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MANAGING UNCERTAINTIES IN BROADBAND INVESTMENTS

— CASE STUDIES OF REAL OPTIONS FOR RURAL AREA ACCESS NETWORKS

Vesa Riihimäki

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Abstract

The aim of the study is to analyze uncertainties in broadband access network investments and determine how real option valuation can be used to manage uncertainties. A rough theoretical model for network investment valuation is summarized. The economics of different broadband access technologies, including WiMAX and ADSL and Flash-OFDM, in rural area is analyzed with the developed methods.

The adaptation of the service is modeled by different S-shaped curves and those models are used to make service penetration forecasts in Finland. The analyses of investments are based on the statistical characteristics of the simulated net present values (NPV) and investment costs of the projects. Different option valuation methods, e.g., Black–Scholes formula and binomial tree and modular Monte-Carlo simulation, are implemented in the study. These valuation methods are compared with scenario based discounted cash flows.

The shape of the NPV distribution for an investment varies depending on the technology and service area of the network. The study shows that the distribution of the network investment costs is log-normal because of the network coverage requirements and the exponential growth of the average network traffic per user. This makes the distribution of the NPV negatively skewed. The shape of the distributions and the valuation models that can theoretically be applied vary from case to case. Thus, there is no single statistical model that could be safely used to optimize the size of the network. The statistical analysis and the comparison of the methods show that Monte-Carlo real option simulation gives the most reliable results.

The simulated service area, a small municipality called Ähtäri, seems to be a challenging business case for operators. With wide coverage and an operation period of 8 years, the expectation of the NPV is negative for WiMAX and positive for ADSL. However, the WiMAX network has an option to extend the service area to summer cottages, too. Another analyzed service case, ITS for railways seems to be profitable and comprehends several technology and growth options for train operator.

Keywords	adaptation of new technology, broadband access, forecasting, Internet, network investment, real option, techno-economics		
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Tiivistelmä

Tämän tutkimuksen tavoitteena on analysoida verkkoinvestoinnin epävarmuuksia ja soveltaa reaalioptioita laajakaistainvestointeihin. Työssä esitetään karkea teoreettinen malli verkkoinvestoinnin arvon määrittämiseksi. Tutkimuksen simuloinneissa paneudutaan tilanteisiin, joissa WiMAX-, ADSL- ja Flash-OFDM-tekniikoihin pohjautuvat laajakaistaiset liityntäverkot rakennetaan harvempaan asutuille seuduille.

Palveluiden käyttöönoton mallintamisessa hyödynnetään erilaisia S-muotoisia käyriä ja niiden avulla tehdään ennusteita Internet- ja laajakaistayhteyksien penetraatioista Suomessa. Investointien analysointi pohjautuu simuloitujen nettonykyarvojen (NNA) ja kustannusten tilastollisiin tunnuslukuihin. Työssä esitellään ja sovelletaan niin Black–Scholesin kaavaa kuin binomipuumenetelmää ja Monte–Carlo -simulointia optioiden hintamäärityksessä. Eri menetelmien soveltuvuutta ja saatuja tuloksia vertaillaan yhdessä skenaariopohjaiseen kassavirtatarkasteluun. NNA:n jakauma vaihtelee riippuen käytetystä teknologiasta ja verkon kohdealueesta.

Tutkimus osoittaa, että peittoaluevaatimuksen ja eksponentiaalisen tietoliikennemäärän kasvun vaikutuksesta verkon investointikustannusten logaritmi on normaalijakautunut. Tämä, samoin kuin kohdealueen talouksien lukumäärän kehitys ja verotuksen huomiointi, aiheuttaa NNA:n jakaumaan negatiivista vinoutumaa. Toisin sanoen, taloudellisten lukujen jakaumien muodot vaihtelevat oleellisesti tapauksesta toiseen. Koska eri optiomenetelmät olettavat jakauman noudattavan tiettyjä muotoja, ei niiden soveltamiseen kaikissa tilanteissa ole teoreettista pohjaa. Erityisesti, jakaumien vaihtelusta johtuen ei verkon kokoa suunniteltaessa voi käyttää pelkästään yhtä tilastollista jakaumaa. Menetelmien vertailu ja tilastollinen analysointi yhdessä osoittavat Monte-Carlo simuloinnin antavan kaikkein luotettavimpia tuloksia.

Työn kohteena ollut palveltava alue, pieni Ähtärin kaupunki, näyttäisi olevan taloudellisesti haastava liiketoimintakohde. Työssä oletetulla lähes koko kunnan kattavalla verkolla ja kahdeksan vuoden toimintajaksolla NNA:n odotusarvo on WiMAX-teknologialla negatiivinen ja ADSL-teknologialla positiivinen. WiMAX-verkolla tosin on laajennusoptio myös alueen kesämökkien palvelemiseksi. Toinen tarkasteltu kohde, junien laajakaistapalvelut, näyttää olevan kannattava liiketoimintamahdollisuus ja tarjoaa useita optioita teknologiaan ja junaoperaattorin sisäisiin tarpeisiin liittyen.

Avainsanat	ennustaminen, Internet, laajakaistayhteys, reaalioptio, teknologian adaptaatio, teknistaloudellinen analyysi, verkkoinvestointi		
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PREFACE

This work has been carried out at the Department of Communications and Networking in the Helsinki University of Technology (TKK, currently Aalto University School of Science and Technology), Espoo, from 2004 to 2008. I have been happy to work under the supervision of professor Patric Östergård. He has supported and helped me doing this thesis, thanks for that. I give many thanks for pre-examiners professor Christer Carlsson from Åbo Akademi University and professor Fotios Harmantzis from Stevens Institute of Technology for their comments during the examination of the dissertation work. I thank also professor Heikki Hämmäinen who gave me helpful comments while examining my Licentiate thesis. At the final phase of the dissertation journey I will meet professor Harri Haanpää as custos and Kjell Stordahl from Telenor as opponent. My studies and work has also been buoyed up — particularly during coffee breaks — by several fellow students and colleagues in Helsinki University of Technology where I have been working from 2001 to 2008.

After graduation in 2004, I received a position in the Graduate School on Network for Information Society (GSNIS). I would sincerely thank the Department of Electrical and Communications Engineering and GSNIS for the financing of my research. My work has also been partially financed by the Academy of Finland under grants number 100500 and 110196.

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I cannot leave unmentioned other friends and neighbors and relatives that are important for me. I want to apologize to my little boys Julius and Ilari and Tuukka and my Maria for spending my time with this thesis in lieu of playing with them. However, their love has encouraged me during the work. Thanks for it!

5th of April 2010, Kirkkonummi

Vesa Riihimäki

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AP	access point (page 10)
APON	ATM based passive optical network (page 7)
ARPU	average revenue per user (page 45)
AT	access terminal (page 10)
ATM	asynchronous transfer mode (page 7)
BPON	broadband passive optical network, see APON
BS	base station (page 10)
CAPEX	capital expenditures (page 13)
CAPM	capital asset pricing model (page 51)
CM	connection manager (page 95)
CPE	customer premise equipment (page 10)
DCF	discounted cash flow (page 51)
DL	downlink, from operator to subscriber (page 5)
DNS	domain name system (page 7)
DSL	digital subscriber line (page 8)
DSLAM	DSL access multiplexer (page 9)
EDGE	enhanced data rates for GSM evolution (page 10)
EGPRS	enhanced general packet radio service (page 10)
EPON	Ethernet based passive optical network (page 7)
ETSI	European Telecommunications Standards Institute (page 10)
FROV	fuzzy real option valuation (page 59)
FTTB	fiber to the building (page 8)
FTTC	fiber to the curb (page 8)

FTTH	fiber to the home (page 8)
FTTN	fiber to the node/neighborhood (page 8)
FWA	fixed wireless access (page 10)
Gbps	gigabits (1 000 000 000 bits) per second
GGSN	gateway GPRS support node (page 10)
GPON	gigabit passive optical network (page 7)
GPRS	general packet radio service (page 10)
GVC	ground-to-vehicle communications (page 95)
G.SHDSL	symmetric high-speed digital subscriber line (page 8)
HALO	high altitude operational aircraft (page 11)
HAP	high altitude platform (page 11)
HFC	hybrid fiber coax (page 8)
HiperLAN	High performance radio LAN (page 10)
HiperMAN	High performance radio MAN (page 10)
HSDPA	high speed downlink packet access (page 10)
HSPA	high speed packet access (page 10)
HTTP	hypertext transfer protocol (page 7)
ICT	information and communications technology (page 95)
IEEE	Institute of Electrical and Electronics Engineers (page 10)
IM	instant messaging (page 21)
IP	Internet protocol (page 7)
IPTV	Internet protocol television (page 22)
IRR	internal rate of return (page 52)
ISDN	integrated services digital network (page 63)
ITS	intelligent transportation system (page 95)
ITU	International Telecommunication Union (page 13)
HDTV	high-definition television (page 22)
kbps	kilobits (1 000 bits) per second
LAN	local area network (page 6)

MAN	metropolitan area network (page 6)
Mbps	megabits (1 000 000 bits) per second
ME	mobile equipment (page 10)
MSE	mean square error (page 30)
MTTF	mean time to failure (page 16)
NPV	net present value (page 51)
OFDM	orthogonal frequency division multiplexing (page 10)
OLT	optical line terminal (page 7)
ONU	optical network unit (page 7)
OPEX	operational expenditures (page 13)
OVC	on-board vehicle communications (page 95)
P2P	peer-to-peer (page 22)
PLC	power line communications (page 8)
PON	passive optical network (page 7)
POTS	plain old telephone network (page 8)
RITS	railways' intelligent transportation system (page 95)
RNC	radio network controller (page 10)
SDSL	symmetric digital subscriber line (page 8)
SGSN	serving GPRS support node (page 10)
TAM	technology acceptance model (page 26)
TCP	transmission control protocol (page 7)
TS	train server (page 95)
UL	uplink, from subscriber to operator (page 5)
UMTS	universal mobile telecommunication system (page 10)
VAM	value based adaptation model (page 26)
VaR	value at risk (page 1)
VDSL(2)	very-high-rate digital subscriber line (page 8)
VoD	video on demand (page 22)
VoIP	voice over Internet protocol (page 21)

VS	vehicular station (page 95)
WACC	weighted average cost of capital (page 52)
WAN	wide area network (page 6)
WCDMA	wide-band code division multiple access (page 10)
WiMAX	worldwide interoperability for microwave access (page 10)
WLAN	wireless local area network (page 10)

SYMBOLS

See page number in parenthesis for the main reference.

β	adaptation state parameter (page 35)
b	adaptation shape parameter (page 35)
$B(t)$	total revenues in year t (page 48)
c_d	growth parameter for throughput demand (page 15)
c_N	growth parameter for potential subscribers (page 23)
c_S	reduction parameter for ARPU (page 45)
c_T	reduction parameter for technology prices (page 15)
$c_{T,F,M}$	balancing parameter for maintenance costs (page 19)
$C(t)$	total network costs in year t (page 19)
$C_{OA}(t)$	operational and administration costs in year t (page 18)
$C_M(t)$	maintenance costs in year t (page 18)
$d_a(t)$	throughput demand per subscriber at time t (page 14)
$D(t)$	total capacity demand at time t (page 18)
$\mathcal{E}xp(\lambda)$	exponential distribution with mean λ (page 16)
f	the value of a call option (page 57)
I	the size of the planned investment (page 18)
k	adaptation growth parameter (page 23)
$K_{T,A}$	coverage area coefficient (page 18)
$K_{T,D}$	capacity coefficient (page 18)
$K_{T,F,OA}$	coefficient for fixed operational and administration costs (page 18)
$K_{T,F,M}$	fixed maintenance costs coefficient (page 18)
$K_{T,I}$	coefficient for network installation costs (page 18)
$K_{T,V,I}$	coefficient for new subscriber costs (page 18)
$K_{T,V,OA}$	coefficient for variable operational and administration costs (page 18)

$K_{T,V,M}$	variable maintenance costs coefficient (page 18)
L	asymptotic penetration level (page 23)
$\mathcal{LN}$	log-normal distribution (page 119)
$M(t)$	profits before taxation for year t (page 51)
$N(t)$	the number of subscribers at time (or year) t (page 23)
$\mathcal{N}(\mu, \sigma)$	normal distribution with mean μ and deviation σ (page 119)
$p_S(t)$	average yearly revenue per user at year t (page 45)
$p_T(t)$	price of the technology (or component) T at time t (page 15)
$P(i)$	profits for year t (page 51)
$\rho(t)$	penetration at time t (page 23)
r	discount rate (page 51)
r_f	interest rate (page 51)
r_m	market growth rate (page 51)
R	discount factor, $1 + r$ (page 51)
R_c	cell radius (page 11)
R_S	service rate (page 14)
σ	deviation or volatility (page 56)
s	investment time (page 53), or sample deviation
S	stock value or discounted profit sum (page 102)
V	net present value (page 53)
X	investment costs (page 18)
z	planning horizon (page 18)

1. INTRODUCTION

1.1 Motivation and Methods

The need for telecommunications and the need for higher capacity in networks is rising all over the world. For rural area households, the bottleneck for the capacity is often the first mile (or miles) from subscriber to operator, i.e., the *access network*.

Broadband access networks require considerable investments. Before investment decision, different technological solutions must be compared and the network needs to be planned and dimensioned for the demanded traffic. However, communications technology develops rapidly producing different opportunities and uncertainties in technologies and investment projects. The difficulty of forecasting the number of subscribers and other critical parameters affecting the value of the network makes the investment decision a very hard task. This thesis contributes to methods for managing those uncertainties in broadband investment decisions.

The economics of technologies, or *techno-economics*, evaluates the value and business opportunities of new commercial products. In telecommunications, techno-economics has been studied a few decades: especially scenario based forecasting and cash flow calculations have been considered [125]. Theoretical models for different parts of the broadband access techno-economics exist but this study combines them in one model.

Forecasting in scenario based valuation meet the challenge of future uncertainties. There might be different *risks*, i.e., phenomena having negative effects, and new *possibilities* related to the uncertainties. To make successful investment decisions, the valuation methods used should handle uncertainties and the investors' possibility to change plans. The first stage for the management of uncertainties in investment decision is the identification and estimation of risks and future options [55]. In addition, the effects of the uncertainties should be illustrated and combined in the investment valuation model. The valuation of uncertainties can be combined in the investment valuation using the theory known as *Capital Asset Pricing Model* (CAPM) [32]. This theory relies on the assumption of perfect markets, which may be valid for backbone networks but is not true in the case of broadband access investment projects. The risks can be estimated and illustrated for example using *sensitivity analysis*, or *value at risk* (VaR) methodology. Other methods for managing uncertainties include decision analysis with *decision trees* [55], *portfolio selection* [109] and *real options analysis* [2, 7]. These three methods also take into account the investor's possibility to change plans.

This study uses traditional scenario based techno-economics combined with Monte-Carlo simulation and option theory [30]. Real options analysis in communications has been requested [3], and some studies have been done, e.g., in capacity upgrade [111, 113] or 3G mobile upgrade [116]. Portfolio analysis has also been introduced for mobile phone network investments [22, 21], but usually the management of uncertainties is based on sensitivity analysis and VaR methods [125, 131]. However, the uncertainties are often analyzed only in end-user markets, as in [22, 251, 252], and not much attention has been paid to probability distribution shapes. Moreover, the few real option cases studied in the field include only one step investment options, but no other types of real options. In this thesis, some new examples of the use of options theory in communications are introduced. The implemented cases are in rural area where the need for managing the critical uncertainties is emphasized. The study does not introduce new data, but the research is based on existing statistics and practical data.

1.2 Aim and Scope of the Thesis

The thesis is focused on real option methods and broadband access technologies. Several current technologies are described and some of them, e.g., WiMAX, Flash-OFDM and ADSL, are analyzed in practical cases. The service areas in the cases are in rural areas of Finland. To be precise, broadband networks are considered for the municipal of Ähtäri, situated in the middle of Finland, and for the Finnish railroads.

The economics of network investments is simulated and analyzed statistically to get a view of the suitability of different valuation techniques in the studied cases. The study shows that the distributions of the broadband access network investments are not log-normal in general. As some option theoretic models rely on strict assumptions, the statistics of the underlying projects must be determined and suitable models used. The results are used to develop an actual real option valuation tool.

In summary, the aim of the thesis is (A) *to develop a theory and models for managing broadband investment uncertainties* by

1. finding out the critical parameters of network investments,
2. analyzing possible theoretical models for network investments, and
3. introducing a rough theoretical model for the valuation of network investments.

To tackle this, it is necessary (B) *to adapt and evaluate market models for the penetration of telecommunication services in households.* In addition, the aim is (C) *to adapt and evaluate the use of real options theory in telecommunications* by

1. making case studies of the possible uses of real options, and
2. comparing different investment valuation methods in network investments.

Moreover, the study gives (D) *some insights in the digital divide* by analyzing the economics of different broadband access technologies in rural area cases.

The focus of the study is on the valuation methods and rural network investments. Mainly broadband networks are considered, but some insights in mobile and narrow-band technologies are included where essential.

This thesis does not comprehend competition issues. Rural area connections are often provided by monopolistic operators, which are owned by subscribers, municipals or incumbent POTS network operators. The absence of competition makes the analysis more straightforward, but for potential subscribers it may lead to weaker market power.

1.3 Author's Contribution and Structure of the Thesis

This study consists of six chapters, see Figure 1.1. After the introduction, the basics of broadband communications and networks are described in Chapter 2. Services, markets and regulation of broadband are studied and adaptation of services is analyzed in Chapter 3. Scenario based valuation and real option models are introduced in Chapter 4.

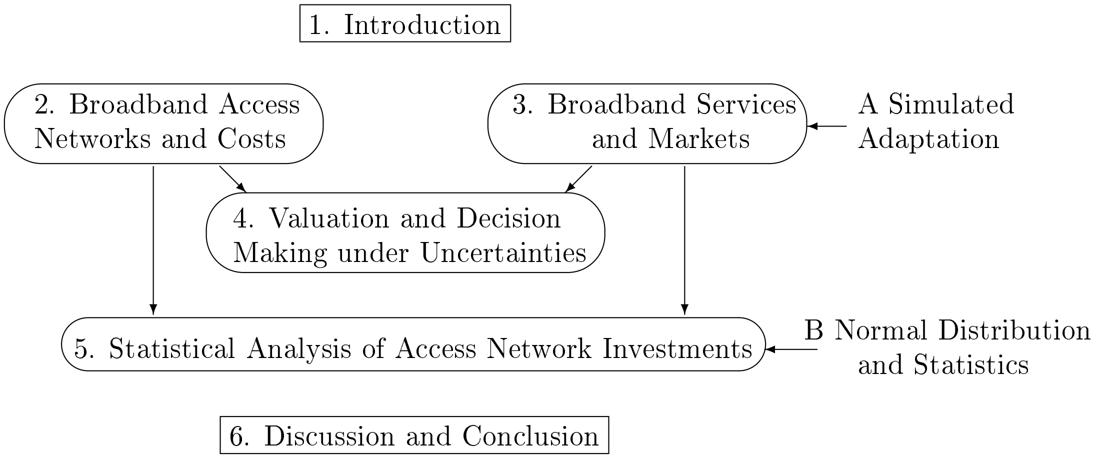

Figure 1.1: The structure of the thesis

In Chapter 5, the statistics of different simulated network investments are analyzed. The implementation of the valuation methods is developed from the version introduced in the author's Licentiate Thesis [225]. Also, a comparison of different valuation methods is done in Chapter 5. The methods and the results of the thesis are discussed in Chapter 6.

The main scientific contributions of the author are the acceptability and adaptation analysis in Chapter 3, the shaping of the theoretical investment model in Chapter 4, and the analysis of the investment cases in Chapter 5.

2. BROADBAND ACCESS NETWORKS AND COSTS

Network users send messages to each others. A message travels through a *channel* between two destinations A and B, see Figure 2.1. In this study, broadband communications is considered and thus it can be assumed that A and B are machines, e.g., servers or personal computers or mobile phones. Certainly, those machines (or computers) are often used by humans. Moreover, the information is sent in two directions, from A to B and from B to A. This means that broadband channels provide possibilities for *two-way communications*. The messages may contain any information, e.g., images, documents, voice, or video.

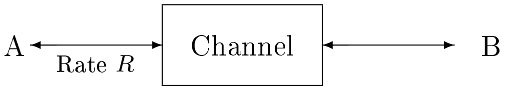

Figure 2.1: A communication channel between two destinations A and B

A *communication network* is a set of *nodes*, e.g., computers, and *links*, i.e., channels, between some pairs of nodes. This study concentrates on *access networks*, which are communication networks with one main node, i.e., *operator*, and a large amount of low level nodes, i.e., *subscribers* (or *users*), see Figure 2.2. In this case, the channel from subscriber A to the operator B is called *uplink* (UL) and the channel from the operator to the subscriber is called *downlink* (DL).

Several definitions of broadband access networks have been proposed. Often, broadband communication channels are defined as channels that can carry real video. Surely, this is not a precise definition since the amount of information in a video signal depends on the quality of the video. Here, channels with high enough *theoretical data rate* are considered as broadband channels. The rate is the limit on how much data, i.e., bits per second, can be delivered in uplink, from the subscriber A to the operator B, and downlink, from the operator to the subscriber. Authorities that collect statistics use different definitions for broadband data rate. In United States, Federal Communications Commission (FCC) uses 200 kbps as the lower bound for the theoretical rate of high-speed services [89]. Other definitions are up to 8 Mbps [255]. A widely used lower bound for the theoretical rate is 256 kbps [52, 159, 243]. The channel capacity of 256 kbps is not enough for many current and future broadband services, but we will here define it as minimum theoretical rate of broadband access networks.

In addition to technological properties, i.e., the sufficient uplink and downlink rates, we need some usability viewpoints. Firstly, subscriber should not log on for the broadband access channel every time, i.e., broadband communication channel is *always-on* [142, 243]. This does not mean that the channel should be always active, but whenever the subscriber's computer is turned on, a message might be sent. Secondly, subscriber must be provided connection to the *Internet* via the broadband channel and the service charge should be based on *monthly tariff*, without charges on transferred data [142, 243]. In summary, the broadband access network in this study means:

- two-way,
- always-on communication channel
- that delivers different kinds of data
- and provides connection to Internet
- with a theoretical data rate of at least 256 kbps
- and with charging not based on the amount of transferred data.

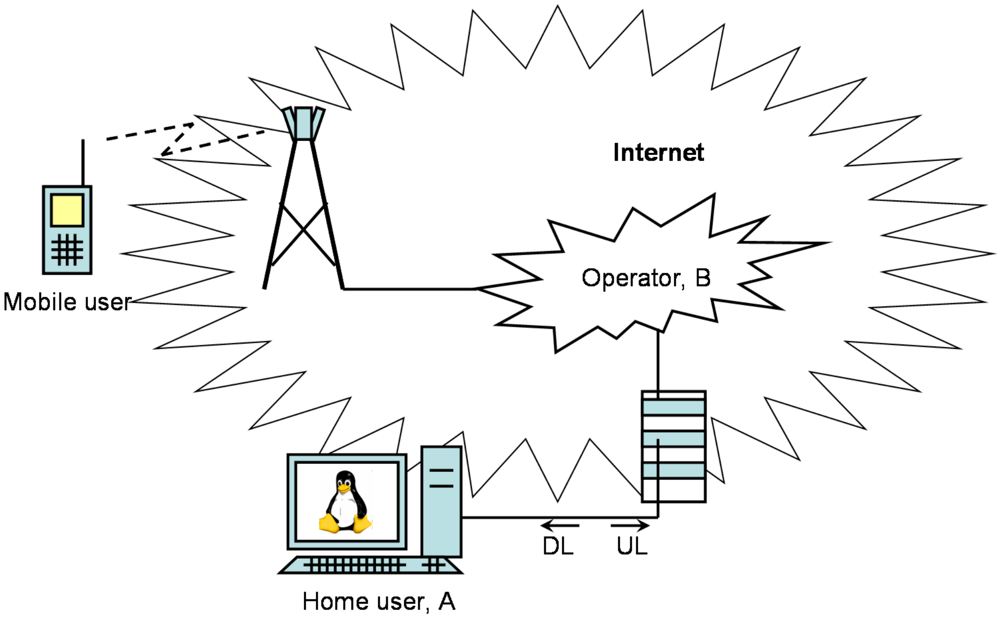

Figure 2.2: An access network with one operator and two subscribers (or users)

2.1 Technology and Network Topology Design

Broadband communication networks use different *technologies*. The general structure of the broadband subscriber's access to the Internet can be modeled as in Figure 2.3 [206]. The subscriber is connected via the *access network* to the *transport network*, which provides connections to different servers. The *Internet* is a network of networks consisting the servers and links connected together all around the world. Some of the subnetworks are *wide area networks* (WAN) covering large areas with high data speeds. The access networks and some of the transport networks may be called *metropolitan area networks* (MAN) covering city areas. The subscriber may divide the connection for a group of *users* as for example, in companies. In that case a *local area network* (LAN) is placed between the user and the access network. Actually, the user's computer is considered part of such a LAN [245].

Internet traffic uses the *Internet protocol suite*, which is a set of different telecommunication protocols [207, 245]. Those protocols can be divided into four layers [206]. This so-called TCP/IP

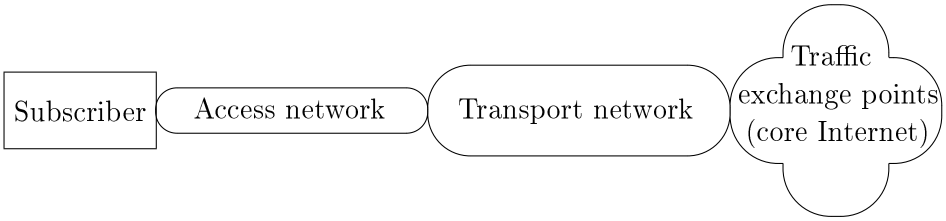

Figure 2.3: Broadband subscriber's connection to the Internet

model is widely used when considering computer networks, but a bit wider OSI-model with seven layers can also be considered [245]. The topmost layer is the *application layer* and it uses protocols like *hypertext transfer protocol* (HTTP) and *domain name system* (DNS). A user's software needs only to know how to use this layer, while different protocols take care of the other layers. The next layer is the *transport layer* and the most important technology in it is the *transmission control protocol* (TCP). This layer manages the data transfer from source to destination. The *internetwork layer* uses the *Internet protocol* (IP) and takes care of the delivery of a packet across a single network. The lowest layer in the model is the *link layer*, which delivers data from a single network device to the next device. It may use different protocols and physical mediums.

The transport network part, see Figure 2.3, usually utilizes optical fiber and is often based on *asynchronous transfer mode* (ATM) [48, 245], which can transfer both Internet and voice traffic. This means that broadband channel goes in optic fiber at some point between the subscriber and the targeted server [128]. Still, the broadband access technology remains open, i.e., the first mile (or miles) from subscriber to the operator. The following sections describe different technologies used in access networks. The technology solutions define, for example, the use of multiplexing, modulation schemes and coding methods for the link layer [245]. These in combination with power, attenuation and interference, determine the performance properties of different link layer technologies.

2.1.1 Optical Fiber

The transport networks and the most demanding access networks are based on fiber optics. A general architecture of an access network on optical fiber consists of an *optical network unit* (ONU) on the subscriber site and an *optical line terminal* (OLT) on the network operators site [128, 207, 238], see Figure 2.4.

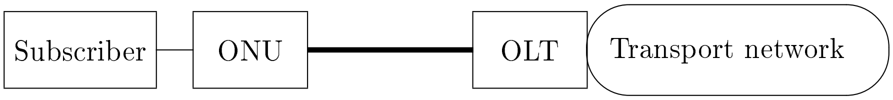

Figure 2.4: The structure of an access network based on optical fiber

Different fiber technologies can be used. For example, the optical line may be active or passive [259]. Active optical networks have amplifiers and other electronic components between OLT and ONU. *Passive optical networks* (PON) have no other active components expect OLT and ONU. Only passive components like splitters are used. PON provide a platform for different protocols. For example ATM or Ethernet protocol can be used [128, 238, 259]. ATM based PON is called APON, or BPON. Ethernet based PON is called EPON (100 Mbps Ethernet) and a multi-service PON with 1 Gbps speed is named GPON. Common name for fiber access networks is *fiber to the home* (FTTH) network, as they use optical fiber all the way from the network operator to the home.

2.1.2 COPPER

Most households in developed countries are connected to one or more copper wire networks. These connections can be used to provide broadband access network. Coaxial cables were originally used for TV services and copper-pair for telephones. In addition, power lines can be used for communication purposes. We describe below technologies that use these wires.

FTTH is an access network solution fully based on fiber, as described in Section 2.1.1. If fiber is not used all the way from the operator to the subscriber, but to the network node near the subscriber's home, we call it *fiber to the node* (or neighborhood) (FTTN) [128, 129, 207]. See Figure 2.5 where an example of such network is drawn. The connection from ONU to the subscriber uses copper wires. In Figure 2.5, coaxial cable completes the access network. Such coaxial cable connections are called *hybrid fiber coax* (HFC) solutions [128, 207, 259].

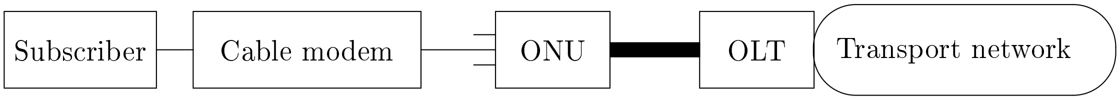

Figure 2.5: FTTB or FTTN access network with coaxial cable extension

The FTTB or FTTN solution can also be based on a paired cable of *plain old telephone service* (POTS) [128, 129, 136, 259]. The technologies making the phone line digital are called *digital subscriber lines* (DSL). DSL technology is used to transfer data up to the telephone central office, which is directly connected to the ATM-based transport network, see Figure 2.6. DSLAM in the central office uses paired cables to communicate with the xDSL modem. The most popular DSL service is the *asymmetric DSL* (ADSL) for which the downlink rate is greater than the uplink rate [128, 136]. ADSL is more popular than *symmetric DSL* (SDSL), which has equal downlink and uplink data rates. One of the other enhanced DSL technologies is *very-high-rate DSL* (VDSL) which uses wider frequency bands than ADSL or SDSL. The enhancements make the data rates bigger and service distances shorter [136, 259, 275].

The development of the technologies would lead to the extension of ATM (or plain fiber) moving the DSLAM nearer to the subscriber's end and yielding *fiber to the building* (FTTB) or *fiber to the curb* (FTTC) solution. ADSL, SDSL, and VDSL technologies have been enhanced to ADSL2(+) [259], G.SHDSL, and VDSL2 [54, 275], respectively. Actually VDSL2 is backward compatible with both VDSL and ADSL technology families.

Practically all homes, but not summer cottages, in developed countries are connected to electricity networks. Those wires can also be used for broadband communication purposes [15, 259]. The *power line communication* (PLC) technology is not widely used, but some energy companies utilize it [1]. The structure of such PLC networks is quite similar to the ones which use coaxial or paired copper technologies [8, 62], see Figures 2.5 and 2.6.

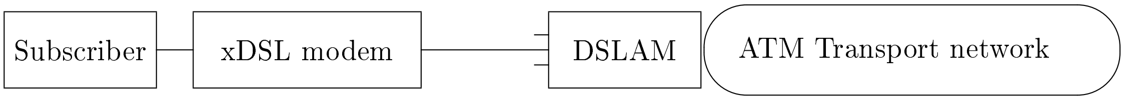

Figure 2.6: The structure of a DSL access network

DSL Coverage

The coverage area of wired broadband technologies depends on the topology of the existing cable networks for homes, i.e., the PSTN network for DSL and the Cable TV network for HFC. The cable length between DSLAM (usually located in the telephone central office) and xDSL modem is called the *loop length.* The practical data rate of a certain DSL technology in each case depends on the general properties of the technology (i.e., frequency bandwidth, multiplexing, transmission power, symbol rate, modulation and coding), but also on the loop length and quality properties of the copper pairs in use [68]. The length and quality properties affect the attenuation of the transmitted signal and are thus related to the service distance of the technology [54].

Figure 2.7 combines some theoretical downlink data rates as a function of loop length for some DSL technologies. The curves are based on experiments and calculations done for SHDSL in [68], for ADSL in [44], and for ADSL2 and ADSL2+ in [73, 271, 275]. The uplink data rate for SHDSL is equal to downlink, whereas ADSL uplink data rate is currently at most 1 Mbps. VDSL has maximum uplink data rate 15 Mbps and VDSL2 100 Mbps.

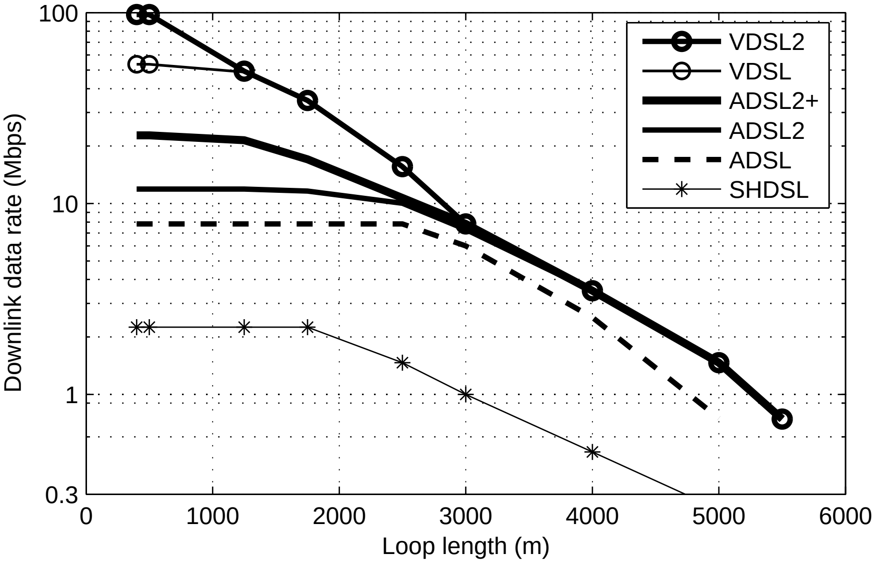

Figure 2.7: Theoretical downlink data rates for different DSL technologies [44, 68, 73, 271, 275]

Since xDSL and HFC technologies use existing networking infrastructures the coverage of each technology is restricted on the current topology of the wires. The service distances in networks can only roughly be estimated by loop lengths and data rate expectations.

2.1.3 Radio Frequency

Radio frequencies provide different communication possibilities. Radio techniques have been used for years for broadcasting. Nowadays, there are efficient mobile systems for personal communications and so called *fixed wireless access* (FWA) systems.

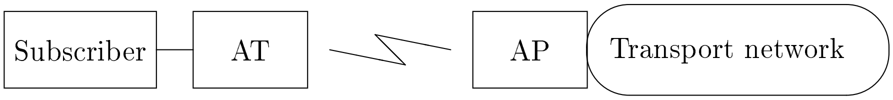

Figure 2.8: WLAN connection (Wi-Fi or HiperLAN)

Wireless local area network (WLAN) is comparable to the traditional Ethernet LAN [206]. WLAN can be used to implement short radio links. The *Institute of Electrical and Electronics Engineers* (IEEE) has published WLAN standard 802.11, which is called Wi-Fi. The European standard by ETSI (*European Telecommunications Standards Institute*) for a WLAN is called HiperLAN [132]. The general structure of WLAN connection is shown in Figure 2.8. A radio link is used between the *access termination* (AT) and the *access point* (AP).

For longer distances, ETSI standard HiperMAN or IEEE 802.16 WiMAX [243, 277] can be used. The subscriber has a *customer premise equipment* (CPE) which connects to the *base station* (BS) as shown in Figure 2.9. Another wireless technique called Flash-OFDM (*orthogonal frequency division multiplexing*) uses a very similar network structure [165].

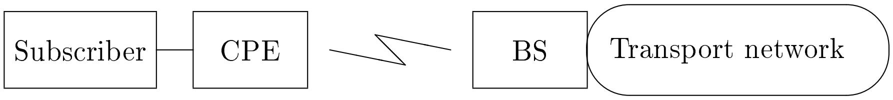

Figure 2.9: WMAN connection (WiMAX or HiperMAN or Flash-OFDM)

The mobile telephone connections with higher bandwidths are supported by the *universal mobile telecommunication system* (UMTS). The *wide-band code division multiple access* (WCDMA) and *high speed (downlink) packet access* (HSDPA and HSPA) technologies for UMTS provide sufficient bandwidth for broadband and *general packet radio service* (GPRS) makes Internet connection possible. Thus, UMTS can be used for broadband access, too [206]. *Enhanced GPRS* (EGPRS) on *Enhanced Data rates for GSM Evolution* (EDGE) networks may reach broadband data rates in some service areas and EGPRS can fulfill the definition of the broadband technology. The structure of the UMTS data connection is more complex than for other radio access techniques, see Figure 2.10. The *mobile equipment* (ME) connects to Node B base station (BS) and the Node B is controlled by the *radio network controller* (RNC). GPRS traffic is transported via the *serving GPRS support node* (SGSN) and the *gateway GPRS support node* (GGSN) to the general transport network.

[194].

Radio frequencies can be used in the outer space, too. Communication satellites can offer a link to rural areas with high data rate. Figure 2.11 shows the structure of a general satellite access [87, 241]. The subscriber terminal connects to a satellite and this satellite connects to a

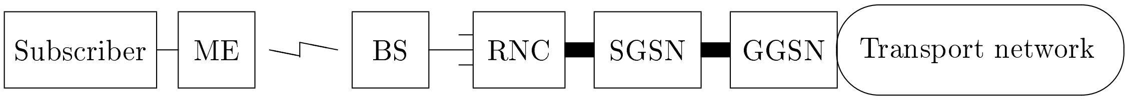

Figure 2.10: GPRS traffic in WCDMA/UMTS mobile network

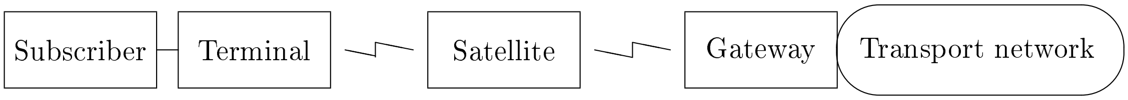

Figure 2.11: Satellite broadband system

gateway. However, some satellite broadband subscribers have another return channel which may be narrow-band

We have here introduced different broadband technologies that use radio frequencies. Terrestrial wireless transmitters are commonly placed in masts and satellite transmitters in satellites in orbits. Yet another place for radio transmitters can be imagined. An aircraft or an airship in the stratosphere (about 20 km away from sea level) can serve quite a big area. These *high altitude long operational* (HALO) aircrafts or *high altitude platforms* (HAP) could be unmanned and use solar power. The structure of such a communication system is similar to satellite broadband systems, see Figure 2.11 [102]. The HALO and HAP have been studied but not yet largely implemented [179, 260, 265]. Especially, satellite and HAP systems can provide good coverage extension for terrestrial systems [87].

Radio Network Coverage

Based on the network topology and performance properties of technology components, the amount of components needed for a network can be estimated. Firstly, each subscriber needs some of the components (base station and its sector antenna) close enough to have an access. Secondly, the topology of the network and forecasted communication traffic guides the amounts of the other network components.

The cell radius R_C, which defines the maximum service distance from base station, is one of the key technology parameters when determining the network coverage. The cell radius can be estimated using path loss calculations [242]. As described for DSL coverage in Section 2.1.2, the general properties of technologies play an important role in these calculations. Among the main topics for cell radius are the central frequency and frequency bandwidth [107, 169, 197]. The central frequency determines the propagation of the signal in general, see Figure 2.12 for estimated maximum frequencies for specified cell ranges with the ITU-T IMT-2000 system. For the cell radius calculations in Figure, the capacity is assumed to be 384 kbps for pedestrian use.

Other relevant technology properties are, for example, multiplexing methods, transmission power, symbol rate, modulation, coding, and antenna gains [101, 117]. The cell radius is not constant and the coverage is affected by overall communications traffic in the sector at a time due the channel interferences as well as location and velocity of the user terminals [119]. Moreover, the service distance can be temporarily extended and lower data rates used by adaptively changing

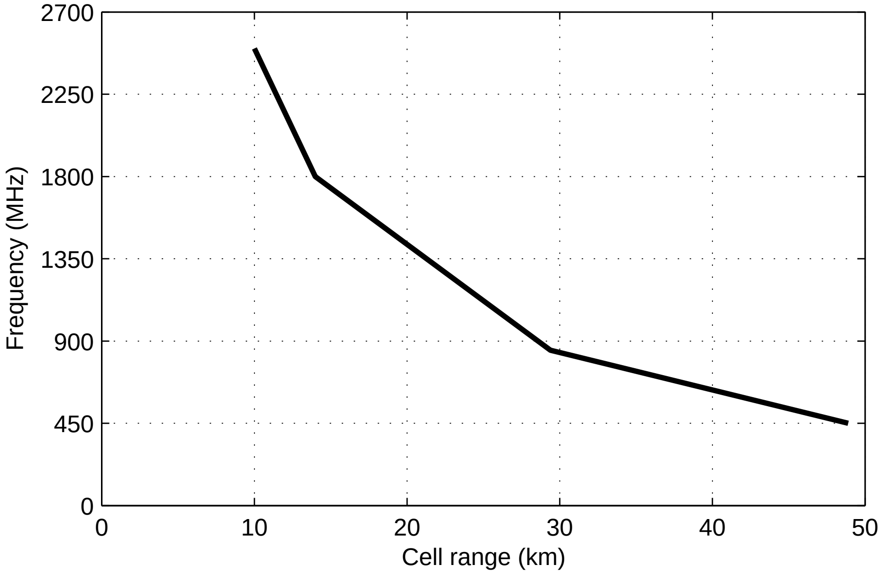

Figure 2.12: Estimated frequencies to achieve ITU-T 3G standard as a function of cell range [45, 197]

modulation and coding schemes.

Once the service distance is known, the service area A must be covered by deploying base stations around the land [125]. The minimum amount of base stations can be estimated by dividing the service area by the coverage area of one base station. The maximum coverage area of a base station is calculated by using the circle area with service distance as radius. More sophisticated estimates are achieved when the overlapping of base station service areas are taken into account. Since the circles cannot cover the service area without overlapping each other, the adjustment of the base station places affects on the overlapping areas. In Figure 2.13 some possible coverage patterns are shown. Table 2.1 presents the efficiencies of these coverage patterns. To count the efficiency, the pattern area with unit distance from center to corners is divided by the circle area with unit radius. The coverage area of the pattern can be calculated by multiplying the pattern area values in Table 2.1 with the square of the service distance.

