Paper IV

Robust prediction of full creep curves from minimal data and time to rupture model

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Robust prediction of full creep curves from minimal data and time to rupture model

S. Holmström* and P. Auerkari

A description of creep strain evolution is frequently needed in design or life assessment of components subjected to service at high temperatures. Unfortunately, long term creep strain data are not as easily available as rupture data or rupture models. One of the most demanding tasks in this context is to predict the creep behaviour reasonably accurately beyond the range of available data. Much effort has been invested into developing reliable methods to extrapolate creep rupture data, for example in the recommended procedures of the European Creep Collaborative Committee (ECCC) and PD6605 of BSI. However, for strain no such tools are currently available. Here a new and robust creep strain model is suggested to provide the whole creep curves based on the corresponding creep rupture model. This logistic creep strain prediction (LCSP) model defines the creep curve only with three additional parameters to those of the corresponding rupture model. In its basic form the LCSP model optimises a non-linear asymmetric logistic transition function fitted in logarithmic strain against a time temperature parameter (TTP), giving time to specified strain. Unique features of the model include its simple inverted expression for strain and strain rate. The fitting effectiveness of the new method is shown to match all the contesting creep strain models of the ECCC intercomparison activity on a single heat data set of the steel P22 (10CrMo9-10). The model is also shown to produce accurate predictions of the stress to 1% strain up to 100 000 h, when compared to the values given in DIN 17243.

Keywords: Creep, Strain, Model, Ferritic steel, ECCC

Introduction

Creep strain sets design limits for many components in power engineering, for example in the case of blading, bolts and casings of steam and gas turbines. In addition, creep strain is considered to be an important quantity to be monitored in the in-service inspections and life assessment of turbines, boilers and steam pipes, with typical recommended limits up to 1 or 2% of strain. However, predicting creep strain is frequently less than straightforward. This is partly because there is much less creep strain data available even on common well established engineering steels than creep rupture data, and partly because most publicly available good models to describe creep behaviour only apply to creep rupture.

Intercomparison activities[1,2] of different strain modelling alternatives have shown that effective prediction of time to specified strain can be achieved from time–temperature parameter formulations.[3,4] The most challenging task, however, is to predict realistic strain response outside the range of available strain data. Creep strain data are often hard to come by, and in the most interesting long term range scarce or lacking. In contrast, rupture data and models for time to rupture are much easier to access. The rupture models are well established and considered to be relatively reliable at least for common engineering alloys with sufficient long term test data. Therefore, it could be an advantage if creep strain models can utilise such data and models as a baseline.

Another problem area in any materials modelling is related to model robustness, i.e. how resilient it is against error or instability due to scatter or when extrapolating outside the range of data. A model is unlikely to be robust if it includes a large number of free fitting constants, although some are inevitably needed to achieve good fit. Robustness is particularly needed for small data sets, but robustness is also inherently related to the mathematical formulation of the model. In this respect, differences also exist between rupture models, but by using strain modelling with rupture models as a baseline, the main contribution to robustness is likely to be limited to the additional fitting constants of the strain model.

Finally, realistic creep strain models should be physically plausible. For example, it is mathematically possible but not really physically plausible to model for creep strength (for strain or rupture) that continuously increases in time, shows discontinuity in its evolution rates, or turns back in time when extrapolated outside

VTT Technical Research Centre of Finland, Kemistintie 3, 02044 VTT, Finland

*Corresponding author, email stefan.holmstrom@vtt.fi

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the range of data. Such behaviour can be also prevented or limited if strain evolution can be suitably related to rupture models that are known not to suffer from these drawbacks at least within the range of interest in terms of time, stress and temperature.

To avoid complex creep strain models with too many fitting constants, and to achieve robust extrapolation with minimal data, a new creep strain model, the logistic creep strain prediction (LCSP) model, has been developed and will be described below. The method relies on the time to rupture (true data or master curve predicted) to provide the end point of each creep curve. The shape of the creep curve is modelled with three additional parameters as output from data fitting for each curve or curve family. The LCSP model is a logistic non-linear asymmetric transition function fitted in logarithmic strain versus time–temperature parameter (TTP) at specified stress and temperature. The model offers an improved sensitivity and robustness to the strain prediction closely related to the work performed on rupture assessments[5] such as the post-assessment tests of the European Creep Collaborative Committee (ECCC),[6] and of PD6605.[7] With confidence to the shape parameters, the LCSP can be used for rupture predictions similarly to the Monkman–Grant[8] expression, except that the LCSP approach is not limited to predictions using the minimum strain rate. Consequently, the LCSP model could be used in assessing tests still running or discontinued. Even for rupture models, this is so far only possible with the PD6605 rupture assessment tool. An estimate of the long term creep ductility[5,9,10] can also be extracted from the model.

