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Thermopower Induced by a Supercurrent in Superconductor–Normal-Metal Structures

Pauli Virtanen and Tero T. Heikkilä*

Low Temperature Laboratory, Helsinki University of Technology, P.O. Box 2200, FIN-02015 HUT, Finland
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We examine the thermopower Q of a mesoscopic normal-metal (N) wire in contact with superconducting (S) segments and show that even with electron-hole symmetry Q may become finite due to the presence of supercurrents. Moreover, we show how the dominant part of Q can be directly related to the equilibrium supercurrents in the structure. In general, a finite thermopower appears both between the N reservoirs and the superconductors and between the N reservoirs themselves. The latter, however, strongly depends on the geometrical symmetry of the structure.

Thermoelectric effects in electrical conductors typically result from the asymmetry of the Fermi sea between the electron ($E > E_F$) and holelike ($E < E_F$) quasiparticles. This is illustrated by the Mott relation [1] for the thermopower

$$Q \equiv \frac{\Delta V}{\Delta T}\bigg|_{I=0} = -\frac{\pi^2}{3}\frac{k_B^2 T}{e}\frac{d\ln\sigma(E)}{dE}\bigg|_{E=E_F}. \quad (1)$$

This relates the potential difference ΔV generated by the temperature difference ΔT to the energy dependence of the conductivity σ due to the asymmetry above and below the Fermi sea.

In metals, the electron-hole asymmetry is governed by the parameter $k_B T/E_F$ arising from the next-to-leading term in the Sommerfeld expansion. At sub-Kelvin temperatures, this leads to a very small Q, typically below 10 nV/K. However, recent experiments [2–4] measuring the thermopower in normal-metal wires connected to superconducting electrodes indicate that it exceeds this prediction at least by one order of magnitude and, moreover, show that Q oscillates with the phase difference between the two superconducting contacts.

The Mott relation is expected to fail in the presence of the superconducting proximity effect when the geometrical symmetry in the measured sample is broken [5]. Our aim is to show that with nearby superconductors, normal-metal circuits can show a thermoelectric effect independent of electron-hole symmetries, since the proximity effect couples the temperatures to the potentials through the supercurrent. This effect is at least two orders of magnitude larger than that predicted by Eq. (1) (cf. Fig. 1).

We discuss the system shown in the inset of Fig. 2, with a supercurrent flowing between the two superconducting elements. Our main result [Eq. (8)] states that the thermopower Q_{NS} between the N and S parts of the structure is proportional to the difference in the supercurrents at the temperatures T_1, T_2 of the two N electrodes. Moreover, we obtain a similar result [Eq. (10)] for the thermopower Q_{NN} between the normal parts in a geometrically asymmetric structure. This can be understood phenomenologi-

cally as follows. If $T_1 \neq T_2$, the temperature-dependent [6] equilibrium supercurrent $I_S(T_1)$ in wire 3 is different from $I_S(T_2)$ in wire 4. (For this qualitative picture, we approximate these wires to be at the temperatures T_1, T_2.) Thus, a compensating effect must arise to guarantee the conservation of currents. Should the N reservoirs be kept at the same potential as the superconductors, a quasiparticle current $I_{qp} \propto [I_S(T_1) - I_S(T_2)]$ from them to the superconductors would balance the difference. However, when no current is allowed to flow in wires 1 and 2, a compensating N-S potential difference $V_N - V_S \propto R[I_S(T_1) - I_S(T_2)]$ is induced instead. The induced potentials oscillate with phase differences, similar to the supercurrent, and may differ in the two N reservoirs, especially in asymmetric structures.

In the following, we concentrate on the diffusive limit, and model the setup with the Keldysh-Usadel equations [7]. These equations assume electron-hole symmetry, under which Eq. (1) predicts a vanishing Q.

