P. Virtanen and T. T. Heikkilä. 2006. Circuit theory for noise in incoherent normal-superconducting structures. New Journal of Physics, volume 8, number 4, 50.

http://stacks.iop.org/njp/8/50

New Journal of Physics
The open-access journal for physics

Circuit theory for noise in incoherent normal-superconducting structures

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New Journal of Physics **8** (2006) 50
Received 26 January 2006
Published 7 April 2006
Online at http://www.njp.org/
doi:10.1088/1367-2630/8/4/050

Abstract. We consider the current fluctuations in a mesoscopic circuit consisting of nodes connected by arbitrary connectors, in a setup with multiple normal or superconducting terminals. In the limit of the weak superconducting proximity effect, simplified equations for the second-order cross-correlators can be derived from the general counting-field theory, and the result coincides with the semi-classical principle of minimal correlations. We discuss the derivation of this result in a multi-node case.

Contents

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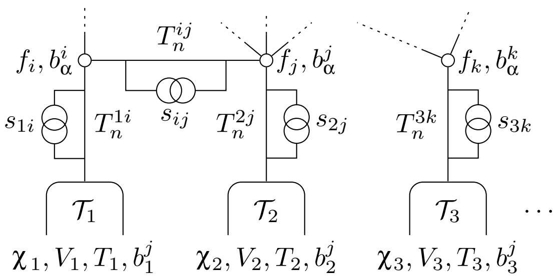

Figure 1. A schematic circuit consisting of nodes, terminals and connectors, specified by the transmission eigenvalues T_n^{ij}, voltages V_α and temperatures T_α at the terminals. The Langevin noise generators s_{ij}, characteristic potentials b_α^i, and distribution functions f_i arise when Green's function is expanded in the counting fields χ_α.

1. Introduction

Fluctuations of charge current in mesoscopic structures are in general sensitive to the interactions and the fermionic nature of electrons. In multi-terminal setups, the geometry of the circuit is important for the cross-correlations, and in superconducting heterostructures, also the Andreev reflection, the superconducting proximity effect and transmission properties of NS interfaces need to be accounted for.

The general theory for the full counting statistics of current fluctuations in multi-terminal structures was outlined in [1]. The calculation of the second-order correlators using this theory can be simplified, from complicated 4×4 matrix equations to a Kirchhoff-type system for scalar parameters (see figure 1), using an approach discussed also, for example, in [2, 3]. In the incoherent case [2, 4, 5], the result coincides with the semi-classical principle of minimal correlations [2, 3]. We show a derivation of this result in arbitrary multi-node systems.

The theory considers a network of normal ($\mathcal{T}_N$) and superconducting ($\mathcal{T}_S$) terminals ($\mathcal{T} = \mathcal{T}_N \cup \mathcal{T}_S$) and nodes ($\mathcal{N}$), connected by connectors. Each connector (i, j) is described by its transmission eigenvalues T_n^{ij} [6], and each node j is characterized by a Keldysh Green function $\hat{G}_j$, which is a 4×4 matrix in the Keldysh($\check{\ }$) $\otimes$ Nambu($\hat{\ }$) space. In the quasi-classical approximation, assuming stationary state and isotropicity, these are only functions of energy, $\hat{G}(\varepsilon)$.

The statistics of the current in the circuit is connected to the generating function $S(\{\chi_k\}_{k\in\mathcal{T}})$ of charge transfer, which can be found by solving transport equations for the Green functions. In the stationary case at zero frequency, the noise correlations $\tilde{S}_{kl}$ between the fluctuations $\delta I_k = I_k - \langle I_k \rangle$ of currents flowing into the terminals $k, l \in \mathcal{T}$ relate to it through [1, 7]

$$\tilde{S}_{kl} \equiv \int_{-\infty}^{\infty} \frac{\mathrm{d}t}{2} \langle\{\delta I_k(t), \delta I_l(0)\}\rangle = -\frac{e^2}{t_0} \frac{\partial^2 S}{\partial\chi_k \partial\chi_l}\bigg|_{\{\chi_j\}=0}. \tag{1}$$

The equality applies provided the duration t_0 of the measurement is much larger than the correlation time of the fluctuations.