The model is unique in the sense that the function of time to specific strain, strain at specified time and strain rate at specified time is all algebraic and can therefore readily be for instance incorporated into finite element (FE) codes. It also makes the extraction of the constants of the Norton creep expression easy for using them in common FE codes.

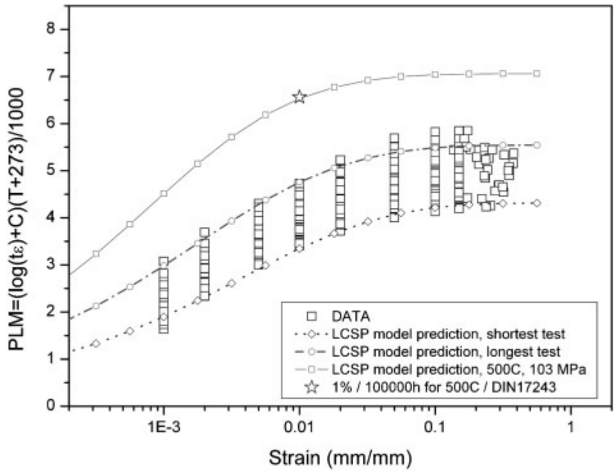
The fitting effectiveness and extrapolation robustness of the method is shown by comparing LCSP results to both more complicated[11] and simpler creep strain models[1,5] with the same single heat data set in the ECCC intercomparison on steel P22 (10CrMo9-10, with a maximum rupture time of only 3000 h) and incorporating time to 1% strain at stress levels of the DIN 17243 standard[12] up to 100 000 h.

LCSP model and its validation for steel P22 (10CrMo9-10)

The modelling aims to produce representative creep strain curves at specified stress and temperature with the help of available creep strain and rupture data. In this paper, the time to rupture in fitting is represented by the actual rupture time, but in extrapolation exercises it is replaced by a master curve prediction for rupture.

If the availability of creep strain data is very poor, the curve shape modelling can be extended with a standard strength value for rupture together with corresponding strengths for a specified strain such as 1%.

The LCSP method introduces a transition in a temperature compensated time parameter (such as the Larson–Miller or Manson–Haferd parameter) as a



1 P22 data at 0·1, 0·2, 0·5, 1, 2, 5, 10, 15% and ε_f, presented in TTP form as defined by LCSP (30 curves): predicted strain curve at 500°C and 103 MPa is also presented together with 1%/100 000 h standard value at same temperature and stress

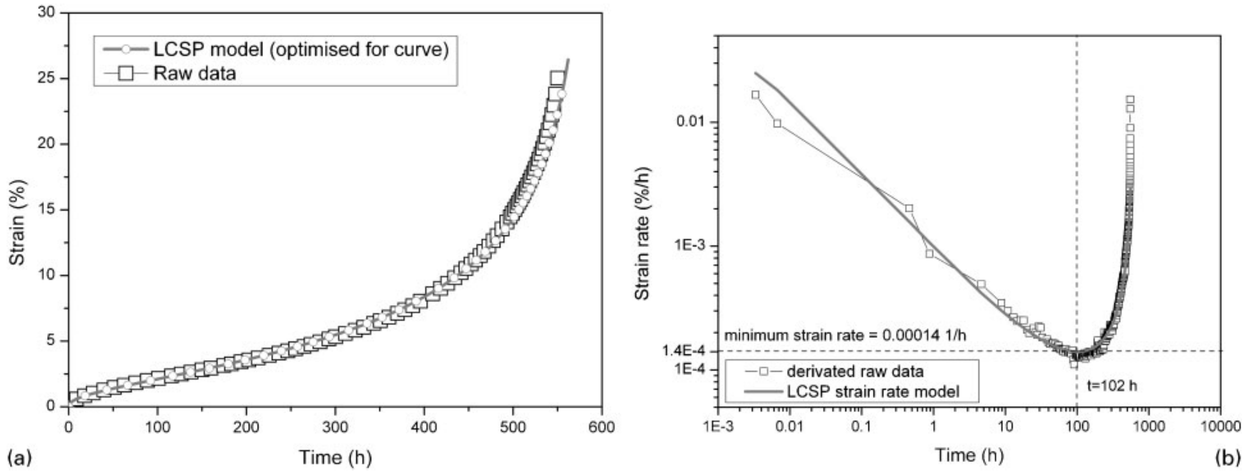
function of creep strain at specified temperature and stress. In this work, the predicted time to specified strain is based on the Larson–Miller parameter, i.e. the TTP has a transition over strain between a parametric value of zero for very small strain (and very short time) to a maximum value restricted by the rupture life and the parameter β as described in equation (1). The function is a non-linear asymmetric transition function with a steepness regulated by the variables p and x_0. It is to be noted that the LCSP approach can be used with any growth function that is shown to accommodate the data, but the possibility of having an algebraic solution for predicting the strain as a function of time (equation (2)) might then be compromised. The LCSP model states that knowing the time to rupture (maximum TTP level) and the material specific parameters p and x_0 is sufficient to describe the whole creep curve at specified temperature and stress. Inversely it is possible to predict time to rupture by knowing only one point on the strain curve (and again the material specific parameters p and x_0). This feature however must be investigated further for limits of applicability.