The Keldysh-Usadel equations are formulated in terms of the quasiclassical Green's functions (which are matrices in the Keldysh-Nambu space), but here we use their θ parametrization [7] for convenience. It reduces the problem to two sets of equations, the spectral equations and the kinetic equations. In terms of the parametrizing functions χ and θ, the phase and the proximity effect strength, the spectral equations in normal metals are

$$D\nabla^2\theta = -2iE\sinh(\theta) + \tfrac{1}{2}D(\nabla\chi)^2\sinh(2\theta), \quad (2a)$$

$$D\nabla \cdot j_E = 0, \qquad j_E \equiv -\sinh^2(\theta)\nabla\chi. \quad (2b)$$

Here, the factor D is the diffusion constant of the normal metal, and E is the energy with respect to the superconductor potential. The kinetic equations are expressed using the symmetric and antisymmetric parts, f_T and f_L [7], of the electron distribution function $f(E, \vec{r})$:

$$D\nabla \cdot j_L = 0, \qquad j_L \equiv \mathcal{D}_L\nabla f_L - \mathcal{T}\nabla f_T + j_S f_T, \quad (3a)$$

$$D\nabla \cdot j_T = 0, \qquad j_T \equiv \mathcal{D}_T\nabla f_T + \mathcal{T}\nabla f_L + j_S f_L. \quad (3b)$$

The equations imply that the spectral current densities j_L

and j_T are conserved (we neglect inelastic scattering). Thus the observable charge and energy current densities,

$$j_c = \frac{\sigma^N}{2e} \int j_T dE \quad \text{and} \quad j_Q = \frac{\sigma^N}{2e^2} \int E j_L dE, \quad (4)$$

are also conserved.

The coefficients $\mathcal{D}_L$, $\mathcal{D}_T$, $\mathcal{T}$, and j_S appearing in the kinetic equations are obtained from the spectral equations:

$$\mathcal{D}_{L/T} \equiv \tfrac{1}{2}\{1 + |\cosh\theta|^2 \mp |\sinh\theta|^2 \cosh(2\operatorname{Im}[\chi])\}, \quad (5a)$$

$$\mathcal{T} \equiv \tfrac{1}{2}|\sinh\theta|^2 \sinh(2\operatorname{Im}[\chi]), \qquad j_S \equiv \operatorname{Im}[j_E]. \quad (5b)$$

Here, $\mathcal{D}_L$ and $\mathcal{D}_T$ are the local spectral energy and charge conductivities, and j_S is the spectral density of the supercurrent [8]. The factor $\mathcal{T}$ arises in the formalism and has an effect on the thermopower. The normal-state values of these coefficients are $\mathcal{D}_L = \mathcal{D}_T = 1$ and $j_S = \mathcal{T} = 0$.

At nodes of wires, assuming clean metallic contacts, the functions θ, χ, and f are continuous, and Kirchhoff-like "spectral current conservation laws" [9] imply that $A\sigma^N j_{L/T}$ and $A\sigma^N j_S$ are conserved. Here, A is the cross-sectional area of a wire and σ^N the normal-state conductivity. At clean metallic reservoir contacts, most of the functions get their bulk values [7]. However, for energies below the superconducting energy gap Δ, the valid boundary conditions at superconductor interfaces are $j_L = 0$, prohibiting the energy flow, and $f_T = 0$, assuming no charge imbalance in the superconductors.

The coefficients j_S and $\mathcal{T}$ couple the energy and charge currents together, and give rise to a finite thermopower. Moreover, these coefficients oscillate with the phase difference in the system, and thus the value of the thermopower should also oscillate. When there is no phase difference, $\mathcal{T} = j_S = 0$, and the thermopower vanishes.

The energy scale of temperatures and potentials is specified by the Thouless energy

$$E_T \equiv \frac{\hbar D}{L^2} \approx \begin{Bmatrix} 13\ \mu\text{V} & e \\ 0.15\ \text{K} & k_B \end{Bmatrix} \frac{D/(200\ \text{cm}^2/\text{s})}{(L/\mu\text{m})^2}, \quad (6)$$

corresponding to a wire of length L [8]. Moreover, E_T of the link between the superconductors is a natural energy scale for the spectral equations. As long as $E_T \ll \Delta$, the results can be scaled to fit all systems with similar ratios of wire lengths and areas.

Since there are no general analytical solutions to the problem, we solve the spectral equations numerically and make a few approximations to solve the kinetic equations. However, the data shown in the figures are obtained numerically without any approximations.