The boundary conditions for transport are assumed such that the terminals are in an internal equilibrium, where the Green function has the form

$$\check{G}_{\mathrm{eq}} = \begin{pmatrix} \hat{R} & \hat{K} \\ \hat{0} & \hat{A} \end{pmatrix}, \qquad \begin{array}{ll} \hat{R} = u\hat{\tau}_3 + v\mathrm{i}\hat{\tau}_2, & \hat{K} = \hat{R}\hat{h} - \hat{h}\hat{A}, \\ \hat{A} = -\hat{\tau}_3\hat{R}^\dagger\hat{\tau}_3, & \hat{h} = f_L + \hat{\tau}_3 f_T. \end{array} \tag{2}$$

Here, $u = |\varepsilon|/\sqrt{\varepsilon^2 - \Delta^2}$, $v = \mathrm{sgn}(\varepsilon)\sqrt{u^2 - 1}$ are the coherence factors and Δ is the superconducting pair amplitude. The functions $f_T(\varepsilon) = 1 - f_0(-\varepsilon) - f_0(\varepsilon)$ and $f_L(\varepsilon) = f_0(-\varepsilon) - f_0(\varepsilon)$ are the symmetric and antisymmetric parts of $f_0(\varepsilon) = [\mathrm{e}^{(\varepsilon - eV)/(k_{\mathrm{B}}T)} + 1]^{-1}$, where T is the temperature and V the potential of the terminal. We assume $V = 0$ in all S terminals to avoid time-dependent effects. For calculation of the statistics of the current, the counting field theory additionally specifies the rotation

$$\check{G}_k(\chi_k) = \mathrm{e}^{\mathrm{i}\chi_k\check{\tau}_K/2}\, \check{G}_{k,\mathrm{eq}}\, \mathrm{e}^{-\mathrm{i}\chi\check{\tau}_K/2}, \qquad \check{\tau}_K \equiv \check{\tau}_1 \otimes \hat{\tau}_3, \tag{3}$$

at each terminal k, which connects the 'counting fields' χ_k to the Green functions.

In circuit theory [8], transport is modelled by the conservation of the matrix current at each node i

$$\sum_{j\in\mathcal{C}} \check{I}^{ij} = \check{0}, \qquad \check{I}^{ij} = \frac{2e^2}{\pi\hbar}\sum_n \frac{T_n^{ij}[\check{G}_j, \check{G}_i]}{4 + T_n^{ij}(\{\check{G}_i, \check{G}_j\} - 2)}. \tag{4}$$

The sum runs over all nodes and terminals ($\mathcal{C} = \mathcal{T} \cup \mathcal{N}$): we assume the convention that $T_n^{ij} = 0$ for $i = j$ and nodes that are not directly connected. The resulting matrix is related to the observable charge and energy currents by

$$I^{ij} = \frac{1}{8e}\int_{-\infty}^{\infty} \mathrm{d}\varepsilon \mathrm{Tr}\left[\check{\tau}_K \check{I}^{ij}\right], \qquad I_E^{ij} = \frac{1}{8e^2}\int_{-\infty}^{\infty} \mathrm{d}\varepsilon\, \varepsilon\, \mathrm{Tr}\left[\check{\tau}_1 \check{I}^{ij}\right]. \tag{5}$$

Their dependency on $\{\chi_i\}$, in turn, describes the generating function of charge transfer: [1]

$$\mathrm{d}S(\{\chi_l\}) = -\frac{t_0}{e}\sum_{k\in\mathcal{T}}\sum_{j\in\mathcal{C}} I^{jk}(\{\chi_l\})\, \mathrm{d}(\mathrm{i}\chi_k). \tag{6}$$

Determining the Green functions at the nodes from (3) and (4) and finally applying (5) and (6), one can in principle find the distribution of the fluctuations in the current. However, the problem becomes considerably simpler if one is interested only in the second moment of this distribution, i.e., the current noise as given in (1).