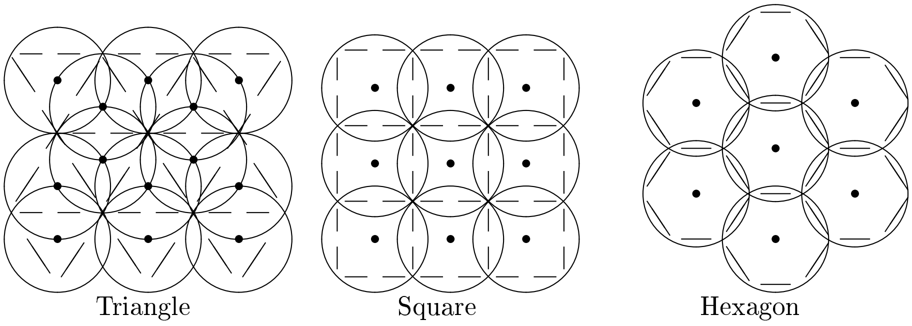

Figure 2.13: Coverage patterns

The actual amount of base stations needed to cover a certain area can be estimated only if the demographic distribution and geography for the case are known. The shape of the terrain

Table 2.1: The efficiencies of different coverage patterns

Pattern	Circle area	Pattern area	Efficiency
Triangle	3.14	0.75	0.24
Square	3.14	2.00	0.64
Hexagon	3.14	2.60	0.83

and obstacles in the area affect the network performance [10]. Moreover, the deployment costs can be reduced by using current pylons, but this restricts the locations of the base stations.

2.2 TRAFFIC AND COST MODELING

2.2.1 CLASSIFICATION OF NETWORK COSTS

The investment costs appear at the beginning of and during the network construction. After that, different costs need to be covered during the use of the network. International Telecommunication Union (ITU) defines different actions producing costs such that [130]

Operations include the operation of support centers/systems, test equipment, methods and procedures, as well as the personnel and training required to install and maintain all the elements.

Provisioning is the process of making available various telecommunications resources for telecommunication services. Provisioning thus includes the design of the network from the demand forecasting to the installations.

Administration covers a broad group of functions that sustain telecommunication services once they have been established. These includes for example the confirmation of the efficient use of network (*network administration*) as well as billing (*service administration*).

Maintenance sets up the operations from the network condition measurements and fault management to the replacement of network components.

The operations cover some of the functions in other definitions, but merely it coordinates the whole process of network implementation and use [198]. The costs of provisioning are discussed in Sections 2.2.2 and 2.2.3. Maintenance costs as well as operation and administration costs can also be modeled as described below and in Section 2.2.4 [125].

The investment costs are often referred to as *capital expenditures* (CAPEX) and they are spread over several years in the annual accounts using deductions for taxation. The costs aimed only at the occurrence year are called *operational expenditures* (OPEX). Another dimension for classification of costs is the dependability on the amount of subscribers (or users). Some of the costs are *fixed* and they are not affected by the amount of subscribers. Other costs may be *direct variable* costs that can be aimed at each subscriber directly. Between these extremes are *indirect variable* costs that cannot be directly aimed at any subscriber but that monotonicly increase as the number of subscribers in or the amount of the use of the network grows. Figure 2.14 points out different types of costs. Investment costs (or CAPEX) emerge before the network is running and they can be divided in *coverage related costs* (i.e., fixed) and *capacity related costs*. Note that some of the coverage related costs could be directly aimed at certain subscribers, but we still analyze them as fixed investment costs. Once the network is built, the administration of the system and maintenance of it needs some fixed costs — see *fixed running costs* in Figure 2.14. *Variable running costs* are the most fluent and they depend on the amount of subscribers (and users) and use of the network at a time. Running costs in common equals OPEX.

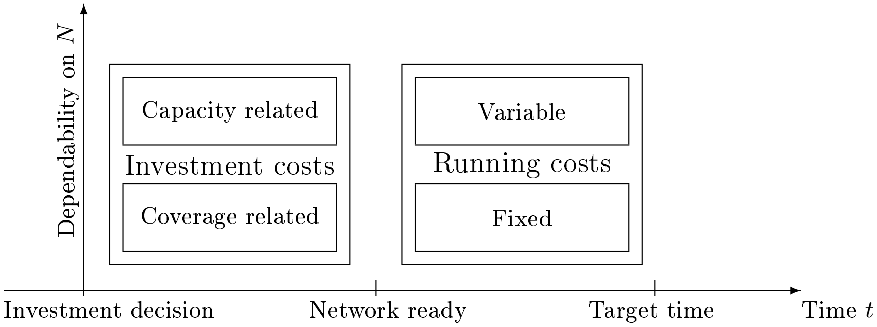

Figure 2.14: Network costs

The operations and administration costs for techno-economic evaluations in telecommunications are often modeled based on the planned infrastructure. The fixed yearly amounts are used [125, 242]. The maintenance costs are often modeled using a mean time to failure or lump sum proportional to the size of invested network infrastructure. This is discussed more and extended in Section 2.2.4. The affect of these cost components in the overall costs tend to be moderate or low [225].

2.2.2 TRAFFIC AND NETWORK CAPACITY

The network acts as a mediator for communication information and thus the information generate traffic in the network. The traffic and networks can be divided in to two types: circuit-switched and packet-switched. The circuit-switched communication need a certain bandwidth when active. However, the amount of active connections through a certain network point seems to be a random variable [10]. On the other hand, the packet-switched traffic needs network capacity only short periods of time, but the intervals between those periods may be quite short, when the transmission between two points is active. The packet-switched traffic can also be modeled statistically [10]. The network operator may, and always do, design the capacity of the network based on the fact that all the subscribed channels are not always active. Thus the capacity does not need to be as high as the sum of subscribers nominal data rates. This is called statistical multiplexing [10].

We define the concept of *throughput demand* $D(t)$ to handle the statistical distribution of subscriber traffic and the limited capacity of the network. The *average throughput demand* for a user group $d_a = R_S\kappa$ is the product of the marketed end-user service rate R_S and the overbooking factor κ. The use of the channel bandwidth by the user group can be estimated by the number of the subscribers $N(T)$ and by the average throughput demand per user. The network infrastructure is planned so that the capacity for targeted user groups are achieved. The total amount of network components needed to provide the forecasted traffic can be estimated based on coverage and capacity calculations. Note that throughput demand $D(t) = N(t)d_a(t)$ tends to grow quite rapidly over time. This is because of the network services used and the amount of users connected in the network. The throughput calculation have been used in the field for years and in telecommunication techno-economics, too [10, 242]. However, the single scenarios can be planned without the concept of throughput demand and the concept becomes necessary only with the simulations of the network dimensioning, as done later in this thesis.

Some different parameters for the growth of the downlink peak rate have been proposed. The rate was stated to double every 90 days in 1999 and every year in 2004 [16]. Other studies propose some 30% to 50% growth every year in the long run [40, 80, 264]. Usually, the rate growth is proposed to be exponential $d_a(t) = d_a(0)e^{c_d t}$, where c_d is the average througput demand growth rate. In summary, we model the throughput demand as

$$D(t) = N(t)d_a(0)e^{c_d t}. \tag{2.1}$$

2.2.3 MODELING THE COMPONENT PRICES

The prices of the network components are key factors when comparing technologies. The price of a new component is fixed or discrete function of time, when there is only few suppliers. If the number of component manufacturers and distributors increases, the market price curve becomes more continuous. As the amount of produced components grows, the research, development and other fixed costs can be allocated more widely and the price of one component tends to decrease. Moreover, the production methods develop, which decreases the price of the components, too.

Wright's learning curve for the average production time T_n of n components is

$$T_n = T_1 n^b, \tag{2.2}$$

where b is a model parameter [203, 281]. Crawford's learning curve uses the same formula, but T_n is defined as the production time of the nth component [203]. When estimating the prices of network components, that curve (2.2) can be used with the production amount based on the total components produced in the world. This extended learning curve is used in [203, 250, 256]. If $p_T(t)$ is the price at time t and $n(t)$ the relative proportion of produced components at time t,

$$p_T(t) = p_T(0)\left[n(0)^{-1}\left(1+\exp\left\{ln(n(0)^{-1}-1)-\frac{2\ln 9}{\Delta t}t\right\}\right)^{-1}\right]^{\log_2 K},$$

where Δt is the time interval between 10% and 90% penetration and K the relative decrease in the cost by the double production. The equation can be modified to

$$p_T(t) = \frac{p_T(0)}{[n(0) + (1-n(0))9^{-2t/\Delta t}]^{\log_2 K}}. \tag{2.3}$$

The TITAN project made a database of the prices of the network components. The database made possible to analyze models and estimate the model parameters for different types of components [205]. The development of the prices of the components is studied in [71], too.

The mentioned price models needs several parameters. However, when analyzing local network investments, the local penetration or the amount of components needed does not affect so much on the amount of network components in the world. In that case the component supply side can well be forecasted without connection between penetration and component prices and the component price is estimated using the exponential curve

$$p_T(t) = p_T(0)e^{-c_T t} = p_T(0)K_{T,p}^t, \tag{2.4}$$

where $c_T > 0$ and $0 < K_{T,p} = e^{-c_T} < 1$ [226, 227, 228, 242]. For electronic components, the yearly reduction in component price is from 0% up to some 30%, depending on the life-cycle phase of the product.

2.2.4 MAINTENANCE INTENSITY MODELING

Let us define the *maintenance intensity* $I_m(t)$ for a system (or network) as the number of maintenance operations in a time unit. The intensity is

$$I_m(t) = \lim_{\Delta t \to 0} \frac{N_r(t+\Delta t) - N_r(t)}{\Delta t},$$

where $N_r(t)$ is the number of replaced components (or maintenance operations) until time t. The maintenance intensity is affected by the lifetime distributions of the system components. Let us define the *survival time* $T_{s,i}$ of a component (or system) as the time of component i successfully working without maintenance. The probability density function for the survival time is $f_{T_{s,i}}(t)$. It is practical to assume the time between maintenance stops for a component i to be distributed as the first survival time $T_{s,i}$. The *mean time to failure* (MTTF) is the expectation of the survival time

$$MTTF_i = E[T_{s,i}].$$

For a new network, the components are new ($t < MTTF_i$) and the maintenance intensity follows the distribution of the survival times, i.e., $I_m(t) \approx \frac{N_r(t)}{t}$. In the long run, the last replacement times for components are scattered and the shapes of the survival time distribution functions do not affect on the maintenance intensity so much, i.e.,

$$I_m(t) \to I_m, \tag{2.5}$$

where the constant I_m is the *asymptotic maintenance intensity.* For such a system the time between maintenance stops can be modeled using exponential distribution, which is often used for the maintenance modeling in practice [234].

The shape of the survival time distribution depends on the physical properties of the system or component. The survival time distribution can be analyzed by using *hazard function*

$$h_i(t) = \frac{f_{T_{s,i}}(t)}{1 - F_{T_{s,i}}(t)},$$

where $f_{T_{s,i}}(t)$ is the probability density function of the survival time and $F_{T_{s,i}}(t)$ the cumulative distribution [234]. The hazard function defines the instantaneous failure intensity given that the component has operated without failure time t. The hazard function for exponential survival time is constant, i.e., the failure intensity of the component with exponential survival time is not affected by the time after the maintenance. This yields to constant maintenance intensity $I_m(t) = I_m$. In practice, the instantaneous failure intensity of a component is often growing [234]. Probability distributions like *Weibull*, *Chi–Squared*, and *Gamma* have a growing hazard function with lowering growing rate, see Figure 2.15. Moreover, the hazard functions of Chi–Squared and Gamma distributions are bounded, i.e., the tail distribution approaches exponential distribution and the instantaneous failure intensity has a finite asymptote.

The survival time distributions other than exponential make the modeling of maintenance costs for an investment more sophisticated. As analyzed earlier, the maintenance intensity of a system with several components tends toward exponential. However, the maintenance costs for a new network may be affected by different component survival time distributions. Some simulated maintenance intensities with different survival time distributions are drawn in Figure 2.16. The shifted exponential distribution have probability $F(t) = 0$ for $t \leq 2$ and $F(t) = 1 - \exp(-\lambda(t-2))$

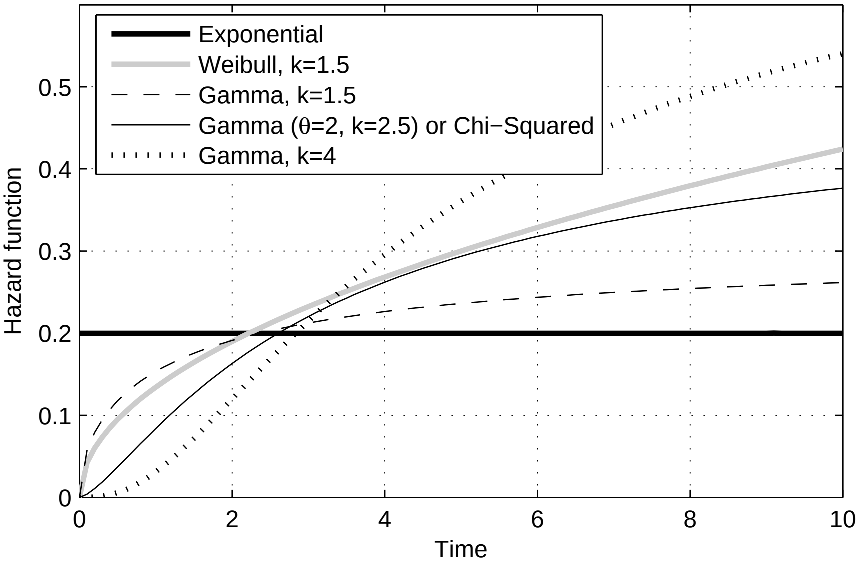

Figure 2.15: Hazard functions for distributions with mean 5.0

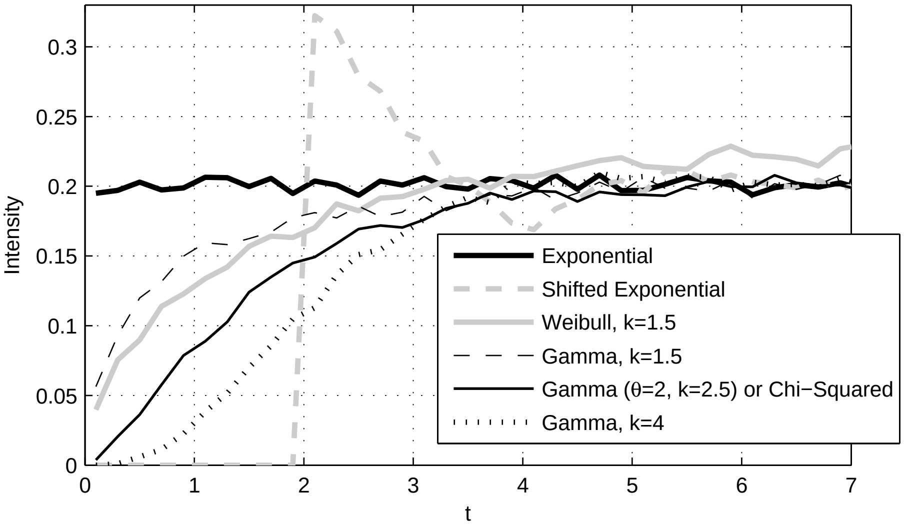

Figure 2.16: Simulated maintenance intensity for a system with 50 000 components

for $t > 2$. All the distributions in Figure 2.16 have parameters so that mean is 5.0. As can be seen, the system maintenance intensity tends to follow the model

$$I_m(t) = (1 - \exp(-kt))I_m.$$

Though this model is not truly validated for the network investment cases, we propose using it in spite of the fixed proportional model (2.5), which have mainly been used in the earlier studies in the field.

2.3 COMBINED INVESTMENT AND COST MODEL

An overview of a modular network investment cost model is drawn in Figure 2.17. The investment costs $X(s)$ at investment time s depend on the demography of the service area and technology to be used as well as subscriber and throughput demand forecasts. The running costs $C(t)$ for each year depend on the infrastructure and the usage of the network.

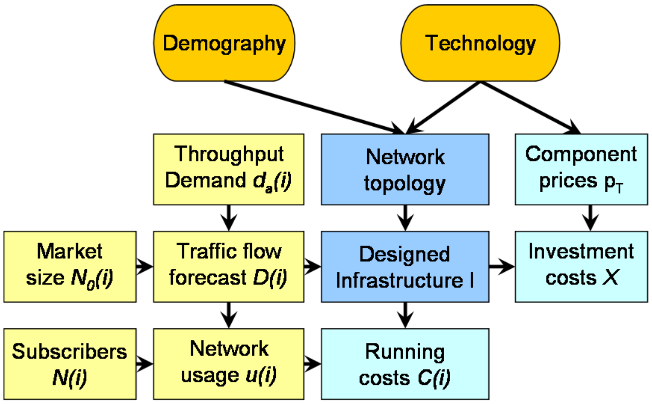

Figure 2.17: Network costs model

The area from which the network should be accessible is considered in coverage calculations. The coverage demand gives the minimum amount I_A of network components needed. The amount I_A is roughly proportional to the service area A and for wireless technologies inversely proportional to the square of the cell radius R_c. Let us estimate the coverage related investment costs by

$$I_A = \frac{K_{T,A}}{R_c^2} A,$$

where the parameter $K_{T,A}$ depends on the technology to be used.

On the other hand, the traffic flow forecast $D(t)$ define another minimum $I_D = K_{T,D}D(z)$, where z is the planning horizon. Now the traffic flow using the subscriber forecast $N(t)$, the initial throughput demand $d_a(0)$ and the throughput demand growth rate c_d, is (2.1) $D(z) = N(z)d_a(0)e^{c_d z}$. The amount of infrastructure needed is $I = \max(I_A, I_D)$ and the investment cost model with the technology price $p_T(s) = p_T(0)e^{-c_T s}$ (2.4) at investment time s is

$$X(s) = p_T(s)I = \left(p_T(0)e^{-c_T s} + K_{T,I}\right) \max\left(\frac{K_{T,A}}{R_c^2}A; K_{T,D}N(z)d_a(0)e^{c_d z}\right). \tag{2.6}$$

Note that with the more sophisticated price model (2.3) the network component price would be negatively correlated with the minimum amount of infrastructure I_D. Thus the investment costs as a product of negatively correlated factors have smaller proportional variance than the factors directly indicate.

Let us similarly make a rough estimate on the running costs. The fixed costs $C_F(t)$ in one time period t are assumed proportional to the amount I of infrastructure, $C_F(t) = K_{T,F}I$. Alike, some of the administrative costs are fixed, but others are related to the number of subscribers,

$$C_{OA}(t) = K_{T,F,OA}I + K_{T,V,OA}N(t) + K_{T,V,I}\Delta N(t),$$

where ΔN is the amount of new customers in the time period. Similarly, some of the maintenance costs of the network are fixed but a part of them are assumed proportional to the usage of the network. Still, the maintenance costs are assumed to be proportional to the price of the network components.

$$C_M(t) = p_T(0)e^{-c_T t}\left[(1 - e^{-c_{T,F,M}(t-s)})K_{T,F,M}I + K_{T,V,M}N(t)d_a(0)e^{c_d t}\right].$$

The total operational and administration and maintenance costs in one time period t are

$$C(t) = C_{OA}(t) + C_M(t). \tag{2.7}$$

3. BROADBAND SERVICES AND MARKETS

The demand for broadband services has increased rapidly in the recent years. Recall the properties of the broadband services and Figure 2.2 as stated in Chapter 2. The broadband communications network provides a two-way, always-on channel to the Internet capable of carrying real time video. In addition, the charging is based on the monthly tariff. The subscribers are different companies and communities as well as individuals and households.

Broadband subscription is neither a product nor a pure service, but it has characteristics of both of them. The use of subscription requires customer premise equipment (CPE), nowadays usually a part of or connected to a personal computer. Computer as CPE is a product whose acquisition is nowadays more and more related to the Internet connection subscription. However, the characteristics of services in general [105]:

- process consisting of activities,
- produced and consumed simultaneously, and
- customer participates as a co-producer in the service production process,

are fulfilled by broadband subscription itself. In summary, both customer product and service adaptation viewpoints should be considered in the analysis of broadband markets.

3.1 TELECOMMUNICATION SERVICES

Many services are based on broadband communications. Some of these are online services on the Internet servers, but the use of direct services using broadband has risen as well. Services can be classified from a user point of view or from a technology point of view. One classification from the user's viewpoint defines seven main groups: education and learning, newspaper and broadcasting, entertainment, medical care, public services, shopping and banking, and living information [284]. The services are used for different purposes, e.g., to collect information, for personal development, in business, and communications, covering most of the life sectors. These services and applications are used in both professional and private fields.

The technology oriented classification of broadband services is based on the overall bandwidth needed in downlink and uplink, and the real-timeness of communication. Table 3.1 presents some examples of services in different categories as well as defines some names for service groups. Established service group names are in bold and examples of services in italics. *Browsing* provides access to information services like educational information, news and magazines, weather forecasts, and so on [284]. These services do not need much bandwidth or real-timeness, and those can often be used with narrow-band Internet connections, too. However, as the quality of images in web pages has risen, the demand for downlink data rate has increased too. Other basic services are for example electronic mail, chat and *instant messaging* (IM) . Telephones and *voice over Internet protocol* (VoIP) needs a real time channel to work properly. They are examples of narrow-band real-time services.

Table 3.1: Technical classification of network services

	Non-real-time	Semi-real-time (respond in few seconds)	Real-time
Full narrow-band	*e-mail*	*text browsing, IM*	*telephone, VoIP*
Broadband downlink	**Downloading**, *VoD*	**Browsing**	**Streaming**, *IPTV*
Broadband uplink	**Uploading**, *photo pressing*		Up-streaming
Full broadband	**P2P**, *file sharing*	*remote office*	*video conferencing*

When a broadband subscriber is transferring data in the downlink direction, the service group is *downloading*. The transferring a large amount of data needs high data rates but does not need real-time channels. For example *video on demand* (VoD) service can be utilized with non-real-time channel. Nevertheless, if the video is played simultaneously with the downloading, the service class becomes *streaming*. IPTV (Internet protocol television) is one good example of streaming applications. For uplink direction, we may call the services *uploading* and *up-streaming*. For example pressing digital photos in photo laboratories can utilize uploading.

The most demanding broadband services demand full broadband in both directions. Those include different *peer-to-peer* (P2P) applications. P2P service may for instance be remote medical care, video calls (or conferencing), or P2P data sharing.

See Table 3.2 for some characteristics of different service applications. Most of the applications can be used for different purposes and thus the service class may vary. The data rates in Table 3.2 are based on Table 9.3 in [10].

Table 3.2: Characteristics of different network services, data rates from [10]

Application	Service class	Real-timeness	Broadband downlink?	Broadband uplink?
E-mail	Communications	Non-real-time	No	No
Telephone	Communications	Real-time	No	No
Web browsing	News, shopping, etc.	Semi-real-time	1 Mbps	No
Video on demand (MPEG1)	Entertainment, education	Non-real-time	1-2 Mbps	No
Streaming (HDTV)	News, entertainment	Real-time	20 Mbps	No
CD-quality stereo	News, entertainment	Real-time	256 kbps	No
Peer-to-peer	Entertainment, etc.	Non-real-time	>2 Mbps	>2 Mbps
Remote office	Business, medical care, etc.	(Semi-)real-time	>10 Mbps	>10 Mbps
Interactive games	Entertainment	Real-time	1.5 Mbps	1.5 Mbps
Videoconferencing	Communications	Real-time	1-2 Mbps	1-2 Mbps

3.2 ACCEPTABILITY AND ADAPTATION

The *penetration* of a service (or equipment) is the average amount of subscribers (or owners) per one hundred potential users. This concept is commonly used in telecommunications. In broadband communications, two different bases for the number of potential users are used. One considers the human beings and another the households. In this study, mainly the later is used.

When considering the future penetration of a new technical service, we are faced up to *technology adaptation*. Adaptation and diffusion models are mainly used in consumer product industry and in telecommunications [5, 167, 184]. The adaptation of broadband services is the growth of the penetration of broadband subscriptions in some target group as a function of time.

The penetration of a service is related to the overall demand of the service. Jointly with revenue forecasting, see Section 3.3.1, forecasting of the penetration produces an estimate on the demand for the service. In the long run, as the penetration grows near to one hundred, it perhaps no longer is a relevant indicator on the demand of the service and the size of the market should then be calculated based on other indicators. Before discussing different penetration forecasting models, we introduce here some aspects and theoretical models for the adaptation process of technology users.

3.2.1 THE NUMBER OF POTENTIAL SUBSCRIBERS

To estimate the amount of potential subscribers, one must imagine which groups may subscribe to the service: companies and homes, or perhaps everyone individually.

After determining the target groups, their size can be estimated using historical data. Such data can be modeled as a mathematical curve using *the least square method*. Extrapolation with parameter c_N and initial value $N_{pot}(0)$ is

$$N_{pot}(t) = N_{pot}(0)e^{c_N t},$$

which gives an estimate on the potential subscribers in each user group for the following years [213].

Example

Consider the subscribers of fixed broadband access. The potential subscribers are households, companies, communities and public authorities. The households may have one broadband subscription at home and perhaps another at a summer cottage. The companies may have a subscription at each office. The communities that employ people may also have broadband subscriptions. See Figure 3.1 for the overall amount of these potential subscribers in Finland. The public authorities often have their own metropolitan and local area networks and the amount of subscribers among them is hard to estimate.

As can be seen in Figure 3.2, the amount of potential subscribers have increased smoothly. The data, which has been collected from [262], shows that linear or exponential regression may be used. In the figure, the dashed line is exponential model for the sum of subscriber segments and solid line is for household subscribers only. Both models have annual growth rates of 1,1%.

3.2.2 SUBSCRIBER'S DECISION MAKING

The decision to subscribe to the broadband network is affected by several factors. The acceptance of a technology depends for example on the usability, usefulness, costs and social attractiveness of the service or product [199]. It is stated that the attributes in broadband subscriber's decision

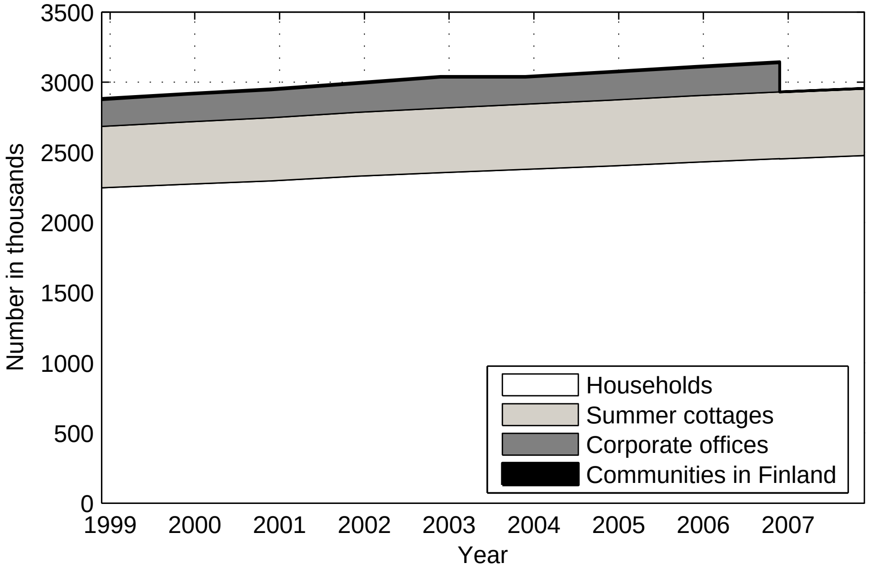

Figure 3.1: Broadband subscriber groups in Finland [262]

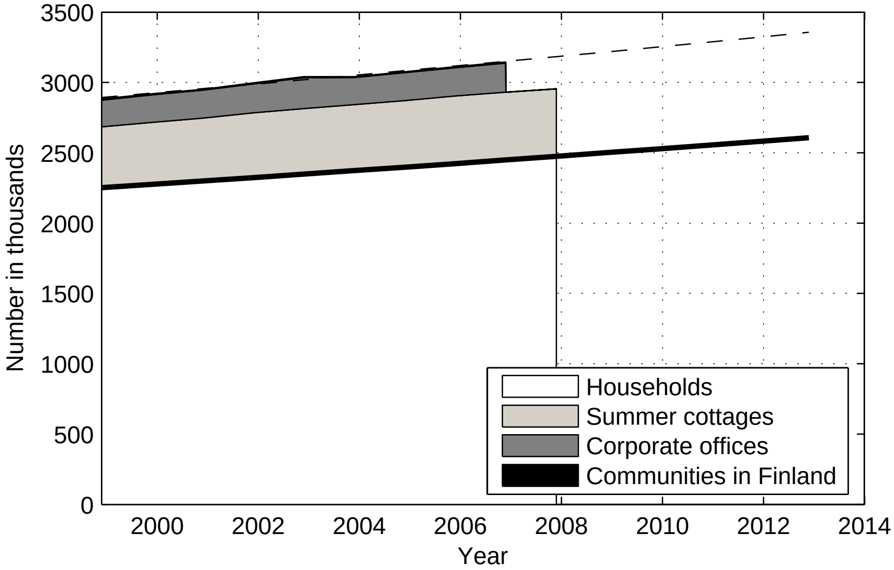

Figure 3.2: The forecasted number of potential broadband subscribers in Finland [262]

making can be combined in *access to the subscription*, *competence for the use of broadband*, and *motivation to subscribe* [96, 201, 273].

For professional use of information technology in different organizations, the decision about adaptation is done in manager level. The decision is affected by the structure and culture of the organization as well as managers' awareness and attitudes towards new technologies [258]. Moreover, costs and new risks if using information technology are more relevant issues for companies than for households [59].

Decision Attributes for Home Broadband Subscription

The attributes for and characteristics of the broadband and other communication service subscribers have been studied from different viewpoints in several research projects [91, 100, 214, 217, 282].

The access to the broadband connection can be seen as an existence of sufficient infrastructure and resources. Physically, the access means that the subscriber's location is near enough to operator's equipments [91, 214]. The physical accessibility can be measured as the coverage area. Financially, the price of the subscription and customer premise equipments should not exceed subscriber's earnings and financial resources. It has been statistically proven that high-income households more likely subscribe to Internet and broadband [100, 282]. Though prices have quite small effects [50, 143], studies show that the most common broadband subscribers are educated people [143]. However, as the penetration grows, this may change. Yet another resource affects the access to the connection: time. Though low-income households have lower possibilities for the connection, they have often more time to use the service [100].

Subscriber's competences need to be sufficient for the use of computer and Internet. It has been stated that age and routines of using technical appliances affect on the physical and technical competences of subscribing Internet connection [214]. Education is strongly related to the willingness to subscribe [100]. Yet cultural habits and cognitive competences play significant roles in the adaptation of the connection. Internet content is multilingual but evidently there are differences in the amount and quality of content for different languages. This may also affect on the subscribers' decisions [217].

The utility of the broadband subscription depends on the subscriber's using habits. Depending on her lifestyle, the Internet may provide access to many infotainment and entertainment services important for the subscriber. The use habits of the connection is related to use habits of other services [76, 121] and the adoption of new technology is affected by the satisfaction from the earlier experiences of similar technologies [28]. Studies further show that trust and fairness play a significant role in broadband business [50, 143]

The networks having different user groups that benefit from each others activity is called to have *two-sided markets*. The quality of the Internet content increases along time and also the number of subscribers affects on the general quality of the content due to two-sided effects [77, 210, 211, 229, 230, 258]. In addition, the direct utility of the Internet and broadband connection for a subscriber grows as the number of friends and relatives online increases [91, 177]. It is because of the communication applications used via connections. The grapevine affects also imitation on consumption decisions, thus increasing the adoption of the services. The relationship between the number of subscribers and the adoption rate are discussed more deeply in Section 3.2.3.

The attributes for broadband access subscription are outlined in Table 3.3. Criteria are general trend-lines and concrete measurable values that are related to the attributes. The characteristics of individuals define the differences between potential subscribers.

3.2.3 SIMULATING THE ADAPTATION OF SERVICES IN SOCIAL NETWORKS

Social Networks and Consumer Decision Making

Product diffusion models and mouth models have earlier been studied in [173, 174]. Three different sets of actions affecting the adoption of IT innovations were found in [137]. Those

Table 3.3: Attributes, criteria and characteristics of broadband access

	Attributes	**Criteria**	**Characteristics**
Access			
- Physical	accessibility	coverage	place [91, 214]
- Financial	costs	price [91, 217]	earnings [100]
- Time	efficiency	bandwidth [75, 78]	time available [100, 120, 121]
Competence			
- Physical	compatibility		working order, age [214]
- Technical	learnability, usability	user-friendliness	technical appliances, age [214]
- Cognitive	effectiveness	literacy, language [217]	education [100]
Motivation			
- Financial	utility	substitutive services [76, 196]	way of life [120, 121]
- Social	social acceptability	interactive services	friends online [91, 177]
- Emotional	satisfaction [153]	service quality	earlier experiences [28]

aspects were contextual, influence and adopter's actions. Here we consider the network effects and some socio-cultural phenomena for broadband subscriptions.

Advertisements of technology innovations and services are traditional ways to raise the demand for the services. However, the user experiences are much more effective incentives for new technological services than for traditional services. This increases the role of the grapevine advertising and social networks [177]. Moreover, potential subscribers may test the online services by visiting friends, schools or libraries having broadband access, thus lowering obstacles for purchasing new service. Thus the social environment and social networks play a role in broadband access decision making. Here we show some theoretical reasoning for social network affects in broadband subscription.

The utility of the network service increases as the number of subscribers increases. This is because of the increased number of friends in Internet and it increases also the value of the network for operators and online service providers. The proposed models for the market value of the network differ from kN to $k2^N$ depending on the service properties. The value for a broadcasting service like TV or traditional newspaper is proportional to N. That model is called *Sarnoff's law*. When considering the services in which any pairs of subscribers can communicate to each other, like a telephone, the network value is stated to follow *Metcalfe's law*, i.e., to be proportional to N^2 [186]. The grow of different online groups make the value of a network even more attractive as the number of subscribers grows: *Reed's law* states the value to be proportional to the number of different groups, i.e., 2^N [222]. These estimates have been criticized and more realistic models like $N \log N$ have been proposed [36, 202]. In addition, the content quality increases as the online service markets grow [77, 210, 211, 229, 230]. This kind of relationship is relevant in two-sided markets. In summary, it seems obvious that the utility of the Internet and broadband connection for a subscriber increases directly and indirectly via the content quality as the overall number of subscribers and the number of friends online increases.

The social network effects are considered in *technology acceptance model* (TAM) and *value based adaptation model* (VAM) . TAM have been used to determine the significancies of the characteristics related to subscriber's broadband access decision making [214]. Some online service adaptations, e.g., for Internet banking, have been studied in [51] and online trading in [168]. VAM model is applied to broadband subscriptions in [153]. TAM and other models for information systems is studied also in [155]

The social network effects have been studied analytically in [177]. The paper analyzes stationary states for certain connectivity distributions, making implications on the critical threshold values for a service to be a success.

The social network effects have implications in the possible marginalization of different social groups. It may be one of the explanatory factors for different penetration levels between races [217]. In addition, it is stated that rural subscribers have different dynamics for the social network effects [91]. The reason for that may be the emphasized role of the utility for the substituting services, e.g., e-commerce or communication with local authorities.

Social Network Model

We define the *social network* as a set of individuals and a set of relations between the individuals. Mathematically, the network can be analyzed as a graph [276]. The two points are related (or the graph has edge between the two points) if the corresponding individuals are friends to each other. The friend relation is symmetric and partly transitive. This means that people having common friends have increased probability of being friends to each other. This also yields to cliques or nearly cliques.

In this section, we generate social networks with randomization so that any two pairs of individuals are friends with probability p. In addition, friends of friends knows each other with probability r. Values $p = r = 0.05$ are later used if nothing is mentioned. To illustrate the adaptation of a service, let us simulate the process with different possible decision models. After an initial set of subscribers $N(0)$ the adaptation process for a single potential subscriber i is affected by the number of friends $N_i(t)$ already using the service and by a randomized individual decision threshold C_i. Let us assume that the potential subscriber i subscribes if

$$kg(N_i(t)) > C_i, \tag{3.1}$$

where the polynomial g defines the decision model as a function of subscribing neighborhood $N_i(t)$ and the parameter k is related to the adaptation rate and asymptotic penetration. Note that this decision model does not take into account possible changes in mass marketing or prices of the service.

Homogeneous Decision Model with Normal Thresholds

Let us simulate the adaptation with homogeneous decision model, i.e., $g(N_i(t)) = N_i(t)$, and normally distributed thresholds, i.e., $C_i \sim \mathcal{N}(N_i, N_i)$ where N_i is the total number of i's friends. This model illustrates the decision model where the probability to subscribe is proportional to the number of friends subscribed, but the thresholds for subscribtions are normally distributed. The rate parameter $k = \frac{1}{1-\theta}$ is used to achieve asymptotic penetration $\max_t N(t) = \theta$. With initial penetration 0.01 and targeted asymptotic penetration $\theta = 0.8$, a network with $N = 500$ potential subscribers were simulated. The results of $K = 1000$ runs are combined in Figure 3.3. Some 900 simulations earned a penetration more than $\frac{\theta}{2}$. The distribution of the asymptotic penetration among those has an expectation of 0.84, deviation of 0.017 and skewness of -0.1.

The simulated growths as a function of the adaptation state $\rho(t) = \frac{N(t)}{\max_u N(u)}$ are drawn in Figure 3.4. As can be seen the maximum growth occurs with penetration more than 0.5. Actually, the expectation of the state for maximum growth is 0.63 with deviation 0.02 and skewness 0.2.

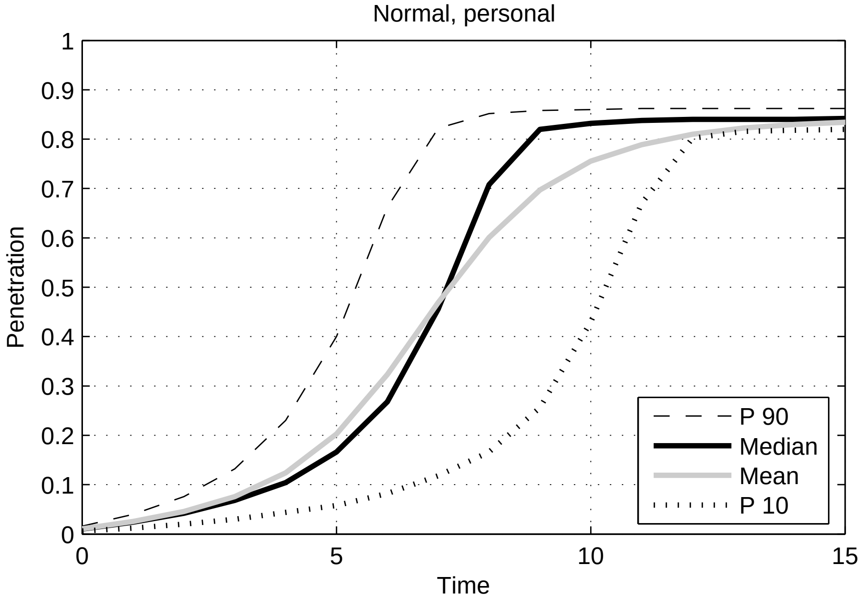

Figure 3.3: Simulated adaptation with homogeneous model and normal thresholds

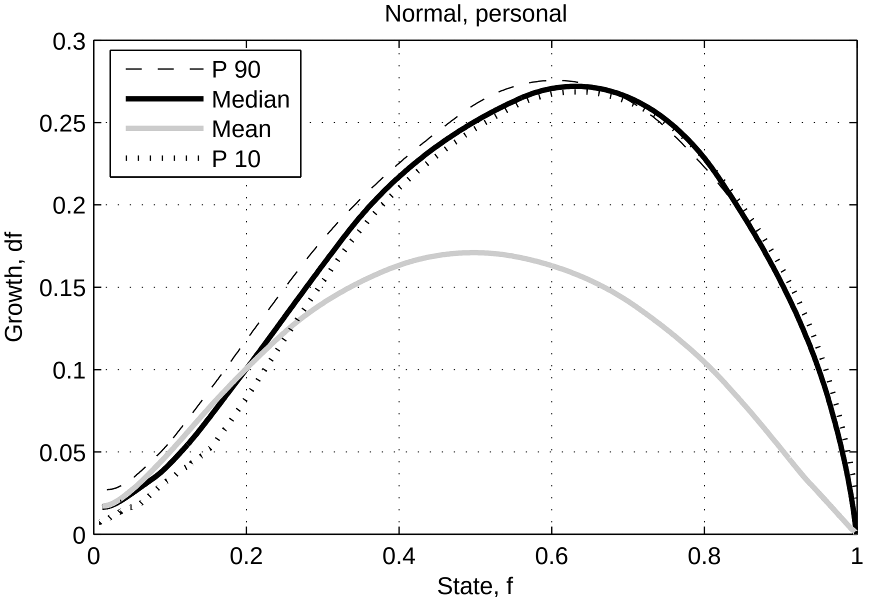

Figure 3.4: Adaptation rate as a function of the penetration

The simulations above had $g(N_i(t)) = N_i(t)$ and $C_i \sim \mathcal{N}(N_i, N_i)$. The results does not change much if the threshold distribution is not a function of the number of i's friends but a function on the average number of friends. See the results with $C_i \sim \mathcal{N}(\bar{N}_\cdot, \bar{N}_\cdot)$ in Appendix A.1. The same occurs with $g = N(t)$ and $C_i \sim \mathcal{N}(N, N)$, where $N(t)$ is the number of subscribers at time t.

Homogeneous Decision Model with Exponential Thresholds

Consider the homogeneous decision model as above, i.e., $g(N_i(t)) = N_i(t)$, and exponentially distributed thresholds, i.e., $C_i \sim \mathcal{E}xp(N_i)$. Now the rate parameter is $k = \frac{1}{\sqrt{1-\theta}}$ with $\theta = 0.8$. Out of the 1000 simulated adaptations for some 980 the penetration increased from 0.01 to $\frac{\theta}{2} = 0.4$ or more. See the simulated adaptations and their derivatives in Appendix A.2. The average of asymptotic penetration was 0.84 with deviation 0.03 and skewness -0.3. Now the maximum growth occur earlier, on average at state 0.53. The deviation was 0.04 and skewness zero. If the thresholds have identical distributions or the decision model is changed to be based on the overall subscribers, the simulation results are quite the same, see Appendix A.2.

Unit Decision Model

If the number of subscribers does not affect one's decision directly or indirectly, the growth is proportional only to the number potential subscribers left

$$\dot{\rho} = k(1-\rho). \tag{3.2}$$

Now the adaptation curve becomes exponential and approaches the level of 1. This model might be relevant for example in modeling the substitution of a service by a new superior technology, i.e., whenever subscriber is purchasing new service she chooses the one with new technology.