The LCSP method defines time to strain ε at engineering stress σ and temperature T as

$$\log(t_\varepsilon)=\frac{\log(\alpha t_r)+\beta}{1+\left(\frac{\log\varepsilon}{x_0}\right)^p}-\beta \quad (1)$$

where t_ε is the time to given strain, t_r is the time to rupture and x_0, p, β and α are fitting factors. In its simplest form, the latter four are all constants. The TTP presentation of equation (1) is shown in Fig. 1 with the assessed P22 strain data from.[1]

In the case of P22, the factors x_0 and p were temperature and stress dependent, and β and α were found to be constants. The model was fitted as time to specific strain at specified temperature and stress (equation (1)). Fitting the function in time makes it possible to use the post-assessment tests for checking fitting accuracies and extrapolation robustness as recommended for creep rupture assessments by ECCC. The inverted equation (1) describing strain as a function



2 Example of *a* LCSP predicted versus measured creep and *b* predicted creep strain rate for P22 at 600°C: model parameters are optimised for curve in question

of time (still algebraic), can be written as

$$\log(\varepsilon_t) = (\mathrm{LTF}-1)^{1/p} x_0 \quad (2)$$

with

$$\mathrm{LTF} = \frac{\log(\alpha t_r) + \beta}{\log(t_\varepsilon) + \beta} \quad (3)$$

The equations (1)–(3) imply that at strain $\varepsilon=1$ the time to rupture is attained ($\alpha \times t_r$). The constant α can correct the strain at time to rupture to correspond to the actual creep ductility but for the case of P22 and other creep ductile materials this is unlikely to be necessary. For the data set available $\alpha=1$ also gave the best overall fit for the master equation incorporating all creep curves.

An example of the LCSP predicted versus measured creep strain for P22 steel is presented in Fig. 2*a*.

By differentiating equation (3) with respect to time, the resulting strain rate as a function of stress and temperature can be written as

$$\dot{\varepsilon} = -\varepsilon \cdot k1 \cdot k2 \cdot x_0 \quad (4)$$

where ε is given by equation (2), and

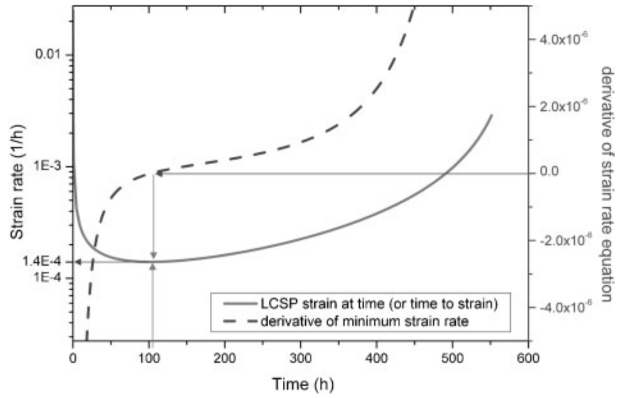
$$k1 = \frac{(\mathrm{LTF}-1)^{1/p}}{p} \quad (5)$$

and

$$k2 = \frac{\log(\alpha \cdot t_r) + \beta}{[\log(t_\varepsilon) + \beta]^2 \cdot t_\varepsilon \cdot (\mathrm{LTF}-1)} \quad (6)$$

Again, this provides an algebraic expression for creep strain rates, and an example of this with comparison to P22 data is shown in Fig. 2*b*.

A second differentiation of the LCSP model equation in time produces a function (still algebraic) of which the root gives the time to minimum creep rate, and thereby the minimum strain rate by using equation (4) (*see* Fig. 3).



3 Strain rate (continuous line) and its derivative (broken line) for creep curve of Fig. 2, showing time to minimum strain rate and minimum creep rate

The LCSP model factors for P22 steel are presented in Table 1, and the shape factors p and x_0 as a function of temperature and stress in Fig. 4.

The accuracy of a creep rupture model can be described by the root mean square (RMS) error calculated from the predicted time to rupture.[5] Similarly, the model accuracy of the predicted time to specified strain can be described as

$$\mathrm{RMS} = \left\{ \frac{\sum \left[\log(t_{\varepsilon-\mathrm{actual}}) - \log(t_{\varepsilon-\mathrm{predicted}})\right]^2}{n} \right\}^{1/2} \quad (7)$$

where n is the number of data points. The better the fitting efficiency, the smaller the scatter factor Z, defined as

$$Z = 10^{2\cdot5\mathrm{RMS}} \quad (8)$$

In a creep rupture assessment, a scatter factor close to 2 is considered to be good. This value can be approached in creep strain assessments for homogenous well behaving creep strain data, but for multicast mixed continuous and interrupted data it can be expected that $Z>6$.