2. Second correlator

We proceed calculating the noise by assuming that the superconducting proximity effect is negligible, so that the anomalous parts ($\propto \hat{\tau}_1, \hat{\tau}_2$) of the functions vanish in each node [2, 4]. Then, one can expand the Green function at node j to the first order in the counting fields $\{\chi_k\}$, in the Nambu-diagonal form [2, 3, 9]:

$$\check{G}_j = \begin{pmatrix} \hat{\tau}_3 & 2\hat{h}_j\hat{\tau}_3 \\ 0 & -\hat{\tau}_3 \end{pmatrix} + \sum_{k\in\mathcal{T}} \mathrm{i}\chi_k \begin{pmatrix} -\hat{h}_j\hat{b}_k^j & 4\hat{c}_k^j - \hat{b}_k^j \\ \hat{b}_k^j & \hat{h}_j\hat{b}_k^j \end{pmatrix} + \cdots, \tag{7}$$

where $\hat{b}_k^j(\varepsilon) = \hat{1}b_k^j(|\varepsilon|)$, $\hat{c}_k^j = c_T^{jk} + \hat{\tau}_3 c_L^{jk}$ and $\hat{h} = f_L + \hat{\tau}_3 f_T$. This satisfies the quasi-classical normalization $\check{G}_j^2 = \check{1}$ up to the second order in $\{\chi_k\}$. For the matrix currents, the above corresponds to the expansion

$$\check{I}^{ij} = \begin{pmatrix} \hat{0} & \hat{I}_0^{ij} \\ \hat{0} & \hat{0} \end{pmatrix} + \sum_{k\in\mathcal{T}} \mathrm{i}\chi_k \begin{pmatrix} \hat{0} & \hat{I}_{c,k}^{ij} - \hat{I}_{b,k}^{ij} \\ \hat{I}_{b,k}^{ij} & \hat{0} \end{pmatrix} + \cdots + \check{I}_{\mathrm{coh}}^{ij} \tag{8}$$

of (4), where $\hat{I}_0$, $\hat{I}_{b,k}$ and $\hat{I}_{c,k}$ have the structure $\hat{I} = \hat{\tau}_3 I(\hat{\tau}_3\varepsilon)$, due to symmetries in the Nambu space. Here, $\check{I}_{\mathrm{coh}}(\{\chi_k\})$ contains the off-diagonal Nambu-elements, present if j corresponds to a superconducting terminal. In what follows, we neglect this coherent part of the current, assuming there are additional decoherence-inducing sink terms in (4). This is valid if the superconductors are weakly connected to the rest of the system, the Thouless energy describing the inverse time-of-flight through the node or the connector is much less than the characteristic energy scales of the problem, or, if there is a strong pair-breaking effect in the node, e.g., due to magnetic impurities or a suitably large magnetic field. Conditions for reaching this incoherent limit are discussed in detail in [2, 4]. The coherent corrections to the current correlators due to the superconducting proximity effect are described, for example, in [9]–[11].

One can then consider expansion (8) in detail, assuming node $i \in \mathcal{N}$ is connected to node or terminal j. This yields four independent equations of conservation:

$$\sum_{j\in\mathcal{C}} I_T^{ij} = 0, \qquad \sum_{j\in\mathcal{C}} I_L^{ij} = 0, \qquad \sum_{j\in\mathcal{C}} I_{b,k}^{ij} = 0, \qquad \sum_{j\in\mathcal{C}} I_{c,T,k}^{ij} = 0, \tag{9}$$

in which I_T corresponds to the spectral charge current, I_L to the energy current, and the last two to a 'noise' current, with the symmetric part defined as $I_{c,T,k}(\varepsilon) = I_{c,k}(\varepsilon) + I_{c,k}(-\varepsilon)$. The corresponding antisymmetric current $I_{c,L,k}^{ij}$ is not needed, as we concentrate on the noise in the charge current. The spectral currents have the form

$$I_T^{ij} = g_{ij}(f_T^j - f_T^i), \qquad I_{b,k}^{ij} = g_{ij}(b_k^j - b_k^i), \tag{10a}$$

$$I_L^{ij} = \begin{cases} 0 & \text{for } j \in \mathcal{T}_S \text{ and } |\varepsilon| < |\Delta|, \\ g_{ij}(f_L^j - f_L^i) & \text{otherwise.} \end{cases} \tag{10b}$$