Linear Decision Model

Combining the unit and homogeneous decision models yields to linear, i.e., $g(N(t)) = aN + N(t)$. The simulations with $k = sqrt(1-\theta)$, $a = 1-\theta$ and exponential threshold distribution $C_i \sim \mathcal{E}xp(6,6)$ are drawn in Figure 3.5. With $k = a = 1-\theta$ and normal threshold distribution $C_i \sim \mathcal{N}(4,4)$, the adaptation is slightly different and growth maximum occurs later.

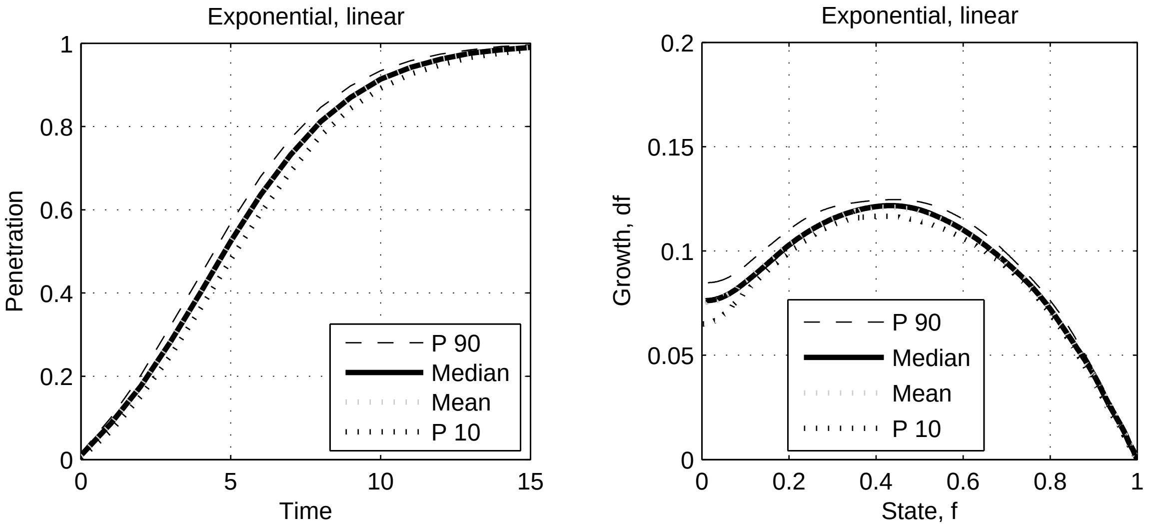

Figure 3.5: Simulated adaptation with linear model and exponential, changing thresholds

3.2.4 GOMPERTZ MODEL FOR PENETRATION FORECAST

The S-shaped curves like demonstrated in Section 3.2.3 are proven models for adaptation process [184]. When forecasting new technology adaptation, the Gompertz model have been used [180, 213]. Let $\rho = Y/L$ be the share of the subscribers Y per the asymptotic level L of the subscribers for the service and

$$\begin{aligned} \rho &= \exp[-\beta \exp(-kt)], && (3.3) \\ \dot{\rho} &= \beta k \exp[-kt - \beta \exp(-kt)] = k\rho \ln(1/\rho), && (3.4) \end{aligned}$$

where k is the growth rate parameter and β the state parameter of the curve.

Estimating the Parameters of Gompertz Curve

The growth rate of the Gompertz-curve (3.3) is proportional to the amount of subscribers and to $-\ln(\rho)$, see (3.4). The curve can be fitted on the penetration data available as in Figure 3.6, where the computer and broadband penetrations among Finnish households are illustrated.

When forecasting brand new services, there is no historical data of penetration. Still, corresponding or substituent services may be considered, and forecasting can be based on the penetrations of such services [166, 269]. The information in the adaptation of other technologies can be used in the forecasting of new technology manually or by using Bayesian updating as in [166]. Furthermore, the amount of subscribers after ten years is hard to forecast although we have historical data about the penetration. The amount of subscribers in the long run may be forecasted by comparing to other services or, for example, by using the Delphi-method [180, 213, 247].

Table 3.4: Some penetrations of home ICT [261, 262]

	1990	1994	1998	2000[a]	2001[a]	2002[a]	2003[a]	2004[a]	2005[a]	2006[a]
Computer	8	20	30	47	51	55	59	63	66	71
Internet		7	16	30	36	41	45	49	57	64
Broadband					2	8	16	26	42	55

[a] the fourth quarter of the year

Let us make forecasts of subscribers based on historical data. Consider a minimum *mean square error* (MSE) problem

$$\min_{L,k,\beta} \Sigma_i (\hat{Y}_i - \rho_{k,\beta}(t_i)L)^2, \qquad (3.5)$$

where $\hat{Y}_i$ are the observed penetrations, $\rho_{k,\beta}(t_i)$ is the predicted adaptation model and L the asymptotic penetration. By using nonlinear programming the parameters L, k and β can be found. The Gompertz equation (3.3) can also be transformed into the time-wise linear equation

$$\ln(\ln(1/\rho)) = \ln(\beta) - kt.$$

Thus, the parameters can be estimated with a linear model and the minimum square error method. Note that because of the logarithmic transformation, the solution is not the minimum square error solution for the original optimization problem (3.5). In summary, using data from [144] we get the forecasts on the computer and broadband penetrations as shown in Figure 3.6. The asymptotic penetrations are assumed to be 90%. The estimated parameters with zero time at year 2000, i.e., time $\hat{t} = t - 2000$, are as shown in Table 3.5. The optimized parameters give asymptotic penetrations $L > 100$ (%), which could not be achieved. This indicate that the the number of data points is quite small, or the models are not practical for this case. The optimized adaptation curve in Figure 3.6 is the solution for the MSE problem (3.5) with constraint $L \leq 100$. The fitted curve tends to be underestimating the forecasted computer penetration. The poor fitting indicates that Gompertz does not describe the computer home penetration in Finland so well.

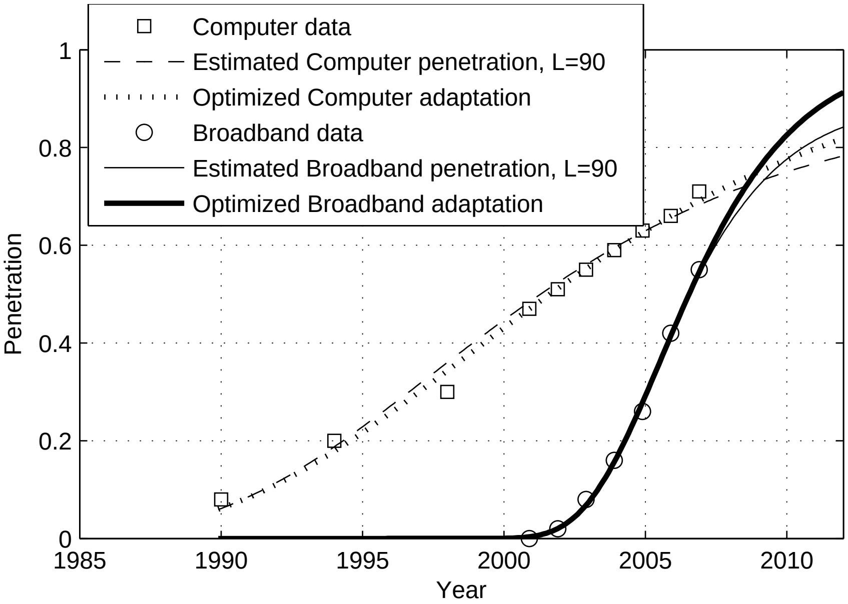

Figure 3.6: Gompertz estimates on the computer and broadband penetrations in Finland [261, 262]

Table 3.5: Estimated parameters for Gompertz model

	L	k	β
Computer Penetration	90 [a]	0.1345	0.700
MSE Computer Adaptation	100 [b]	0.1195	0.842
MSE Computer Adaptation	132	0.0897	1.144
Broadband Penetration	90 [a]	0.4006	8.139
MSE Broadband Adaptation	100 [b]	0.3720	7.906
MSE Broadband Adaptation	126	0.3091	6.974

[a] assumed value

[b] bounded value

3.2.5 FISHER–PRY MODEL FOR PRODUCT SUBSTITUTE

When modeling the substitution of a technology by another superior technology, a Fisher–Pry model have been used [180, 213]:

$$\rho = \frac{1}{2}(1+\tanh(c(t-t_0))), \tag{3.6}$$

$$\dot{\rho} = \frac{c}{2\cosh(c(t-t_0))} = 2c\rho(1-\rho), \tag{3.7}$$

where c is the shape parameter of the curve and t_0 the time for 50% substitution. The time differential of the curve is proportional to the subscribers already using the service and to the number of subscribers not yet served, see (3.7). The shape of the Fisher–Pry curve is symmetrical, see Figure 3.7. It is actually equivalent to the Pearl curve [180]

$$\rho = \frac{1}{1+\beta\exp(-kt)}, \tag{3.8}$$

where β and k (and L) are the parameters of the curve.

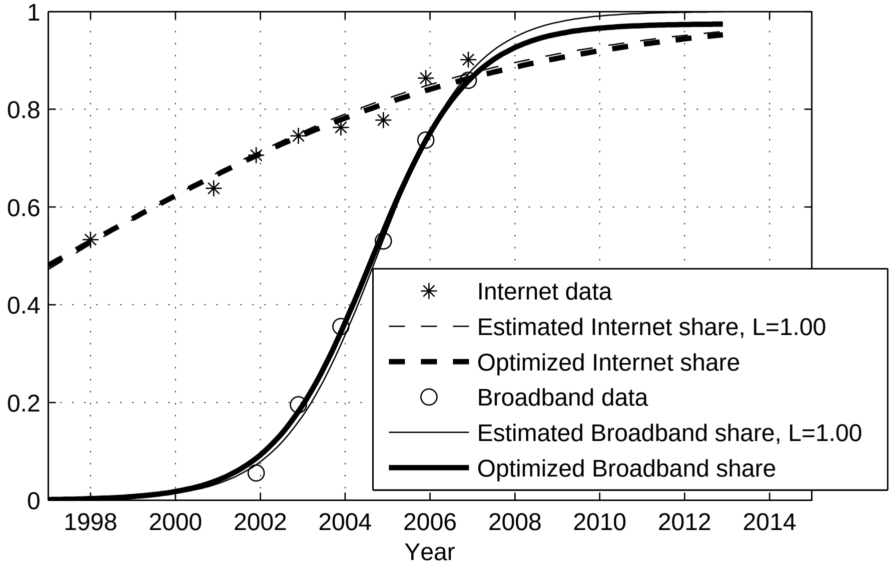

Figure 3.7: Fisher–Pry estimates on the share of the Internet and broadband connections in Finland [262, 261]

The Internet connection cannot be used without some kind of computer. Let us thus model Internet adaptation inside the computer adaptation by Fisher–Pry model. Moreover, the broadband usage can be modeled as a substitute for Internet. The Fisher–Pry model (3.6) traditionally forecasts the substitution of a technology. The equation (3.6) is equivalent to

$$\ln\left(\frac{\rho}{1-\rho}\right) = 2ct - 2ct_0,$$

which is linear equation as a function of time. If the minimum MSE line for $\ln(\frac{\rho}{1-\rho})$ versus time is $y = At+B$, the parameters for the Fisher–Pry model on the data can be estimated as $c = A/2$

Table 3.6: Estimated parameters for Fisher–Pry model

	L	c	t_0
Internet (for computers)	1.00 [a]	0.1027	1997.5
MSE Internet Adaptation	1.20	0.0735	1999.8
MSE Internet Adaptation	1.00 [b]	0.0968	1997.4
Broadband Share	1.00 [a]	0.4468	2004.8
MSE Broadband Share	0.975	0.4333	2004.6

[a] assumed value

[b] bounded value

and $t_0 = -B/A$. Thus, using data from [261, 262], the Internet adaptation and the fraction of Internet subscribers that use broadband can be forecasted. The parameters found are as in Table 3.6 and the adaptation curves are drawn in Figure 3.7. The maximum share for the estimates are assumed to be 100%. The optimized models are estimated using nonlinear programming for the minimum MSE problem

$$\min_{L \leq 1.0, c, t_0} \Sigma_i(\hat{Y}_i - \rho_{c,t_0}(t_i)L)^2. \tag{3.9}$$

The models in Figure 3.7 suit in the data poorly. This is partly because of the changes in the marketing and circumstances of the services, but the models might be wrong to the case, too.

The forecasts for Internet penetration and broadband usage are combined in Figure 3.8. The fitted curves for Internet penetration seems to be underestimating the growth rate of the penetration. The wrong dynamics of the model may yield for example from the unheeded price erosion in subscription tariffs (see Section 3.3.1) or the simultaneous adaptations of computers and Internet. The direct fitting of the Gompertz model seems to be more acceptable for the data, see thick dark curve in Figure 3.8.

The Comparison of Gompertz and Fisher–Pry Dynamics

The dynamics of the Gompertz curve differs a bit from the dynamics of the Fisher–Pry curve. Recall that the time differential of Gompertz curve is proportional to $\rho \ln(1/\rho)$ and the time differential of Fisher-Pry is proportional to $\rho(1-\rho)$. The growth of the Gompertz curve is clearly greater than ρ when ρ is small. Meanwhile, Fisher–Pry grows proportional to ρ, see Figure 3.9. Since $\ln(1/\rho) = (1-\rho) + \frac{(1-\rho)^2}{2} + \frac{(1-\rho)^3}{3}...$, both models tend to grow proportionally to $1-\rho$, when the service under consideration has quite a much subscribers. In summary, when modeling data and forecasting, the substantial difference one should notice is the fact that the growth of the Gompertz curve is greatest when $0.3 < \rho < 0.4$ while Fisher–Pry is symmetric and thus the greatest growth occurs at $\rho = 0.5$. The Fisher–Pry model is a kind of analytical formulation of the simulated adaptation model in social network with homogeneous decision model and exponential thresholds, see Section 3.2.3.

The dynamics of the adaptation models have also been analyzed using hazard functions. It points out the probability for a potential subscriber to subscribe as a function of the number of subscribers adopted and it can be defined as

$$H(\rho, t) = \frac{\dot{\rho}(t)}{1-\rho(t)}.$$

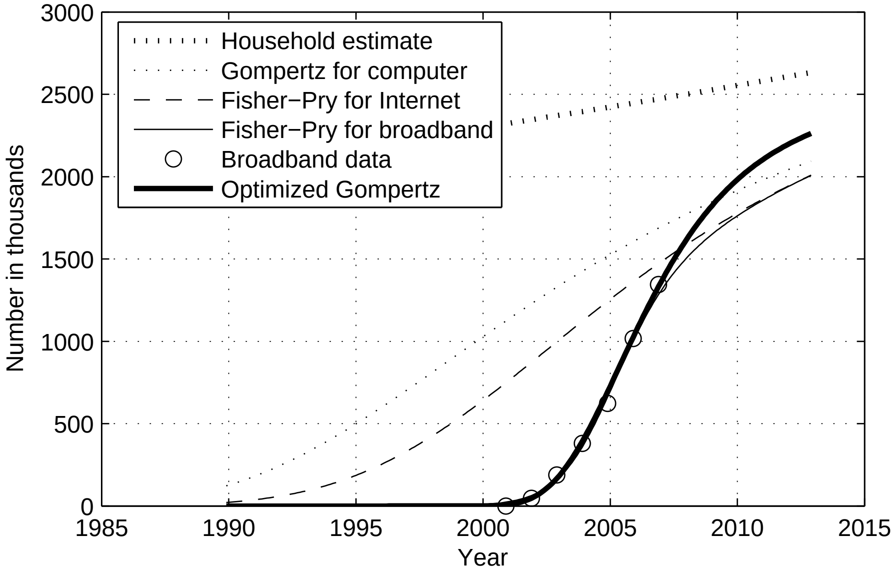

Figure 3.8: Estimates on the broadband and Internet subscribers in Finnish households [262, 261]

The hazard rate of Fisher-Pry model is $k\rho$ and for Gompertz model it is proportional to $\rho + \frac{\rho(1-\rho)}{2} + \frac{\rho(1-\rho)^2}{3} \ldots$. See Figure 3.9 for hazard rate illustration.

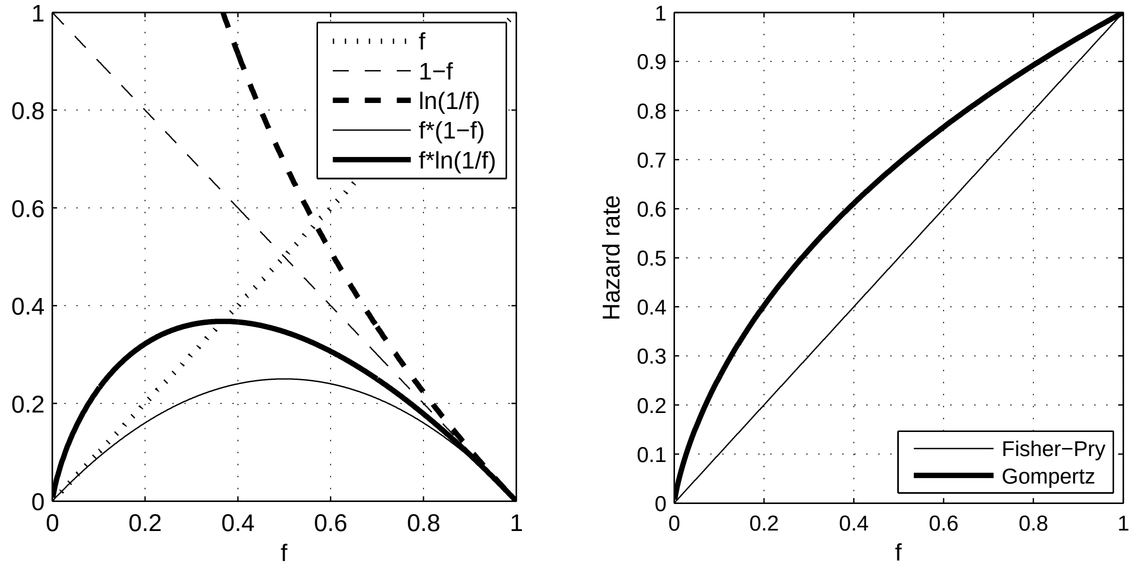

Figure 3.9: The dynamics of the Gompertz and Fisher–Pry curves

3.2.6 Richards Generalized Model

Richards introduced a generalized growth model [224]. It is sometimes called von Bertalanffy–Richards model since it covers also a family of models by von Bertalanffys. The model mainly used in biological growth modeling states [24, 90, 170, 224]

$$\rho = \frac{1}{(1+\beta\exp(-kt))^{\frac{1}{b}}}, \tag{3.10}$$

$$\dot{\rho} = \frac{k}{b}\rho(1-\rho^{b}), \tag{3.11}$$

where $\rho(t) = Y(t)/L$, β is a parameter for the current state of the adaptation, k rate parameter, and b a shape parameter.

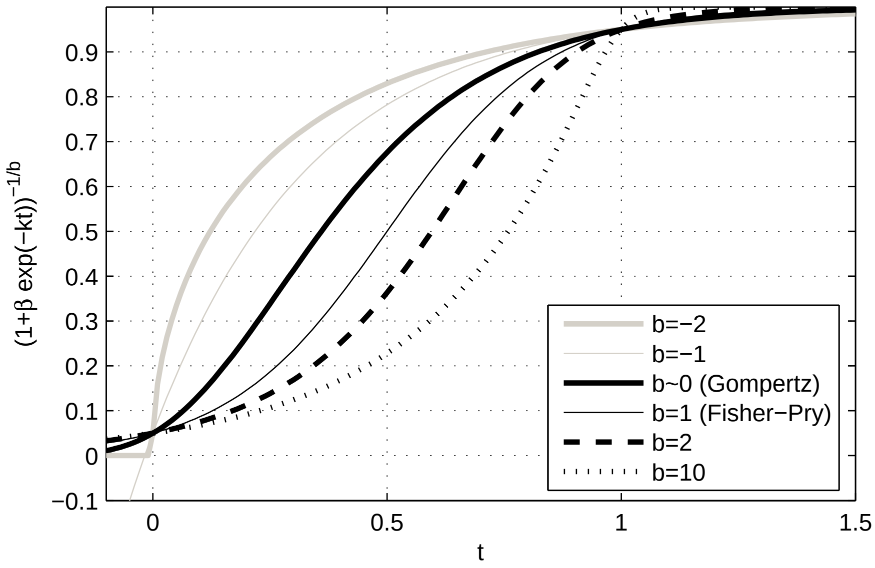

Figure 3.10: A serie of Richards models

Gompertz and Fisher–Pry models are special cases of Richards model [24, 170]. The model approaches Gompertz as b approaches zero. The parameters with $b = 1$ define Fisher–Pry models, see equations (3.6) and (3.8). The Richards model is S-shaped or concave depending on the parameters [170]. For typical growth modeling, the rate k and asymptotic level L are positive and the sign of β equals the sign of b. With $b \geq 0$ the curve has asymptotic levels 0 ($t \to -\infty$) and L ($t \to \infty$), and inflection point at $\rho = 1/(1+b)^{\frac{1}{b}}$ thus being S-shaped. For $-1 < b < 0$ the curve is S-shaped with upper asymptotic level L, but the function is zero at $t = \frac{\ln -\beta}{k}$ and not defined for time before that. For the case $b \leq -1$ the curve is concave with zero point at the mentioned time value. See Figure 3.10 for a series of Richards models with different values of b. The models are scaled to have $L = 1$ and $\rho(0) = 0.1$ and $\rho(1) = 0.9$, i.e., $\beta = \rho(0)^{-v} - 1$ and $k = -\ln\left(\frac{\rho_1^{-v}-1}{\beta}\right)$.

The dynamics for the Richards growth models with different values of parameter b is drawn in Figure 3.11. The smaller b means that the maximum growth rate appears earlier. More accurately, the growth maximum appears at state $\rho = 1/(1+b)^{\frac{1}{b}}$. The hazard function for Richards model with natural number b is $\frac{k}{b}(\rho + \rho^2 + \ldots + \rho^b)$.

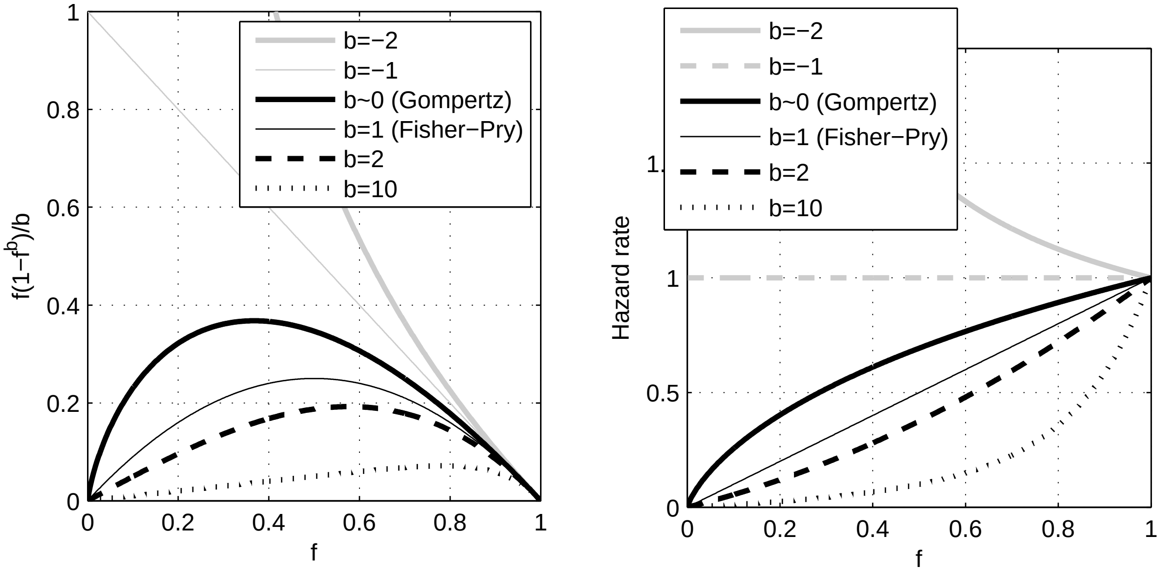

Figure 3.11: The dynamics of the Richards model

Estimating the Parameters of Richards Model

The estimation of the parameters of different growth models is studied in [90, 216]. The observation error may be assumed to be log-normal as in [216]. The problem yielding is solvable and the confidence intervals for the parameters is produced by bootstrapping. This method is practical for cases with much data points, which is not achieved in technological innovations. Instead, the

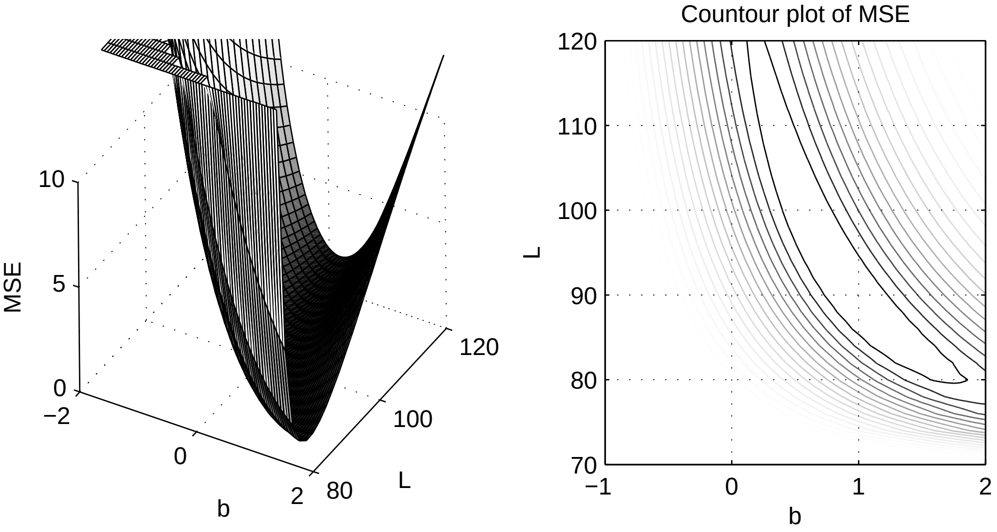

Figure 3.12: Mean square error of the Richards estimate for computer adaptation

problem can be formulated as minimum mean square error problem

$$\min_{L,b,k,\beta} \Sigma_i(\hat{Y}_i - \rho_{b,k,\beta}(t_i)L)^2. \tag{3.12}$$

This can be solved using nonlinear programming with some starting values for parameters [90]. Practically, local search algorithms typically could not step over parameter value $b = 0$ for (3.12). Thus, at least two searches with different starting values must be run for one problem.

The model equation (3.10) can be transformed to linear equation using logarithm, i.e.,

$$\ln \frac{\rho^b}{1-\rho^b} = kt - \ln \beta. \tag{3.13}$$

This linearization can be used to get a rough estimate for the parameters. Note that the solution for the minimum MSE problem with given L, b does not have to be optimal solution for (3.12). Equation (3.13) is only used if the asymptotic level L and shape parameter b are forecasted using other techniques or the solution space is raked for a good starting value for the local search.

Consider the home ICT data stated in Section 3.2.4 and Section 3.2.5. Let us first estimate the computer and broadband penetrations directly. Some values of MSE for the subproblem solutions (3.13) with different values of L and b are in Figure 3.12. In the right hand side of the figure, contour curves are drawn to illustrate the states of equal MSE. The figure shows that the optimal solution for (3.12) have $b > 0$ and good initial guesses may be found from $b = 0.5, L = 110$ to $b = 1.5, L = 85$. Similarly for the broadband penetrations we get Figure 3.13. Now the minimum MSE values occur near $b = 0$ and starting values from $b = -0.2, L > 120$ to $b = 0.3, L = 95$ should be used for nonlinear programming methods. The optimized parameters can be found in Table 3.7 and model curves in Figure 3.14. Solved models are solutions for the problem (3.13), i.e., with $L = b = 1$ the solutions are equivalent to Fisher-Pry models, and MSE refers to the solution of the problem (3.12). Since the asymptotic level L and the model shape b correlate negatively and they partly substitute the effects of each other, the constraint $L \leq 100$ does not change the value of the minimum MSE dramatically. Because of the simpler and more realistic models, the constrained versions are preferred.

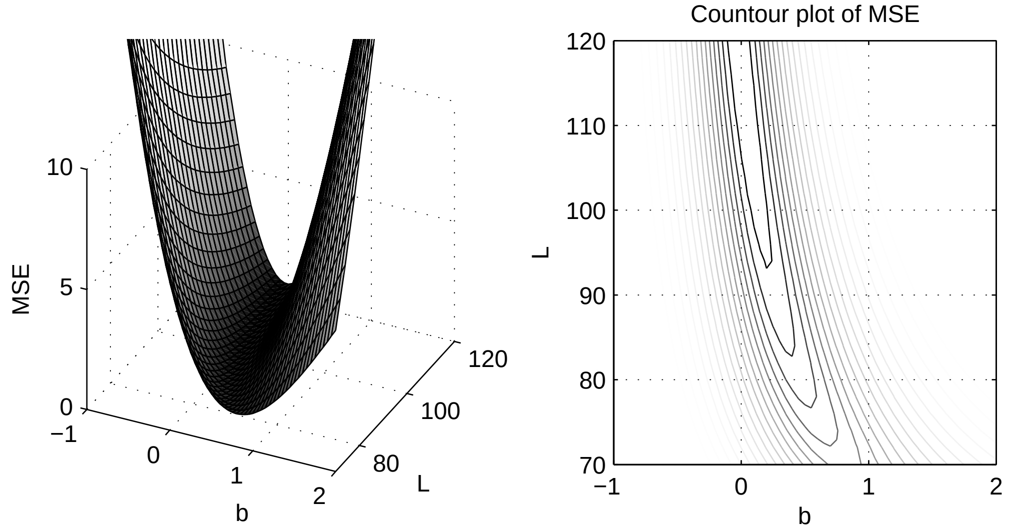

Figure 3.13: Mean square error of the Richards estimate for broadband adaptation

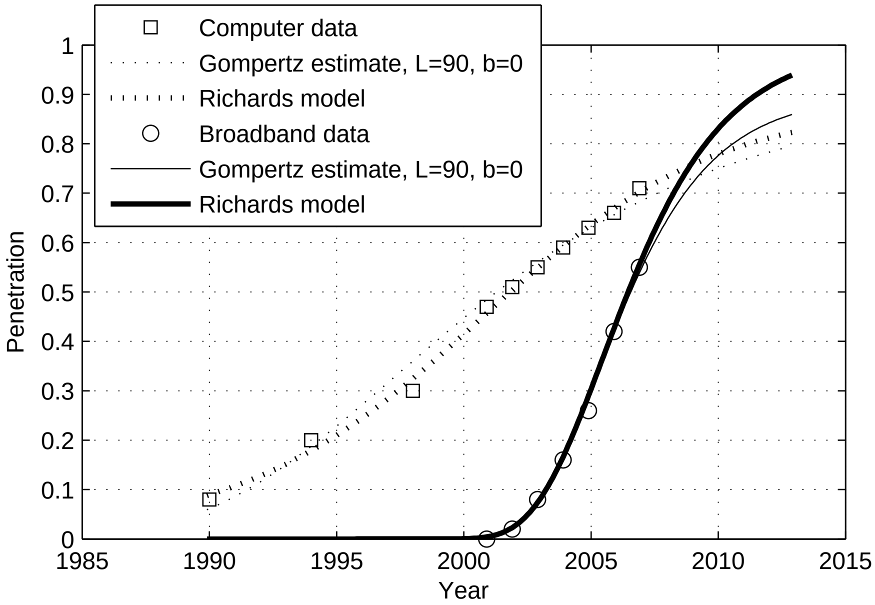

Figure 3.14: Richards estimates on the computer and broadband penetrations

Table 3.7: Estimated parameters for Richards model

	L	b	k	β
MSE Computer Adaptation	87.8	1.146	0.2230	1.362
MSE Broadband Adaptation	100 [b]	0.0016	0.3724	0.0123
MSE Broadband Adaptation	129	0.0017	0.3037	0.0117
MSE Broadband Adaptation	143	-0.099	0.2598	-0.543
Solved Internet Share	1.00 [a]	1.00 [a]	0.2054	0.603
MSE Internet Share	1.00 [b]	2.35	0.2587	2.193
MSE Internet Share	1.31	0.515	0.1131	0.485
MSE Internet Share	2.19	-0.636	0.0353	-0.555
Solved Broadband Share	1.00 [a]	1.00 [a]	0.8935	69.78
MSE Broadband Share	1.00 [b]	0.056	0.5746	0.549
MSE Broadband Share	1.17	0.071	0.4649	0.541
MSE Broadband Share	1.40	-0.283	0.3088	-1.064

[a] assumed value

[b] bounded value

The Internet subscription share among home computer owners and the share of broadband subscriptions were modeled using Fisher-Pry in Section 3.2.5. To make Richards estimates, let us first analyze the contour plots of the MSE for solutions of subproblems (3.13) in Figure 3.15. The proposed starting values for the optimization algorithms are from $b = -0.5, L > 1.6$ to $b = 0.5, L = 1.3$ for Internet share and from $b = -0.5, L > 1.6$ to $b = 0.2, L = 1.1$ for broadband share. The optimized parameters are in Table 3.7 and curves in Figure 3.16. Richards model has more flexibility than Fisher-Pry and thus it suits better in data. The optimized Richards models are combined in Figure 3.17. The constraint $L \leq 1.0$ does not affect on the level of minimum

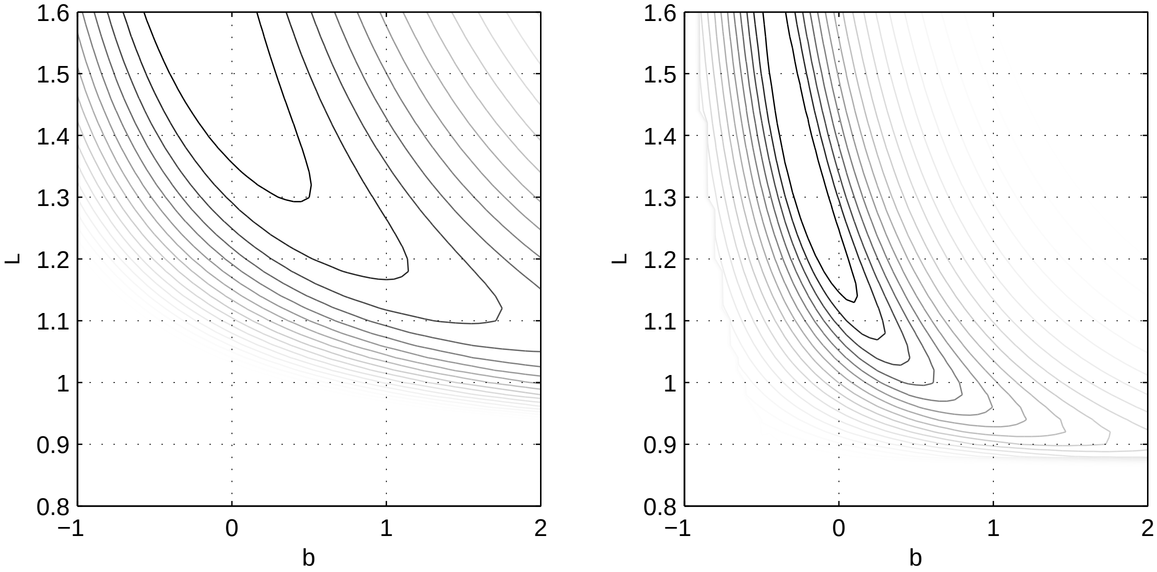

Figure 3.15: Contour plots for the MSEs of Richards estimates for Internet and broadband share

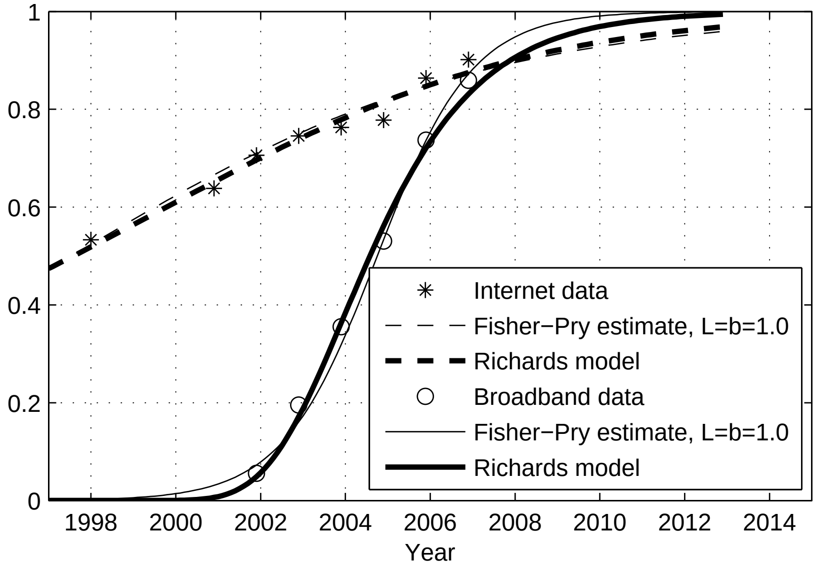

Figure 3.16: Richards estimates on the share of the Internet and broadband connections

MSE for Internet share models, but for the broadband share models the minimum MSE triples. The minimum MSE levels for cases $b < 0$ and $b > 0$ equal.

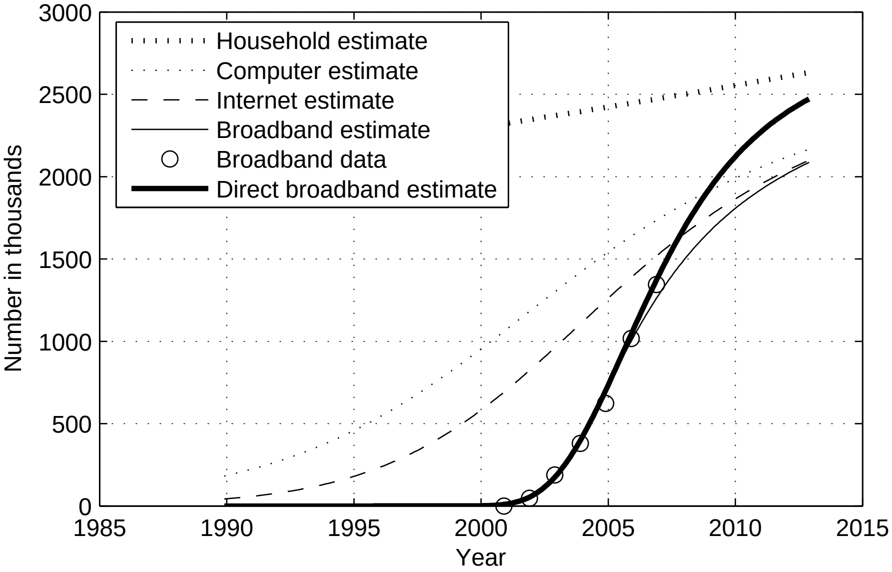

Figure 3.17: Richards estimates on the broadband and Internet subscribers in Finnish households

3.2.7 BASS MODEL

Bass model was introduced in [18, 19, 20]. It defines a series of models from S-shaped logistic to exponential model. Thus it has some coincidences with the Richards model but the dynamics are different. The model for the adaptation is

$$\rho = \frac{1-\exp(-k(1+p)(t-T_0))}{1+\frac{1}{p}\exp(-k(1+p)(t-T_0))}, \tag{3.14}$$

$$\dot{\rho} = k(p+\rho)(1-\rho), \tag{3.15}$$

where the shape parameter is $p > 0$, rate parameter $k > 0$, and T_0 is the time when the growth starts. The model approaches Fisher-Pry as $p \to 0$ and exponential as $p \to \infty$. See Figure 3.18 for a series of Bass models. The growth rate is drawn in Figure 3.19. Note that the hazard function is linear equation $k(p+\rho)$. Bass model is analytical formulation of the simulated network adaptation with linear decision models, see Section 3.2.3.

Estimating the Parameters of Bass Model

The parameters of the Bass model can be estimated similarly to Richards model. The MSE optimization problem is defined as

$$\min_{L,p,k,T_0} \Sigma_i(\hat{Y}_i - \rho_{p,k,T_0}(t_i)L)^2. \tag{3.16}$$

With given L and p, the model can be linearized as

$$\frac{1}{1+p}\ln\frac{1+\frac{\rho}{p}}{1-\rho} = kt - kT_0. \tag{3.17}$$

Equation (3.17) is used to make a Bass MSE contour plot. Figure 3.21 indicates that the optimal solution for problem (3.16) with computer penetration data have parameter values from $L = 90$,

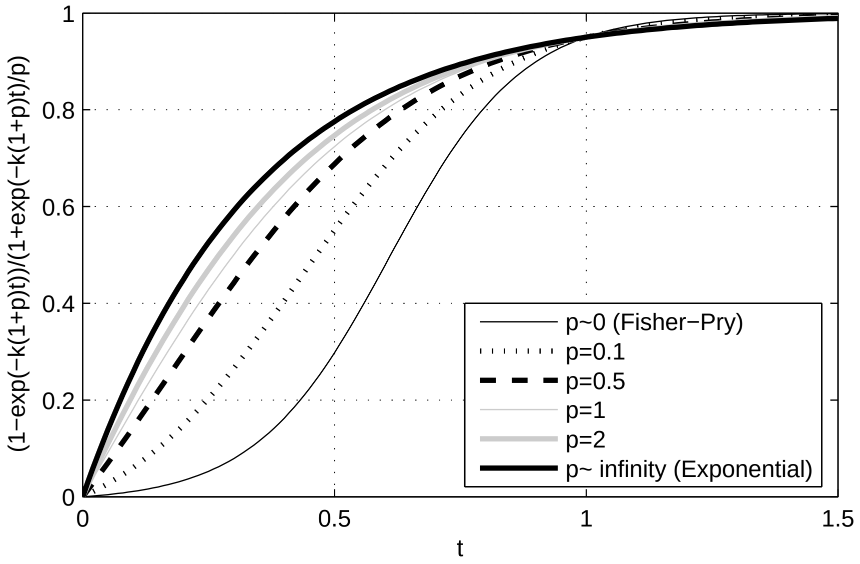

Figure 3.18: A serie of Bass models

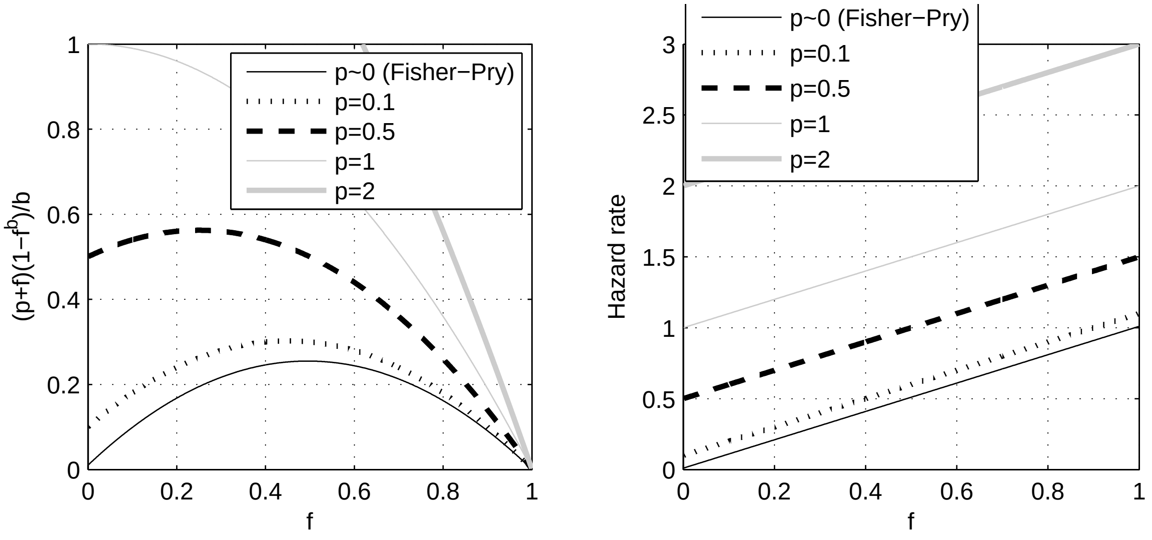

Figure 3.19: The dynamics of the Bass model

$p = 0$ to $L = 100$ to $p = 0.05$. Using these as starting values for the nonlinear programming, parameters as in Table 3.8 are found. The curve with these parameters is drawn in Figure 3.20. Similarly with starting values from $L = 80$, $p = 0.04$ to $L = 110$, $p = 0.07$ we find optimal solution for broadband penetration data, see Figure 3.23.