Table 1 Logistic creep strain prediction fitting factors for steel P22 (10CrMo9-10)

$x_0(\sigma,T)$	$p(\sigma,T)$	C	α
$-0\cdot391+0\cdot696\log(\sigma)-3392\cdot5/(T+273)$	$4\cdot363-2\cdot271\log(\sigma)+3874\cdot9/(T+273)$	3·49	1

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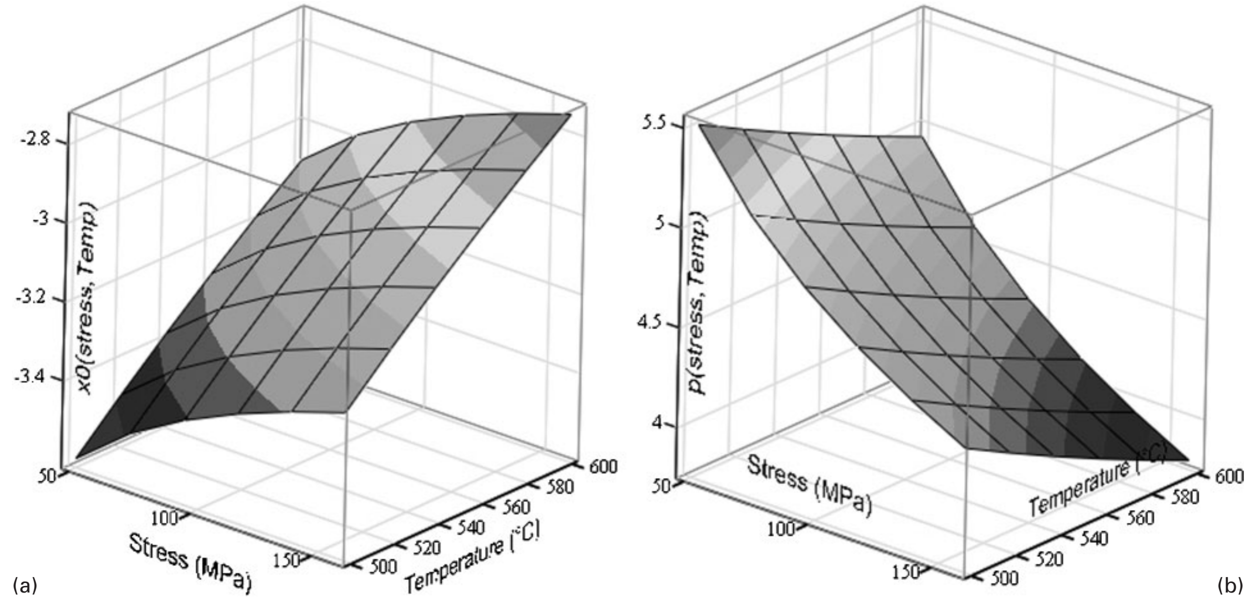


4 Temperature and stress dependence of shape factors *a* x_0 and *b* *p* for P22

To validate the LCSP model the same P22 creep strain data have been used as for the ECCC intercomparison exercise. The single heat data set contained 30 creep curves, with a temperature range of 510–600°C and a stress range of 180–280 MPa. The maximum testing time was ~3000 h. To support longer term prediction, the data were simultaneously fitted with the 10 000 and 100 000 h/1% creep strength values of the DIN 17243 standard.

A comparison of Z values of the LCSP model and those predicted by other models in an ECCC intercomparison of the same data set (Table 2) shows that the LCSP model performs at least as well as the best one from the previous intercomparison. The LCSP value Z=2·38 calculated on all data can be considered to be excellent. It is to be noted that the long term predictions of time to 1% strain for stress levels of the standard are also well defined by the LCSP, namely the predicted times give a Z value below 2, as shown in Fig. 5.