First, we note that the "local potential" f_T is generally small (as shown by the numerical results), as are the induced potentials at the reservoirs. Thus, we can neglect the terms proportional to it in the kinetic equation (3a). Physically this means that we mainly neglect the effect of

supercurrent on the energy currents and the temperatures. (If the potentials were large, the omitted term would be the source for a Peltier-like effect [10].) With this approximation, we integrate the kinetic equations, which yields the connection between the spectral current densities and the distribution functions $f_L(x)$ and $f_T(x)$ at the ends of a wire of length L:

$$j_L = \frac{1}{LM_L}[f_L(L) - f_L(0)], \quad (7a)$$

$$j_T = \frac{1}{LM_T}[f_T(L) - f_T(0)] + j_S f_L(0) + \frac{1}{L^2 M_L M_T}$$
$$\times \left(\int_0^L \frac{\mathcal{T}}{\mathcal{D}_L \mathcal{D}_T} dx + j_S \int_0^L \int_0^x \frac{dx' dx}{\mathcal{D}_L(x') \mathcal{D}_T(x)} \right)$$
$$\times [f_L(L) - f_L(0)]. \quad (7b)$$

Here $M_{L/T} \equiv \frac{1}{L} \int_0^L \mathcal{D}_{L/T}^{-1} dx$ are the dimensionless spectral energy and charge resistances. To simplify the final result, we also approximate $\mathcal{D}_L = 1$ in (7), and $\mathcal{D}_T = 1$ in the latter term in (7b), since the variation in $\mathcal{D}_L$ (away from superconductor interfaces) and $\mathcal{D}_T$ with respect to the energy is smaller than that of the other coefficients. Numerical results verify that this does not affect the result crucially. The energy-dependent $1/M_T$ as a coefficient for f_T causes an important temperature dependence of the conductance, so we retain it.

Using Eqs. (7) and the conservation of spectral currents, we obtain a linear system of equations for the spectral current densities. They can be solved with respect to the given temperatures and potentials in the reservoirs, with different results for $|E| < \Delta$ and $|E| > \Delta$, due to the different boundary conditions. Next, we integrate over the energy to obtain the observable current densities, after which we require the condition $j_{c,1} = j_{c,2} = 0$. To solve the resulting equations for the small induced potentials eV_1 and eV_2, we linearize the distribution functions with respect to them, and obtain a linear equation for the potentials, which can then be solved.

If we proceed with the analytical approximation in the limit $\Delta \gg E_T$, eV, $k_B T$ by neglecting $\mathcal{T}$ and the energy dependence of $\mathcal{D}_T$, we obtain the dominant term:

$$V_{1/2}^0 = \frac{1}{2} \frac{R_5(2R_{4/3} + R_5)R_{3/4}[I_S(T_1) - I_S(T_2)]}{(R_1 + R_2 + R_5)(R_3 + R_4 + R_5)}. \quad (8)$$

Here, $I_S(T) = (A\sigma^N/e) \int_0^\infty j_S f_L^0 dE$ is the observable equilibrium supercurrent flowing in the system when all parts are at the temperature T and there are no potential differences. Moreover, $f_{L,k}^0 \equiv \tanh[E/(2k_B T_k)]$ is a linearized distribution function, and $R_k = L_k/(A_k \sigma_k^N)$ are the normal-state resistances of the wires. Thus a difference in equilibrium supercurrents due to the varying f_L contributes significantly to the thermopower.

Similarly, we can take the effect of $\mathcal{T}$ into account and obtain the correction terms

$$eV^1_{1/2} = \frac{\mp R_{1/2}}{R_1 + R_2 + R_5} \int_0^\infty (f^0_{L,1} - f^0_{L,2})\langle \mathcal{T}_{1/2}\rangle dE$$

$$\mp \frac{R_{3/4}R_5}{(R_1 + R_2 + R_5)R_{SNS}} \int_0^\infty (f^0_{L,1} - f^0_{L,2})\langle \mathcal{T}_5\rangle dE,$$

(9)

where $R_{SNS} = R_3 + R_4 + R_5$. Here, we denoted $\langle \mathcal{T}_k\rangle \equiv \frac{1}{L_k}\int_0^{L_k}\mathcal{T}_k dx$ [shown in Fig. 3(d) compared with j_S]. The correction is necessary, as it compensates for the fast decay of $V^0_{1/2}$ at high temperatures $k_BT \gtrsim 10E_T$, but it is not negligible even at lower temperatures (see Fig. 2).