The contour plots for the MSE of Bass estimates for Internet and broadband data are drawn in Figure 3.22. The left curve indicate that the model is not suitable for the Internet share. The exponential or linear model would fit as well to the data. Using the starting values from $L = 1.05$, $p = 0.1$ to $L = 1.3$, $p = 0.25$, the broadband share optimum model can be found as in Table 3.8. The solutions with constraints $L \leq 100$ are modeled in Figure 3.20.

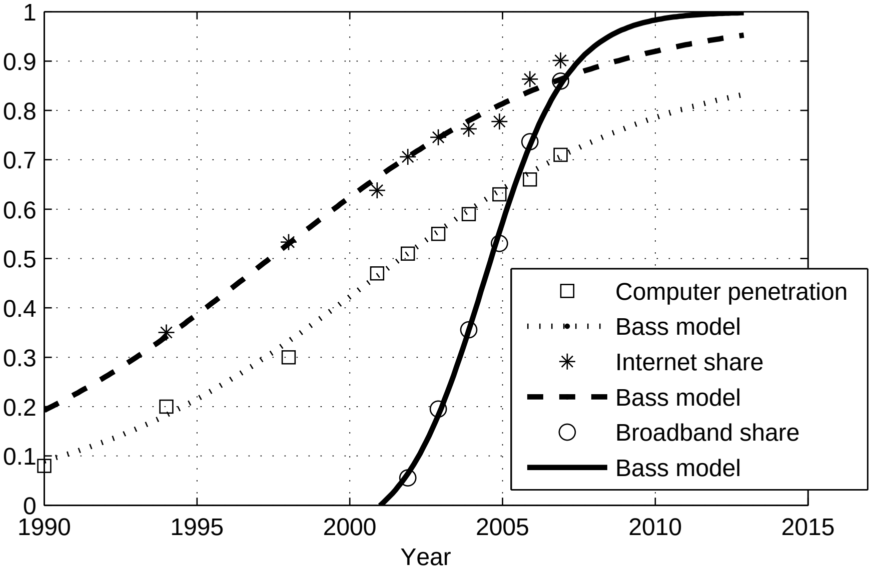

Figure 3.20: Bass computer penetration, Internet share and broadband share estimates

Table 3.8: Optimized parameters for Bass model

	L	p	k	T_0
MSE Computer Adaptation	89.7	0.0017	0.2064	1969.7
MSE Broadband Adaptation	86.2	0.048	0.6100	2001.2
MSE Internet Share	1.00 [b]	0.0029	0.1926	1967.1
MSE Broadband Share	1.00 [b]	0.0768	0.6931	2001.0
MSE Broadband Share	1.19	0.1828	0.4444	2001.4

[b] bounded value

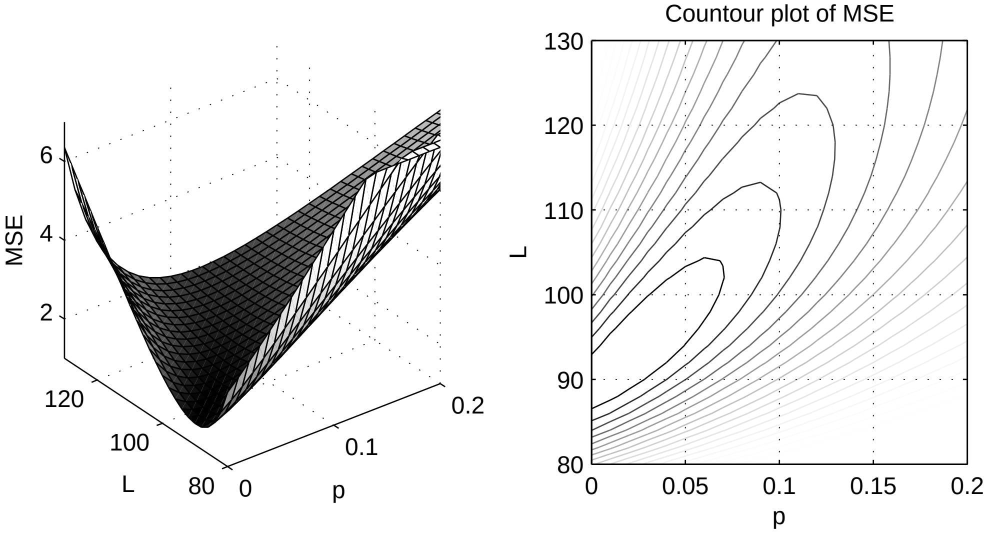

Figure 3.21: Mean square error of the Bass estimate for computer adaptation

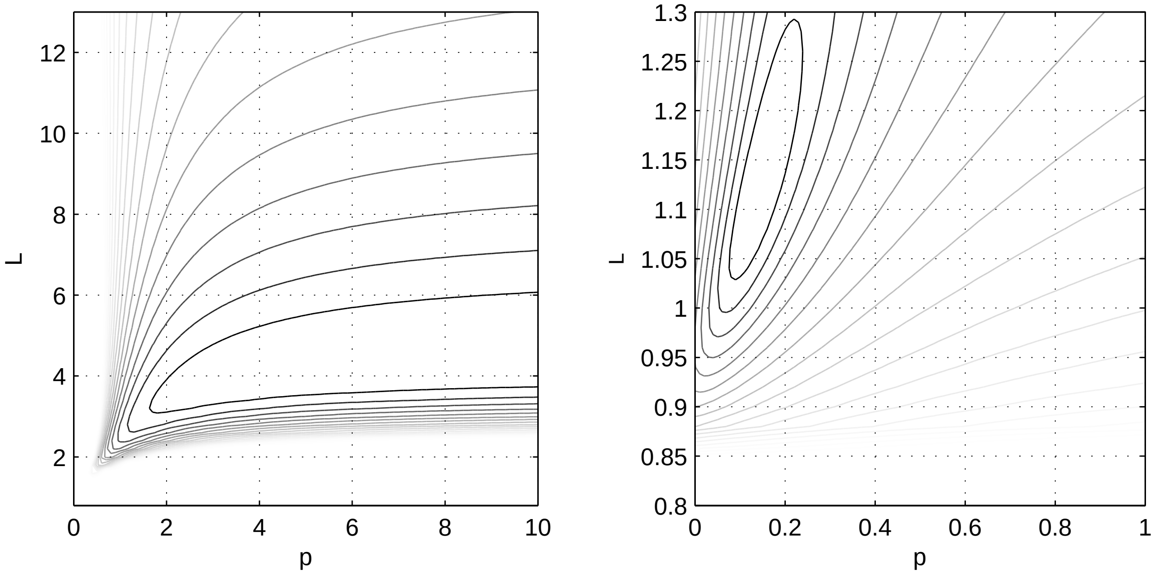

Figure 3.22: Contour plots for the MSEs of Bass estimates for Internet and broadband share

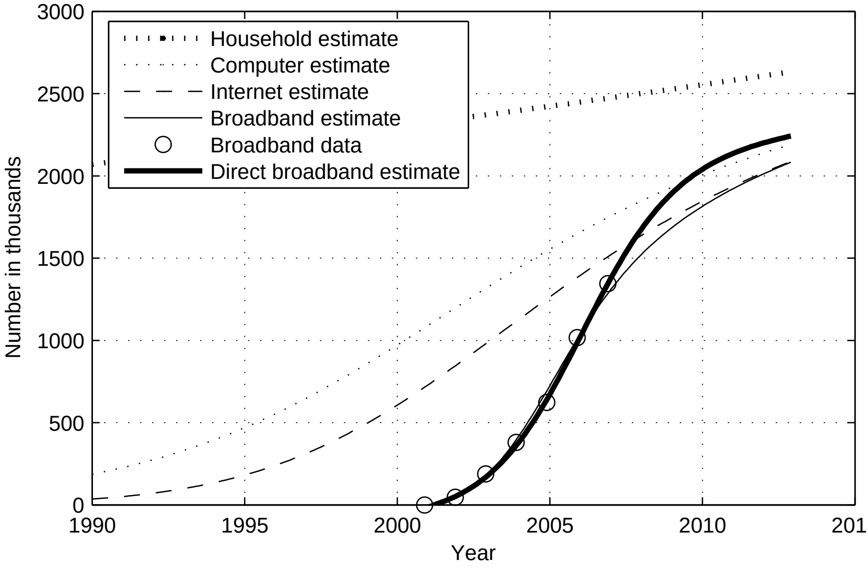

Figure 3.23: Bass estimates on the broadband and Internet subscribers in Finnish households

3.3 Broadband Markets

Figure 3.24 shows the development of broadband subscribers in Finland. The figures include all subscribers, e.g., households, communities, and companies. For a wide introduction in broadband markets, see [248].

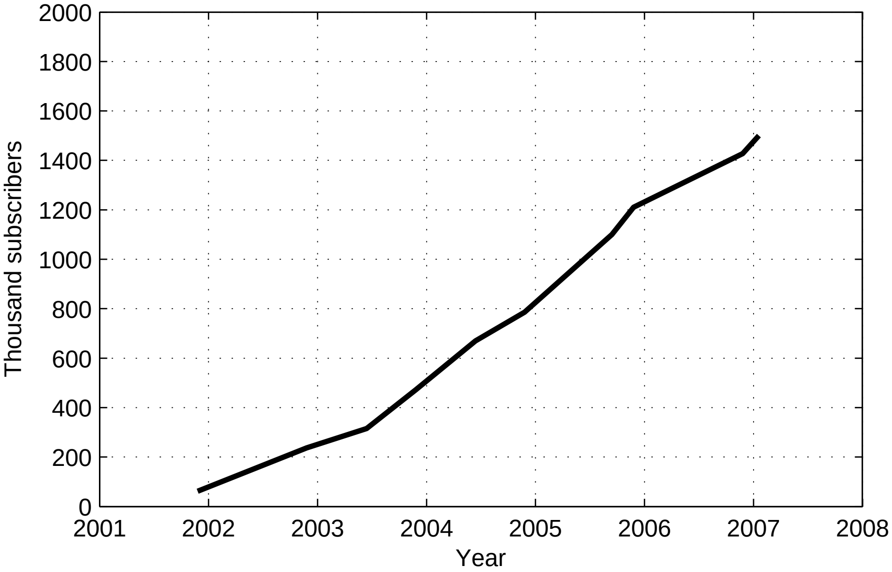

Figure 3.24: Broadband subscribers in Finland [218, 261]

3.3.1 Revenue Modeling

The demand for broadband connection increases when the price decreases. The number of subscribers and the price of the services are nearly inversely proportional to each other. Studies [189, 190, 191] show that the broadband demand curve within the corporate segment is exponential. If x is the number of subscribers and p is the price of the service,

$$\begin{aligned} x &= Ap^b, \\ \ln x &= \ln A + b \ln p, \end{aligned} \tag{3.18}$$

where A and b are parameters of the model. Historical or market survey data can easily be estimated using this model and linear regression.

The exponential model is used in [252, 254, 256]. The model

$$\begin{aligned} x &= \exp[(\alpha + \beta p)^\gamma], \\ \ln x &= (\alpha + \beta p)^\gamma, \end{aligned} \tag{3.19}$$

has three parameters α, β, γ, which can be estimated using the least square method. This model is not linear and cannot be transformed to linear. Thus, nonlinear programming must be used. Even a more complex model for residential subscribers was established in [157]. It uses discrete variables for different ages, areas and income levels, and estimates the demand curve.

The estimated demand curve is used to forecast the corresponding price evolution of the services. If a penetration forecast is available, the price of the service with models (3.18) and (3.19) are

$$\begin{aligned} p &= (x/A)^{1/b}, and \\ p &= \frac{(\ln x)^{1/\gamma} - \alpha}{\beta}, \end{aligned}$$

respectively.

The broadband subscription tariffs in Finland are two-folded. Firstly, the starting fee is used to cover the costs of setting the channel active. Secondly, the monthly fee yields a profit after running costs to cover the investment costs. Though the starting fees are three to six times the monthly fees, the starting fees are usually not gathered but the opening of the channel are offered as free of charge.

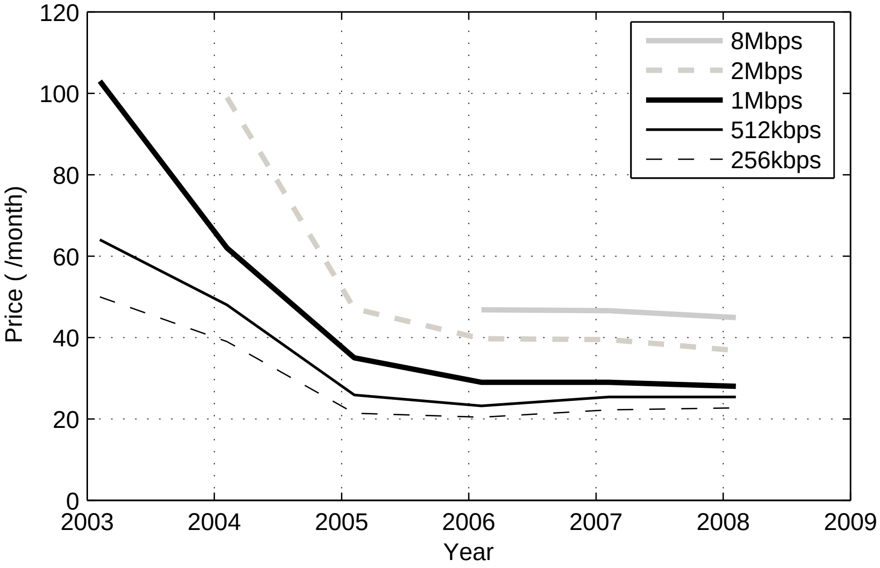

Figure 3.25: Broadband subscription prices in Finland [145, 146, 147, 148, 149]

The Figure 3.25 display the developments of the average monthly fees in Finland. The prices are based on the studies [145, 146, 147, 148, 149]. The service classes are 256 kbps, 512 kbps, 1 Mbps, 2 Mbps and at least 8 Mbps for theoretical downlink data rate. The uplink data rates are 512 kbps except 256 for the lowest and 1 Mbps for the highest service class. As can be seen, the prices went down from 2003 to 2005. After the year 2006 the level of the broadband tariffs have been stabilized. The development of the subscriber prices and costs in Finland have also been studied in [13, 159]. It is stated that prices have quite small effects in broadband subscription of households [50, 143].

It is convenient to model the revenues by estimating *average revenue per user* (ARPU). ARPU multiplied by the number of subscribers gives an estimate on the total revenues for operator. The ARPU is more stable than a single service prices and it can be modeled as a function of time

$$p_S(t) = p_S(0)K_S^t = p_S(0)e^{-c_S t}, \tag{3.20}$$

where $p_S(0)$ is ARPU at time 0 and K_S (and c_S) the erosion parameter.

3.3.2 Competition

The supply side of the broadband markets is strictly based on technological development. Especially, the connection bandwidth grows rapidly. The suppliers vary based on technological changes. The changes make broadband business live and may help newcomers. On the other hand, some technologies are based on the old telephone lines, i.e., subscriber lines. This gives a significant market power to those traditional network operators possessing the lines. They are called *incumbent operators*. New technologies are providing other routes for services too, e.g., radio frequency. In addition to commercial operators, there are different associations and cooperative societies in Finland that are also broadband service operators.

3.3.3 Other Communication Services

The broadband connection can be seen as a substitute for narrow-band Internet connection. The substituent effects have been studied earlier [75, 78, 91]. The cross price elasticities between broadband and narrow-band have been analyzed and the results support them being substituents for each other [91]. Moreover, the price of the broadband subscription fee seems to be significantly affecting the decision to upgrade or not from narrow-band [91, 282]. The broadband subscription also affects on the consumption time and content utilized in Internet [120].

Nowadays, some mobile phones include broadband access premise equipments. However, the charging of mobile broadband is often based on the amount of transferred data. The definition described in Chapter 2 means that broadband connection is always-on and charged monthly. This definition does not include all mobile phone users, but it must be noted that new mobile standards make mobile broadband possible.

The mobile phones and mobile communication services have broken through in developing countries and little by little all over the world. Studies on the price elasticities and demographic issues for mobile should be considered when analyzing broadband access [95, 183, 221]. However, the differences and similarities must be examined with care. It is shown that the mobile is a substitute for wired telephone, but the opposite is not clearly true [94, 95]. This kind of asymmetry exists and is relevant especially in developing countries.

3.3.4 Regulation and Public Authorities

The communication business has been highly regulated but is being liberated step by step in many countries. In Finland, one public authority served as operator until liberalization started in the 1990s. Still, state partly owns a mobile operator and a significant local telephone operator, but fully private companies exist, too.

The role of governmental and municipal ownership is discussed widely, for example, in the USA [108, 267]. Public ownership is supported, because communications is seen as a public service. Opponents call for open communication markets and argue that private companies are more efficient than public actors. In addition to direct ownership, different subsidies by local and governmental authorities are paid all over the world. For example in Sweden, broadband projects are sponsored up to SEK 5.25 billion in total, i.e., some EUR 550 million [83, 240]. As a result, there were residential connections up to 1000 Mbps in southern Sweden in 2005 [161]. Kidokoro has studied the Internet connection market and welfare issues in [151]. He states that for high Internet penetrations both the competition policy could lead to maximum welfare with market price equal to marginal cost. The same could be achieved by regulated monopolic policy. However, for low penetration markets the social welfare maximum is not achieved without subsidies to operators or customers [151].

Regulation in communication business is done on three main levels. The first level is general competition regulation which is done in every business sector [134]. However, because of the nature of networking and because the open competition in communication business has so short history, there exist many local monopolies [122]. This makes the role of the regulator in communications bigger than in other businesses [192]. Another level of regulation is service controlling, which include consumer protection and quality controlling. The third level of regulation is the network operator specific. This includes for example security issues and resource specific issues, e.g., radio bandwidth. Further information on telecommunications regulation can be found in [154, 192, 272]

3.4 COMBINED DEMAND AND REVENUE MODEL FOR FINNISH HOUSEHOLDS

The adaptation of home ICT has been rapid process in Finnish households. The process can be seen to contain three partly overlapping phases. Firstly, the introduction and penetration growth of personal computers begun before the year 1990. The Internet became popular during the next decade 1990-2000 and broadband penetration has grown in the last years, after 2001. These three phases are so simultaneous that they cannot be taken fully apart from each others. The early adaptation of computers is quite a typical product adaptation process, but nowadays Internet and broadband subscriptions play a significant role in home computer adaptation as well. Moreover, the adaptation of broadband begun as a service substitute process over Internet but latterly the adaptation is more related to the home computer adaptation. In summary, the product adaptation process of computers and the service adaptation of Internet have escalated to a single product-service adaptation. This explains partly why the traditional S-shaped curves does not fit in the Finnish home ICT penetration data.

The amount and quality of services in the Internet has increased as the penetration of Internet subscribers grows. The penetration of broadband connections for market with relatively high initial Internet penetration may differ from the traditional S-shaped curve for new technology penetration. The rate of the penetration growth tend to be higher than the traditional model would express. This is because the broadband subscriptions can be updated quite easily from the plain Internet subscription without large investments. On the other hand, as the broadband adaptation process is more in combination with computer adaptations, the penetration growth is still moderate. We prefer to model the adaptation of the broadband subscriptions using Richard's model (3.10). It is flexible and yet practical. The number of subscribers is modeled in this study with equation

$$N(t) \quad = \quad h(t)\rho(t)N_{pot}(t) = \frac{h(t)N_{pot,0}e^{c_N t}}{(1+\beta e^{-kt})^{\frac{1}{b}}},$$

where $h(t)$ is the market share of the operator.

The operator revenues depend on demography, markets, competition and company actions, see Figure 3.26. Because the service adaptation is only modestly sensitive to price [50, 143] and to keep our model simple, we use exponential time dependent model (3.20) for the average revenue per user. Discrete price models might be more realistic for smaller markets where operators have strong impact. Still, the exponential model as a forecasted expectation of the ARPU is reasoned.

The forecasted total revenues for operator are thus

$$B(t) = p_S(t)N(t) = \frac{p_S(0)N_{pot,0}e^{(c_N - c_S)t}}{(1+\beta e^{-kt})^{\frac{1}{b}}}. \quad (3.21)$$

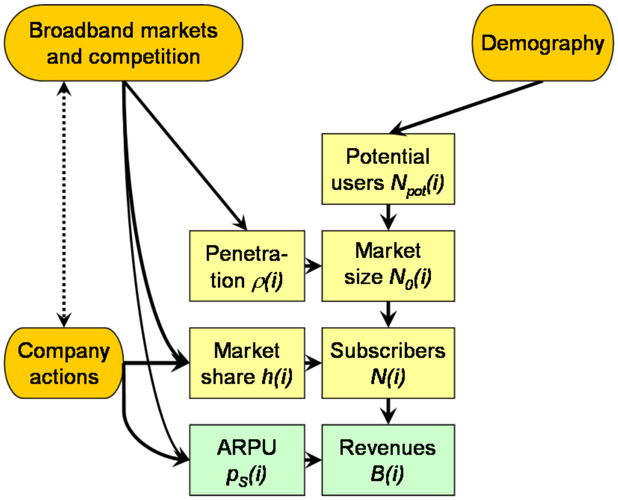

Figure 3.26: Broadband network operator revenue modeling

4. VALUATION AND DECISION MAKING UNDER UNCERTAINTIES

Let us consider a situation where a network operator is aiming to launch a new broadband access network in Finland. The access network will be located at a certain place in Finland where the operator is not an incumbent telephone operator. The network investment is medium sized but not crucial for the operators other businesses, i.e., the investment decision can be done based on economic evaluation without immediate strategic implications.

4.1 Managing Uncertainties in Decision Making

Decision making consist of different tasks from the identification of decision context via the analysis and modeling to choosing the alternative. Decision making may be hard because of [55]

- the complexity of the problem,
- the uncertainty in the situation,
- the multiple objectives of the decision maker(s), and
- different perspectives relevant for the decision maker(s).

The uncertainty lies in different parts. Firstly, the current markets, technology and solution alternatives may be uncertain for the decision maker, i.e., the *information is not complete.* Secondly, the *impacts of decisions are uncertain.* And thirdly, the *future is uncertain*, i.e., different events may occur in the future regardless of the decision makers actions.

Different methods for managing the uncertainties in decision making are used. The starting point for successful decision is to *gather information* from different sources from literature to experts. The researchers may have a common narrow vision on future applications. Current information may be mentally restricted and new ideas should be sought, too. Different alternatives and uncertainty in the future can be worked out by *creativity tools* like brainstorming and checklists [55, 215].

When drawing the relations of current situation, decision maker's actions and possible future events, one gets *influence diagrams* [55]. To reveal the impacts of investment decision *net present valuation* can be used. More about basic valuation methods is written in Section 4.2 and practical cases can be found in Chapter 5. Basic valuation gives result for single one scenario and *sensitivity analysis* should be made to trace impacts of parameter uncertainties [55], see Chapter 5 or [82, 250] for examples.

The uncertainties on the values of the parameters may mislead the scenario based valuation. For example in broadband investments, the amount of subscribers and the market situation is hardly forecasted correctly and that makes calculations open to doubt. However, the uncertainty can be considered as a continuous distribution of the parameter value, like *fuzzy numbers.* The distributions make it possible to choose several scenarios randomly. Thus, by calculating profitability in different scenarios new distributions are generated representing the value of the network investments. This method is called *Monte-Carlo simulation*. Monte-Carlo simulation has been used for example in [81, 86, 129, 252, 249]. More cases of practical use are analyzed by simulation in Chapter 5. When the *risk profiles* of different decision alternatives are found, they can be compared to each other [55]. That way the decision maker can choose between alternatives with her risk carrying capacity and risk willingness in mind. Simulation examples and some risk profiles can be found in Chapter 5.

Some quantitative decision analysis methods take into account decision maker's several plans or alternatives. *Decision tree* is applicable to different situations where there is one or several consecutive decision points [55]. In investment decisions, *portfolio analysis* can be used for comparing several alternatives and optimizing the combination of different investments [109]. Yet *real option analysis* is effective method for analyzing the investor's possibility to change plans. More on real options is written from Section 4.3 forward.

The methods mentioned are not complementary but each of them are helpful for decision maker. They are recommended to be used in management of uncertainties. Some of the methods are presented in more detail below.

4.2 Valuation of Network Investments

The valuation of investments is essential for decision makers [249]. The valuation process assess how much the investment is worth. Furthermore, it is important to know how much capital the project needs and how quickly the planned investment pays itself back. Valuation methods are used to find out whether it is possible for a company to invest in a project or not, and to compare different solutions to each other. The most common valuation methods are based on cash flows and general financing theory [32].

4.2.1 Scenario Based Techno-Economics

Techno-economics studies the economics of technologies. The main purposes are the valuation of technological investment projects and to answer such questions as what the cost structures of different technological solutions are like. Often, techno-economics is future oriented and uses different forecasting tools. It must be noted, however, that economics is not the only viewpoint to consider when choosing technology. It has been proposed that the overall user needs must be satisfied to be dominant business player [53].

Scenarios

Scenarios are plans or projections on circumstances [27]. For example, the demographic scenario may be such that there lives 1 000 people in 10 km^2 area, i.e., the population density is 100 per km^2. The set of scenarios on different environmental factors — i.e., on demographic and service and regulation and on technology — is called a *case*. Scenario based techno-economics uses scenarios and cases to estimate the profitability of a certain technology in different circumstances. In communications, scenarios are used in many cases [46, 125, 126, 127, 142, 162, 204, 205, 243, 285].

An example on the use of scenario based techno-economics can be found in a study by Smura [243], where WiMAX technology is compared to ADSL. This study concludes that WiMAX is not competitive with ADSL in dense areas. However, indoor WiMAX may be feasible in the most densely populated areas. In rural areas that are not connected to a fiber backbone, there are no clearly profitable broadband technologies, Smura argues. Still, WiMAX may be the best alternative to provide solution for those areas.

Another WiMAX analysis is by the WiMAX forum [277]. The paper considers urban, suburban, ex-urban and rural areas in three different scenarios. The rural area is represented by a small city far from other cities and the ex-urban is area nearby a city. The population density in rural area may be relatively high but in ex-urban areas, population density is always very low. The analysis concludes that WiMAX may be a profitable technology in urban and suburban areas. However, this study does not compare WiMAX to ADSL. In addition, the WiMAX forum generates an attractive business model for a rural town such that the cumulative cash flow becomes positive in four years.

Forecasting Broadband Communications

Forecasting is a demanding process. However, successful future analysis may produce great benefits for persons or companies, even if the forecasts are incorrect. This is because methods push forecasters to think about the future. Several forecasting methods are developed for different purposes [88, 213]. The methods vary from creative group sessions to detailed calculations.

4.2.2 DISCOUNTED CASH FLOW

Net Present Value

Operators and users behave rationally. When choosing communication technologies they try to minimize cost and maximize utility. The revenue of an investment should exceed the costs of it. Let $B(n)$ be the revenues and $C(n)$ the costs in period n. The *profits* in the period is

$$P(n) = B(n) - C(n). \tag{4.1}$$

However, the investment needs capital, which could be invested in other businesses. Moreover, we prefer money today to money in the future. Thus the amount X now is worth of $RX = e^r X$ after one period with the gowth rate r. The amount Y after n periods is worth of $YR^{-n} = Ye^{-rn}$ now. This calculation is called *discounting* and r the discount rate. Summing up the cash flows from different periods results in the *discounted cash flow* (DCF), or *net present value* (NPV), of an investment [32]:

$$V_{z,r} = \sum_{n=0}^{z} \frac{B(n) - C(n)}{e^{rn}}, \tag{4.2}$$

where z is the planning horizon. See Figure 4.1 for a net present value model.

The discount rate r in perfect world is related to the riskyness of the future cash flows and their correlation to the market fluctuations. If the expectation of the whole market growth rate is r_m and risk-free interest rate if r_f, then the *risk-neutral* discount rate r of a portfolio fulfills [32, 123]

$$r - r_f = \beta(r_m - r_f), \tag{4.3}$$

where β is the sensitivity coefficient for the portfolio in the market. This formula is known as *capital asset pricing model* (CAPM). If the covariance $Cov(r, r_m)$ of the portfolio and variance $Var(r_m)$ in the market is known, the beta can be estimated as

$$\beta = \frac{Cov(r, r_m)}{Var(r_m)}.$$

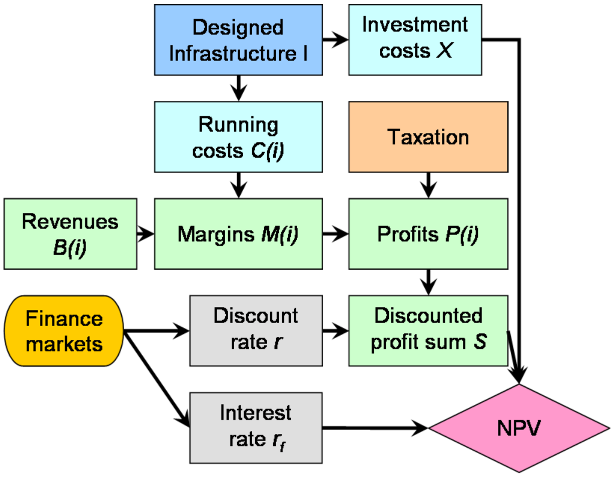

Figure 4.1: General NPV model

Internal Rate of Return and Payback Period

In addition to the net present value, the investor is interested in the time the investment pays itself back. The time

$$\min_{V_{T,r}>0} T \tag{4.4}$$

with given discount rate r is called *payback period*. Yet the investor may have other investment alternatives. The *internal rate of return* (IRR) is the discount rate r that solves the equation

$$V_{z,r} = 0 \tag{4.5}$$

with given planning horizon z. Shareholders determine required rates of return that guide the management in discounting. The CAPM model still may be behind the guidelines.

Weighted Average Cost of Capital

The investments are often financed by both shareholders equity and finance dept. Ones the cost of dept D and shareholders' equity E is known by the interest rate r_f and required rate of return r_r, respectively, the *weighted average cost of capital* (WACC) can be calculated as

$$r = \frac{E}{E+D} r_r + \frac{D}{E+D} r_f (1-t), \tag{4.6}$$

where t is the corporate tax rate.

4.2.3 THEORETICAL BROADBAND NETWORK INVESTMENT MODEL

Modular Scenario Based Simulation Model

The household density and other surroundings are the basis for planning of the structure of the network. In addition, the number of operators and the market situation affects revenues and costs, see Figure 4.1. The revenues and costs of the investment can be estimated using the methods described in Chapter 2 and Chapter 3. In Figure 4.2 a combined scenario based model for network investment is drawn.

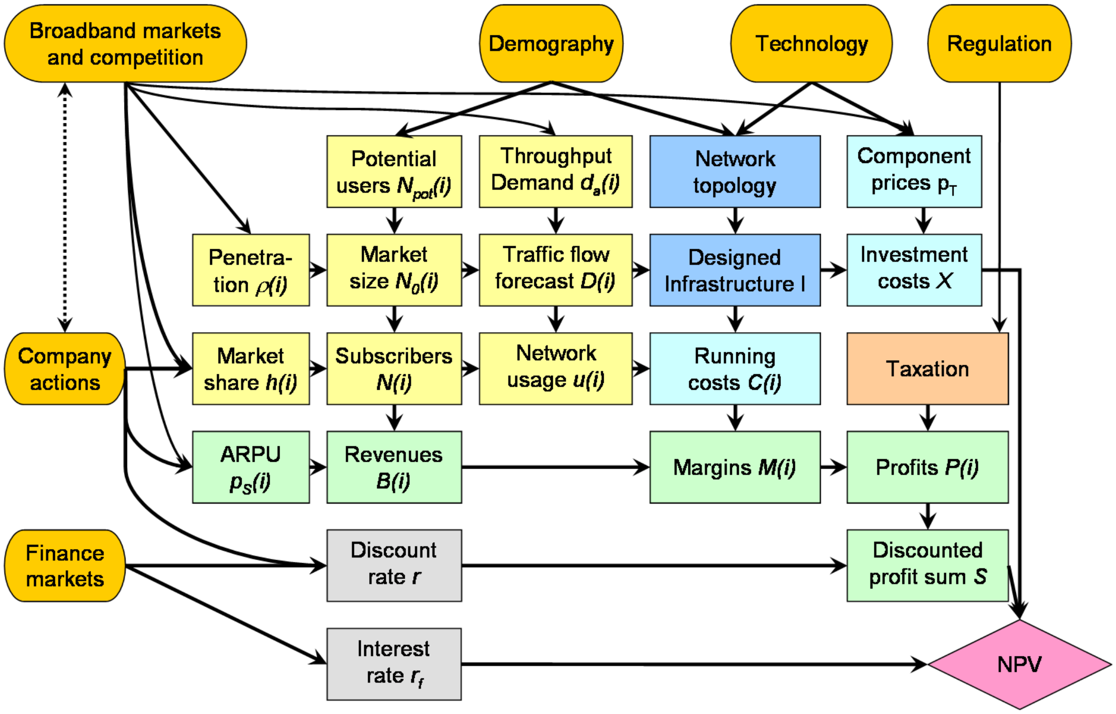

Figure 4.2: A broadband investment valuation model

Dynamics in Finnish Household Segment

The scenario based DCF method has been used for years in telecommunication techno-economics, for example in [86, 126, 127, 142, 162, 193, 204, 205, 253, 285]. In addition, the same methods are used in [243, 277]. The detailed scenario models for different parts of the overall model have been demonstrated but no complete dynamic equations are stated. Using the models reasoned in Section 2.3 and Section 3.4 the net present value for an investment is

$$V = \sum_{n=0}^{z} \frac{B(n) - C(n)}{e^{rn}}.$$

The revenues and costs are assumed zero until the period s when the investment costs X (2.6) are paid. After that the revenues B follow (3.21) and costs C follow (2.7). Thus the net present value of the investment becomes

$$\begin{aligned}
V &= \sum_{t=s}^{z} \frac{B(t) - C_{OA}(t) - C_M(t)}{e^{rt}} - \frac{X(s)}{e^{rs}}, && (4.7)\\
X(s) &= \left(p_T(0)e^{-c_T s} + K_{T,I}\right) I, \\
C_{OA}(t) &= K_{T,F,OA} I + K_{T,V,OA} N(t) + K_{T,V,I}(N(t) - N(t-1)), \\
C_M(t) &= p_T(0)e^{-c_T t}\left[(1 - e^{-c_{T,F,M} t})K_{T,F,M} I + K_{T,V,M} N(t) d_a(0) e^{c_d t}\right], \\
B(t) &= p_S(0)e^{-c_S t} N(t), \\
I &= \max\left(\frac{K_{T,A}}{R_c^2} A; K_{T,D} N(z) d_a(0) e^{c_d z}\right), \\
N(t) &= \frac{h(t) N_{pot,0} e^{c_N t}}{(1 + \beta e^{-kt})^{\frac{1}{b}}}.
\end{aligned}$$

The model above assumes that investment is designed for the right network traffic level and the network capacity does not constraint the number of subscribers. Actually, the capacity may become a constraint after the network is built. In this case the amount of users can be modeled as

$$N(t) \quad = \quad \min\left(\frac{h(t)N_{pot,0}e^{c_N t}}{(1+\beta e^{-kt})^{\frac{1}{b}}}; \frac{I}{K_{T,D}d_a(0)e^{c_d z}}\right). \tag{4.8}$$

Other parts of the dynamic models have been used in field but the application of rising maintenance costs is new.

4.3 REAL OPTIONS THEORY

Large investments do not produce benefits in one year. Instead, the pay-back period is often up to 5 or even 10 years. During that time the operator can adjust its strategy if the market situation changes. In other words, the investor have an option to modify or stop the project. Investor decision alternatives in the future are called *real options* [178]. If new information about future events does not support some alternatives, those alternatives can be abandoned. That is why future real options increase the value of the project.

The idea of modeling investments as options arose in the 1970s [195]. Since then, real options have been used mainly in natural resource projects, i.e., oilfield and mining projects [7, 34, 163]. Other subjects with real option include infrastructures like power plants, start-up companies, drug development and closing of paper mills [2, 7, 118]. Dos Santos started to apply the methods to information technology [72].

4.3.1 VANILLA OPTION TYPES

The real option theory is partly analogic to the theory of financial options. The typical option types used in financial markets are call and put options. Real options have the same characteristics, but are a bit more complex.

Call

A *call option* is the right to invest. The financial call option is its holder's option to buy a security — typically a share of stock — at a fixed price, i.e., *exercise price* [25, 37]. The focused stock or security is called as *underlying assets*. The time of the exercising is normally bounded, see Section 4.3.2. The result from the exercise of a call option is

$$\Phi(S_t) = \max[0; S_t - X],$$

where S_t is the stock price at *exercise time* (or *maturity date*) t and X the exercise price.

A typical real option that has the nature of financial call options is the *expand option* [3, 11, 86]. Expand option is an option to make the business larger. Shortly speaking, a real call option is an option to invest [200].

Put

A *put option* is the right to sell at fixed price. The exercise yields a profit

$$\Phi(S_t) = \max[X - S_t; 0],$$

where S_t is the stock price at exercise time t and X the exercise price [25, 37]. The real put option is an option to abandon a project already running or an option to reduce costs [3, 200].

4.3.2 Expiration of Options

The maturity date (exercise time) and exercise price are the main characteristics of an option, together with the option type and the underlying asset of an option. The exercise time may be fixed or it can be bounded more or less strictly.

European	options can be exercised only at the fixed maturity date [25, 37].
American	options can be exercised at any time before the maturity date [25, 37].
Bermudan	options can be exercised at any of the fixed expiration times [74].

The most popular financial options can be exchanged in stock markets without expiring the options themselves. This is why the price of an option related to an underlying assets is very important. Real options normally does not have such markets which makes them differ from financial options.

4.3.3 Exotic Options

American and European call and put options are often named as plain options or vanilla options or even plain vanilla options [37, 235, 280, 283]. This is because they are simple in their nature and payoff functions. In contrast, there are many *exotic options* that have more complex payoff functions [235].

For example, a *look-back option* gives the holder possibility to buy (or sell) at the minimum (or maximum) price the stock has reached [235]. The name *look-back* means that the exercise price yields from the price history. Furthermore, a *Russian option* is a look-back option with no maturity date, i.e., it runs for perpetuity [237]. Moreover, the payoff of an *Asian option* is not dependent on a single price but on the average price over a pre-defined period of time [38, 93, 235, 283].

Another group of exotic options is *barrier options*, which are activated or deactivated if the price breaks a fixed threshold [23, 235]. In addition, the *Parisian option* is activated or deactivated only if the price is under (or over) a fixed threshold long enough [23].

In the *game option* or the *Israeli option*, the writer of the option has an opportunity to cancel the option and pay a penalty sum in addition to normal payoff [79, 152].

The above exotic options have different payoff functions and expiration dates, but the underlying assets is typically a stock share. The option does not have to be focused on stock shares. For example, a *compound option* is an option to buy or sell another option. In this case, the underlying asset is an option.

4.3.4 Challenges in Real Option Models

The use of option theory on real assets and investment projects yields some problems and possibilities to critiques. Black and Scholes discussed some problems for valuation of warrants, and some of them are relevant to real option pricing as well [26]

1. The time under consideration is long and thus the variance of the growth may vary,

2. Dividends may be paid during the study period,
3. The exercise price may change,
4. Companies may merge.

One of the most important things in option valuation is the shape of the distribution of the underlying assets. Traditional option theory assumes a log-normal distribution, i.e., exponential growth, which suits badly in some real investment projects [98].

The discount rate used in valuation is somehow problematic. The use of the risky discount rate and the interest rate, even opportunity costs [110], are be argued. Moreover, the interest rate does not need to be modeled as a constant value [43, 65].

The taxation models affect option valuation [171, 172]. The taxation may be simply excluded from evaluations, which may mislead the study [209]. If taxes are taken into account they affect asymmetry in the distribution of underlying assets [156]. The asymmetry and other properties of taxation makes the valuation with some methods hard [200].

Some other critical issues can be found in [41, 163]. After all, the gift from real options to the valuation of investments is the modeling of the flexibility of the investor [140]. Moreover, the real option modeling offers a professional method to handle uncertainties [69]. The methods were commented in [239]:

> *It makes little sense to use a numerical technique to calculate the option price accurate to 1% or 2% when the underlying asset price is only known to an accuracy of 10%, as in real options.*

4.4 VALUATION OF OPTIONS

The valuation of options has been a famous problem. Merton and Scholes were awarded the Nobel Price in Economics in 1997 for their valuation methods introduced in the 1970s. There are many different valuation methods, nowadays [37]. They can basically be ordered in three groups [7]:

1. Differential equation solutions (Black–Scholes formula and variants),
2. Discrete event and decision models (binomial tree),
3. Simulations (e.g. Monte-Carlo and quasi Monte-Carlo).