Discussion

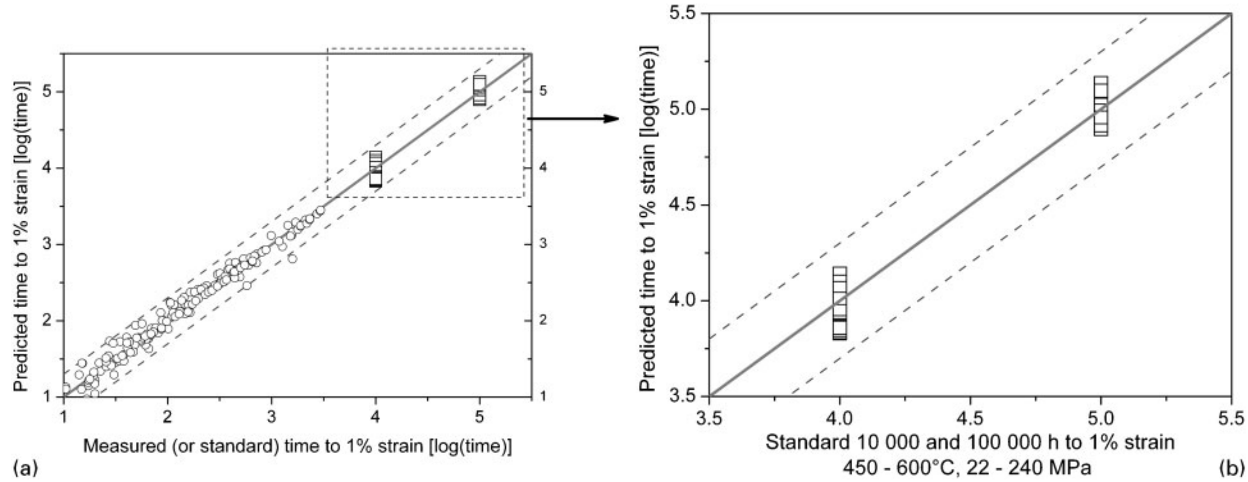
A particular strength of the LCSP model is its linkage to the rupture time and the robustness of the rupture model. The small number of fitting parameters when compared to the other models with similar Z values in Table 2 is also favourable. The LCSP model is robust and simple mainly because of relatively low number of degrees of freedom (constants to be fitted), and because no complex numerical operations such as numerical integration are needed for fitting. In this case, the LCSP model performs as well as the best model in the intercomparison (MHG-10, modified Garofalo model, Modified Graham–Walles) but with only eight fitting constants. The model is also predicting well the time to 1% strain at specified stress levels up to 100 000 h with $Z<2$ (Fig. 5). According to the Norton law (equation (9)) parameters A and n, and the minimum creep rate can be easily extracted from the LCSP model, as a function of stress and temperature (Figs. 6 and 7). Also the parameters of Nadai's (equation (10)) strain hardening model, the continuum damage mechanic model (equation (11),[14] the Omega[15] and modified Omega model[16] can readily be calculated utilising the LCSP model.

The uniaxial steady state or minimum rate of creep strain ε under stress σ and at temperature T can be

Table 2 Scatter factor *Z* for strain models[1–3] on steel P22 (single heat data)*

Model equation	Creep regimes (param)	*Z* for time to specified strain (all temperatures)				
		0·2%	0·5%	1%	2%	5%
Omega model	S/T (8)	468	39	10		
BJF model	P/S	15	6	4		
Theta model	P/S/T	17	3	2		
Modified theta model	P/S/T	10		4		
Creep strength ratio model	P/S/T	4	8	7		
NPL model	P/S/T	12	7	3		
Baker–Cane model	P/S/T	5		2		
Bartsch model	P/S	3	2	4		
Modified Garofalo model[11]	P/S/T (27)	2	2	2		
Modified Graham–Walles	P/S/T	3	3	2	2	2
Modified Nadai	P/S/T (11)	3	2	2	2	2
MHG-4	P/S/T (4)	3	3	3	3	3
MHG-10	P/S/T (10)	3	2	2	2	2
LCSP	P/S/T (8†)	3	2	2	2	2

*In creep regime column, P/S/T=primary/secondary/tertiary (number of free parameters in the model).
†The parameters needed for $t_r(T,\sigma)$ are not accounted for.



5 Predicting time to 1% creep strain for ECCC data and 10 000 and 100 000 h DIN 17243 standard values using data set specific material rupture models and the shape parameters presented in Table 1: for DIN 17243 time to 1% strain predictions LCSP model utilised rupture model based on EN 10216[13] standard values for rupture; acquired scatter factor for standard values is Z=1·8

described by the Norton law

$$\dot{\varepsilon}=B\exp\left(-\frac{Q}{kT}\right)\sigma^{\mathrm{n}}=A\cdot\sigma^{\mathrm{n}} \tag{9}$$

where $\dot{\varepsilon}$ is the minimum strain rate, B is a rate coefficient, Q is the apparent activation energy for creep, k is the Boltzmann constant and n is the Norton creep exponent. Here $B\exp(-Q/kT)$ is treated as a constant for given temperature

To predict primary to secondary creep deformation the Nadai form can be used. In this work, the formulation is fitted as used in the ABAQUS software (Abaqus Inc.)

$$\dot{\varepsilon}=\{A\sigma^{\mathrm{n}}[(m+1)\varepsilon]^{\mathrm{m}}\}^{1/\mathrm{m}} \tag{10}$$

where $\dot{\varepsilon}$ is the strain rate at specified creep strain and A, n and m are fitting factors. The Norton model (equation (9)) is obtained from equation (10) by setting m=0.