The induced $Q_{NS,2} \equiv V_2/(T_2 - T_1) \approx Q_{NS,1}$ is shown in Fig. 1. The magnitude of Q_{NS} is of the order $\sim \frac{\mu V}{K}$ at the highest, but there is also a strong temperature dependence. Figure 2 shows cross sections of Fig. 1, compared with the approximation (8).

We see that Eqs. (8) and (9) predict an induced N-N potential difference

$$\Delta V \equiv V_2 - V_1 \approx (V^0_2 - V^0_1) + (V^1_2 - V^1_1). \quad (10)$$

In a left-right symmetric structure both terms vanish, but Q_{NN} may still be finite since $\mathcal{D}_T$ is energy dependent:

$$\Delta V \approx \int_0^\infty \tilde{R}^{-1}\left(\frac{\mathrm{sech}^2(\frac{E}{2k_BT_1})}{2k_BT_1} - \frac{\mathrm{sech}^2(\frac{E}{2k_BT_2})}{2k_BT_2}\right)dE[R_1 + (R_3R_5)/(2R_3 + R_5)](V^0_1 + V^1_1), \quad (11)$$

valid for a symmetric structure $R_1 = R_2$, $R_3 = R_4$. Here the coefficient $\tilde{R} = \tilde{R}_1 + (\tilde{R}_3\tilde{R}_5)/(2\tilde{R}_3 + \tilde{R}_5)$, where $\tilde{R}_k = M_{T,k}R_k$ are the spectral resistances of the wires. The voltage (11) can be understood to be caused by the proximity-effect–induced temperature dependence of conductances [11], which creates asymmetry in resistances. However, in reality the asymmetry of the structure likely causes a more significant effect (see Fig. 4), especially at $T_1 \approx T_2$, where Eq. (11) predicts $\Delta V \sim (T_2 - T_1)^2$ [12].

Equation (8) implies that the thermopower should oscillate as a function of the phase difference ϕ between the two superconducting elements, because the equilibrium supercurrent oscillates roughly as $\sin(\phi)$. Numerical simulations show [see Fig. 3(c)] that also the exact solution oscillates similarly, vanishing at $\phi = 0$.
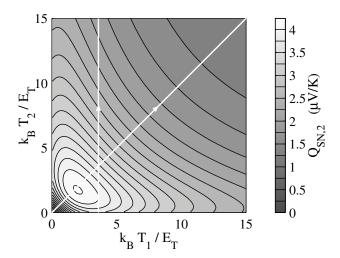
Besides changing the prefactors in Eqs. (8) and (9), varying the resistances of the wires from the near-symmetric presented in the figures changes the behavior in the supercurrent [8] and the coefficient $\mathcal{T}$. In general, large departures from such symmetry decrease the potentials.

A finite value for Δ causes two distinct modifications to the thermopower. First, the coefficients (5) are modified, but changes are mostly only quantitative, e.g., sharpening of peaks [see Fig. 3(a)]. Second, there is also a contribution from energies $E > \Delta$, which couples the superconductor temperature T_0 to the system. The latter effect is weaker than those predicted by Eqs. (8) and (9) at least for
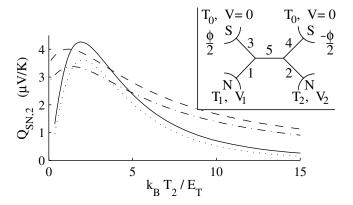

FIG. 1. N-S thermopower at different temperatures T_1 and T_2 of the normal electrodes, for a setup with L_k and A_k assumed equal for all wires, and in the limit $\Delta \gg E_T$, eV, k_BT. The analytical approximation (8) with $\mathcal{T}$-correction (9) deviates at worst 15% from these results, but the difference is over 5% only where Q_{NS} is small, i.e., $k_BT_1, k_BT_2 \gtrsim 10E_T$. In all of the figures, $E_T \equiv \hbar D/(L_3 + L_4 + L_5)^2$, and we plot the results for $\phi = \pi/2$ yielding a near-maximal supercurrent.