Fuzzy sets and fuzzy logic have been used in 2000's in developing a theoretical real option valuation, see [56, 57, 118].

4.4.1 PARTIAL DIFFERENTIAL EQUATION SOLUTIONS

Black–Scholes Formula

Financial studies usually assume that stock prices follow *geometric Brownian motion* [25]. Geometric model is the exponential version of the *Wiener process* W_t (or plain *Brownian motion*) which assumes that the continuous process has independent random increments $W_{t+dt} - W_t$ with normal distribution $N[0, \sqrt{dt}]$ [25]. Consider a stock price S_t at time t and Wiener process W_t. Geometric Brownian assumption for stock price means that

$$dS_t = \alpha S_t dt + \sigma S_t dW_t,$$

where drift α and volatility σ are parameters of the model. This model means that the growth of stock price S_t/S_0 is log-normally distributed.

The use of Brownian motion in financing started in 1900 [14, 64]. Different valuation models for derivatives were developed, but no analytical closed form appeared until 1970s. Black and Scholes published their well known results concerning option pricing in 1973 [26]. They assumed stock markets with

1. Constant and known interest rate,
2. The price of the underlying assets changes continuously and it is distributed log-normally with known variance,
3. No dividends are paid during the period,
4. No transaction costs,
5. It is possible to borrow at interest rate and buy any fraction of securities,
6. No penalties for short-selling.

With these assumptions the price $f(S,t)$ of an European option on the stock S follows the differential equation [26]

$$\frac{\delta}{\delta t}f(S,t)+\frac{1}{2}\sigma^2S^2\frac{\delta^2}{\delta S^2}f(S,t)+rS\frac{\delta}{\delta S}f(S,t)-rf(S,t)=0, \tag{4.9}$$

where r is the constant interest rate. Now, the solution for the European call option on this equation with exercise price X at time T is [26]

$$\begin{aligned} f(S,t) &= S\mathcal{N}(d_1)-Xe^{-r(T-t)}\mathcal{N}(d_2), \\ d_1 &= \frac{\ln(S/X)+(r+\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \\ d_2 &= d_1-\sigma\sqrt{T-t}. \end{aligned} \tag{4.10}$$

Assumptions made by Black and Scholes can be reduced lightly [33]. The valuation formula (4.10) does not change. For example, if the investor is assumed averse to risk, the equation (4.10) holds without continuous trading assumption [33, 233]. On the other hand, the interest rate can be modeled by a Wiener process and dividends can be taken into account [33, 185].

The price of an European put option equals $g(S,t)=f(S,t)-S+Xe^{-r(T-t)}$ [185]. In other words, the prices of European call and put options are strictly connected to each other. Meanwhile, the prices of American options are not so simple to determine. Few results were found by Merton. If the dominant securities does not exists and no dividends are paid, the price of American call option equals the price of equivalent European call option [25, 185]. Furthermore, the analytical valuation of American put options have been studied in [35, 97, 160, 212]. Numerical approximations are done in [164].

Equation (4.9) can be solved numerically, too. Such integration and difference methods gives approximations for the option valuation [63].

Valuation by Call Options

Real options have been used in the valuation of investments. The first examples were in valuation of natural resources projects, like oil fields or mining. A few examples can be found in [34, 61, 69, 209].

The use of real options in valuation of projects or investments have risen since 1980s. One practical example by Luehrman simplified the use of Black–Scholes equation [178]. The underlying assets in the model is the NPV ratio, i.e.,

$$NPVq = \frac{S}{Xe^{-r(T-t)}}, \tag{4.11}$$

where S is the discounted sum of operational cash flows and X is the sum of the investment costs. The exercise price of the expand option is 1, because the project has positive net present value if and only if the NPV ratio is more than 1. With these assumptions, the Black–Scholes valuation model can be used straightly to the real options and the value of the option is

$$\begin{aligned} f(NPVq,t) &= NPVq \times \mathcal{N}(d_1) - \mathcal{N}(d_2), \\ d_1 &= \frac{\ln(NPVq) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}, \\ d_2 &= d_1 - \sigma\sqrt{T-t}, \end{aligned} \tag{4.12}$$

where the volatility σ of the NPV ratio is often between 30% and 60% [178] per year and WACC based discount rate r between 0% and 10%. However, these parameters must be determined carefully for use cases individually.

The valuation of different kinds of companies by real options have been introduced in literature. Similar to the natural resources are other investments with large infrastructures and long pay-back periods. For example, toll road investment has been valued by real options [232]. In addition, the research intensive projects are one of the best applications for real options due to the great value of future growth possibilities. For example, biotechnology companies have been valued in [150] and Internet companies in [236]. Other strategic investments can be connected to standards, see [158], or licensing [41].

Brennan Model

The option valuation methods are mainly developed for financial options for which the underlying assets are shares on stock. Those assets are log-normally distributed in a short term. However, the distributions of investments, and thus the assets that real options are based on, differ from log-normal. Brennan studied the assumptions made by Black and Scholes and expanded the model [33]. Moreover, a model for option valuation on normally distributed assets were introduced. The value of option on assets S with exercise price X is [33]

$$f(S) = (S - XR^{-1})\mathcal{N}\left(\frac{SR - X}{\Sigma}\right) + R^{-1}\Sigma n\left(\frac{X - SR}{\Sigma}\right), \tag{4.13}$$

where R is WACC and Σ the deviation of the assets in one period. $\mathcal{N}(.)$ is the cumulative standard normal distribution function and $n(.)$ is the density function. Note that (4.13) holds for a one study period only.

Fuzzy Real Option Valuation

Fuzzy number is such a number for which the specific value of the number cannot be determined. Many of the scenario parameters in an investment case are such numbers. In addition, we cannot determine the current value of the underlying assets of real option, i.e., the net present value, which makes the situation different from the valuation of financial options.

Fuzzy real option valuation (FROV) is stated in [118] as

$$\begin{aligned} FROV &= \max\left\{\overline{S(t)}e^{-\delta(T-t)}\mathcal{N}(d_1) - \overline{X}e^{-r(T-t)}\mathcal{N}(d_2); 0\right\}, \qquad (4.14)\\ d_1 &= \frac{\ln\left(E(\overline{S(t)})/E(\overline{X})\right) + \left(r - \delta + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}},\\ d_2 &= d_1 - \sigma\sqrt{T-t}, \end{aligned}$$

where $\overline{S(t)}$ is the fuzzy net present value of the project at time t, $\overline{X}$ the fuzzy value for fixed costs and δ the value lost for waiting.

4.4.2 Discrete Event and Decision Models

The so called *binomial tree valuation* is one typical model for valuing options [66]. Other discrete event models include *trinomial tree model* [133] and *decision trees* [109, 110].

Binomial Tree Model

A simplified model for valuation of stock options was established in 1979 by Cox and Ross and Rubinstein [66, 223]. The assumptions 1 and 3–6 introduced in Section 4.4.1 for analytical valuation are relevant also here [25]. However, the distribution of the stock changes are different. Firstly, consider a small time intervals $[t_i, t_{i+1}]$. Secondly, assume that stock prices can increase from S to uS or decrease to dS during each interval with probabilities q and $1-q$. Thus we get *binomial model* for stock price changes, see Figure 4.3.

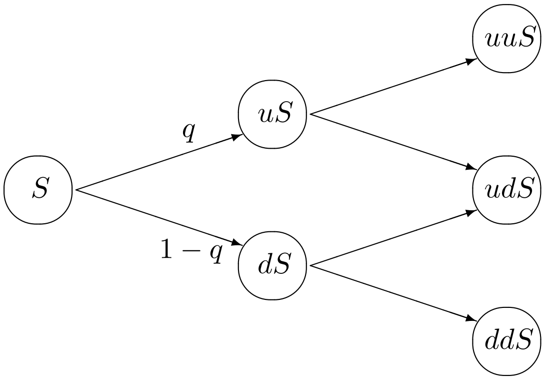

Figure 4.3: Two stage binomial model for stock price

With these assumptions, the price of European option $V_i(k)$ at time t_i — until which stock has increased k times — follows the equations [25, 66, 223]

$$\begin{aligned}
V_i(k) &= \frac{1}{1+R}[q_u V_{i+1}(k+1) + q_d V_{i+1}(k)], \qquad (4.15)\\
V_N(k) &= \Phi(Su^k d^{N-k}),\\
q_u &= \frac{1+R-d}{u-d},\\
q_d &= \frac{u-(1+R)}{u-d},
\end{aligned}$$

where R and N are the discount rate for each time interval and the amount of time intervals, respectively. The expiration function at time $t_N = T$ is $\Phi(Y) = \max[0; Y - X]$ for a call option and $\Phi(Y) = \max[X - Y; 0]$ for a put option. The model can easily be modified for American options by changing (4.15) to [223]

$$V_i(k) = \max\left\{\Phi(Su^k d^{i-k}); \frac{1}{1+R}[q_u V_{i+1}(k+1) + q_d V_{i+1}(k)]\right\}.$$

The hedging assumptions made above can be reduced, if one can assume risk-aversion [246]. In that sense, the binomial model have the same characteristics as the continuous analytical model. The binomial tree valuation model can also be presented in fuzzy logics to trace more realistically the uncertainty in the situation, see [2, 118].

The multiplicative binomial distribution, which the model leads to, is an approximation of log-normal distribution. If the standard deviation of the stock price is assumed σ, the upward and downward coefficients should be defined as $u = e^{\sigma\sqrt{T/N}}$ and $d = e^{-\sigma\sqrt{T/N}}$ [66]. As a limiting case, binomial model thus leads to the Black–Scholes formula [66, 223]. On the other hand, the binomial model can approximate Poisson distributed jumps of stocks [66], too.

4.4.3 SIMULATIONS

Monte Carlo Simulation

The option value can be estimated by simulation. While options that are based on one stochastic stock can be solved numerically using the Black–Scholes formula, for real options combined with multi-stochastic surroundings the formula cannot be used. Thus, for example simulation or fuzzy number based techniques must be used.

Monte Carlo simulation is a method to solve the integral

$$\int_A g(a)f(a)da = \overline{g},$$

where f is probability density function — i.e., $\int_A f(a)da = 1$ and $f(x) \geq 0$ — and g is the function, whose mean is to be verified. To estimate $\overline{g}$ by using Monte-Carlo one takes n random samples $y_1, y_2, ...y_n$ from $f(y)$. The estimate of $\overline{g}$ is [29]

$$\hat{g} = \frac{\sum_{i=1}^{n} g(y_i)}{n}$$

and the variance of the estimate is

$$\hat{s}^2 = \frac{\sum_{i=1}^{n}(g(y_i) - \hat{g})^2}{n-1}.$$

The Monte Carlo method for options was first established by Boyle in 1977 [29]. In the method, the series of stock processes are simulated to get the samples of the stock maturity price S_T. The expectation of $\Phi(S_T) = \max(0, S_T - X)$ is calculated and discounted to get the value of the European call option. The value of the European (or American) call option is thus

$$e^{-rT} E[\max(0, S_T - X)]$$

and it can be estimated using Monte–Carlo simulation by

$$e^{-rT} \frac{\sum_{i=1}^{n} \max(0, S_{T,i} - X)}{n}, \tag{4.16}$$

where $S_{T,i}$ are the simulated stock prices at expiration time T and n the amount of simulations. In addition, the European put option value equals $e^{-rT} E[\max(X - S_T, 0)]$ which can be estimated by

$$e^{-rT} \frac{\sum_{i=1}^{n} \max(X - S_{T,i}, 0)}{n}$$

Accuracy of the estimate can be increased by producing more simulations or by using so called *variance reduction techniques* [29]. The variance is inversely proportional to the number of simulations, which may lead to the huge number of simulation to get the variance reduced sufficiently. On the other hand, variance reduction techniques use somehow structured random samples or quasi-random samples that represent the event space [6, 29, 30, 31, 49, 279]. This helps to increase the accuracy without increasing the simulation time.

American put options were not valued using Monte-Carlo methods until 1993. Since then, different method for the valuation have been proposed [9, 30, 39, 74, 92, 99, 176, 231, 263, 266]. The methods solve optimal exercise problem of the option. Some studies have recently concentrated on Monte-Carlo valuation of American type real options [60, 61, 106, 232], too.

Fuzzy Payoff Method

With fuzzy numbers the real option value becomes [57]

$$ROV = \frac{\int_0^\infty A(x)dx}{\int_{-\infty}^\infty A(x)dx} E(A_+),$$

where A is the fuzzy NPV, $E(A_+)$ the fuzzy mean value for the positive NPV, upper integral the weight for positive NPV and lower the total weight of fuzzy NPV. The fuzzy payoff method simplifies the simulation to some critical events depending on the fuzzy number representing current project NPV.

4.5 REAL OPTIONS IN COMMUNICATIONS

An option on real assets or project is a right but not an obligation to do something within preset boundaries, e.g., not later than on the maturity date. Large investments and contracts are often signed with some options. The value of a typical real option depends on the NPV of the operations and on the uncertainty in the NPV, see Figure 4.4.

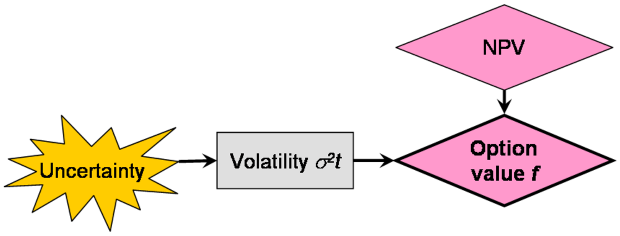

Figure 4.4: Real option model

4.5.1 REAL OPTION TYPES

Theoretically, a number of real option types are introduced in literature. Perhaps the most popular real options are *investment options*. They are call options with investment costs being as the exercise prices and the discounted operational profit sums being the prices of the investments, see Table 4.1.

Table 4.1: The concepts of investment options and call options

Investment option	**Symbol**	**Financial call**
Discounted operational profit sum	S	Stock price
Investment costs	X	Exercise price
The last date to make investment	T	Maturity date
Time-value of money (WACC)	r	Interest rate
Uncertainty	σ	Standard deviation of stock

One type of investment option is called *expand option*, which can be modeled as an European call option. Shortly, an expand option is an option to expand the dimensions of corporate businesses [11, 163]. Similar to expand options are *options to defer*, i.e., the possibility to delay for new information before investing [11, 163]. An option to defer is a kind of American, or Bermudan call option. *Growth options* are nearly the same as expand options, but they have greater strategic importance for companies [11]. Normally, growth options have two stages and they are kind of compound options with two stages. The first option is for modeling pilot investment, and the second option is a normal expand or growth option. Other compound options are *time-to-build options*, which are series of call options [11].

An *option to abandon* is opposite to the expand or defer option, i.e., it is an American put option [11, 163]. An option to abandon leaves a whole unit of production, but often there is the possibility to reduce the production slightly. Those *options to contract* are typically put options on some parts of an investment [11, 257]. Once infrastructure is ready, the company has the possibility to operate or not to operate a certain period. Those possibilities are named as *shutdown-and-restart options* and they can be modeled as a set of European call options.

Yet another real option type has been introduced. We call it here as an *switch option*. It is an option to alter the input and output of the production [11, 163, 257]. They are kind of compound options.

Table 4.2: Real option types and their natural models

Type	**Model**
Expand option	European call option
Option to defer	American (or Bermudan) call option
Growth option	Compound call option
Time-to-build option	Sequence of call options or compound option
Option to abandon or option to contract	American put option
Shutdown-and-restart option	Set of European call options
Switch option	Compound call option

4.5.2 THE USE OF REAL OPTIONS IN COMMUNICATIONS

The use of real options in communications started in [72] with ISDN *investment valuation*. Since then, real options have slightly been used in communications [3, 86, 116]. For example, the *capacity upgrading* of base network is analyzed in [111, 112] and the investments of rural access networks in [141]. *Spectrum licensing* and 3G investments have been studied in [17, 114, 115] and *optimal pricing* of network traffic in [4]. The *technology evolution* and *existing infrastructures* offer many possibilities to utilize real options, see Figure 4.5. Some practical cases are introduced and analyzed more deeply in Chapter 5.

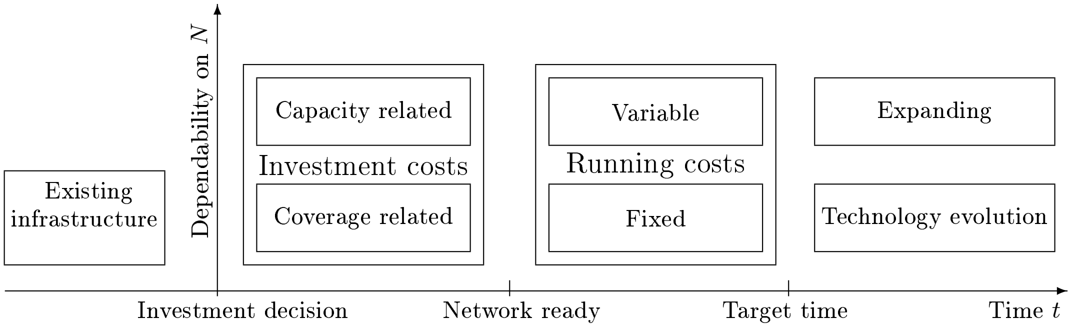

Figure 4.5: Network costs and options

The value of investment can be analyzed similarly to financial options using Black–Scholes model or other techniques. However, the models have different assumptions. Those assumptions should be examined carefully. In general, the options increase the value of the original investment [141].

The techno-economics of broadband communications (Section 4.2.2) and the real option valuation (Section 4.3) can be summarized as in Figure 4.6. In summary, the option value of future investment depends on the markets and technology, as well as uncertainties and opportunities on those.

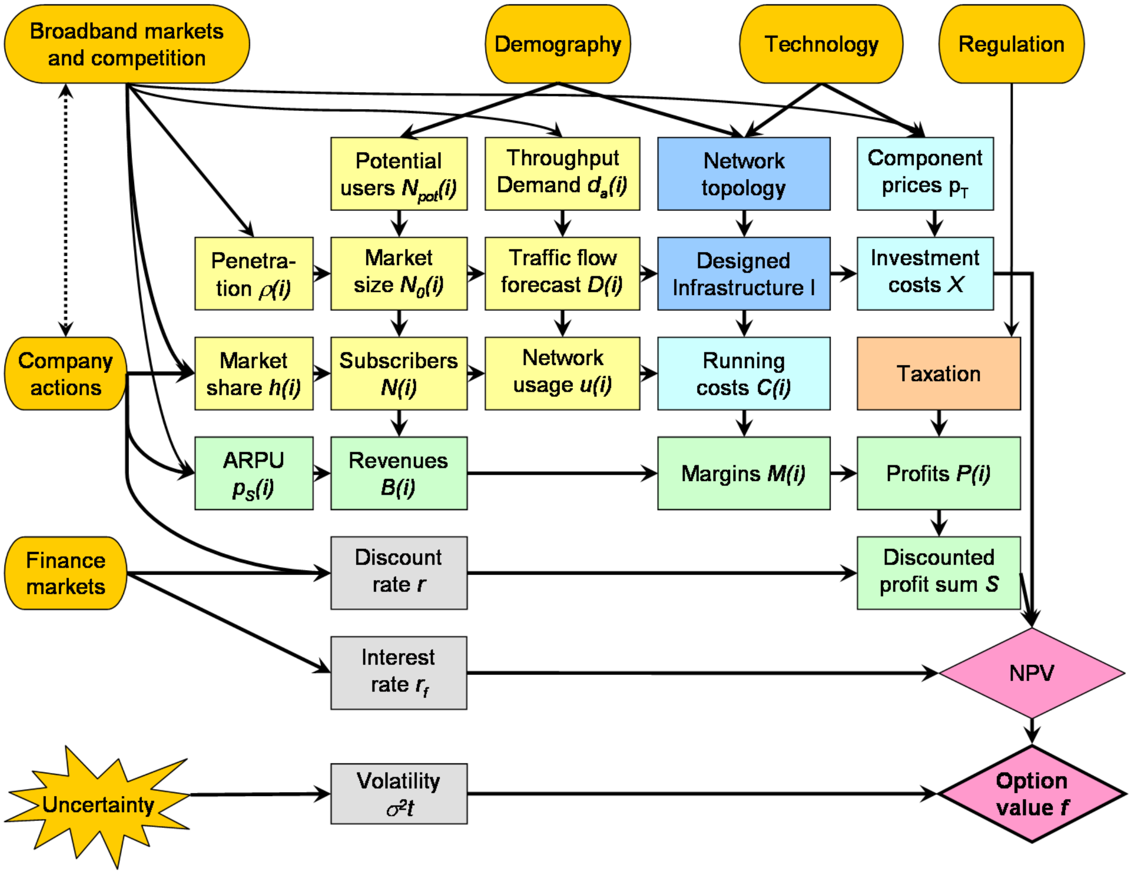

Figure 4.6: Communication investment real option model

5. STATISTICAL ANALYSIS OF ACCESS NETWORK INVESTMENTS

This chapter includes several broadband investment cases for example markets with different networking technologies. The investment costs and values are simulated using scenarios and Monte Carlo simulation. The aim of the chapter is to study the statistical distributions of the investment costs and values as well as to touch different practical real option examples. Some of the cases are extended and updated examples from [225]. All the calculations are done using a desktop computer with an AMD Athlon 1.0 GHz processor and 512 MB of RAM. The simulation times are at maximum just few minutes.

5.1 Simulation Model Implementation

The simulation model used in this chapter is based on the models defined in previous chapters. This section shortly describe the modular model implemented for Matlab. The earlier version of this simulation model was introduced in [225].

5.1.1 Direct DCF Implementation

Let us consider a single communication investment. The implementation of the simulation model consists of routines defining scenario parameters and functions for calculating the value of the investment. The relationships of the functions for the simulation model are drawn in Figure 5.1, and the connections to the model presented in Section 4.2.2 are described in Figure 5.2.

The main function `calculateNPV` calls for scenario functions and capacity functions as well as forecasting functions. The investment costs and running costs are calculated within the subfunctions `simInvestment` and `simCosts`. After revenue calculations, the costs are cut down and the operating profit is determined. The subfunctions `simTaxation` and `simDiscounting` handle with taxation and discounting, respectively.

Parameters and Forecasting

The model parameters can be loaded modularly using scenario functions. Service parameters, i.e., average revenue or traffic demand per user, are defined in a service scenario function, e.g., `broadband`. Demography parameters, i.e., number of households and summer cottages, as well as local penetration forecasting parameters are defined in one function, e.g., `ahtari`. Technology scenario function, e.g., `wimax` or `adsl`, returns the names and technical properties of components. Moreover, the function forecasts component prices with extended learning curve model stated in Section 2.2.3.

The function `simPenetration` forecasts the broadband penetration with Richards model. The main function `calculateNPV` implements the other forecasting actions using exponential models.

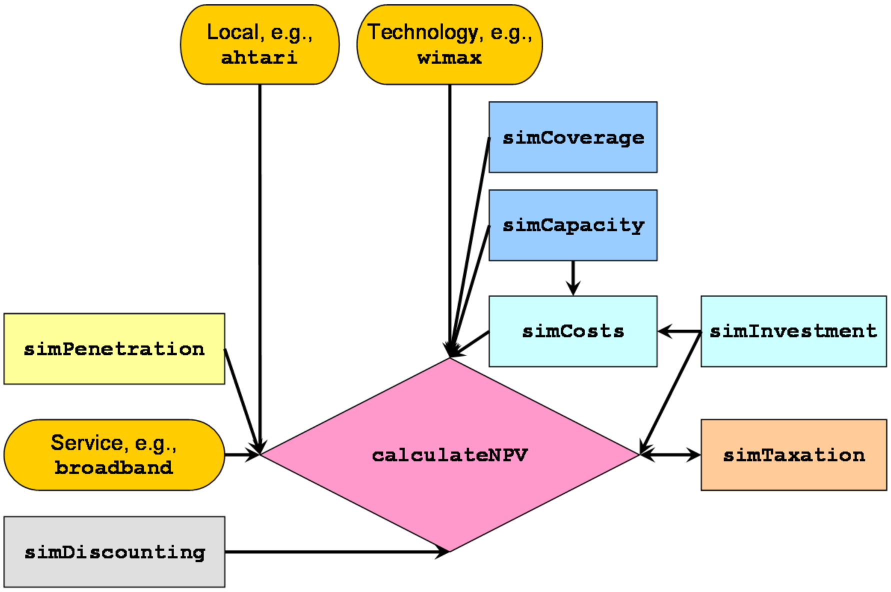

Figure 5.1: The functions to calculate NPV

Coverage and Capacity

Based on the properties of technology components, the amounts of the components needed are calculated in functions `simCoverage` and `simCapacity`. Each subscriber needs some of the components close enough. The technology parameters define the maximum service distance. Thus the minimum infrastructure needed to cover the service area can be calculated, as in function `simCoverage`.

The use of the channel bandwidth is simulated by the number of the subscribers and by the average throughput demand per user. The network infrastructure is planned so that this capacity is achieved.

Estimating Costs

Based on the coverage and capacity calculations, the function `simInvestment` evaluates the investment costs, i.e., material and installation costs. The running costs calculated by `simCosts` consist of operation and maintenance and administration costs. Furthermore, customer premise equipments and installation costs need to be simulated as well, and the implementation of `simCosts` checks for capacity, too.

Markets and Taxation

Operator competition in the broadband markets is not modeled. The market share is currently assumed to be 100%, i.e., no competition between operators. Our model, which is built from the investment point of view, does not restrict competition between virtual service operators. The model only assumes an overall penetration and ARPU for our network.

Taxation is modeled with depreciations and deductions. The tax rate 26% is counted on the profit with depreciations reduced. Negative profit for a year is possible and no tax is paid in such years. However, the negative profits can be deducted in the next 5 years.

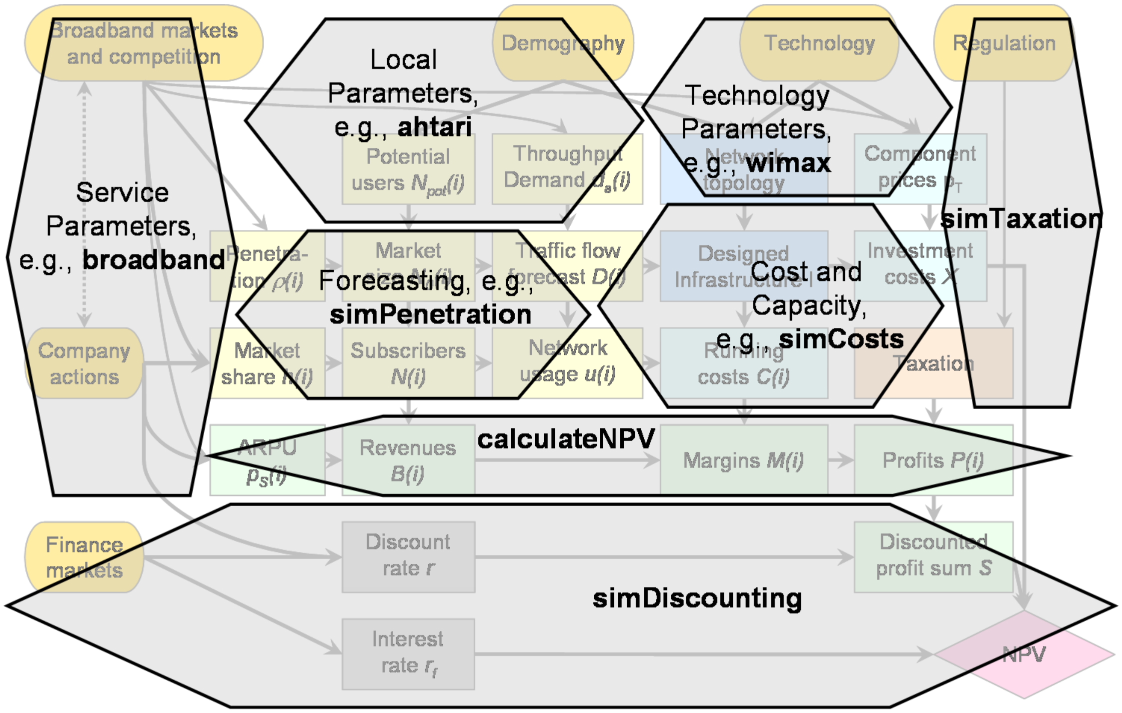

Figure 5.2: The implementation of NPV calculations fulfill the model, compare with Figure 4.2

Discounting

Let us assume interest rate of $r_f = 2.5\%$ and expected stock market rate of $r_m = 10.0\%$ which are reasonable levels in practice. For telecommunications field with 40% corporate taxation some beta levels of $\beta_a = 0.4$ for assets have been calculated [112]. With tax rate of 26% this would mean $\beta_a = 0.33$. Now using (4.3) we have

$$r = r_f + \beta(r_m - r_f) = 2.5\% + 0.33 \cdot (10.0\% - 2.5\%) \approx 5.0\%.$$

For a company with dept to equity level of $D/E = 3.0$ the expected return on equity as in (4.6) could then be

$$r_r = \frac{(E + D)r - Dr_f(1-t)}{E} = (1+3) \cdot 5.0\% - 3 \cdot 2.5\% \cdot (1 - 0.26) \approx 14.5\%.$$

Based on the above calculations, investor has set a goal for return on equity as 15%. Thus the cash flows are discounted to the year of the investing with discount rate $r_r = 15\%$. Note that depreciations affect only the taxation and the cash flow is based on the revenues and running costs and taxation. The net investment value — i.e., the difference between discounted cash flow from operations and the investment costs — is discounted to present time with WACC rate $r = 5\%$.

5.1.2 Monte Carlo Implementation for Real Options

Monte Carlo real option implementation uses the subfunctions of the traditional discounted cash flow implementation described in Section 5.1.1. The relationships between the simulation functions can be found in Figure 5.3.

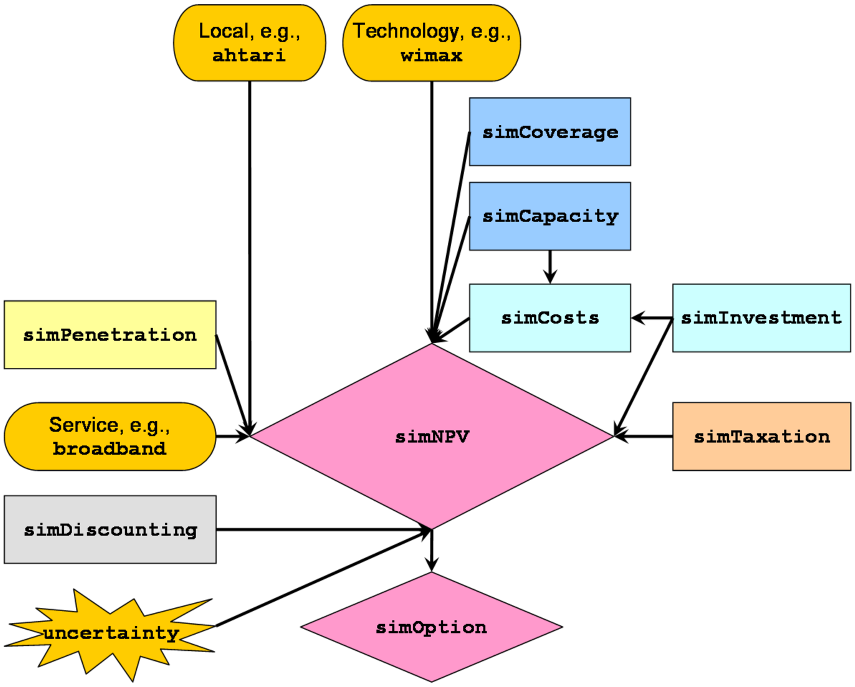

Figure 5.3: The functions to simulate option value

The main function `simOption` simulates possible future projections and calls function `simNPV`. The called function differs from `calculateNPV` so that in Monte Carlo simulation the NPV is simulated. This is to model the real life. The option can be executed at the time of investing, but the value of the investment is not precisely known. The main function `simOption` uses the strategy to exercise the option if the expectation of the NPV is positive. The uncertainty after the investment time is simulated in `simNPV` using random numbers generated in the function `uncertainty`. This yields the valuation of the real option. The connections between the model and the functions are described in Figure 5.4.

5.2 CASE: WIMAX FOR ÄHTÄRI HOUSEHOLDS

This section explains studies on a WiMAX network investment in rural municipality in Finland. The analysis is focused on the general shapes of the rural WiMAX investment. In addition, a practical defer option is demonstrated. The main research foundings in this section are published allready in [227].

5.2.1 SERVICE AREA AND DEMAND FORECASTS

Consider a broadband access network investment in a rural area. The service area is 906 square kilometers, i.e., the area of Ähtäri, a sparsely inhabited municipality in western Finland [262], see Figure 5.5. The number of households was some 3000 at the end of the year 2007 [262]. The number of summer cottages in municipality was some 1300 [262]. The number of households is quite stable, but growth rate of cottages is 1% a year, see Figure 5.6. Revenues are based on flat

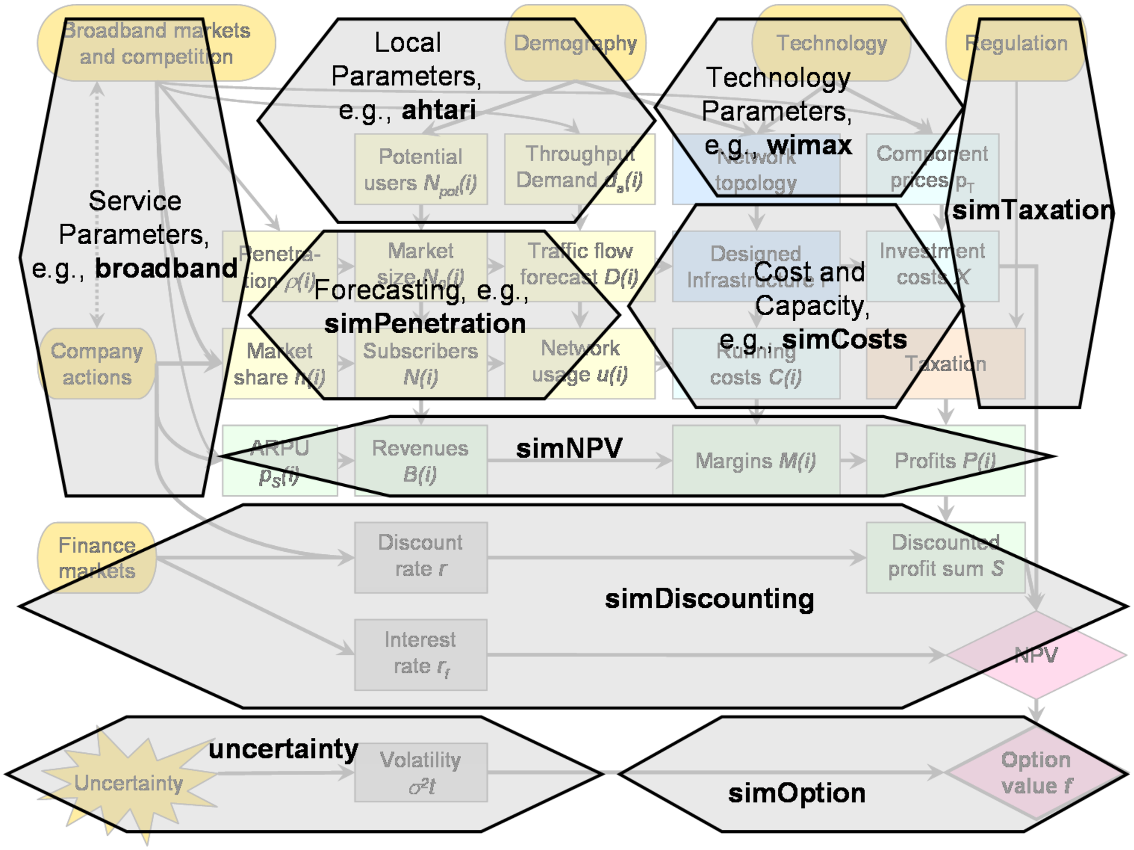

Figure 5.4: The simulation functions fulfill the valuation model, see Figure 4.6

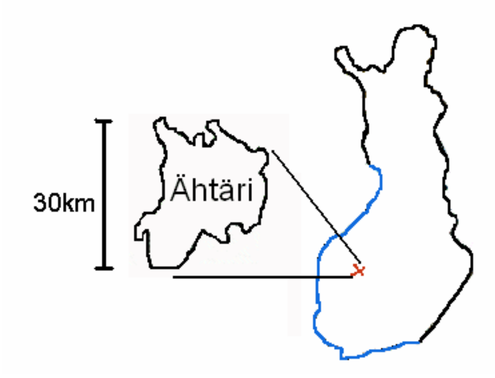

Figure 5.5: Ähtäri is a sparsely inhabited municipality in western Finland

monthly tariffs. The start up fee is decided by investor to be EUR 0 but subscriber buys the premise equipment. The installation costs for a new user is assumed to be EUR 100 and variable cost to be EUR 20 per year. These estimated parameters for the broadband service and service area are collected in Table 5.1

Let us assume the penetration of broadband subscription to be 20% at the start of the study period, i.e., in 2008, and 50% in year 2011. A Richards adaptation model is used with

Table 5.1: Estimated parameters for broadband service and area

	Initial value	**Exponential model factor**
Area	$906 km^2$	-
Potential subscribers	2 800	1.0
Connection charge	0	-
Subscribers proportion on CPE costs	100%	-

shape parameter -0.5. The nominal subscriber forecast is drawn in Figure 5.7. We assume the penetration growth to be achieved using the initial average yearly revenue per user (ARPU) EUR 240 (without VAT). The ARPU is modeled to slide down with factor 0.95 every year. The network infrastructure is designed for traffic with 0.05 Mbps throughput per user, i.e., with marketed service rate 1 Mbps and an over-booking factor of 20. Let us assume the growth ratio of throughput demand to be 1.20. Since the average traffic demand per user is increasing exponentially, the network is initially dimensioned for the whole demand growth in the study period. The assumed service market parameters are gathered in Table 5.2.

5.2.2 WiMAX Network Topology and Cost Structure

The WiMAX network infrastructure and costs are estimated based on traffic forecasts and component prices. Let us forecast the network component prices using exponential curves. See Table

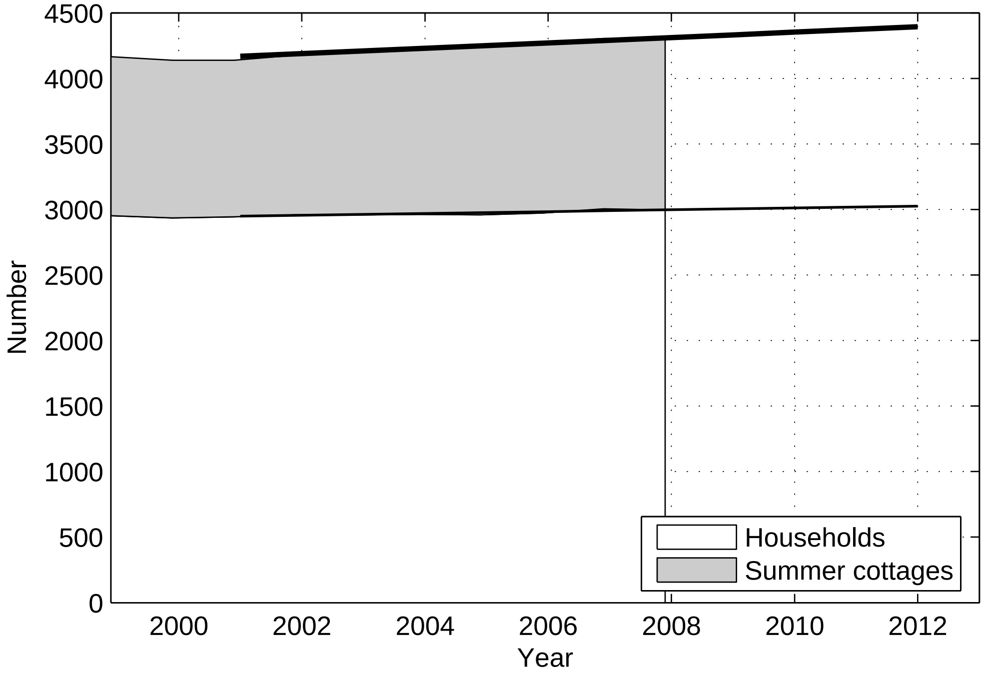

Figure 5.6: The number of households and summer cottages in Ähtäri

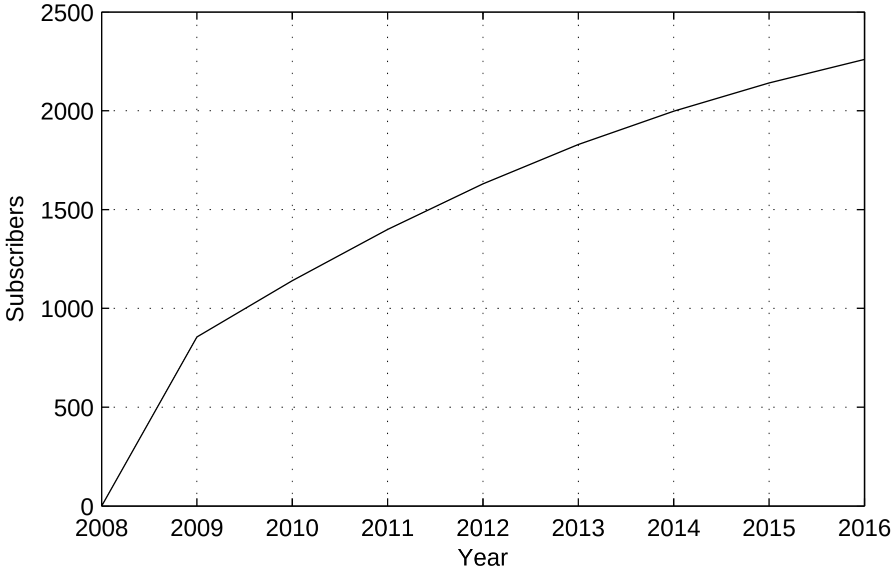

Figure 5.7: Subscriber forecast for the WiMAX network

Table 5.2: Parameters for the broadband service

	Initial value	**Exponential model factor**
ARPU	240€/a	0.95
Initial penetration	20%	-
Time for 50% penetration	2011	-
Richards shape	-0.5	-
Throughput demand	0.05 Mbps	1.2

Table 5.3: Assumed mean financial parameters for a WiMAX network [243]

	Price	**Model factor**	**Installation**	**Operating**
Antenna	1 000 €	0.90	200 €	600 €/a
Radio	6 000 €	0.85	300 €	600 €/a
Base Station	10 000 €	0.85	4 000 €	1 800 €/a
Link	25 000 €	0.90	2 000 €	2 400 €/a
Spectrum	-	-	-	7 €/km^{2}a

5.3 for the assumed price parameters as well as installation and operating costs. The growth parameters for exponential models are 0.85–0.90. We assume the stabilized maintenance costs for a single component to be 20% of the component price at a time.