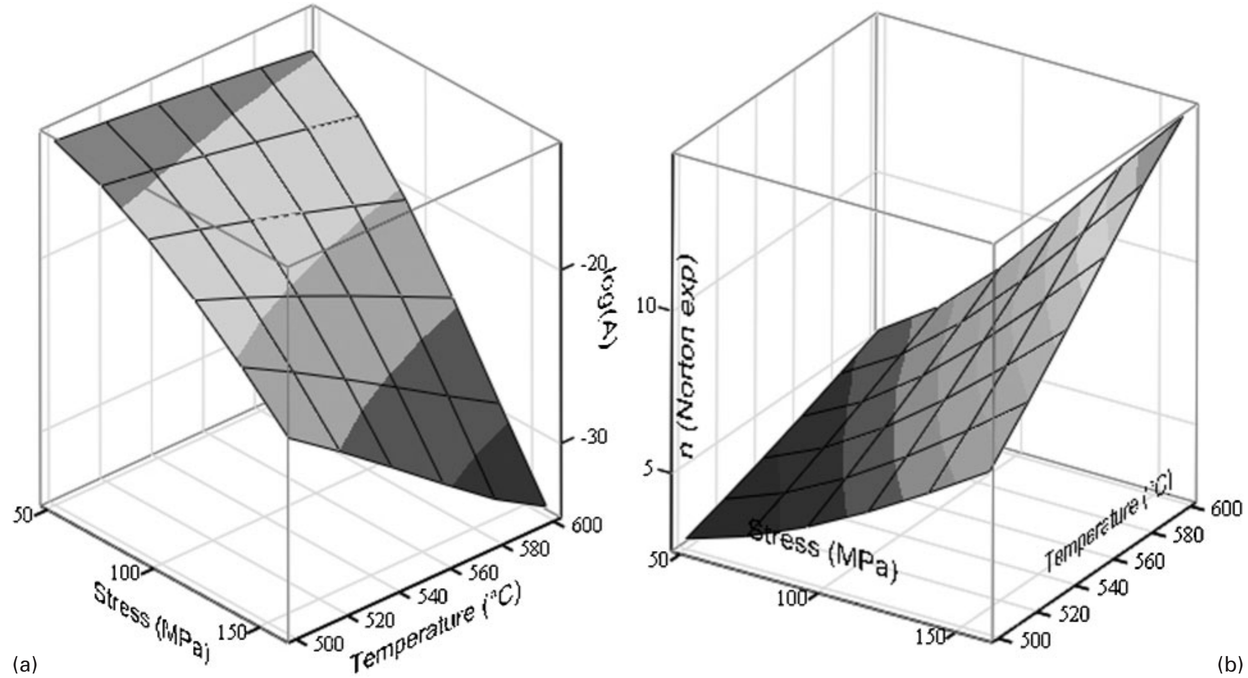
The LCSP model predictions can also be compared to a classical simple strain model based on continuum damage mechanics (CDM). The time to specified strain of the CDM model is defined as

$$t_{\varepsilon}(\sigma,T)=\left[1-\left(1-\frac{\varepsilon}{\alpha_0}\right)^{1/\alpha_1}\right]t_{\mathrm{r}} \tag{11}$$

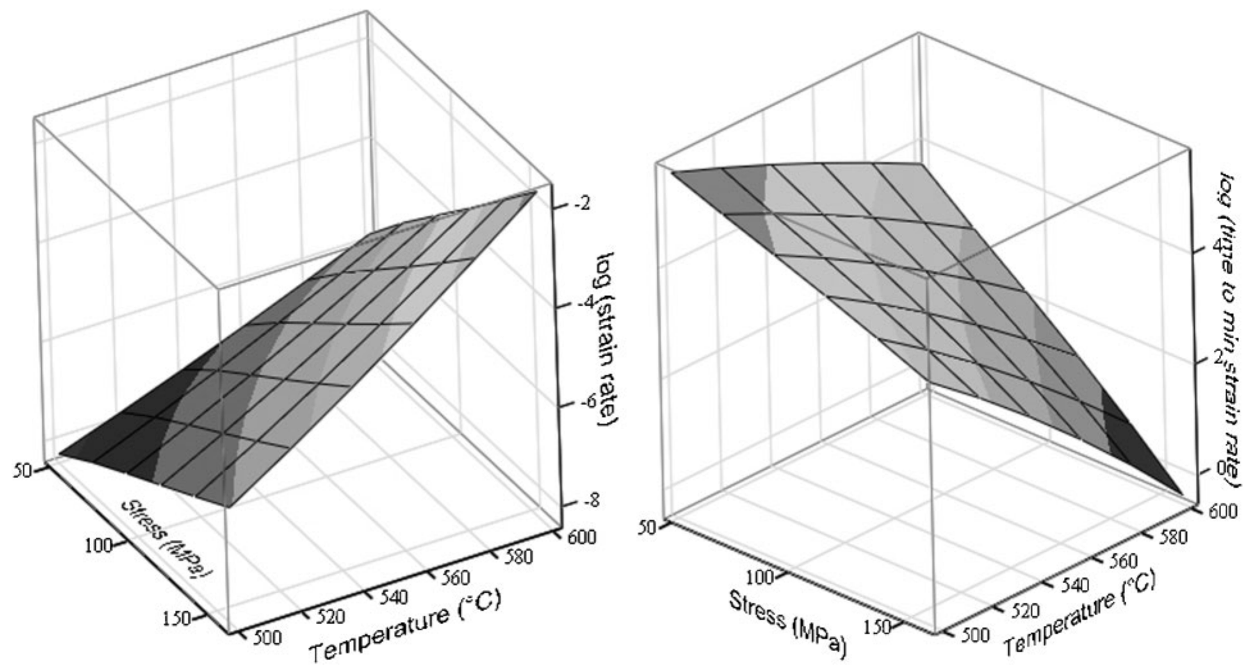
$$\varepsilon_{\mathrm{t}}(\sigma,T)=\left[1-\left(1-\frac{t}{t_{\mathrm{r}}}\right)^{\alpha_1}\right]\alpha_0 \tag{12}$$