FIG. 2. N-S thermopower along the cross sections of Fig. 1. Solid line: Q_{NS} at $T_1 \approx T_2$. Dotted line: Approximation (8). Correction (9) accounts for most of the difference. Dashed line: Q_{NS} at $k_BT_1 = 3.6E_T$ with varying T_2. Dash-dotted line: the corresponding approximation. Inset: The setup under consideration: two superconducting (S) reservoirs with phase difference ϕ connected to two normal-metal (N) reservoirs through diffusive normal-metal wires. We assume that the lengths L_k, $k = 1, \ldots, 5$ of the wires satisfy $\xi_0 \lesssim L_k \ll l_\phi, l_E$, where $\xi_0 = \sqrt{\hbar D/(2\Delta)}$ is the superconducting coherence length, l_ϕ the phase-coherence length, and l_E the energy-relaxation length. We also consider the wires as quasi-1D structures [8].
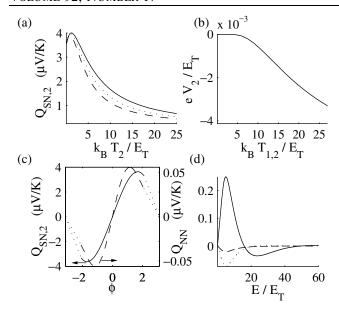
FIG. 3. (a) N-S thermopower at $k_B T_1 = 3.6 E_T$ for various values of Δ: ∞ (solid line), $54 E_T$ (dotted line), $27 E_T$ (dashed line). Superconductor temperature $T_0 = 3.6 E_T$ is fixed. (b) Potential eV_2 induced due to the energies above Δ, with $\Delta = 27 E_T$. Here, $k_B T_0 = 3.6 E_T$ but $T_1 = T_2$ vary. (c) Phase oscillation of Q_{NS} (solid line) and Q_{NN} (dashed line), at $k_B T_1 = 4.5 E_T$, $k_B T_2 = 1.8 E_T$. Because of a numerical convergence problem, there are no data for $|\phi| > 2.1$. (d) Spectral variables: $L_3 j_S$ (solid line), $\langle \mathcal{T}_1 \rangle$ (dashed line), $\langle \mathcal{T}_3 \rangle$ (dotted line). In each figure, all the lengths L_k and areas A_k are assumed equal.

$\Delta > 30 E_T$. Although the coupling of T_0 is weak, it induces finite potentials even for $T_1 = T_2$ [Fig. 3(b)].

Our predictions agree quantitatively with the experimental results with the correct order of magnitude for both the linear thermopower [3] and the temperature scale [2]. The flux dependence (antisymmetric about $\phi = 0$—this holds for the exact as well as the approximate solutions) is in accord with most of the measurements in [2–4]. However, we cannot explain the symmetric oscillations with respect to zero flux, seen in the "house" interferometers in Ref. [2]. Moreover, the main result, Eq. (8), cannot describe a sign reversal of Q_{NS} [4], but there is no principal reason forbidding such an effect in a suitable structure. Nevertheless, further experiments are required to quantitatively demonstrate the connection between the thermopower and the supercurrent.

Results of a similar type as presented in this Letter for the N-S thermopower have been obtained for small temperature differences in Refs. [13,14], assuming high tunnel barriers at the N-S contacts. However, the direct connection between the thermopower and the supercurrents as in Eq. (8) has not been shown. Moreover, Ref. [14] discusses a finite N-N thermopower from the energies above Δ. Our results show that for an appreciable temperature difference between the N reservoirs, this effect is washed out by the asymmetry effects, at least for $\Delta \gtrsim 30 E_T$.
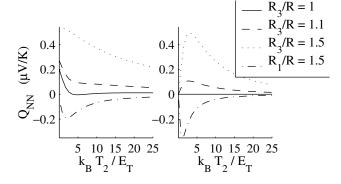

FIG. 4. Left: N-N thermopower induced by setup asymmetry, at fixed $k_B T_1 = 3.6 E_T$. One wire (1 or 3) is assumed to have a different resistance than the others. Right: N-N thermopower with the same parameters, but with $T_1 \approx T_2$.

In summary, we have obtained a relation linking the voltages induced by a temperature difference to the supercurrent in a mesoscopic structure. The phase-oscillating N-S thermopower is mostly induced by the temperature dependence in the supercurrent, and the N-N thermopower can be attributed to left-right asymmetries in a structure. These effects are independent of electron-hole asymmetry, and can be much larger in magnitude than the thermopower due to electron-hole symmetry breaking.

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*Electronic address: Tero.T.Heikkila@hut.fi
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