The network is designed so that the existing pylons and base station masts are used. Figure 5.8 presents the location of four masts that are decided to be used for the network in outskirts. The central town with 50% of the targeted households is served by lower pylons in the central area. The coverage area of the network is estimated using the cell radius of 10 kilometers for

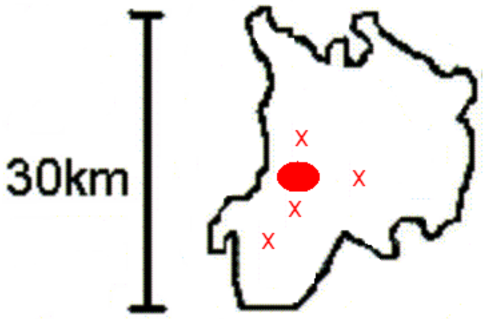

Figure 5.8: Central area and existing masts in Ähtäri to be used

WiMAX [12]. The capacity of each WiMAX base station and radio link is 155 Mbps. However, the effective capacity is 120 Mbps since one base station may contain 8 sectors at maximum and each sector radio is assumed to have a capacity of 15 Mbps. The central town seems to be capacity limited yielding to two base stations and 14 sectors. The outer parts of the service are coverage limited with four base stations and 14 sectors. The cost structure of the WiMAX network with subscriber forecast as in Section 5.2.1 is drawn in Figure 5.9.

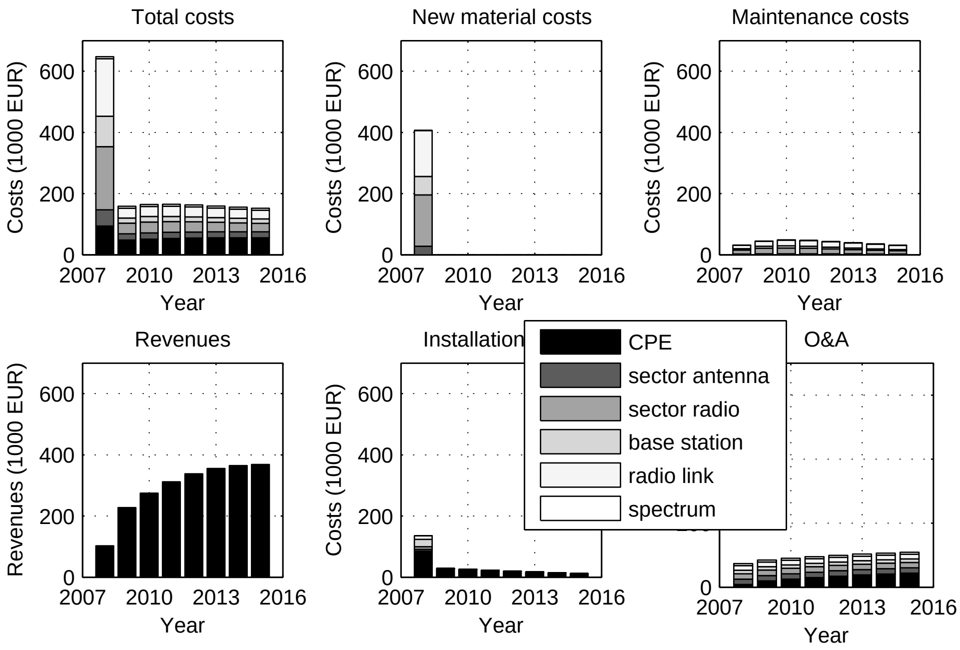

Figure 5.9: Cost structure for the WiMAX network

5.2.3 DCF Valuation and Sensitivity Analysis

Consider the model described in Section 5.2.1 and Section 5.2.2. The tax rate in Finland is 26% and we depreciate the infrastructure investments within 5 years. For valuation simulations, the 8-year study period is used. The simulated net present value for the network is EUR 6 000 negative. The payback period would be 7.1 years, i.e., it takes until the first quarter of the year 2015 for the investment to pay itself back. The internal rate of return is 14.7%. The theoretical model described in Section 4.2.3 with the parameters corresponding to simulation parameters yields NPV of EUR 71 000. Note that this calculation does not take into account taxation and the coverage is estimated using direct hexagonal mapping. Other differences between simulation and the theoretical model are the technology dependencies and capacity calculations.

An investor is interested in the sensitivity of the parameters in NPV. Table 5.4 presents simulated and calculated NPV with altered parameters. The changes in growth parameters e^{c_X} and especially in household growth e^{c_N}, throughput growth e^{c_d} and ARPU growth e^{c_S} have strong effects. As can be seen in Table 5.5 the same holds for the investment costs. The wider perspective on the sensitivity on the growth parameters of the number of households and throughput as well as the effects of ARPU $p_S(0)$ and cell radius R_c parameters on NPV is drawn in Figure 5.10. The case analyzed seems to be on the edge between coverage and capacity constrained. The optimal household growth for our case would be 0.95 to 1.0, i.e., a slight decrease in the number of households. This provides the best capacity efficiency in network for the study period as the growth of the average throughput demand is diminished by the decrease in the growth of customers in the end of study period.

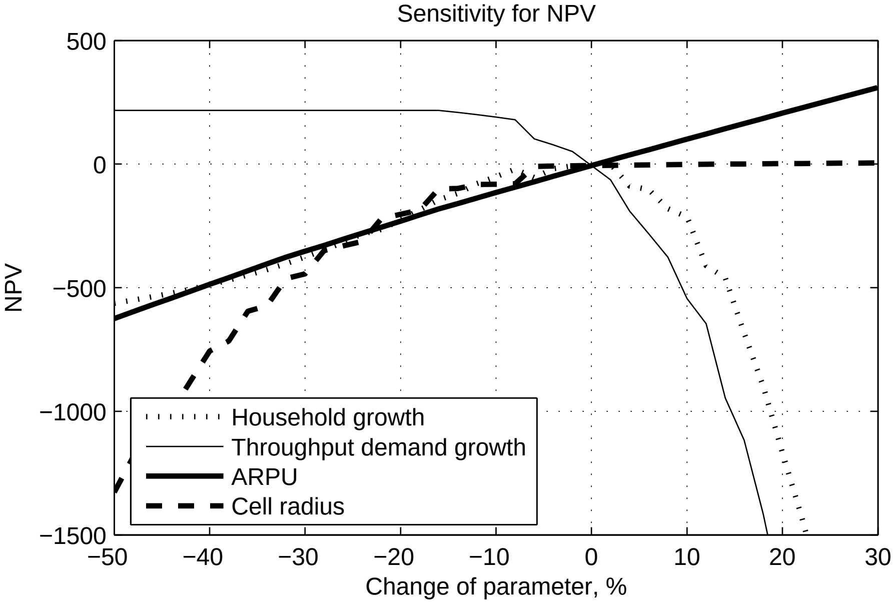

Figure 5.10: Sensitivity analysis for the WiMAX network investment

5.2.4 Monte-Carlo Simulation

Let us model uncertainties in the parameters. To simulate uncertainty, the parameters, except the area size, are multiplied by normal random numbers with mean 1.0 and standard deviation $\sigma = 0.1$. However, the taxation parameters are not randomized and growth parameters c_X are multiplied by random numbers with deviation $\sigma = 0.05$.

Monte-Carlo simulation is used to model NPV and investment cost distributions. Some 1000 events are calculated for an investment cost distribution. Each simulation step contains another simulation with 30 events for estimating the uncertainty after the investment time. So a net present value distribution is affected by 30 000 simulated cases. The model is implemented using

Table 5.4: Sensitivity analysis for NPV

Parameter	**Sim -10%**	**Sim +10%**	**Calc -10%**	**Calc +10%**
No changes	-6 k€	-6 k€	71 k€	71 k€
Households, $N_{pot}(0)$	-61 k€	48 k€	64 k€	78 k€
Households growth, e^{c_N}	-51 k€	-214 k€	-61 k€	-323 k€
Penetration, β			156 k€	-2 k€
Penetration growth, k			46 k€	98 k€
Adaptation shape, b	-7 k€ [a]	-5 k€ [a]	169 k€	-38 k€
Initial penetration, $\rho(0)$	-9 k€ [a]	-3 k€ [a]		
Time to 50% penetration	7 k€ [a]	-14 k€ [a]		
Throughput, $d_a(0)$	22 k€	-35 k€	156 k€	-14 k€
Throughput growth, e^{c_d}	190 k€	-544 k€	221 k€	-889 k€
ARPU, $p_S(0)$	-115 k€	101 k€	-49 k€	191 k€
ARPU growth, e^{c_S}	-335 k€	429 k€	-321 k€	652 k€
Technology prices, $p_T(0)$	45 k€	-58 k€	121 k€	20 k€
Price growths, e^{c_T}	31 k€	-58 k€	77 k€	65 k€
Installation, $K_{T,I}$	13 k€ [b]	-26 k€ [b]	75 k€	67 k€
New user costs, $K_{T,V,I}$	13 k€ [b]	-26 k€ [b]	87 k€	55 k€
O&A costs, $K_{T,F,OA}$	31 k€ [c]	-44 k€ [c]	101 k€	41 k€
Subscription costs, $K_{T,V,OA}$	31 k€ [c]	-44 k€ [c]	83 k€	59 k€
Maintenance, $K_{T,F,M}$	4 k€ [d]	-15 k€ [d]	81 k€	60 k€
Maintenance, $c_{T,F,M}$	4 k€ [d]	-15 k€ [d]	76 k€	67 k€
Maintenance, $K_{T,V,M}$	4 k€ [d]	-15 k€ [d]	72 k€	69 k€
Cell radius, R_c	-81 k€	-2 k€	55 k€	71 k€
Capacity, $1/K_{T,D}$	-35 k€	22 k€	-22 k€	147 k€

[a] The forecast assumes the shape parameter and fits curve to two parameter defined points

[b] Parameters $K_{T,I}$ and $K_{T,V,I}$ changed

[c] Parameters $K_{T,F,OA}$ and $K_{T,V,OA}$ changed

[d] Parameters $K_{T,F,M}$, $c_{T,F,M}$ and $K_{T,V,M}$ changed

Matlab.

The development of revenues, profits and discounted cash flows are drawn in Figure 5.11. The dotted, strong solid and dashed lines are the first, second and third quartiles and the light solid line is the mean value line.

5.2.5 THE INFLUENCE OF THE ERROR SHAPES

Consider the model described in Section 5.2.1 and Section 5.2.2. To study the effects of parameter error shapes on resulting distributions, we run the simulation model with different parameter shapes. Three different shapes are used — normal, log-normal and uniform distributions. The parameters are multiplied by an random error with mean 1.0 and standard deviation $\sigma = 0.1$ (or $\sigma = 0.05$ for growth parameters).

The distribution parameters for normally distributed errors are 1 and σ. For log-normal distribution, we use parameters $-\sigma^2/2$ and σ to get mean 1. The standard deviation then becomes $\sqrt{e^{\sigma^2} - 1}$ which is slightly — but not significantly — greater than σ. Uniform errors are generated using parameters $1 - \sqrt{3}\sigma$ and $1 + \sqrt{3}\sigma$ which yields mean 1 and deviation σ.

Table 5.5: Sensitivity analysis for investment costs

Parameter	**Sim -10%**	**Sim +10%**	**Calc -10%**	**Calc +10%**
No changes	456 k€	456 k€	427 k€	427 k€
Households, $N_{pot}(0)$	441 k€	471 k€	384 k€	469 k€
Households growth, e^{c_N}	340 k€	763 k€	352 k€	832 k€
Penetration, β			434 k€	419 k€
Penetration growth, k			408 k€	442 k€
Adaptation shape, b	456 k€ [a]	456 k€ [a]	439 k€	409 k€
Initial penetration, $\rho(0)$	456 k€ [a]	456 k€ [a]		
Time to 50% penetration	471 k€ [a]	441 k€ [a]		
Throughput, $d_a(0)$	441 k€	471 k€	384 k€	469 k€
Throughput growth, e^{c_d}	340 k€	733 k€	352 k€	915 k€
Technology prices, $p_T(0)$	415 k€	497 k€	388 k€	465 k€
Price growths, e^{c_T}	456 k€	456 k€	427 k€	427 k€
Installation, $K_{T,I}$	451 k€ [b]	461 k€ [b]	423 k€	431 k€
New user costs, $K_{T,V,I}$	451 k€ [b]	461 k€ [b]	427 k€	427 k€
Cell radius, R_c	497 k€	456 k€	435 k€	427 k€
Capacity, $1/K_{T,D}$	471 k€	441 k€	474 k€	388 k€

[a] The forecast assumes the shape parameter and fits curve to two parameter defined points

[b] Parameters $K_{T,I}$ and $K_{T,V,I}$ changed

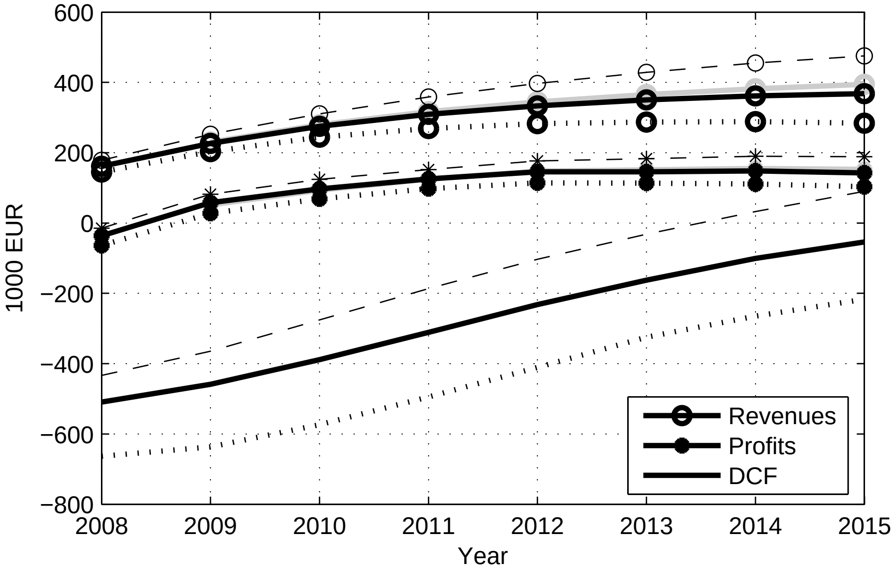

Figure 5.11: Discounted cash flows for the WiMAX network investment

The simulated 1000 cases for each error shape are used to compare the influences on the distributions of investment costs, NPV, and NPV ratio. The cumulative distribution of the simulation results are analyzed and compared. We use the Kolmogorov–Smirnov test for the difference between two distributions [181] and the normality tests by Lilliefors [175] and Jarque–Bera [135]. We examine log-normality of the samples by testing normality of the logarithmic values.

Table 5.6: Statistics of the investment costs with different parameter error shapes

	Mean	**Deviation**	**Skewness**	**Kurtosis**	**Shape** (-/$\mathcal{N}$/$\mathcal{LN}$)
Log-normal	539 k€	194 k€	2.1	6.4	-
Normal	548 k€	206 k€	2.7	15.0	-
Uniform	543 k€	190 k€	1.5	2.7	-

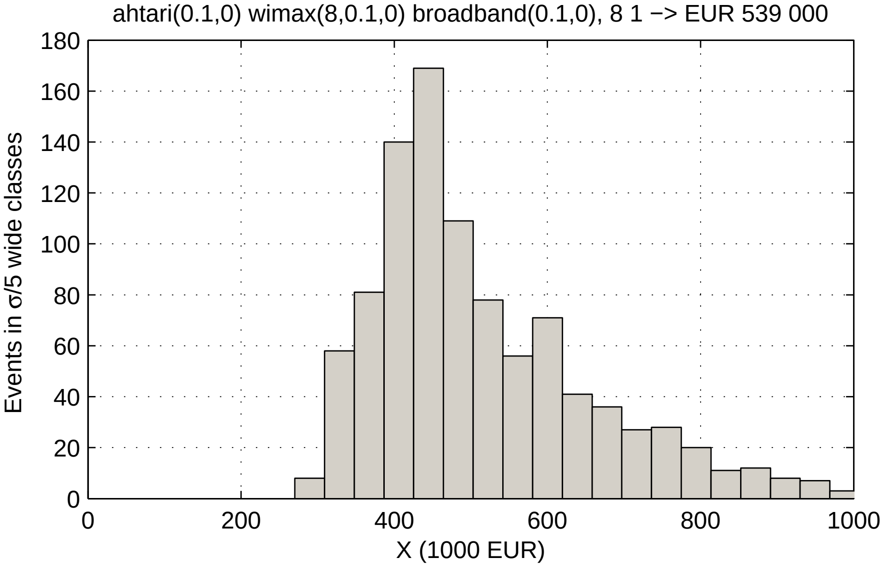

Figure 5.12: The histogram of the investment costs with log-normal parameters

The Influence of the Error Shapes on Investment Costs

The distribution of the simulated investment costs is positively skewed; see Figure 5.12, where the histogram of the investment costs with log-normal parameters are drawn. This is due to the exponential traffic growth that was assumed in the network planning phase. The mean values for investment costs with different error shapes on parameters are 539 k€ and 547 k€ and 543 k€, see Table 5.6, where the deviations, skewnesses, and kurtosis excesses are recorded. The last column *shape* is $\mathcal{N}$ if the distribution is normal and $\mathcal{LN}$ if it is log-normal. Parentheses indicate a borderline case.

The investment cost distributions do not differ from each other significantly, see Figure 5.13 where the cumulative distributions with different error shapes are drawn. The null hypothesis of Kolmogorov–Smirnov statistical test for two sets is that test samples are from the same distribution [181]. In our case, this test does not reject the hypothesis for simulated investment costs samples. The P-values for the pairs log-normal errors vs. normal errors, log-normal errors vs. uniform errors, and normal errors vs. uniform errors are 0.26, 0.16, and 0.33, respectively. However, the deviation, kurtosis, and skewness are the least for the sample with uniform parameter errors.

The kurtosis excesses of the simulated investment costs are for all error shapes positive, which means that density distributions of investment costs have sharper peaks than standard normal distribution. Thus, the Jarque–Bera test [135] and Lilliefors test [175] reject the log-normality with P-values less than 0.01.

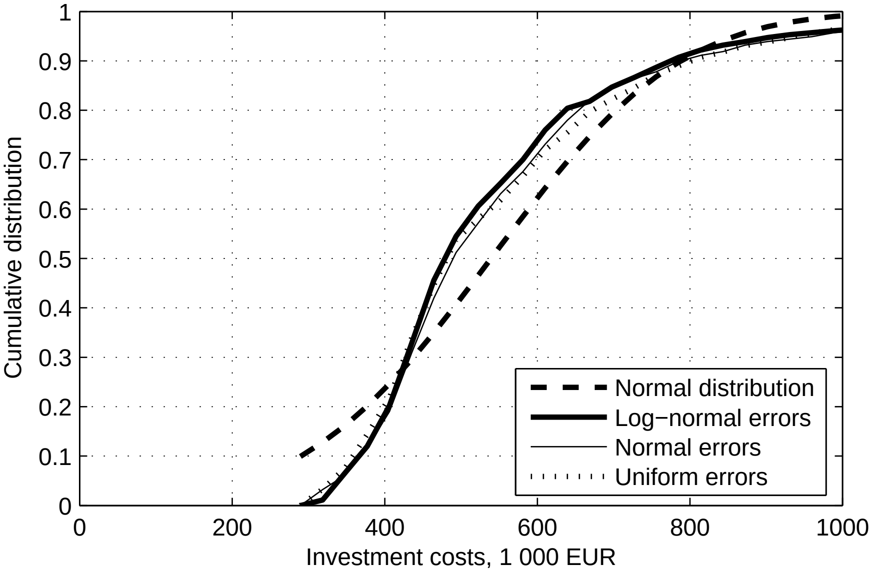

Figure 5.13: Cumulative distribution functions of the investment costs with different error shapes

The Influence of the Error Shapes on Net Present Values

Because of the taxation and the log-normal distribution of the investment costs, the distribution of the net present value tend to be negatively skewed. In Table 5.7, we collect the statistics of NPV distributions that were simulated using different error shapes. The shape of the parameter errors does not affect the situation dramatically. The means of the net present values are 103–123 k€ negative. However, the deviation is the least with uniform parameter errors and the kurtosis greatest with normal parameters, like above when analyzing investment costs.

Table 5.7: Statistics of the net present values with different parameter error shapes

	Mean	**Deviation**	**Skewness**	**Kurtosis**	**Shape** (-/$\mathcal{N}$/$\mathcal{LN}$)
Log-normal	-103 k€	316 k€	-1.9	7.4	-
Normal	-123 k€	345 k€	-2.6	14.0	-
Uniform	-104 k€	307 k€	-1.3	3.0	-

The samples do not differ from each others significantly. The Kolmogorov–Smirnov test does not reject the null hypothesis that the samples are from a common distribution. The P-values for log-normal vs. normal, log-normal vs. uniform, and normal vs. uniform are 0.40, 0.29, and 0.23, respectively.

The Influence of the Error Shapes on NPV ratios

The analysis of the simulated NPV ratios is concluded in Table 5.8. The mean ratios are 0.87–0.91 with deviation 0.46–0.47. Still, the Kolmogorov–Smirnov test cannot differentiate the samples and the P-values for that are greater than earlier: 0.17, 0.51, and 0.21.

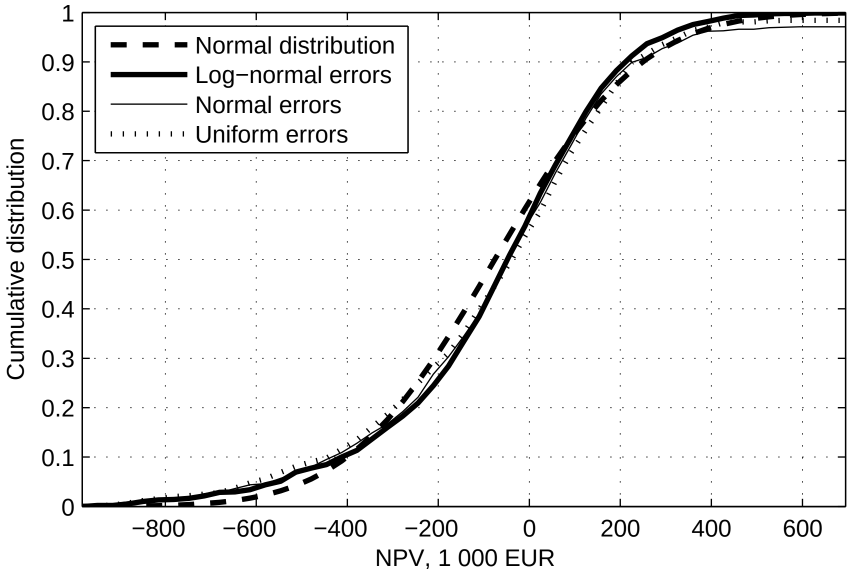

Figure 5.14: Cumulative NPV distribution functions with different error shapes

Table 5.8: Statistics of the NPV ratio with different parameter error shapes

	Mean	**Devi-ation**	**Skew-ness**	**Kur-tosis**	**Shape** (-/$\mathcal{N}$/$\mathcal{LN}$)
Log-normal	0.909	0.462	0.28	0.48	$\mathcal{N}$ (0.07)
Normal	0.879	0.460	0.15	0.07	$\mathcal{N}$ (> 0.15)
Uniform	0.910	0.468	0.20	-0.34	($\mathcal{N}$) (0.04)

The Influence of the Error Shapes on IRR

The descriptive statistics for internal rate of return (IRR) are found in Table 5.9. The mean values vary from 0.128 to 0.135 with standard deviation of some 0.1. The IRR is positively skewed but neither normal nor log-normal. The kurtosis excess is lowest with uniform error shapes.

Table 5.9: Statistics of the IRR with different parameter error shapes

	Mean	**Devi-ation**	**Skew-ness**	**Kur-tosis**	**Shape** (-/$\mathcal{N}$/$\mathcal{LN}$)
Log-normal	0.135	0.100	1.00	0.99	-
Normal	0.128	0.096	0.88	0.49	-
Uniform	0.135	0.102	0.78	0.05	-

The Influence of the Error Shapes on Payback Period

The mean values for payback period with different error shapes vary from 9.7 to 10.7, see Table 5.10. The error shapes seem to affect on the descriptive statistical of the payback period quite dramatically.

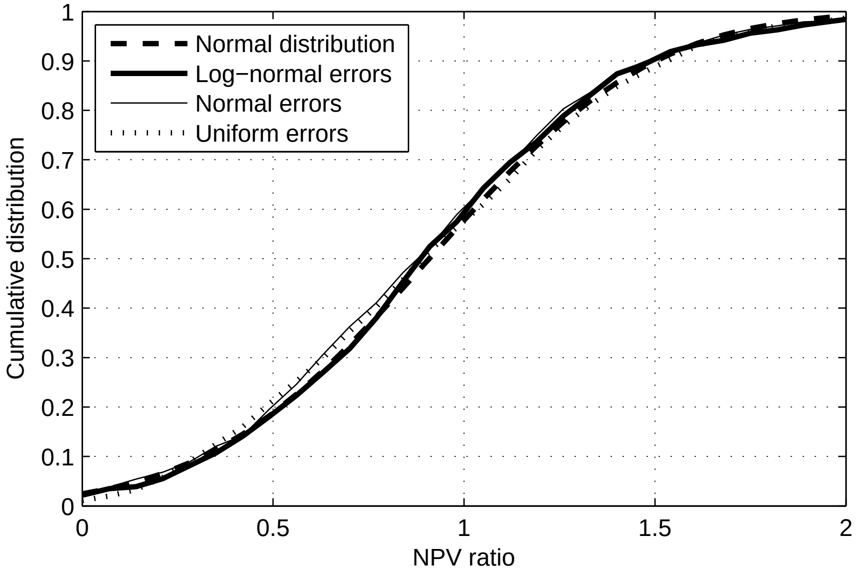

Figure 5.15: Cumulative NPV ratio distribution functions with different error shapes

Table 5.10: Statistics of the payback period with different parameter error shapes

	Mean	**Deviation**	**Skewness**	**Kurtosis**	**Shape** (-/$\mathcal{N}$/$\mathcal{LN}$)
Log-normal	8.7	31.9	-21.0	470	-
Normal	10.7	10.6	7.6	93	-
Uniform	9.8	6.3	2.3	10	-

5.2.6 THE EFFECTS OF UNCERTAINTIES IN WIMAX INVESTMENT

The Shape of the Investment Costs Distribution

The distribution of the investment costs is positively skewed. One reason for that is the exponential traffic growth that is assumed in the network planning phase. Let us now assume the parameters with normally distributed uncertainties, or errors. The effects of different uncertainties in parameters are analyzed by reducing the uncertainties on some parameters. In the analysis in [225], the uncertainty was assumed log-normal and the effect of the exponential traffic growth on the investment costs was reduced by using yearly upgrading construction strategy for network. The results with both methods are quite similar.

The average service rate (i.e., throughput demand) growth factor is one important parameter affecting the investment. The other growth parameters — the household growth factor and the growth factors of the service and network component prices — could also influence skewness in the investment costs. In addition, the penetration growth may lead to skewness and the coverage requirement sets the minimal infrastructure for the network.

Let us switch off the randomization for all these parameters, one at a time. The means, deviations, skewnesses, kurtosis excesses, and shapes from the simulations are presented in Table 5.11. To study the joint effects, the household growth, which have the greatest affects on results, are assumed constant simultaneously with other parameters in some cases.

The skewness and kurtosis would nearly halve — compared to the base case — when the

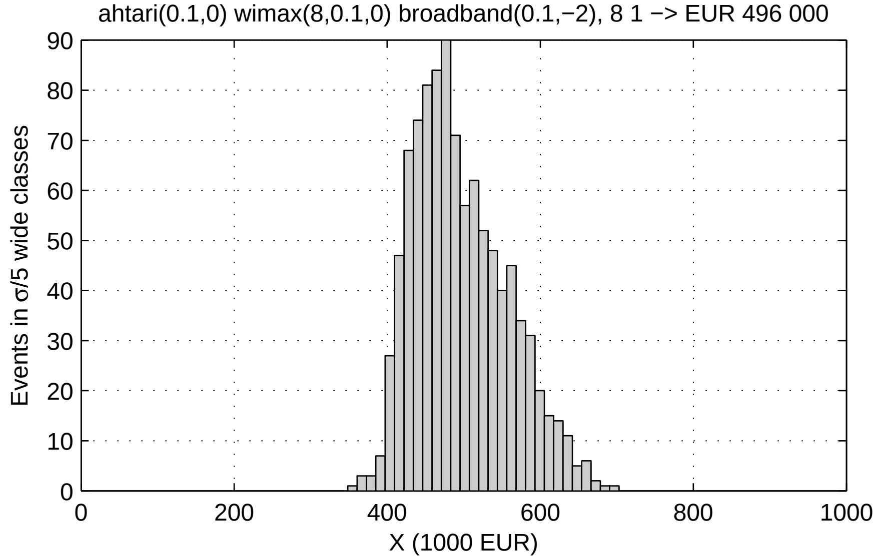

Figure 5.16: Histogram of the investment costs with known service rate and household growths

household growth parameter is assumed to be known. The effects of other parameters are not that dramatic. If the effect of the two most influential parameters (throughput demand growth and household growth factor) are combined, the skewness and kurtosis are reduced further. The resulting distribution approaches normal distribution, see Figure 5.16 where the histogram of the simulated investment costs is drawn.

A network may be coverage limited or capacity limited. In [225, 227] the investments were capacity limited and the uncertainty in the coverage did not make the shape of the investment costs more asymmetric. This is because the WiMAX network in rural areas with outdoor antennas does not seem to be sensitive for the coverage limits [242]. Now that the assumptions are more specific, the coverage is an issue and it affects the shape of the investment cost distribution. As can be seen in Figure 5.17 and Table 5.12, the shape of the investment costs becomes log-normal if the coverage does not have to be achieved.

Table 5.11: Statistics of the investment costs

Reduced uncertainty for growth parameter	**Mean** (k€)	**Deviation**	**Skewness**	**Kurtosis**	**Shape** (-/$\mathcal{N}$/$\mathcal{LN}$)
None	539	194	2.1	6.4	-
Penetration	544	201	2.3	8.7	-
Adaptation shape	559	225	2.3	8.3	-
Service rate	527	157	2.5	12.0	-
ARPU	548	217	2.7	13.0	-
Network Price	543	218	2.5	9.4	-
Household	517	130	1.5	3.9	-
Household & Penetration	524	132	1.7	4.7	-
Household & Adaptation shape	515	118	1.3	3.2	-
Household & Service rate	496	61	0.5	-0.3	-
Household & Network Price	518	129	1.4	3.6	-

Table 5.12: Statistics of the investment costs without coverage demand

Reduced uncertainty for growth parameter	**Mean** (k€)	**Devi-ation**	**Skew-ness**	**Kur-tosis**	**Shape** (-/$\mathcal{N}$/$\mathcal{LN}$)
None	462	254	1.7	4.5	$\mathcal{LN}$ (0.07)
Household	438	158	1.3	3.5	-
Penetration	465	231	1.3	2.0	$\mathcal{LN}$ (0.58)
Adaptation shape	469	259	2.2	8.7	$\mathcal{LN}$ (0.09)
Service rate	448	185	1.3	2.2	$\mathcal{LN}$ (0.11)
ARPU	474	255	1.8	6.0	$\mathcal{LN}$ (0.21)
Network Price	463	256	2.0	7.1	$\mathcal{LN}$ (0.20)

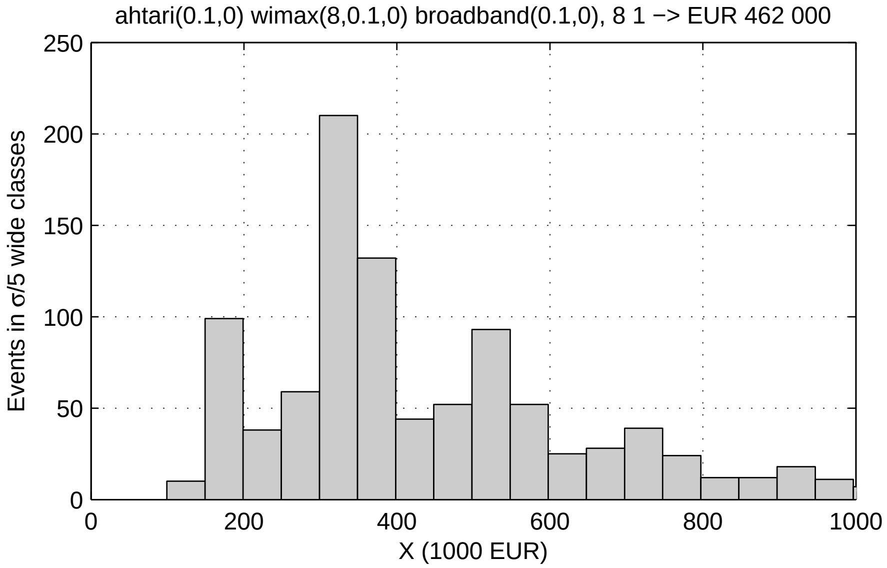

Figure 5.17: The histogram of the investment costs without coverage demand

In summary, the investment costs are positive skewed and nearly log-normally distributed, see Figure 5.18. The normally distributed errors in the growth of the number of households has the greatest impact on the asymmetry. If the parameter errors are not assumed normal but log-normal or uniform, the situation does not change. The simulated samples — assuming known service rate and household growths — cannot be proved to differ from each other.

The Shape of the NPV Distribution

The NPV distribution for network investments is negatively skewed. Let us analyze the distribution using the same modifications as used above. The statistics of the resulting net present values are collected in Table 5.13.

The skewness and kurtosis of NPV reduce within the modifications. If the service rate growth or household growth factor is assumed to be known, the distribution turns towards normal distribution. The uncertainty in the other parameters does not affect similarly the shape of the distribution. When the service rate is assumed to be known simultaneously with known household growth or without taxation simulation, the simulated net present values are normally distributed, see Figure 5.19 where the NPV distributions for the cases are compared to the base case and normal distribution.

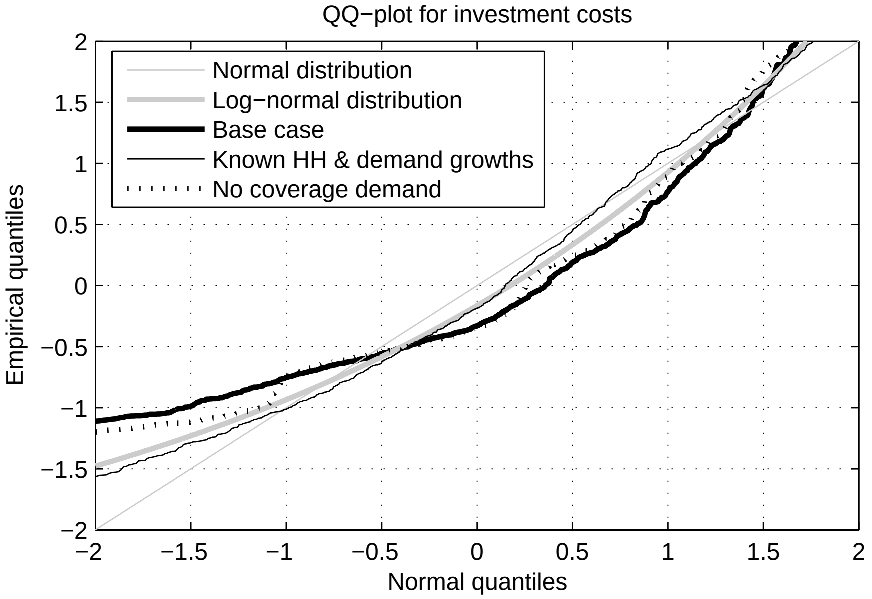

Figure 5.18: The QQ plot of investment costs

Table 5.13: Statistics of the net present values

Reduced uncertainty for growth parameter	**Mean** (k€)	**Deviation**	**Skewness**	**Kurtosis**	**Shape** (-/$\mathcal{N}$/$\mathcal{LN}$)
None	-103	316	-1.9	7.4	-
Penetration	-116	337	-2.4	12.0	-
Adaptation shape	-125	362	-2.2	8.0	-
Service rate	-71	217	-1.5	5.7	-
ARPU	-116	357	-2.9	16.0	-
Network Price	-117	356	-2.6	10.0	-
Household	-75	275	-1.3	3.9	-
Household & Penetration	-83	272	-1.3	3.5	-
Household & Adaptation shape	-63	267	-1.0	2.8	-
Household & Service rate	-25	159	0.1	-0.1	$\mathcal{N}$ (> 0.2)
Household & Network Price	-72	272	-1.1	2.9	-
No taxation	-46	440	-4.6	50.0	-
Service rate & no taxation	4	222	-0.2	1.2	($\mathcal{N}$) (0.02)

Normality is not rejected even if the shapes of the parameter errors are assumed log-normal or uniform. Quite the contrary, the P-values for Lilliefors and Jarque–Bera tests increase. The Kolmogorov–Smirnov test for the difference of the NPV distributions — assuming normal, log-normal, or uniform error shapes — does not reject the null hypothesis of the different samples being from the same distribution, either.

The Shape of the NPV Ratio Distribution

The shape of the NPV ratio distribution is important for the real option valuation methods. The NPV ratio is used to combine two random variables together — the discounted profit sum and

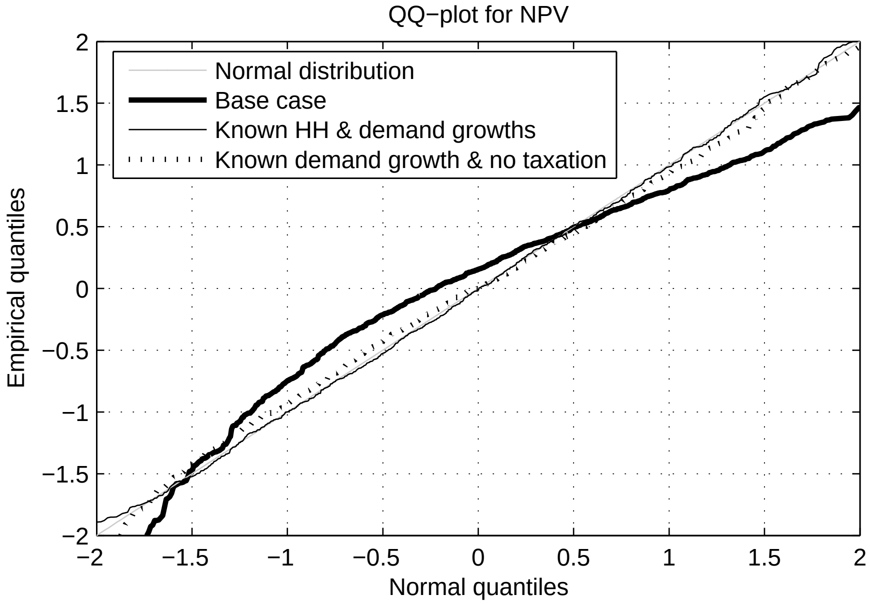

Figure 5.19: The QQ-plot of NPV distributions

investment costs. This must be done when using analytical option valuation methods that are not developed for multiple uncertainties.

The modifications described earlier do not affect the NPV ratio as strongly as the NPV or the investment costs, see Table 5.14 where statistics of NPV ratios in different cases are presented. The skewness is positive, as it is for log-normal distributions. However, the large deviation suggests that the NPV ratio is negative for some events. This can also be seen from Figure 5.21 or from Figure 5.20 where QQ-plots for some cases are drawn.

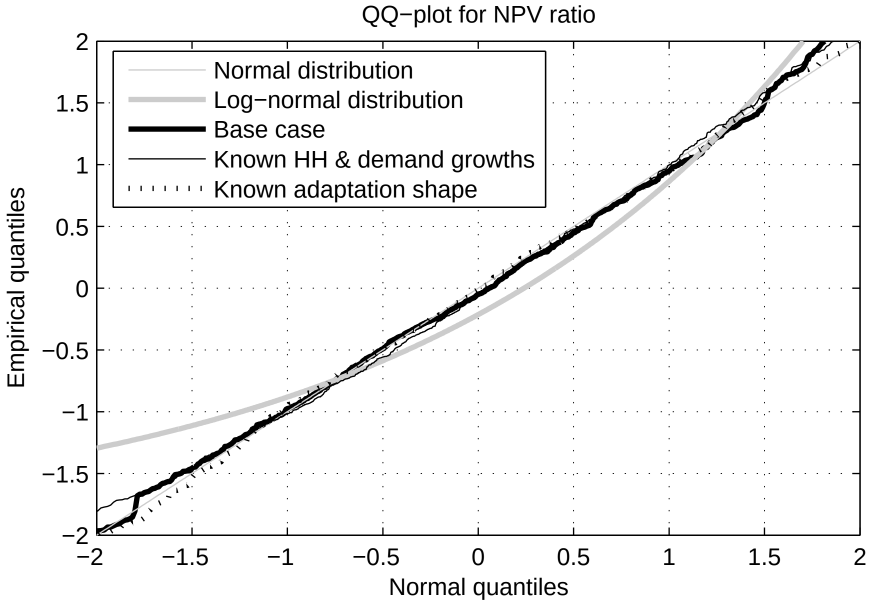

Figure 5.20: QQ-plot for NPV ratios in three cases

Table 5.14: Statistics of the NPV ratio

Reduced uncertainty for growth parameter	**Mean**	**Deviation**	**Skewness**	**Kurtosis**	**Shape** (-/$\mathcal{N}$/$\mathcal{LN}$)
None	0.909	0.462	0.3	0.5	$\mathcal{N}$ (0.07)
Penetration	0.890	0.461	0.2	0.5	$\mathcal{N}$ ($>$ 0.20)
Adaptation shape	0.894	0.473	0.1	0.0	$\mathcal{N}$ ($>$ 0.20)
Service rate	0.910	0.348	0.2	0.2	$\mathcal{N}$ (0.08)
ARPU	0.896	0.462	0.2	0.4	($\mathcal{N}$) (0.02)
Network Price	0.897	0.452	0.0	0.4	$\mathcal{N}$ (0.07)
Household	0.940	0.463	0.3	0.4	-
Household & Penetration	0.920	0.448	0.2	0.1	$\mathcal{N}$ (0.14)
Household & Adaptation shape	0.953	0.474	0.3	0.3	($\mathcal{N}$) (0.05)
Household & Service rate	0.962	0.320	0.3	-0.1	($\mathcal{N}$) (0.02)
Household & Network Price	0.942	0.460	0.2	0.2	$\mathcal{N}$ ($>$ 0.20)

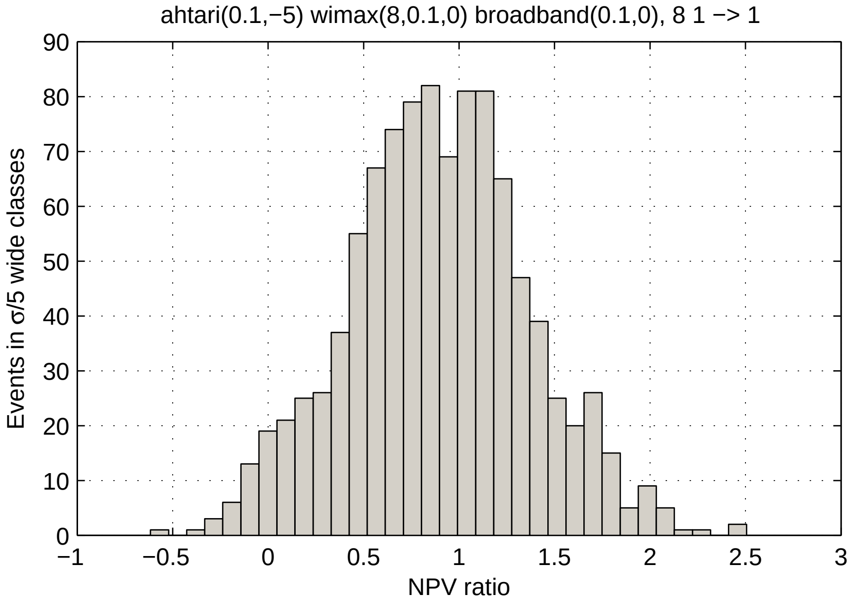

Figure 5.21: The histogram of the NPV ratio when the adaptation shape is known

The adaptation shape and service rate growth have the greatest influences on the skewness. Household growth, on the other hand, does not affect the skewness so much. The NPV ratio in every cases is near to normal distribution. The greatest P-value is with known adaptation shape, see the histogram of the NPV ratio in Figure 5.21.