where t_{ε} is the time to specific strain, ε_{t} is the strain at specific time, t_{r} is the time to rupture, α_0 and α_1 are fitting factors, T is the temperature (K), σ is the engineering stress (MPa) and ε is the creep strain.

A comparison of the predicted strain at specified times for the LCSP, Norton minim strain rate, Nadai and the CDM model is presented in Fig. 8. In the primary creep regime the CDM and the Norton minim strain rate



6 Norton law parameters *a A* and *b n* for steel P22, as predicted by LCSP model



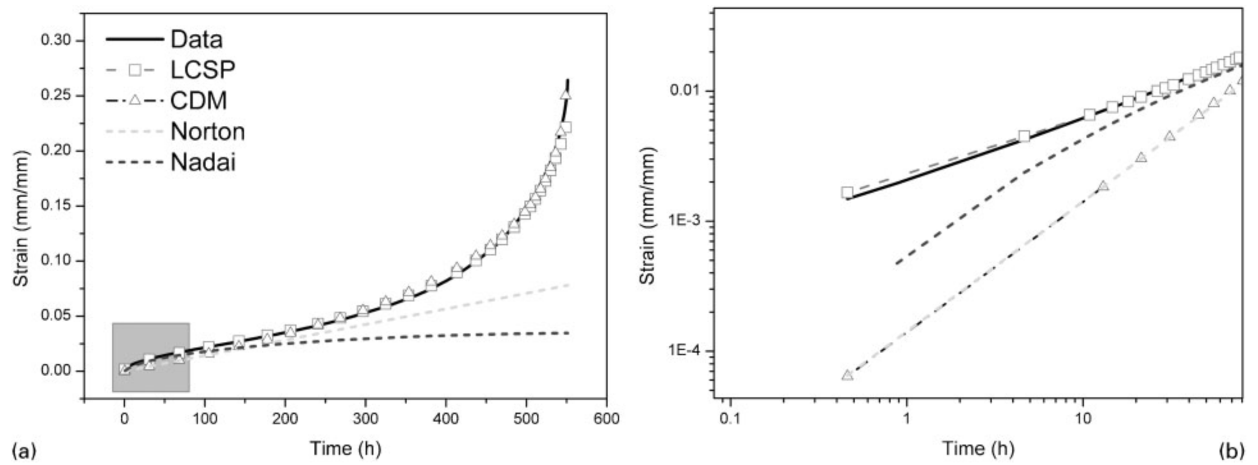
7 Minimum creep rate and time to minimum creep rate for P22 steel, as predicted by LCSP model

models behave similarly (*see* Fig. 8*b*), but the predicted strain is underestimated. The Nadai model improves the primary creep strain prediction, but the model is not flexible enough to accommodate data accurately, at least when fitting to raw data up to time to minimum strain rate. Some configuration of synthetic data or data point weighting in the fitting routine might improve this shortcoming. In the secondary to tertiary creep regimes, the CMD model produces improved strain predictions closing in on the raw data and the LCSP results. The Nadai and the Norton model do naturally not produce good predictions beyond this point since they do not accommodate tertiary creep. It is to be noted that the LCSP creep strain model was able to represents all three regimes accurately. The presented LCSP case is calculated on the true rupture time of the test and using optimal shape parameters, the CDM, Nadai and the basic Norton are also all optimal fits.