Thus, no energy current flows to the superconductors for $|\varepsilon| < |\Delta|$. The fourth current is

$$\tfrac{1}{4} I_{c,T,k}^{ij} = g_{ij}(c_T^{ik} - c_T^{jk}) - (b_k^i - b_k^j)[s_{ij}(\varepsilon) + s_{ij}(-\varepsilon)], \tag{11}$$

but it can be eliminated, see below.

The factors g_{ij} and $s_{ij}(\varepsilon)$ appearing in the expansion can be identified as the conductances and spectral noise densities characteristic of the connectors, and their exact form depends on whether the connector lies between two normal nodes (NN) or between a normal node and a superconducting terminal (NS). The expressions for the NS case are lengthy, so for simplicity we use here only the limits $\varepsilon \ll \Delta$ and $\varepsilon \gg \Delta$ for superconducting Green's functions, effectively neglecting the exact form of the superconducting density of states. In this approximation, for an NS connector at $|\varepsilon| \gg |\Delta|$ or an NN connector,

$$s_{ij}^{\mathrm{NN}}(\varepsilon) = \tfrac{1}{4} g_{ij}^{\mathrm{NN}} [2 - (f_L^i + f_T^i)^2 - (f_L^j + f_T^j)^2 + F_{ij}^{\mathrm{NN}} (f_L^i + f_T^i - f_L^j - f_T^j)^2], \tag{12a}$$

$$g_{ij}^{\mathrm{NN}} = \frac{e^2}{\pi\hbar} \sum_n T_n^{ij}, \qquad F_{ij}^{\mathrm{NN}} = \frac{e^2}{g_{ij}^{\mathrm{NN}} \pi\hbar} \sum_n T_n^{ij}(1 - T_n^{ij}). \tag{12b}$$

The result for an NS connector at $|\varepsilon| \ll |\Delta|$ is, with $i \in \mathcal{N}$, $j \in \mathcal{T}_S$,

$$s_{ij}^{\mathrm{NS}}(\varepsilon) \equiv s_{ji}^{\mathrm{NS}}(\varepsilon) = \tfrac{1}{2} g_{ij}^{\mathrm{NS}} [1 - (f_L^i)^2 - (f_T^i)^2 + F_{ij}^{\mathrm{NS}} (f_T^i)^2], \tag{13a}$$

$$g_{ij}^{\mathrm{NS}} = \frac{e^2}{\pi\hbar} \sum_n \frac{2(T_n^{ij})^2}{(2 - T_n^{ij})^2}, \tag{13b}$$

$$F_{ij}^{\mathrm{NS}} = (g_{ij}^{\mathrm{NS}})^{-1} \frac{e^2}{\pi\hbar} \sum_n \frac{16(T_n^{ij})^2}{(2 - T_n^{ij})^4} (1 - T_n^{ij}), \tag{13c}$$

as found through an expansion of (4). Naturally, the results above agree with expressions for the noise generated between two terminals, with F_{ij} being the differential Fano factor [6, 12].

The above equations are supplied with the boundary conditions

$$b_k^l = \delta_{kl}, \qquad c_T^{kl} = 0, \qquad f_k(\varepsilon) = f_0(\varepsilon, V_k, T_k), \tag{14}$$

where k and l are indices of terminals. These can be found by comparing expansion (7) to (3) (for N terminals), and by examining the expression for $\check{I}$ (for S terminals).