The Shape of the Payback Period Distribution

The expectation of the payback period is 8.7 years. The distribution is skewed and has large deviation, see Table 5.15. The service rate growth is the most important factor affecting the uncertainty and skewness in the distribution.

Table 5.15: Statistics of the payback period

Reduced uncertainty for growth parameter	Mean	Deviation	Skewness	Kurtosis	Shape (-/$\mathcal{N}$/$\mathcal{LN}$)
None	8.7	32	-21	470	-
Penetration	11.1	15	10	150	-
Adaptation shape	10.7	12	10	170	-
Service rate	8.9	5.1	1	30	-
ARPU	11.1	30	28	850	-
Network Price	189.0	5 610	32	1000	-
Household	9.6	8	5	43	-
Household & Penetration	9.6	6	3	15	-
Household & Adaptation shape	9.1	19	-17	5 300	-
Household & Service rate	8.2	3	1	3	-
Household & Network Price	9.7	12	18	4 600	-

The Shape of the IRR Distribution

The internal rate of return has expectation 13.5%. The distribution is slightly positively skewed, see Table 5.16. The most influential parameters are service rate growth and household growth.

Table 5.16: Statistics of the internal rate of return

Reduced uncertainty for growth parameter	Mean	Deviation	Skewness	Kurtosis	Shape (-/$\mathcal{N}$/$\mathcal{LN}$)
None	0.135	0.10	1.0	1.0	-
Penetration	0.131	0.10	1.0	1.0	-
Adaptation shape	0.133	0.10	0.9	0.6	-
Service rate	0.130	0.08	0.9	0.8	-
ARPU	0.131	0.10	0.9	0.8	-
Network Price	0.132	0.09	0.9	0.7	-
Household	0.140	0.10	0.9	0.7	-
Household & Penetration	0.136	0.10	0.9	0.6	-
Household & Adaptation shape	0.144	0.10	0.9	0.5	-
Household & Service rate	0.138	0.07	0.7	0.2	-
Household & Network Price	0.141	0.10	0.8	0.5	-

5.2.7 OPTION TO DEFER

The network value may increase if the network investment is delayed. This is mainly because of the recovered penetration information, cost savings in fixed operational costs and more rapid adaptation for later launch as the services are more mature. Such an option to defer is analyzed here as a Bermudan call option, which means that the exercise may occur at any of a finite number of given times. The American call option model is not used because of its computational challenges. The simulated option values at different exercise times are drawn in Figure 5.22. The option value is greatest at years 2012 and 2013.

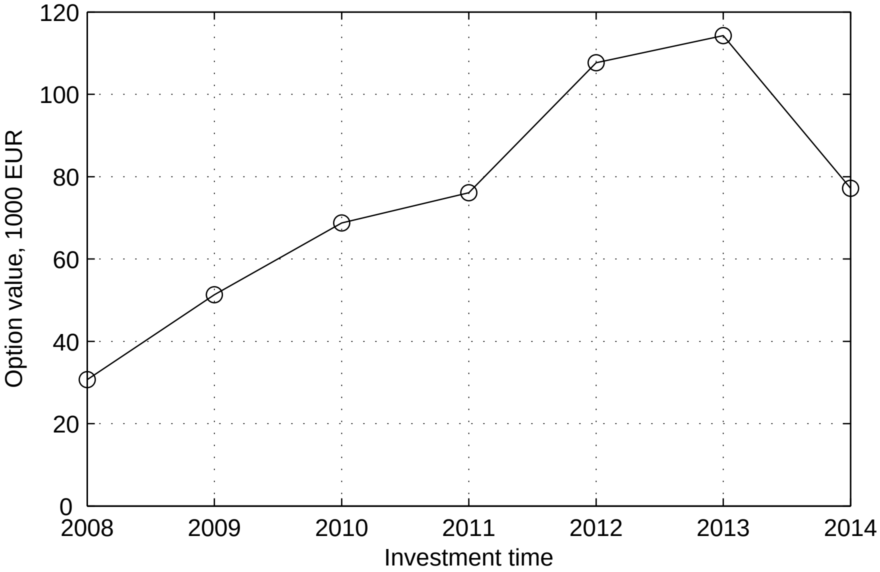

Figure 5.22: Real option values for the WiMAX network

WiMAX Investment in Year 2010

Let us analyze more deeply the WiMAX investment in year 2010. Table 5.17 presents the descriptive statistics for investment costs, NPV, NPV ratio, IRR and payback period. The investment has expected net present value EUR 38 000 negative, but the option value is some EUR 70 000 positive. NPV distribution is negatively skewed and NPV ratio is nearly normally distributed, see Figure 5.23.

Table 5.17: Statistics for the WiMAX investment in year 2010

	Mean	**Deviation**	**Skewness**	**Kurtosis**	**Shape** (-/$\mathcal{N}$/$\mathcal{LN}$)
Investment costs	437 k€	169 k€	2.0	5.6	-
NPV	-38 k€	251 k€	-1.2	5.3	-
NPV ratio	1.0	0.56	0.7	0.9	-
IRR	0.20	0.22	1.3	1.2	-
Payback period	9.34	54.1	-2.8	140.0	-

The influence of the error shapes on option values

The analyzed distributions above used 1 000 simulated outcomes, or scenarios, for each case. Note that only one option value is calculated using (4.16) for each error shape. To estimate the accuracy of the simulated option value, let us simulate some 1 000 option values based on 1 000 000 simulated events for each error shape type. As a result, we get the cumulative distributions for option values drawn in Figure 5.24. The distributions are normally distributed with means and standard deviations EUR 69 000 and EUR 3 500; EUR 69 600 and EUR 3 500; and EUR 69 800 and EUR 3 500 for normal, log-normal and uniform error shape types, respectively. Now the option values with different error shapes get values near each other. Actually

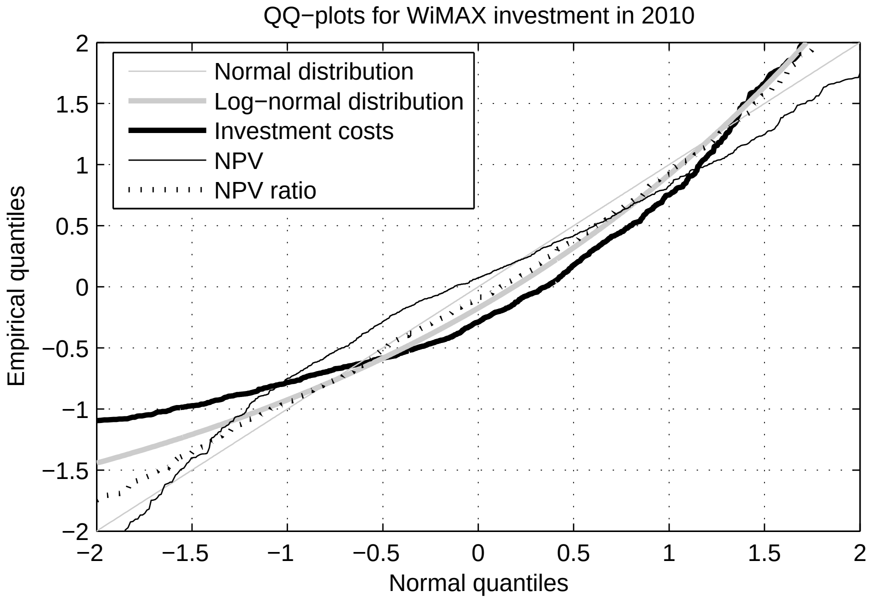

Figure 5.23: QQ-plots for the WiMAX investment in year 2010

the differences of the means are smaller than the deviations of the option values within each error shape. Thus it seems that the error shapes do not have a real impact on the simulation results. However, the distributions of the option values differ from each other as can be seen in Figure 5.24. The Kolmogorov–Smirnov significance test for two samples does not reject the null hypothesis of samples with log-normal and uniform errors. The P-value is 0.05. The other pairs of samples are rejected and the P-values for the Kolmogorov–Smirnov test are less than 0.01.

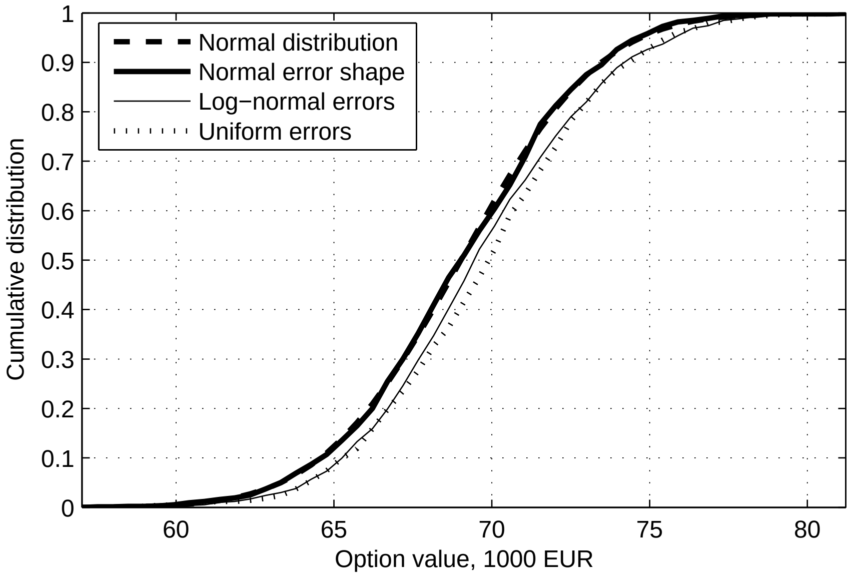

Figure 5.24: The comparison of the option value distributions with different error shape types

5.3 CASE: WIMAX FOR ÄHTÄRI COTTAGES

5.3.1 EXPAND OPTION

Consider the WiMAX network analyzed in Section 5.2. Let us analyze the economics of the network expansion for summer cottages in 2011. One new base station is needed to achieve coverage for 1000 new potential subscribers.

The ARPU for summer cottage users is assumed EUR 15 per month, i.e., EUR 180 a year. The service rate is assumed the same as for household users but an overbooking factor of 30 is used, yielding one third lower throughput demand, i.e., currently 0.033 Mbps. The descriptive statistics for the investment are summarized in Table 5.18 and discounted cash flows in Figure 5.25. The dotted, strong solid and dashed lines are the first, second and third quartiles and the light solid line is the mean value line. The revenues are stabile since the customer growth and price erosion in ARPU have opposite effects. The QQ-plots in Figure 5.26 indicate that the investment costs are nearly log-normally distributed. The expectation of investment costs is EUR 60 000 and NPV in the planning horizon is negative. However, the payback period and IRR indicates that the investment could be still attractive, see Table 5.18. The option value is EUR 25 000.

Table 5.18: Statistics for the WiMAX cottage investment

	Mean	**Devi- ation**	**Skew- ness**	**Kur- tosis**	**Shape** (-/$\mathcal{N}$/$\mathcal{LN}$)
Investment costs	60 k€	56 k€	2.8	12.0	-
NPV	-36 k€	87 k€	-2.2	9.9	-
NPV ratio	1.80	7.89	8.0	79.0	-
IRR	0.20	0.27	1.5	1.2	-
Payback period	13.7	262	23.0	660.0	-

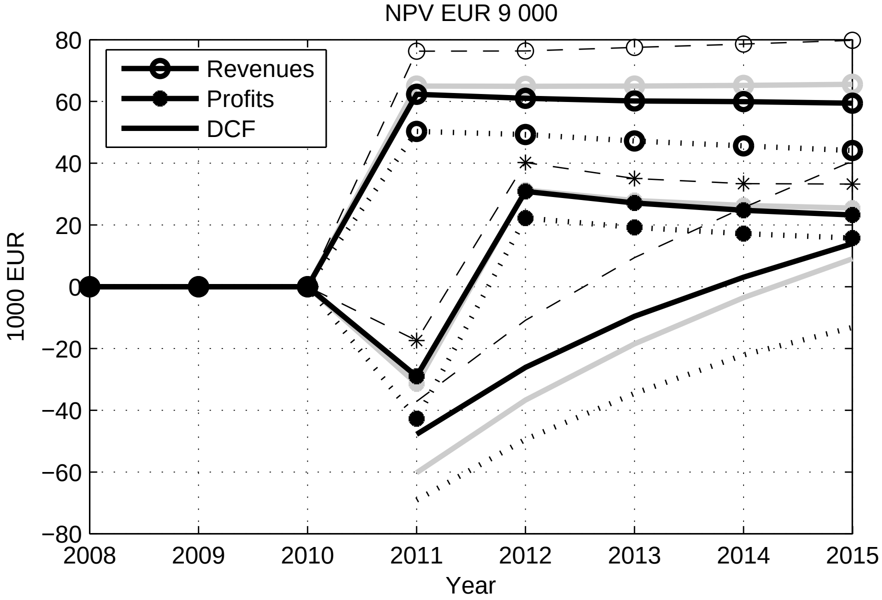

Figure 5.25: Discounted cash flows for the WiMAX network expansion in year 2011

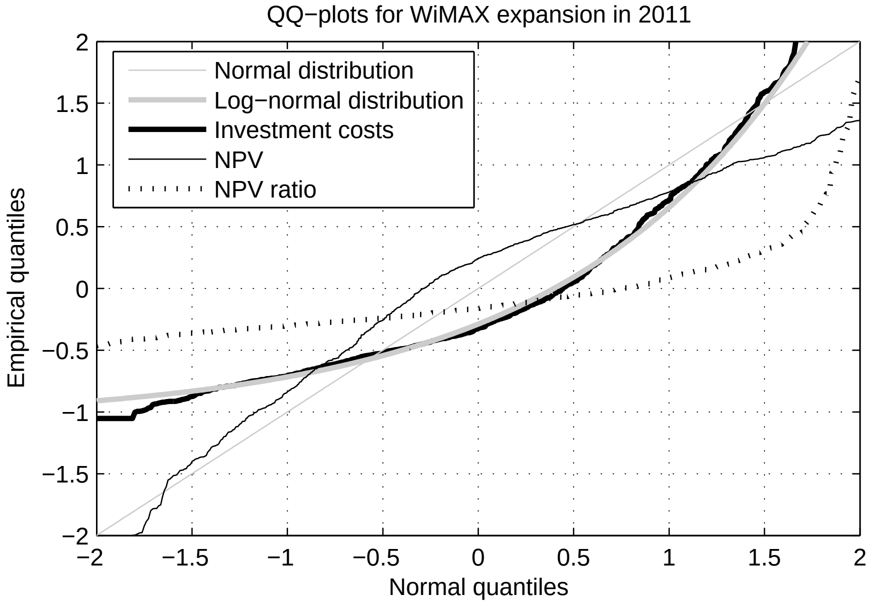

Figure 5.26: QQ-plots for the WiMAX expansion in year 2011

5.3.2 Option to Defer

The network extension does not have to be done in 2011. Current option values for different exercise times are drawn in Figure 5.27. The optimal time would be in 2011 or 2012 with value of EUR 27 000. The smallest value is at 2008 with EUR 3 000.

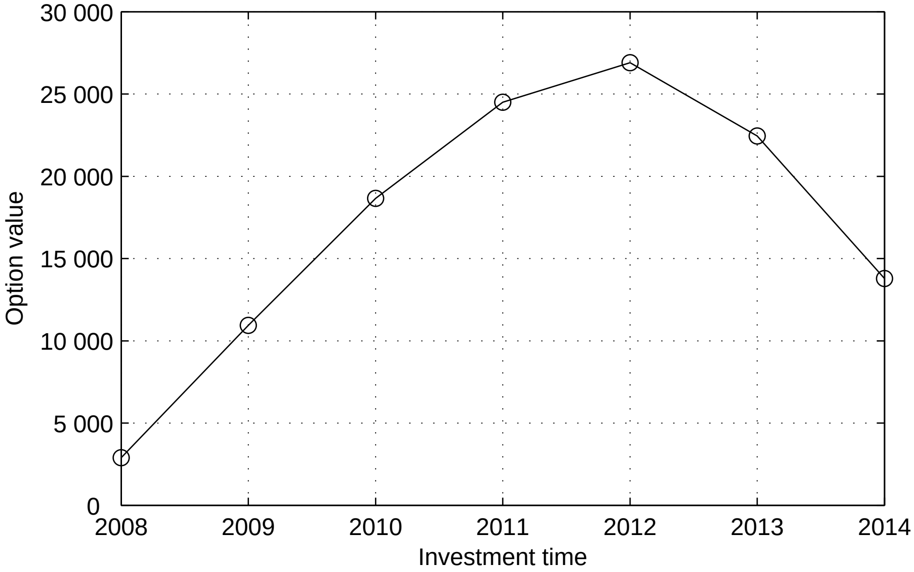

Figure 5.27: Real option values for the cottages

5.4 Case: ADSL@Ähtäri

Consider the service area and demand scenarios presented in Section 5.2.1. Let us now estimate the value of an ADSL network investment on the Ähtäri households.

5.4.1 ADSL Network Parameters and Cost model

ADSL coverage is predicted using a radius of 6 kilometers for each DSLAM. The access lines are assumed to be wired already, but the broadband network operator has to rent it from the incumbent operator and install it to a line card. One line card may serve 64 lines and one DSLAM 16 line cards. From DSLAM there is fiber connection to the transfer network with 155 Mbps capacity.

Table 5.19: Assumed parameters for ADSL networks [13, 243]

	Price	**Model Factor**	**Installation**	**Operating**
Access line	0 €	1.0	100 €	100 €/a
Line card	3 200 €	0.9	100 €	100 €/a
DSLAM	3 000 €	0.9	1 000 €	1800 €/a
Ground works	0	1.0	4 000 €/km	0
Fiber	1 000 €/km	0.9	0	100 €/km

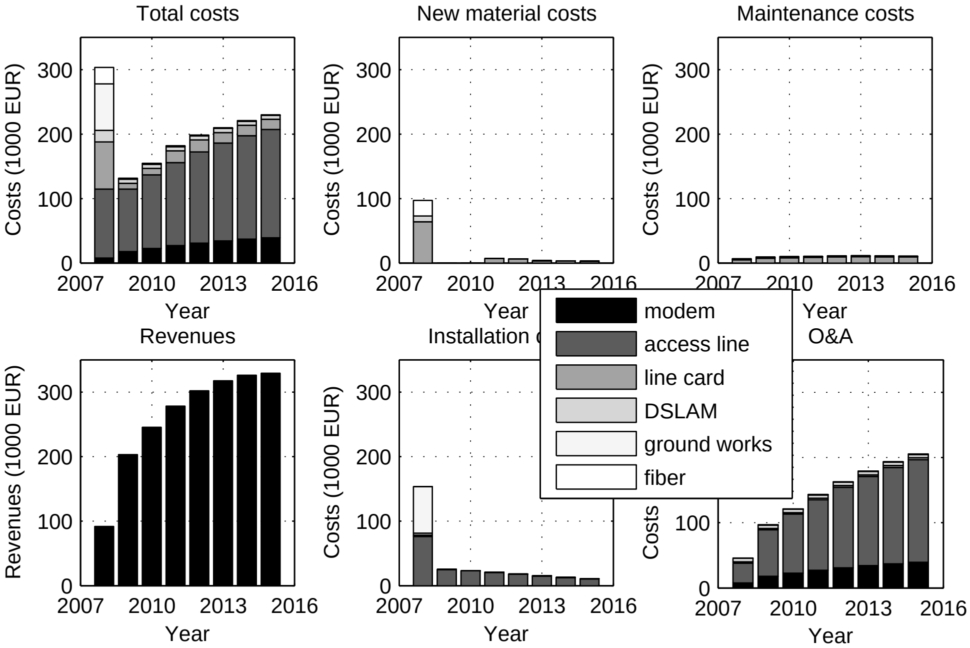

Figure 5.28: Cost structure for the ADSL network

The town central area having some 1500 households is covered with one DSLAM. For the outer areas 20% coverage is used to achieve some 1000 more households. Now the total of 2500 households are served with three DSLAMs and 20 line cards.

The assumed prices and the parameters for the ADSL component price models are combined in Table 5.19. The stabilized maintenance rate for electronics is assumed to be 20% and for fiber 5%. The network costs follow the values in Figure 5.28. The rental of access lines is the biggest cost for the network. The tax rate is assumed to be 26% and the infrastructure investments are depreciated within 3 (CPE) or 5 (electronics) or 20 (wires) years.

5.4.2 DCF VALUATION AND SENSITIVITY ANALYSIS

The simulated net present value for the ADSL case above is EUR 113 000 and the calculated theoretical value EUR 25 000. The simulated payback period for the investment is 4 years and internal rate of return 28 %.

The sensitivity of each parameter on NPV is depicted in Table 5.20 and on investment costs in Table 5.21. Household, throughput and ARPU growth parameters have the most significant effects on NPV, and household and throughput growths on investment costs.

Table 5.20: Sensitivity analysis for NPV of the ADSL investment

Parameter	**Sim -10%**	**Sim +10%**	**Calc -10%**	**Calc +10%**
No changes	113 k€	113 k€	25 k€	25 k€
Households, $N_{pot}(0)$	89 k€	136 k€	23 k€	28 k€
Households growth, e^{c_N}	61 k€	77 k€	182 k€	-360 k€
Penetration, β			106 k€	-45 k€
Penetration growth, k			21 k€	32 k€
Adaptation shape, b	113 k€ [a]	112 k€ [a]	117 k€	-71 k€
Initial penetration, $\rho(0)$	110 k€ [a]	115 k€ [a]		
Time to 50% penetration	133 k€ [a]	101 k€ [a]		
Throughput, $d_a(0)$	117 k€	107 k€	107 k€	-56 k€
Throughput growth, e^{c_d}	135 k€	1 k€	444 k€	-903 k€
ARPU, $p_S(0)$	20 k€	204 k€	-68 k€	119 k€
ARPU growth, e^{c_S}	-172 k€	497 k€	-266 k€	457 k€
Technology prices, $p_T(0)$	124 k€	101 k€	35 k€	15 k€
Price growths, e^{c_T}	125 k€	95 k€	27 k€	24 k€
Installation, $K_{T,I}$	132 k€ [b]	93 k€ [b]	41 k€	9 k€
New user costs, $K_{T,V,I}$	132 k€ [b]	93 k€ [b]	25 k€	25 k€
O&A costs, $K_{T,F,OA}$	162 k€ [c]	63 k€ [c]	80 k€	-30 k€
Subscription costs, $K_{T,V,OA}$	162 k€ [c]	63 k€ [c]	35 k€	16 k€
Maintenance, $K_{T,F,M}$	115 k€ [d]	110 k€ [d]	28 k€	23 k€
Maintenance, $c_{T,F,M}$	115 k€ [d]	110 k€ [d]	26 k€	24 k€
Maintenance, $K_{T,V,M}$	115 k€ [d]	110 k€ [d]	26 k€	25 k€
Cell radius, R_c	73 k€	113 k€	25 k€	25 k€
Capacity, $1/K_{T,D}$	107 k€	117 k€	-65 k€	99 k€

[a] The forecast assumes the shape parameter and fits curve to two parameter defined points

[b] Parameters $K_{T,I}$ and $K_{T,V,I}$ changed

[c] Parameters $K_{T,F,OA}$ and $K_{T,V,OA}$ changed

[d] Parameters $K_{T,F,M}$, $c_{T,F,M}$ and $K_{T,V,M}$ changed

The sensitivity of household growth, throughput demand growth, ARPU, operational and administration costs and cell radius are depicted in Figure 5.29. The most relevant parameters

Table 5.21: Sensitivity analysis for investment costs of the ADSL investment

Parameter	**Sim -10%**	**Sim +10%**	**Calc -10%**	**Calc +10%**
No changes	174 k€	174 k€	236 k€	236 k€
Households, $N_{pot}(0)$	167 k€	181 k€	213 k€	260 k€
Households growth, e^{c_N}	135 k€	287 k€	115 k€	460 k€
Penetration, β			245 k€	228 k€
Penetration growth, k			223 k€	248 k€
Adaptation shape, b	174 k€ [a]	174 k€ [a]	250 k€	217 k€
Initial penetration, $\rho(0)$	174 k€ [a]	174 k€ [a]		
Time to 50% penetration	174 k€ [a]	167 k€ [a]		
Throughput, $d_a(0)$	167 k€	181 k€	213 k€	260 k€
Throughput growth, e^{c_d}	135 k€	273 k€	115 k€	506 k€
Technology prices, $p_T(0)$	164 k€	184 k€	229 k€	244 k€
Price growths, e^{c_T}	174 k€	174 k€	236 k€	236 k€
Installation, $K_{T,I}$	166 k€ [b]	182 k€ [b]	220 k€	252 k€
New user costs, $K_{T,V,I}$	166 k€ [b]	182 k€ [b]	236 k€	236 k€
Cell radius, R_c	208 k€	174 k€	236 k€	236 k€
Capacity, $1/K_{T,D}$	181 k€	167 k€	263 k€	215 k€

[a] The forecast assumes the shape parameter and fits curve to two parameter defined points

[b] Parameters $K_{T,I}$ and $K_{T,V,I}$ changed

seems to be ARPU and O&A costs.

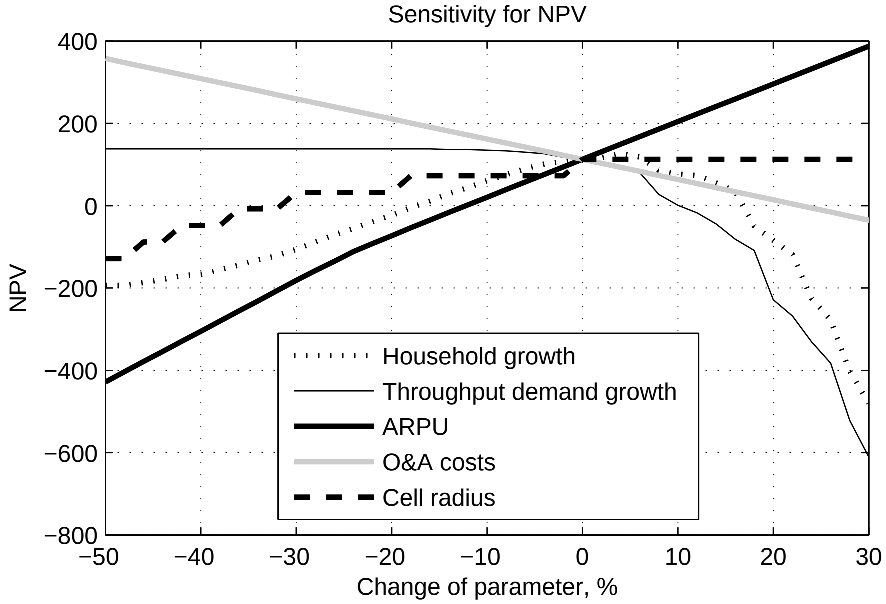

Figure 5.29: Sensitivity analysis for the ADSL network investment

5.4.3 Monte-Carlo Simulation

The statistics of the ADSL network investment differs from the WiMAX network investment. The simulated NPV is almost normally distributed with expectation EUR 97 000 and deviation EUR 126 000, see Figures 5.30 and 5.31. The skewness and the kurtosis of the distribution are -0.3 and 2.3, respectively. The normality tests reject normality with P-values less than 0.01.

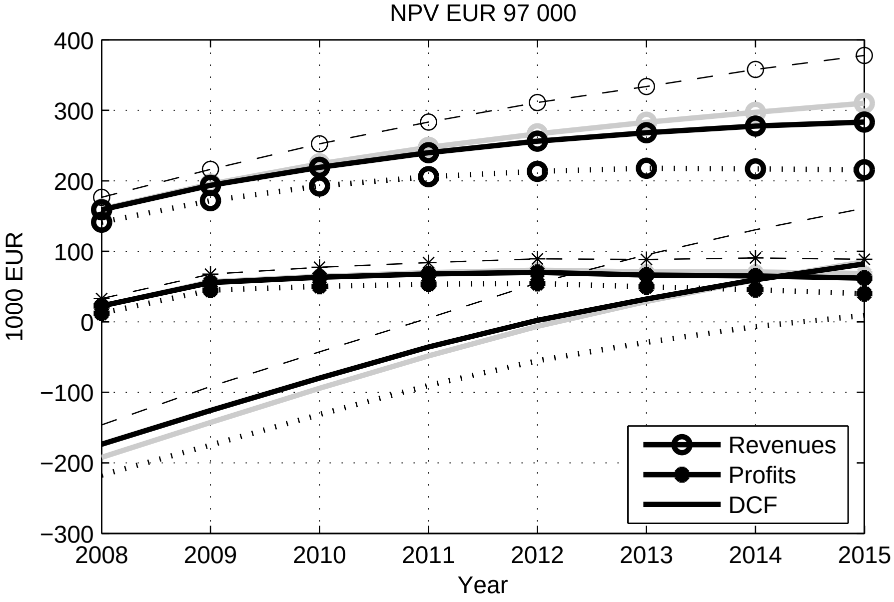

Figure 5.30: The discounted cash flows for an ADSL investment

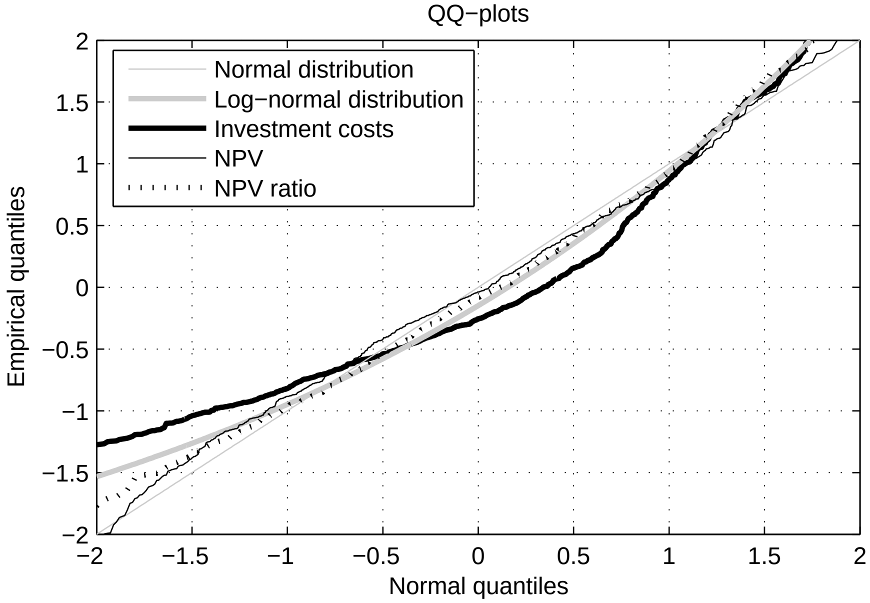

Figure 5.31: QQ-plots for ADSL

The distribution of the simulated investment costs is positively-skewed and it is nearly log-normal, see Figure 5.31 and Figure 5.32. Normality tests reject normality with P-value less than 0.1. The expectation of the investment costs is EUR 215 000. The weight of investment costs on the NPV is lower than in WiMAX network, see Figure 5.9 and Figure 5.28. This is why NPV is nearly normally distributed though costs are not. The descriptive statistics of the ADSL investment are combined in Table 5.22

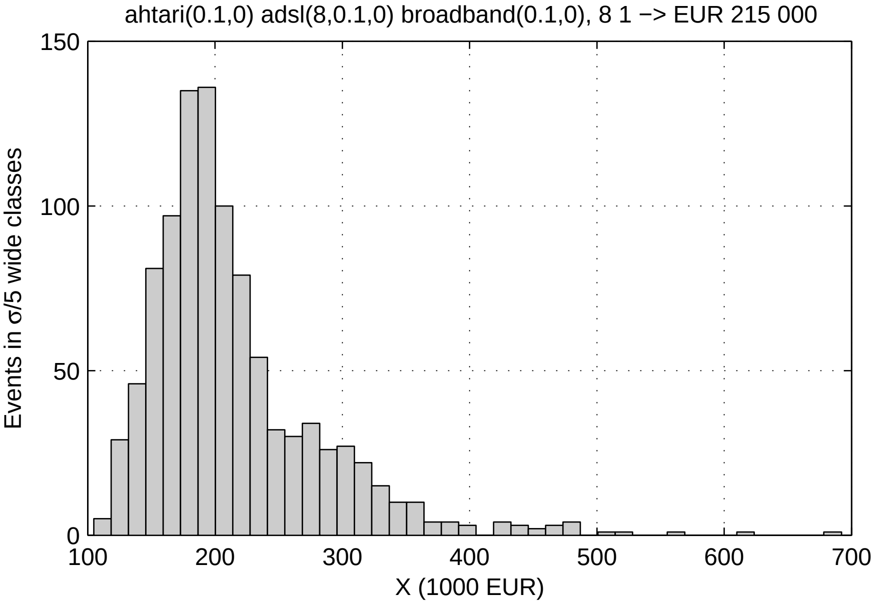

Figure 5.32: The distribution of the ADSL investment costs

Table 5.22: Statistics for the ADSL investment

	Mean	**Deviation**	**Skewness**	**Kurtosis**	**Shape** (-/$\mathcal{N}$/$\mathcal{LN}$)
Investment costs	215 k€	68 k€	1.9	6.1	-
NPV	97 k€	126 k€	-0.3	2.3	-
NPV ratio	1.53	0.60	0.5	0.4	-
IRR	0.29	0.16	0.7	-0.1	-
Payback period	6.4	38.2	22.0	630.0	-

5.5 CASE: INTELLIGENT RAILWAY TRANSPORTATION SYSTEM

Intelligent transportation systems (ITS) is a concept for developing the efficiency and versatility of transportation using information and communications technology (ICT) [85]. ITS can be considered in any transportation method or combination of them. Using ITS for railroads is here defined as railway with intelligent transportation system (RITS). The RITS end-users are travelers, the railway operator in-house users and cargo companies. The service groups for RITS include the wide scope of broadband services as well as telemetrics, billing and security for trains. The business model and benefits of RITS are introduced in [47, 138, 182] and analyzed in [268, 270]. This section is an extended techno-economical analysis for RITS based on the author's analysis in article [228].

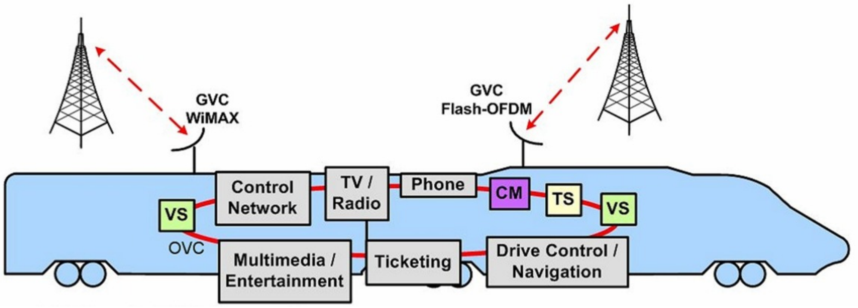

Figure 5.33: RITS system architecture

A general RITS network architecture is depicted in Figure 5.33. The RITS system is often divided in two network layers. The on-vehicle communications (OVC) transfers the data inside the train between the communication manager (CM), the train server (TS) and vehicle stations (VS). CM uses ground to vehicle communications (GVC) to connect the OVC network to the backbone network. The RITS end users communicate via a VS with TS or CM and backbone network. The modular network system in the RITS framework could utilize different communication technologies. Nowadays, the ultimate solution for end user junction points (VS) is WLAN, but the solution for the GVC network may differ from case to case. For example, in [104] De Greve introduced a solution with WiMAX network in GVC network and WLAN in train. Existing RITS solutions in Belgium, France, Sweden and UK utilize satellite connections and GPRS traffic in GSM and UMTS network [58].

5.5.1 RITS MODEL WITH FLASH-OFDM AND WLAN

Consider a RITS model for Finnish electrified railways with Flash-OFDM GVC network. The main potential user segment is the regular travelers with laptop computer. The number of long distance railway trips was 12.9 million in 2007 [274].

Let us assume that some 90 % of these use electrified railroads and 10 % of the travelers have laptop with them. Thus we have some 3200 potential RITS passenger subscribers a day. This can be assumed to grow some 5 % a year. Let us assume that the penetration grows from 10 % at the launching time in 2008 to 50 % in two years. The Richard's adaptation model is used with shape parameter -0.5. The average revenue per subscriber in a trip is some EUR 3. The assumed parameters for RITS service is summarized in Table 5.23.

The length of Finnish electrified railroads is some 2800 kilometers. The main parts of the

Table 5.23: Assumed parameters for the RITS service

	Initial value	**Exponential model factor**
Laptop trips per day	3000	1.05
Connection charge	0	-
Subscribers proportion on CPE costs	100%	-
ARPU	3€/a	1.0
Throughput demand	0.04 Mbps	1.2

Table 5.24: The assumed financial parameters for a RITS Flash-OFDM network

	Price	**Model factor**	**Installation**	**Operating**
Vehicular station (WLAN)	100 €	0.90	200 €	0 €/a
Train connections (per VS)	100 €	0.90	200 €	0 €/a
Management unit	2 000 €	0.85	2 000 €	1 600 €/a
Flash-OFDM transceiver	300 €	0.90	100 €	1 000 €/a

railways are covered by the Digita 450 MHz Flash-OFDM network [70]. The coverage will be expanding and tightening in future. The Flash-OFDM network offers service rate of some 1 Mbps download and 512 kbps upload even for high speed trains with speed of some 200 km/h [268]. Flash-OFDM transceivers cost EUR 300 per train and the subscription in Digitas network costs EUR 1 000 per train a year.

The management unit in a train with connection manager and train server costs are assumed EUR 2 000. WLAN transceiver and connections for VS costs EUR 100 and installation EUR 200 for both articles. It is assumed that one vehicular station can serve 20 subscribers and a capacity of 54 Mbps. The stabilized maintenance costs are 20% of the equipments price at a time. The price erosion is 0.90 per year except for management unit which has price erosion 0.85. The assumed financial parameters are combined in Table 5.24.

The sole Finnish railway operator VR has some 722 carriages and they produce some 270 scheduled turns a day. We equip each carriage with a vehicular station and one fourth of them will have a management unit and Flash-OFDM transceiver in them. Thus the trains operating must have at least one carriage with management unit and some number of other carriages. The network costs for RITS with Flash-OFDM and WLAN follows the amounts in Figure 5.34. The total investment costs are EUR 1 165 000. Table 5.25 states sensitivities of some parameters for investment costs.

Table 5.25: Sensitivity analysis for RITS investment costs with Flash-OFDM and WLAN

Parameter	**Sim -10%**	**Sim +10%**
No changes	1165 k€	1165 k€
Technology prices, $p_T(0)$	1114 k€	1216 k€
Price growths, e^{c_T}	1165 k€	1165 k€
Installation, $K_{T,I}$, $K_{T,V,I}$	1100 k€	1230 k€

The RITS with Flash-OFDM GVC network is capacity limited after 2012 if the subscriber forecast realizes. Thus the number of subscribers must be restricted as in (4.8). This scenario of

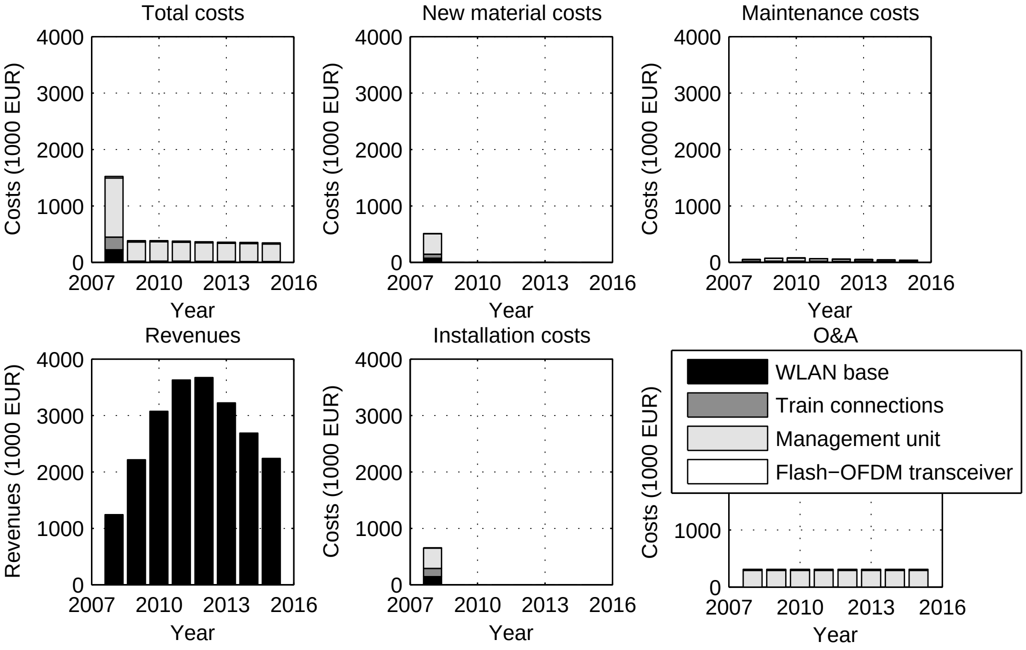

Figure 5.34: Cost structure for the RITS Flash-OFDM network

RITS subscribers can be found in Figure 5.35.