One of the main features of the LCSP is the temperature and stress dependence on the curve shape. As the rupture life becomes longer (low temperature or low stress) the shape of the creep curve changes. The differences in the shape of the predicted short and longer term creep curves for steel P22 are demonstrated in Fig. 9.

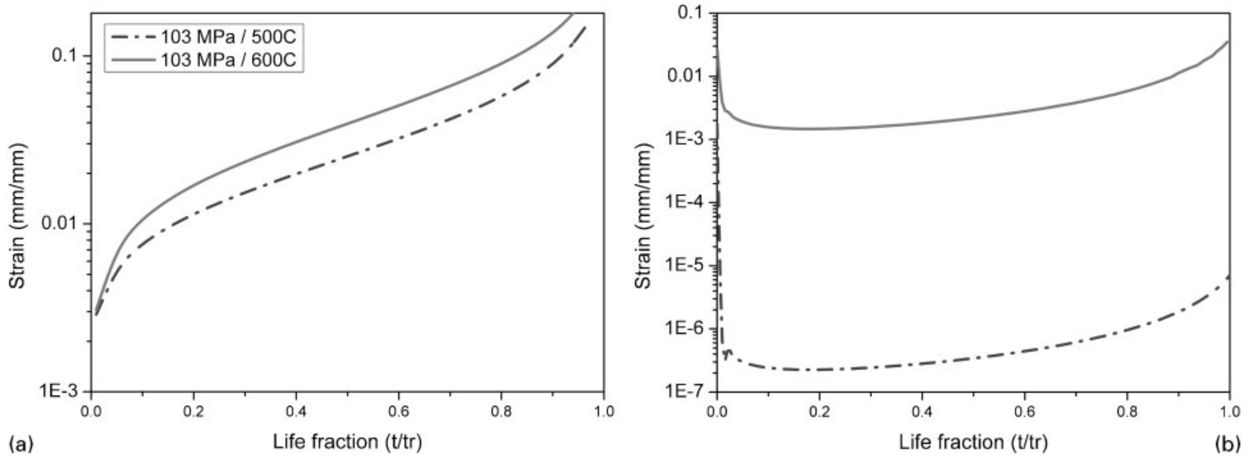
Many common creep related design and assessment problems can be solved by using FE software packages that typically include some creep models to describe strain evolution in time. However, the corresponding generic materials data in e.g. EN standards are usually expressed in terms of time to rupture rather than time to given strain. Even though generic strain related data exist, it tends to be limited to few fixed points such as time to 1% strain, without information of important preceding strain evolution. For example for many design problems the strains <1% are the most important, as this can be the limit to the end of the design life.

Here a new simple and robust creep strain model (LCSP) has been described to include all essential features of typical uniaxial constant load or constant



a linear presentation of creep curve and model predictions; *b* blow up of squared section presented in logarithmic scales for emphasis on primary creep representation

8 Comparison between basic Norton (equation (9), Nadai (equation (10), CDM (equation (12) and LCSP (equation (2) predictions of strain at specified time for creep curve presented in Fig. 2



9 Comparison of *a* normalised short term (rupture at 600°C for 40 h, continuous line) and longer term (rupture at 500°C for 100 000 h, dotted line) creep curves for steel P22 and *b* corresponding strain rates

stress creep curves, with capabilities to predict realistic strain for design and life assessment. When needed, the same model can also predict through simple algebraic formulation time to given, strain rates and minimum strain rates. The LCSP model can be implemented to FE codes directly, or if preferred, by providing the required description of simplified models (such as Norton and Nadai) that are commonly included in the commercial FE packages.

The LCSP model is robust, because it requires minimal numerical transformations and relatively few free fitting constants. The model is also accurate and shows generally very competitive fitting to the creep testing data, as was shown when comparing to the results of the ECCC intercomparison exercise.

A particular advantage of the LCSP approach is its ability to utilise the existing creep rupture models (or data) that are much more widely available than information on creep strain. In principle, a correlation between time to given creep strain and time to rupture can be expected from the often observed correlation among the primary, secondary and tertiary stages of creep.

Conclusions

A robust strain model based on the time to rupture curves has been developed to cover the whole creep curve. This LCSP model makes use of the creep rupture model of the same material, to fully utilise the robustness and usually much more extensive data base of creep rupture information. The strain model itself is simple and defines a creep curve only with three parameters (stress and temperature dependent) in addition to those of the corresponding rupture model. In its basic form the LCSP model optimises a non-linear asymmetric logistic transition function fitted in logarithmic strain against a TTP, giving the time to a specified strain. Unique features of the LCSP model include simple inverse expressions for strain and strain rates. The new model has been shown to be very competitive when compared with any other creep strain models of the ECCC intercomparison activity on a single heat data set of the steel P22 (10CrMo9-10). By comparing with the stress values to 1% creep strain as given in DIN 17243 for the same material type, the model was shown to provide accurate predictions at 10 000 and 100 000 h.

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