation).

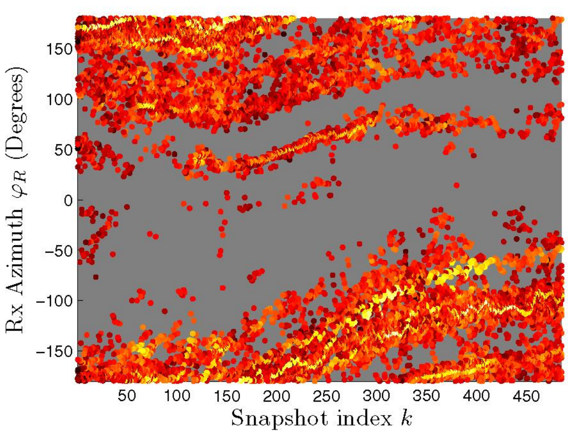
B. Estimation of Measurement Data

In spring 2004 an indoor MIMO measurement campaign was conducted at Helsinki University of Technology, in the building of Electrical and Communications Engineering Department. The measurement was conducted at 5.3 GHz carrier frequency with semispherical antenna arrays of 15 dual-polarized antenna elements at both the static transmitter and the moving receiver. The excitation signal was a pseudo random BPSK signal of code length $L_c = 255$ and a bandwidth of $B_m = 60$ MHz [7].

Figure 8(a) shows the azimuth DoA estimates for a L-shaped Rx measurement route inside an office room, with the transmitter situated in another room about 10 m down the corridor. For comparison, Figure 8(b) shows estimates generated by a commercial ISIS/SAGE based estimator. From



(a) Estimates produced by EKF implementation.



(b) Estimates produced by a reference estimator (ISIS/SAGE).

Fig. 8. DoA azimuth angle estimates processed from measurement data. Lighter color denotes stronger paths (the color scales are not comparable). Due to differences in path dropping criterion, the EKF (Figure 8(a)) doesn't produce estimates of the weakest (darkest tones) values seen in Figure 8(b), but otherwise the results have a clear resemblance.

Figures 8(a) and 8(b) it can be observed that the DoA azimuth estimates have good resemblance. The main difference comes from the fact that Figure 8(a) (EKF) shows less estimates of weak paths due to the conservative path dropping criterion. Also, many of the estimates seen in Figure 8(b) are probably caused by the DMC component, which is not modeled as a separate component in the ISIS. The number of paths in the reference estimator is limited by a fixed number for the maximum number of paths, in this case 30.

The benefit of the proposed propagation path tracking procedure can be observed from Figure 9. With estimators such as ISIS/SAGE, the snapshots were processed independently, and it was very tedious to pair parameters belonging to one path over time. Figure 9 shows that by using the proposed algorithm, many paths are tracked over tens of snapshots, what is useful in dynamic channel modeling research.

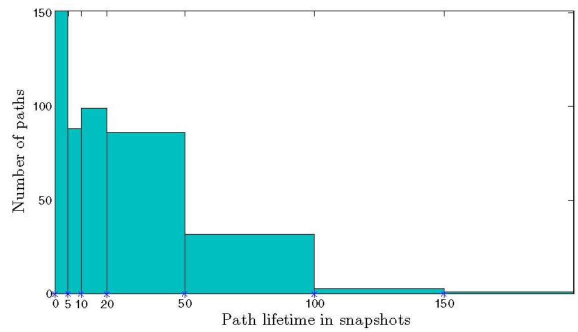


Fig. 9. Histogram of the lifetime of tracked (with EKF) propagation paths (total number of snapshots was 486). Tens of paths were tracked for over 50 snapshots, some even over 100.

V. Conclusion

In this paper we propose a procedure for radio propagation channel parameter estimation and tracking using a state-space model with varying state dimensionality. The method uses the EKF algorithm for the nonlinear tracking problem. The algorithm has good performance in simulations, as well as in experiments with real world measurement data. The implemented EKF solution is able to track propagation paths over time in a reliable manner. The estimator is also computationally less demanding compared to observation-by-observation based, e.g., iterative maximum likelihood based estimation algorithms.

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