We now note that the solution to the conservation equations for $I_{c,T}$ can be written directly,

$$c_T^{il} = \sum_{j \in \mathcal{N}; m \in \mathcal{C}} \gamma_{ij}^{-1} (b_l^j - b_l^m)[s_{jm}(\varepsilon) + s_{jm}(-\varepsilon)] \equiv \sum_{j \in \mathcal{N}; m \in \mathcal{C}} \gamma_{ij}^{-1} x_{jm}^l, \quad \forall i \in \mathcal{N}, \tag{15}$$

in terms of the elements γ_{ij}^{-1} of the inverse of the positive definite conductance matrix $\gamma_{ij} = -g_{ij} + \delta_{ij} \sum_{k \in \mathcal{C}} g_{ik}$. Substituting this solution back to (11) and making use of (1), (5) and (6) finally yields the result

$$\begin{aligned} \tilde{S}_{kl} &= \sum_{m \in \mathcal{C}} \int_{-\infty}^{\infty} \mathrm{d}\varepsilon \tfrac{1}{8} I_{c,T,l}^{mk} = \frac{1}{2} \int_{-\infty}^{\infty} \mathrm{d}\varepsilon \left(\sum_{i,j \in \mathcal{N}; m \in \mathcal{C}} g_{ki} \gamma_{ij}^{-1} x_{jm}^l + \sum_{m \in \mathcal{C}} x_{km}^l \right) \\ &= \frac{1}{2} \int_{-\infty}^{\infty} \mathrm{d}\varepsilon \left(\sum_{i \in \mathcal{N}; m \in \mathcal{C}} b_k^i x_{im}^l + \sum_{i \in \mathcal{T}; m \in \mathcal{C}} b_k^i x_{im}^l \right) \\ &= \sum_{(i,j)} \int_{-\infty}^{\infty} \mathrm{d}\varepsilon (b_k^i - b_k^j)(b_l^i - b_l^j) s_{ij}(\varepsilon), \end{aligned} \tag{16}$$

for the correlations between terminals k and l. The last sum runs over all connectors (i, j) in the circuit. To assemble this expression, we made use of the boundary conditions (14) and the facts that $\sum_{j \in \mathcal{N}} \gamma_{ij}^{-1} g_{jk} = b_k^i$ and $\gamma_{ji}^{-1} = \gamma_{ij}^{-1}$.

3. Discussion

3.1. Equivalence to Langevin formalism

It is illustrative to note that the above quantum-mechanical counting-field result agrees with the well-known principle of minimal correlations, which is often used in semi-classical calculations [2, 6]. In a typical model, assuming only N-terminals, one has the Langevin equations

$$\sum_{j\in\mathcal{C}} I^{ij} = 0, \qquad I^{ij} = g_{ij}(f_j - f_i) + \delta I^{ij}, \tag{17}$$

where f_i are the electron distribution functions at the nodes, and δI^{ij} are the microscopic fluctuations of the current generated in connector (i, j) within the energy interval $[\epsilon, \epsilon + \mathrm{d}\epsilon]$. Assuming the distribution functions f_i do not fluctuate at the terminals, one can perform a similar calculation as above and find the result

$$\delta I_k = \sum_{(i,j)} (b_k^i - b_k^j)\delta I^{ij}, \tag{18}$$

for the fluctuations δI_k in the current flowing to terminal k. The variables b_k^i coincide exactly with the characteristic potentials often used in semi-classical multi-terminal calculations [13].

Assuming δI^{ij} are stochastically independent, one finds an analogue to (16):

$$\begin{aligned}\tilde{S}_{kl} &= \int_{-\infty}^{\infty} \mathrm{d}\varepsilon \tfrac{1}{2} \left\langle \{\delta I_k, \delta I_l\} \right\rangle, \\ &= \sum_{(i,j)} \int_{-\infty}^{\infty} \mathrm{d}\varepsilon\, (b_k^i - b_k^j)(b_l^i - b_l^j) \tfrac{1}{2} \langle \{\delta I^{ij}, \delta I^{ij}\} \rangle.\end{aligned} \tag{19}$$

The result coincides exactly with the prediction from the counting field theory, for an arbitrary circuit, showing that s_{ij} correspond to correlators of the microscopic fluctuations that should be evaluated using the (average) distribution functions at the nodes (see figure 1). The distribution functions may in general be in non-equilibrium, and should be obtained from a kinetic equation. Moreover, in the incoherent limit, the Langevin equations can be applied also in the presence of superconducting terminals [2, 4, 5].

The above discussion also clearly shows that an attempt to evaluate the higher correlators of noise using the principle of minimal correlations fails, as this corresponds to truncating expansion (7) after the first two terms. The higher-order semi-classical corrections needed to fix this are discussed for example in [3].

We implicitly assumed above that there is no inelastic scattering which would drive the system towards equilibrium. However, following [14], a strong relaxation of the distribution function in a node may be modelled by assuming that f_j has the form of a Fermi function. In the case of relaxation due to strong electron–electron scattering, the corresponding potential V_j and temperature T_j can be determined by taking the two first moments, $\int \mathrm{d}\varepsilon$ and $\int \mathrm{d}\varepsilon\varepsilon$ of (9):

$$\sum_{j\in\mathcal{C}} g_{ij}(V_i - V_j) = 0, \qquad \mathcal{L} \equiv \frac{k_{\mathrm{B}}^2\pi^2}{3e^2}, \tag{20a}$$

$$\sum_{j\in\mathcal{C}\setminus\mathcal{T}_S} g_{ij}[V_i^2 - V_j^2 + \mathcal{L}(T_i^2 - T_j^2)] = 0. \tag{20b}$$

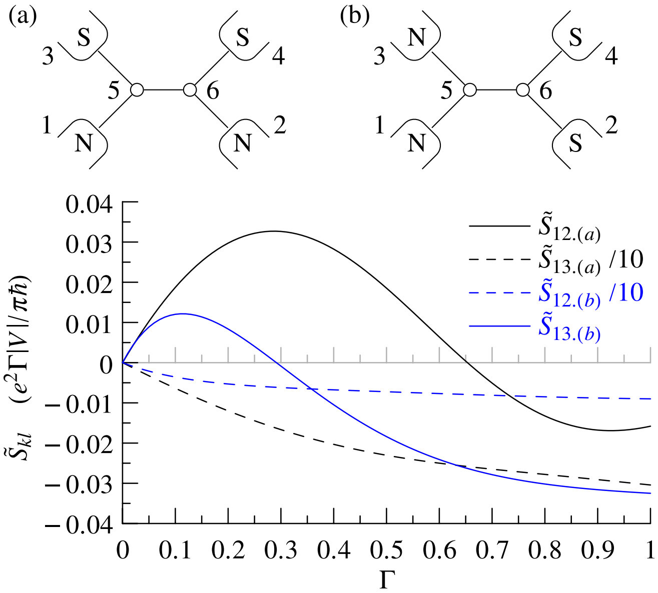

Figure 2. Two multi-node circuits and the cross-correlations $\tilde{S}$ between the normal terminals when both are biased to voltage V and are at temperature $T = 0$. Connectors are assumed to be single-channel conductors with transparency Γ.

These describe the conservation of charge and energy currents. If some of the nodes are in non-equilibrium, one can define the effective voltages and temperatures so that the above equations still apply for the whole circuit. In addition, relaxation due to strong electron–phonon coupling can be modelled by forcing T_i coincide with the lattice temperature, so that only V_i need be determined.

3.2. Application to two-node circuits

As an example of multi-node structures, consider the two circuits in figure 2, consisting of two normal and superconducting terminals joined by two nodes. It is straightforward to calculate the cross-correlation between the normal terminals, and the result in a simple case is shown in figure 2. When the setup (a) is assumed symmetric on an interchange of the two normal or the two superconducting terminals, we get from (9)

$$b_1^5 = b_2^6 = \frac{r}{R} b_3^5 = \frac{r}{R} b_4^6 = \frac{r(2r+R)}{(r+R)(3r+R)}, \tag{21a}$$

$$b_1^6 = b_2^5 = \frac{r}{R} b_3^6 = \frac{r}{R} b_4^5 = \frac{r^2}{(r+R)(3r+R)}, \tag{21b}$$

where $r = 1/g^{\mathrm{NS}}$, $R = 1/g^{\mathrm{NN}}$. The correlators can now be evaluated using the knowledge that in general $f_T^i = \sum_{\alpha\in T} b_\alpha^i f_{T,\alpha}$, and here $f_L^5 = \frac{2}{3} f_L^1 + \frac{1}{3} f_L^2$, $f_L^6 = \frac{1}{3} f_L^1 + \frac{2}{3} f_L^2$. The integration over the energy is convenient to perform by writing (16) in terms of f_L, f_T in terminals and collecting the quadratic terms. At $T = 0$, this results in $\int \mathrm{d}\varepsilon\, f_L^i f_L^j \mapsto -\left|V_i + V_j\right| - \left|V_i - V_j\right|$

and $\int d\varepsilon\, f_T^i f_T^j \mapsto \left|V_i + V_j\right| - \left|V_i - V_j\right|$, so that the N–N correlator reads

$$\tilde{S}_{12,(a)} = -z^2 \frac{d_1(|V_1| + |V_2|) + d_2\,|V_1 + V_2| + d_3\,|V_1 - V_2|}{18R(z+1)^4(3z+1)^4}. \tag{22}$$

Here, $z = g^{\mathrm{NN}}/g^{\mathrm{NS}}$ and the full expressions for the coefficients d are

$$d_1 \equiv z(2z+1)(4z+1)(21z^2+20z+5) + F\left(48z^4+169z^3+146z^2+49z+6\right) \\ -18fz^2(2z+1)(5z^2+4z+1), \tag{23a}$$

$$d_2 \equiv 2z(2z+1)(39z^3+44z^2+16z+2) + F\left(33z^4+128z^3+106z^2+32z+3\right) - 36fz^3(2z+1)^2 \tag{23b}$$

$$d_3 \equiv 2z(2z+1)(4z+1)(9z^3+12z^2+8z+2) \\ +F\left(54z^6+288z^5+537z^4+416z^3+160z^2+32z+3\right) + 36fz^3(2z+1)^2, \tag{23c}$$

with $f = F^{\mathrm{NS}}$ and $F = F^{\mathrm{NN}}$. The cross-correlation can be positive due to the presence of the superconductors: while the coefficient d_3 is always positive, d_1 and d_2 can be negative for large z and f, i.e., when the coupling to the superconductors is weak and the NS contact produces enough noise [15]. The largest effect is achieved for a voltage configuration where all normal terminals are biased at the same potential.

The setup (b) in figure 2 can be reduced to a cross [16] or Y-shaped [15, 17] one-node system, if we are not interested in cross-correlations between the superconductors. Setup (a), however, cannot. This is simply due to the fact that in (a) the distribution functions at 5 and 6 are linearly independent combinations of $f_{T/L,1}$ and $f_{T/L,2}$, whereas in (b) the two enter in the same ratio at both nodes. For example, if the setup is required to be symmetric with respect to an interchange of the normal terminals, in (b) the terms $|V_1|$, $|V_2|$, and $|V_1 + V_2|$ have the same prefactor. This does not occur in (a), resulting in a different voltage dependence.

4. Conclusions

In conclusion, we discuss a simple model for the transmission of noise in multi-node incoherent normal–superconducting structures, applying the microscopic counting field theory. We show that the formalism coincides exactly with the semi-classical principle of minimal correlations also in arbitrary multi-node circuits.

Acknowledgments

We thank W Belzig for discussions, and P Samuelsson and M Büttiker for pointing out their previous work in [2]. TTH acknowledges funding by the Academy of Finland.

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