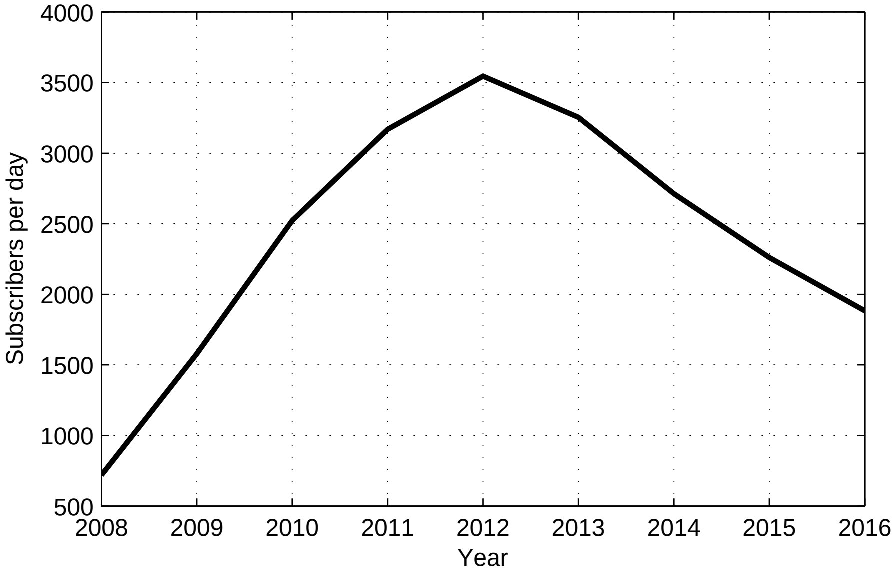

Figure 5.35: Subscriber forecast for the RITS passenger services

The net present value of the RITS investment with Flash-OFDM and WLAN is EUR 7 368 000. The sensitivities of the parameters for NPV can be found in Table 5.26. It seems that parameters for passenger growth, time to 50% penetration, throughput growth, ARPU and capacity have the greatest effect on NPV. The sensitivity curves for some of these parameters are drawn in

Figure 5.36.

Table 5.26: Sensitivity analysis for RITS NPV with Flash-OFDM and WLAN

Parameter	**Sim -10%**	**Sim +10%**
No changes	7368 k€	7368 k€
Passengers, $N_{pot}(0)$	6723 k€	8014 k€
Passengers growth, e^{c_N}	5193 k€	8566 k€
Adaptation shape, b	7368 k€ [a]	7368 k€ [a]
Initial penetration, $\rho(0)$	7365 k€ [a]	7372 k€ [a]
Time to 50% penetration	7770 k€ [a]	6980 k€ [a]
Throughput, $d_a(0)$	7737 k€	7067 k€
Throughput growth, e^{c_d}	8986 k€	5833 k€
ARPU, $p_S(0)$	6391 k€	8346 k€
ARPU growth, e^{c_S}	4747 k€	11052 k€
Technology prices, $p_T(0)$	7429 k€	7308 k€
Price growths, e^{c_T}	7417 k€	7303 k€
Installation, $K_{T,I}$, $K_{T,V,I}$	7434 k€	7303 k€
O&A costs, $K_{T,F,OA}$, $K_{T,V,OA}$	7483 k€	7254 k€
Maintenance, $K_{T,F,M}$, $c_{T,F,M}$, $K_{T,V,M}$	7391 k€	7346 k€
Cell radius, R_c	7368 k€	7368 k€
Capacity, $1/K_{T,D}$	7022 k€	7700 k€

[a] The forecast assumes the shape parameter and fits curve to two parameter defined points

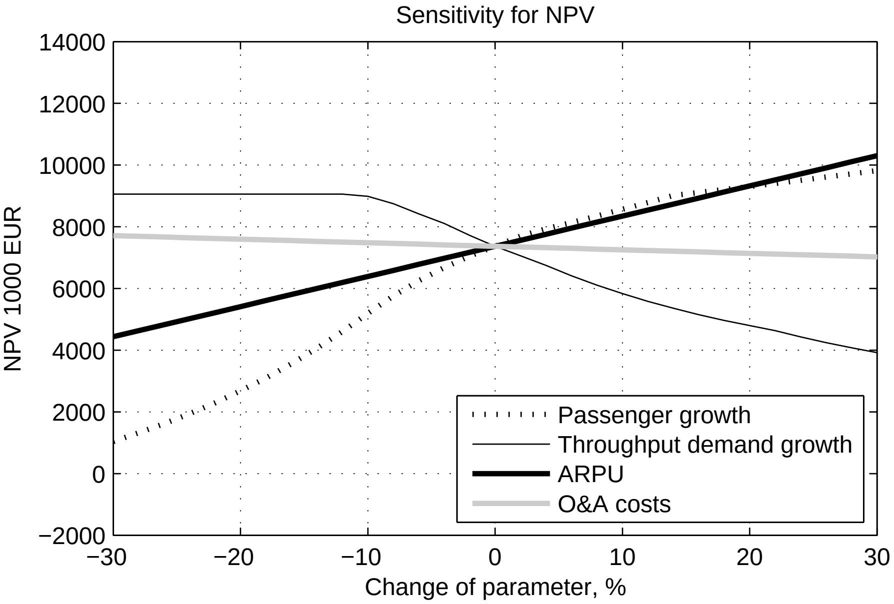

Figure 5.36: Sensitivity analysis for the RITS investment

Table 5.27: Statistics for the WiMAX investment for RITS in year 2011

Technology	**Max terminal speed**	**Max downlink data rate**	**Estimated data rate at speeds of 200 km/h**
Flash-OFDM	250 km/h	5.3 Mbps	1 Mbps (DL) / 512 kbps (UL)
EGPRS	250 km/h	1.0 Mbps	236 kbps (DL, UL)
UMTS/HSPA	250 km/h	14.4 Mbps	2 Mbps (DL) / 1 Mbps (UL)
3G LTE	350 km/h	100 Mbps	-
802.16e-2005	120 km/h	70 Mbps	5 Mbps (DL) / 3 Mbps (UL)
802.16 m	350 km/h	100 Mbps	-

Table 5.28: The mean financial parameters for a RITS WiMAX network

	Price	**Model factor**	**Installation**	**Operating**
In-train components	400 €	0.9	100 €	2000 €/a
WiMAX sector	6 000 €	0.85	500 €	1200 €/a
WiMAX base Station	10 000 €	0.85	4 000 €	1 800 €/a
Radio link	25 000 €	0.90	2 000 €	2 400 €/a
Spectrum	-	-	-	7 €/km^2a

5.5.2 TECHNOLOGY OPTIONS

The modularity of the RITS model gives a set of technology options for the railway operator. The GVC network can be prepared for using different technologies simultaneously. The connection manager then switches for a right connection whenever needed.

One alternative for Flash-OFDM would be 3G mobile networks. These were analyzed shortly in [228]. The 3G network could be updated to LTE technologies in the future. However, the coverage and capacity of mobile networks seem to be sufficient for RITS use only in the main city areas of Finland. The railway stations could also be equipped with WLAN networks such that the connection manager switches to it near the stations.

Consider a mobile WiMAX technology for RITS use. It offers a promising technology option for upgrading the RITS to serve the growing throughput demand beyond year 2011 [103, 124, 278]. The capacity of the mobile WiMAX technology with speeds of 200 km/h is debatable, yet. See Table 5.27 for the estimates on the data rates of different technologies. Though the current mobile WiMAX standard 802.16e-2005 does not support terminal speeds of more than 120 km/h, there are modifications allowing it [84, 219]. Such modifications may lead to incompatibilities with standards. We assume a downlink capacity of 5 Mbps with cell radius 5 km. The base stations are built in existing pylons which reduces the costs considerably. Let us estimate the mobile WiMAX network component costs using the fixed network costs. Table 5.28 summarizes the financial parameters.

The railway leg between Helsinki and Tampere is some 200 km long and some 20 000 passengers use the route every day. This is some 60 % of all long distance trips in Finland. The capacity of the Flash-OFDM network beside the route becomes bottleneck at latest in 2012. Upgrading the capacity for this leg makes it possible to increase the number of RITS subscribers to over 3500 per day, see Figure 5.35. The mean investment costs of the WiMAX expansion between Helsinki and Tampere are EUR 1 880 000 and deviation EUR 178 000, see Table 5.29. The investment costs are log-linearly distributed with P-values 0.2 and 0.03 from Lilliefors and

Table 5.29: Statistics for the WiMAX investment for RITS in year 2011

	Mean	**Deviation**	**Skewness**	**Kurtosis**	**Shape** (-/$\mathcal{N}$/$\mathcal{LN}$)
Investment costs	1 880 k€	178 k€	0.4	-0.1	$\mathcal{LN}$ (0.03)
NPV	2 640 k€	2 350 k€	0.9	1.5	-
NPV ratio	2.66	1.5	1.1	2.3	-

Jarque–Bera tests, respectively. The net present value has mean EUR 2 640 000 and deviation EUR 2 350 000. The mean NPV ratio is 2.66 and deviation 1.5. The expansion option has a value of some EUR 2 700 000. See Figure 5.37 for QQ-plots of the distributions and Figure 5.38 for the mean and first, second and third quartiles for revenues, profits and discounted cash flows.

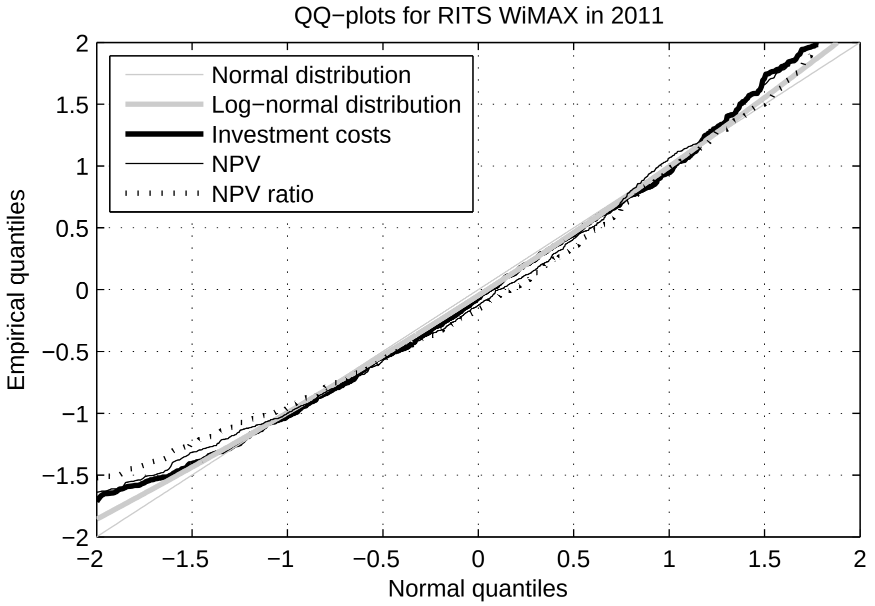

Figure 5.37: QQ-plots for the WiMAX investment for RITS in year 2011

Option to Defer

The railway operator has the option to start the WiMAX expansion between 2009 and 2014. Figure 5.39 shows a curve on the value of the launch of WiMAX network between Helsinki and Tampere at different possible exercise years. It seems that the years from 2010 to 2012 are the most promising years for the technology upgrading.

5.5.3 INTERNAL OPTIONS FOR MAINTENANCE

In addition to the passengers, the RITS network can be used by train operator internal users. One group that benefits from the communications is maintenance operations. The capital tied in train operator's machines was EUR 880 000 000 in the end of the year 2007 book value or EUR 1 795 000 000 as acquisition costs [274]. Over half of these figures fall on locomotives and carriages.

The maintenance of locomotives and carriages does not currently know the exact location

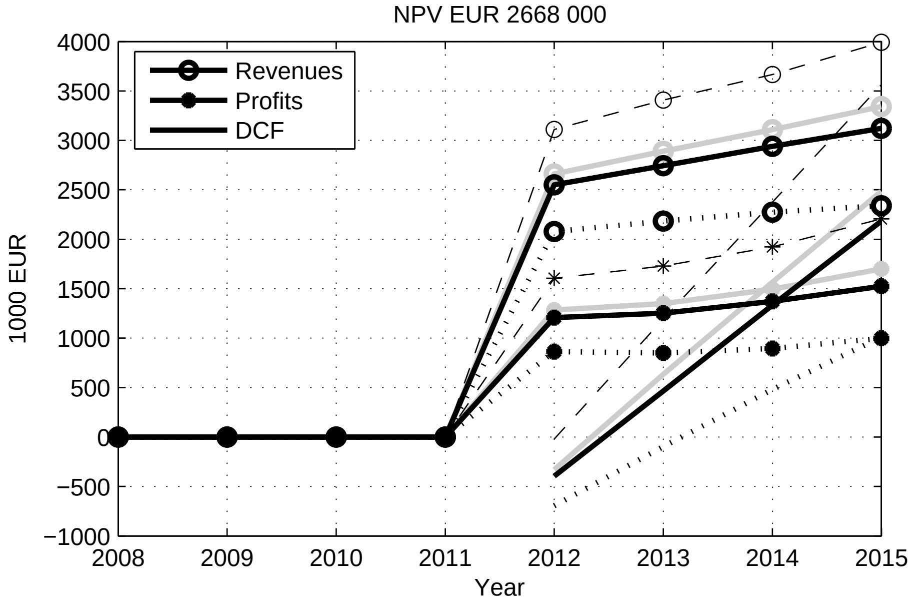

Figure 5.38: Discounted cash flows for the WiMAX investment for RITS in year 2011

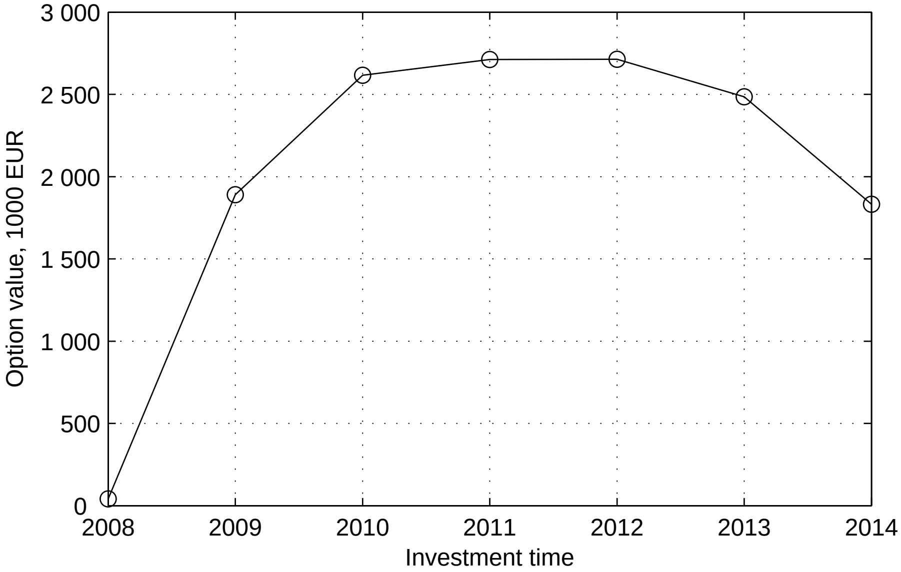

Figure 5.39: Real option values for the WiMAX network

and general condition of each car. Thus pit stops cannot be effectively planned or prepared [270]. The RITS network would make it possible to monitor and analyze the condition of carriages and plan the maintenance services according to the needs. This would reduce the number of needed locomotives and carriages of the train operator. The RITS investor has an option to sell the cars or decrease the number of purchased new cars in the future.

If the reduction would be 1%, the revenues from selling the cars would be worth of some EUR 5 000 000. The deferring of the purchase of new cars may be worth of EUR 10 000 000. Note that the options is exercised when the costs for the maintenance system developments are launched. After that the savings could not be modeled using options but they are estimated as expectation values with uncertainty in the real savings.

5.5.4 Option to Change Conductors Work

The conductors do their work in train and after the trips. One visible part of their work is checking and selling of tickets. In some parts of Finland there are no active stations but the conductors sell the tickets for passengers. In RITS framework, the checking of credit cards and selling of seat tickets could be done more efficiently if the connection from the train to the train operator's ground systems were operating. In addition to ticketing, conductors make travel and fault reports. The reports are done in trains and re-written in electronic form and/or copied to operator's central systems after trips [270]. These reports could be made directly to the ground system if the RITS network were operating.

Let us assume that the work of conductors could be made 5 to 10% more efficient in the RITS framework. In addition, we assume that 5 to 10% of the total groups personnel salaries of EUR 450 000 000 are for conductors [274]. In summary, we estimate the value of the switch option for conductors work with some 0.5% of salaries. The cost savings are in the magnitude of EUR 2 000 000.

5.6 Comparison of Valuation Methods

Different valuation methods are presented in Section 4.2.2 and Section 4.4. Consider the discounted cash flow calculations and different real option valuation methods. Let us compare them theoretically and apply them to WiMAX and ADSL network valuations.

5.6.1 Theoretical Comparison of Valuation Methods

There are five option valuation methods and two NPV methods compared here. The option valuation methods are

- Monte-Carlo simulation (See Section 4.4.3 and Section 5.1.2),
- The Brennan method (See Section 4.4.1),
- The Luehrman method (See Section 4.4.1),
- The Black–Scholes formula (See Section 4.4.1), and
- The binomial tree method (See Section 4.4.2).

The main result from those methods is the value of option to invest in future. Instead, the results from the NPV methods

- DCF, or discounted cash flow (See Section 4.2.2 and 5.1.1) and
- Monte-Carlo simulation (See Section 5.1.2)

are the net present values of investments, or the expectations of them.

The Monte-Carlo methods and DCF calculations are based on scenarios and thus they need dozens of parameters. If the uncertainties are modeled, the amount of parameters increases.

Theoretically, if all the parameters are modeled to have uncertainty, the amount of parameters doubles. The non-scenario models need just a few parameters, but the parameters are such that they must be calculated somehow. For example, the variance is hard to estimate correctly without simulations and still it has a significant effect on the result from the method. Overally, the absence of required data is one of the main problems in real cases. It is not clear whether it is easier or harder to find values for multiple input parameters, since some of them might be easy to handle. Furthermore, the shape of the distribution influences the result, too. That is why the assumptions on the Brennan, Luehrman, Black–Scholes, and binomial models may be in conflict with the applied case.

The direct scenario based DCF calculation is the only method that cannot handle the uncertainties. Instead, Monte-Carlo simulations handles multiple uncertainties. On the other hand, the benefits from option valuation methods are the capabilities to value opportunities. This flexibility makes it better than DCF [3]. The best results can be expected from the combination of scenarios and real options [187], i.e., Monte-Carlo based option valuation. The summary of the comparison of the methods can be found in Table 5.30.

Table 5.30: Theoretical comparison of valuation methods

Method	**Inputs**	**Main results**	**Handles uncertainties**	**Handles opportunities**	**Assumed distribution**
Monte-Carlo option	Dozens	Option value, distributions	Multiple	Multiple	Any
Brennan	4	Option value	OK	OK	Normal
Luehrman	3	Option value	OK	OK	Log-normal
Black–Scholes	5	Option value	OK	OK	Log-normal
Binomial	6	Option value	OK	OK	Log-normal
DCF	Dozens	NPV	No	No	None
Monte-Carlo simulation	Dozens	E[NPV], distributions	Multiple	Some	Any

5.6.2 WiMAX Network Investment

Consider the WiMAX network investment in Ähtäri as described in Section 5.2. The function `calculateNPV` implements the traditional discounted cash flow valuation method with the parameters described above. The distribution of costs and the total revenues can be found in Figure 5.9. As can be seen, the initial network investment costs play major role in the investment.

Yearly revenues and profits and the discounted cash flows are drawn with the simulated distribution of the NPV in Figure 5.40. The DCF method results in NPV EUR 6 000 negative and theoretical model EUR 71 000 positive for the WiMAX investment in 2008, see Table 5.31. The simulation shows that depending on the future, the NPV for the investment may vary between EUR -1 000 000 and EUR 500 000, see Figure 5.14. This results expectation EUR 103 000 to EUR 123 000 negative and the value of investment option EUR 55 000 to EUR 63 000, depending on the assumed parameter error shapes. The three different simulated values for the base case give frames for the option values. The shape of the underlying assets affects the option value. The simulated values are the best approximates for the real option values of the WiMAX investment projects. For the deferred WiMAX investment in 2011 or for WiMAX extension (to serve cottages) have simulated NPV EUR -38 000 and EUR -56 000, and option values EUR 69 000 and EUR 10 000, respectively.

Table 5.31: The NPV values with different methods in WiMAX cases

Method	**WiMAX investment in 2008**	**WiMAX investment in 2011**	**WiMAX expansion in 2011**
Theoretical	71 k€	-50 k€	-
Scenario DCF	-6 k€	90 k€	-
Simulated NPV			
- normal errors	-123 k€		
- log-normal errors	-103 k€	-38 k€	-56 k€
- uniform errors	-104 k€		

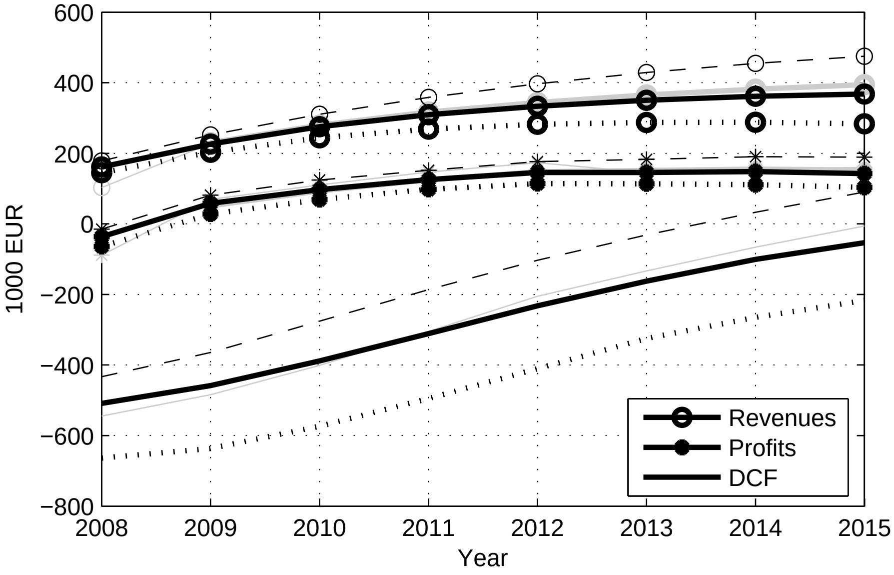

Figure 5.40: Cash flows of WiMAX network investment

The real option values can be estimated by using the analytical option formulas (4.10) and (4.13) if the statistical characteristics for investment costs, net present values, and NPV ratios are known. The mean and deviations for the mentioned quantities in the base case are: $\overline{X} =$ 539 000 € and $s_X = 194\ 000$ €, $\overline{NPV} =$ -103 000 € and $s_{NPV} = 316\ 000$ €, and $\overline{NPVq} = 0.91$ and $s_{NPVq} = 0.46$. For deferred investment, we have $\overline{X} = 437$ 000 € and deviation $s_X = 169$ 000 €, $\overline{NPV} =$ -38 000 € and $s_{NPV} = 251\ 000$ €, and $\overline{NPVq} = 1.0$ and $s_{NPVq} = 0.56$. And for WiMAX extension, we have $\overline{X} = 69$ 000 € and deviation $s_X = 61\ 000$ €, $\overline{NPV} =$ -56 000 € and $s_{NPV} = 104\ 000$ €, and $\overline{NPVq} = 1.34$ and $s_{NPVq} = 8.2$.

Now, let us use two analytical methods, Black–Scholes (4.10) and Brennan (4.13), in three different ways. The results with different analytical methods are presented in Table 5.32.

Firstly, the direct NPV style uses the characteristics of the net present value and estimates the option value using these parameters only. The underlying assets are the net present values and exercise prices zero. Because the Black–Scholes formula assumes log-normality — which does

Table 5.32: The option values with different methods in WiMAX cases

Method	**WiMAX investment in 2008**	**WiMAX investment in 2011**	**WiMAX expansion in 2011**
Simulated option			
- normal errors	55.3 k€		
- log-normal errors	58.9 k€	68.7 k€	9.56 k€
- uniform errors	63.1 k€		
Brennan (NPV or S style)	81.4 k€[b]	73.3 k€[b]	14.6 k€[b]
Black–Scholes (S style)	-	84.6 k€[a]	3.15 k€ [a]
Simulation (NPV ratio)	14.1%	21.7%	122%
Luehrman (NPV ratio)	-	22.2% [a]	134% [a]
- using total mean	-	97 k€	92.4 k€
- weighted mean	-	78.2 k€	11.6 k€
Brennan (NPV ratio)	14.2%	22.5% [b]	345% [b]
- using total mean	76.8 k€	98.2 k€	238 k€
- weighted mean	59.7 k€	79.1 k€	29.8 k€

[a] The method incorrectly assumes log-normality.

[b] The method incorrectly assumes normality.

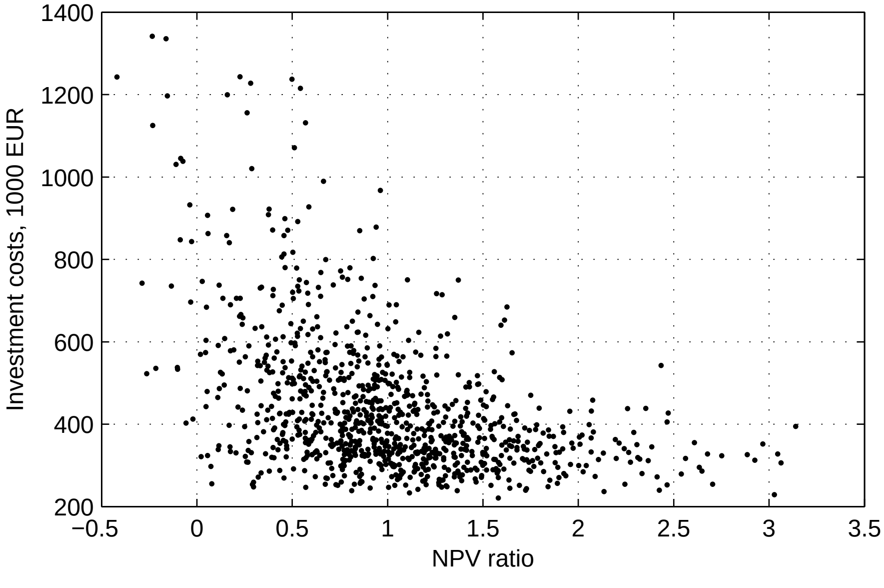

Figure 5.41: NPV ratio and investment costs

not take into account the negative values — and it needs a non-zero exercise price, see (4.10), the method cannot be used. However, the option value approaches the mean of the NPV while the exercise price approaches zero from the positive side. The Brennan model (4.13) assumes normality and suits the real option valuation quite well. With NPV as the underlying assets, the results are EUR 81 400, EUR 73 300 and EUR 14 600. The results are in the same level as the results from the simulation.

The second way of using the valuation formulas is to calculate the mean discounted surplus sum S by adding the mean of the NPV and the mean investment costs. The sum S is the underlying asset and the mean of the investment costs $\overline{X}$ is the exercise price. Because the absolute deviations equal those of the direct NPV style, the Brennan model gives the same option values. On the other hand, the Black–Scholes formula gives the sums EUR 84 600 and EUR 3 150 for WiMAX investment in 2011 and WiMAX expansion in 2011, respectively. For the base case with investment in 2008, the method cannot give a result as there actually is no option (the exercise time is right now).

Thirdly, the NPV ratio transformation (4.11) is used like in [178]. Now that we operate on the ratio dimension, the risk-free rate of return is zero. The NPV ratio serves as underlying asset and the exercise price is 1.0. In Table 5.32, the Black–Scholes method with NPV ratio style is called *Luehrman* since it follows (4.12). The results are in NPV ratio scale, i.e., the surpluses from the investment projects are about 22.2% and 134% of the investment costs. The Brennan model gives the results 14.2%, 22.5%, and 345% while the simulated values are 14.1%, 21.7% and 122%. Let us transform the percentages to euros by multiplying them with the investment costs. If the mean over the whole sample set is used, the result is bigger than the simulated option value. This is because the biggest NPV ratios derive from the lower investment costs, whereas large investment costs lead to lower NPV ratio, see Figure 5.41 where the correlation in the investment time 2011 is pointed out. The transformation of the option value from the NPV ratio scale to the currency units can also be done using weighted mean. In that case the means are weighted so that only events with NPV ratio greater than 1.0 are used and the means are weighted by the margin between NPV ratio and 1.0, i.e., if W was the set of events i with $NPVq_i > 1$, the mean would be

$$\overline{X_W} = \frac{\sum_{i \in W} X_i (NPVq_i - 1)}{\sum_{i \in W} (NPVq_i - 1)}.$$

With weighted means, the results from the Luehrman and Brennan (using NPV ratio style) methods are closer to simulated values, see Table 5.32. The weighted investment costs were EUR 419 000, EUR 352 000 and EUR 8 600. In Table 5.32, note that Luehrman method is more reliable than Brennan in the expansion option valuation as the NPV ratio is quite skewed.

5.6.3 ADSL Network Investment

Consider the ADSL network investment simulated in Section 5.4. The discounted cash flow with the parameters in Section 5.4 is drawn in Figure 5.30. The NPV of the project is EUR 113 000 as simulated or EUR 25 000 when using theoretical model, see Table 5.33. The simulated NPV varies from EUR -50 000 to EUR 200 000.

The simulated mean NPV is EUR 97 000 and the option value is EUR 112 000 with the expected NPV ratio 1.53 and deviation 0.62. The expectation of the investment costs is EUR 215 000. With these parameters the different methods give option values from EUR 112 000 to EUR 128 000, see Table 5.34. The correlation of NPV ratio and investment costs are drawn in Figure 5.42 and weighted mean investment costs are EUR 191 000.

Table 5.33: NPV values for ADSL investment with different methods

Theoretical	25 k€
Scenario DCF	113 k€
Simulated NPV	97 k€

Table 5.34: Option values for ADSL investment with different methods

Simulated option	112 k€
Black–Scholes	97 k€[a]
Brennan	113 k€[b]
Simulation (NPV ratio)	58.3%
Brennan (NPV ratio)	59.7% [b]
- using total mean	128 k€
- weighted mean	114 k€

[a] The method incorrectly assumes log-normality.

[b] The method incorrectly assumes normality.

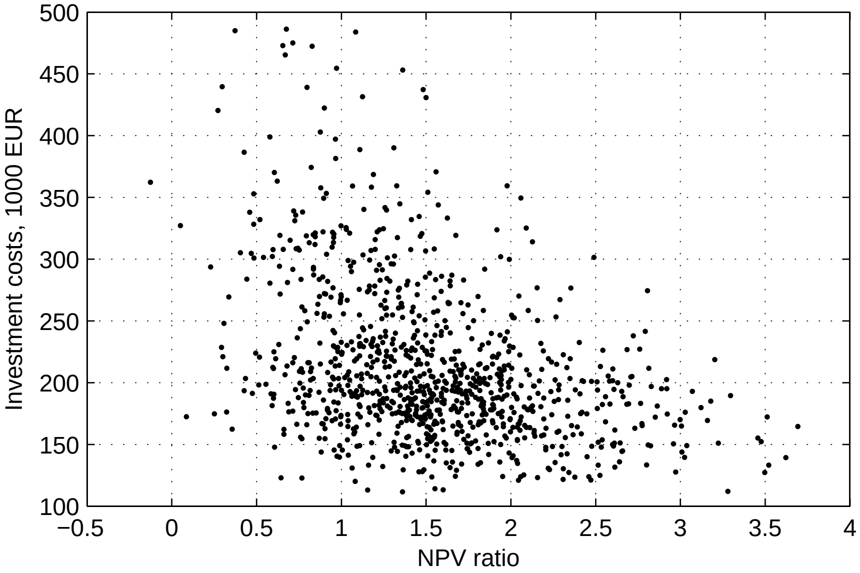

Figure 5.42: NPV ratio and investment costs for ADSL case

5.6.4 Intelligent Railway Transportation System

Railways intelligent transport system (RITS) was analyzed in Section 5.5. The WiMAX network investment for the leg Helsinki–Tampere was simulated to be worth of EUR 2 640 000 with deviation EUR 2 350 000. The option value was EUR 2 710 000. The simulated mean NPV ratio was 2.66 with deviation 1.5 and mean investment costs EUR 1 880 000. The analytical methods with these parameters give option values from EUR 2 640 000 to EUR 3 720 000, see Table 5.35. Now the NPV ratio and investment costs are not so much correlated, see Figure 5.43, and thus the option values for Luehrman method with weighted mean does not differ so much from a value when using weighted mean.

5.7 Chapter Conclusion

The results in Table 5.32 show that Black–Scholes and Brennan models give both correct and biased values for options, depending on the parameters used. The investment costs are positively

Table 5.35: Values for RITS mobile WiMAX investment with different methods

Simulated NPV	2 640 k€
Simulated option	2 710 k€
Black–Scholes (direct NPV style)	2 640 k€[a]
Brennan (NPV or S style)	2 740 k€[b]
Black–Scholes (S style)	2 660 k€[a]
Simulation (NPV ratio)	170%
Luehrman (NPV ratio)	198% [a]
- using total mean	3 720 k€
- weighted mean	3 660 k€
Brennan (NPV ratio)	176% [b]
- using total mean	3 300 k€
- weighted mean	3 250 k€

[a] The method incorrectly assumes log-normality.

[b] The method incorrectly assumes normality.

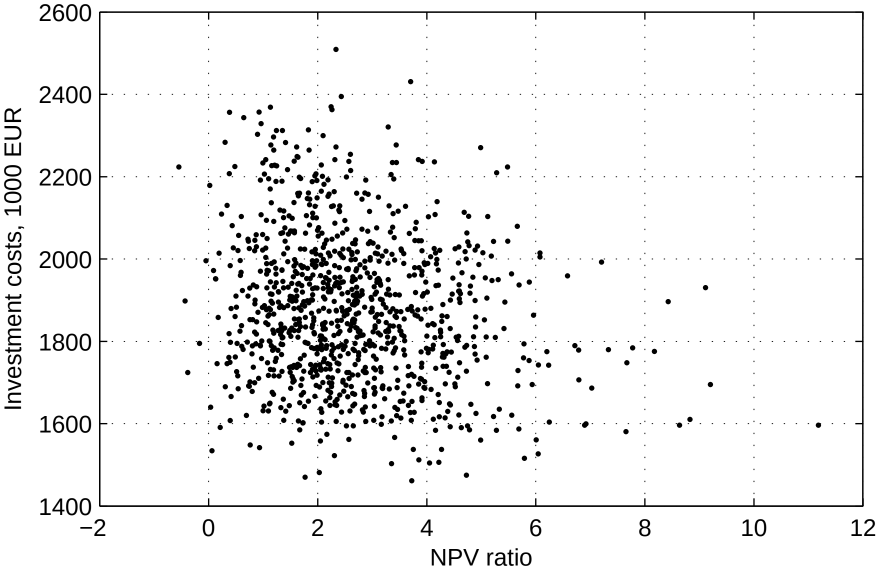

Figure 5.43: NPV ratio and investment costs for RITS WiMAX case

skewed and can be approximated by log-normal distributions. This makes NPV negatively skewed. Furthermore, the relative deviation may be over 100% and thus NPV should not be modeled by log-normal distribution. The NPV ratio is slightly positively skewed but gets also negative values and cannot safely be modeled by log-normal distribution. In addition, NPV ratios are correlated with investment costs yielding biased option values when transforming to euros. The bias can be avoided if the weighted mean over the investment costs with NPV ratio more than 1.0 is present and used instead of the mean over all the events. In summary, the analytical option valuation methods should be developed further to meet the assumptions relevant in WiMAX (or other network) investments.

The analysis shows that the shape of the uncertainty — or error — in the parameters does not affect the shapes of the investment costs, NPV, or NPV ratio distribution. Instead, it matters which of the parameters are modeled with uncertainty in them. Especially, the uncertainty in the service rate growth or household growth factor influence the resulting distributions. The analysis done in this study could be extended to other broadband technologies as well to examine the general characteristics of broadband communications investments.

The network traffic per user is exponentially growing. The exponential growth makes the distribution of the network investment costs log-normal. The log-normally distributed investment costs make the distribution of the NPV negatively skewed. Thus the construction strategy, i.e., the network dimensioning, affects the shape of the NPV distribution. There is no single statistical model which could be used to optimize the size of the network.

6. DISCUSSION AND CONCLUSION

Broadband communications makes it possible to transfer information quickly all around the world. Broadband access has been used in homes only a few years, but the penetration is increasing rapidly, making Internet connection nearly universal. The valuation of broadband communication networks is important for operators, subscribers and regulators alike.

Forecasting and planning are demanding processes. However, the decision makers who need valuation methods have a variety of techniques to use. This study analyzes the uncertainties in investment projects, applies different investment valuation methods, and compares the results. The main contributions of the study are the analysis of the broadband acceptability and adaptation, the shaping of the theoretical investment model and the statistical analysis of the access network investment cases.

6.1 KEY RESEARCH FINDINGS

One aim of the thesis was to develop the theory and models for managing broadband investment uncertainties by

1. finding out the critical parameters of network investments,
2. analyzing possible theoretical models for the network investments, and
3. introducing a rough theoretical model for the valuation of network investments.

One of the most critical parameters for network investment is the *exponentially growing network traffic per user*. This yields several questions for network planners. For which traffic demand should the network be sized? The *changes in the number of potential subscribers* is another critical parameter. It seems that slightly decreasing scenario of the number of potential subscribers may maximize the use of the network and thus net present value of the network. A slight decrease in the number of potential subscribers is a counterpart for the exponential growth of the network traffic per user, thus making the use of invested network more efficient over the whole study period.

Other critical parameters are *ARPU* of the service and *local adaptation* of broadband technology. ARPU was here modeled with exponentially decreasing continuous model, but discrete models could be analyzed too. The adaptation of new service have often been modeled by S-shaped curves. The penetration of broadband connections for market with relatively high initial Internet penetration may differ from the traditional S-shaped curve for new technology penetration. This is because the broadband subscriptions can be updated quite easily from the plain Internet subscription without large investments. Moreover, the amount and quality of the content in the Internet has grown rapidly making the service more attractive. The study shows that Richards adaptation model is more practical for broadband penetration forecasting as it provides more flexibility than the S-shaped curves with only two parameters.

The analysis in this study suggests a *theoretical model* (4.7) of network investments. The model

$$\begin{aligned}
V &= \sum_{t=s}^{z} \frac{B(t) - C_{OA}(t) - C_M(t)}{e^{rt}} - \frac{X(s)}{e^{rs}}, \\
X(s) &= \left(p_T(0)e^{-c_T s} + K_{T,I}\right) I, \\
C_{OA}(t) &= K_{T,F,OA} I + K_{T,V,OA} N(t) + K_{T,V,I}(N(t) - N(t-1)), \\
C_M(t) &= p_T(0)e^{-c_T t}\left[(1 - e^{-c_{T,F,M} t})K_{T,F,M} I + K_{T,V,M} N(t) d_a(0) e^{c_d t}\right], \\
B(t) &= p_S(0)e^{-c_S t} N(t), \\
I &= \max\left(\frac{K_{T,A}}{R_c^2} A; K_{T,D} N(z) d_a(0) e^{c_d z}\right), \\
N(t) &= \frac{h(t) N_{pot,0} e^{c_N t}}{(1 + \beta e^{-kt})^{\frac{1}{b}}}
\end{aligned}$$

includes the subscriber forecast $N(t)$, planned infrastructure I, forecasted revenues $B(t)$, maintenance costs $C_M(t)$, operational and administration costs $C_{OA}(T)$, investment costs $X(s)$, and net present value V of the network.

We analyzed the economics of different broadband access technologies in rural area cases thus giving some insights in the digital divide. The analysis adapted and evaluated the use of real options theory in telecommunications by

1. making case studies of the possible uses of real options, and
2. comparing different investment valuation methods in network investments.

The exponential growth and coverage requirements make the *distribution of the network investment costs log-normal.* The log-normally distributed investment costs make the distribution of the NPV negatively skewed. Thus the construction strategy, i.e., the network dimensioning, affects the shape of the NPV distribution.

The assumptions of the current analytical option valuation methods may not coincide with the statistics of the network investment costs or NPV distribution. The use of the Black–Scholes formula and other option pricing models that assume log-normal distribution for the underlying assets must be questioned. The simulations show that the *NPV ratio is not log-normally distributed in general.* However, it is not necessarily normally distributed either. Thus the Brennan model is not much better than the Black–Scholes formula. In summary, there is *no certain model that suits all the investment projects or even all the broadband access network investments.* There is still demand for new valuation formulas taking into account the real option requirements.

6.2 DISCUSSION ON VALUATION METHODS

The DCF valuation method has been used for years and it is the standard method for valuation of investment projects in the business world. The method itself is quite simple and it gives understandable results. Moreover, the forecasting of revenues and prices as well as diffusion models have shown their capabilities and functionality. However, in many cases DCF calculation is too simple and does not model the uncertainties and new opportunities in innovative investments.

The use of simple option valuation models in the network investments is problematic. The valuation methods have assumptions on the shapes of the NPV or NPV ratio distributions. The shape of those distributions is not static but it changes depending on the technology and service area. Thus a bit more detailed model — as in this study — should be used to approve the investors calculations. Yet the method should be understandable to assure the decision makers.

6.3 Discussion on Results

The simulations and calculations in this study produced many results. These results can be examined from different viewpoints. The values of WiMAX and ADSL network investments in Ähtäri are EUR -100 000 and EUR 100 000, respectively. The profitabilities of WiMAX and ADSL technologies in our demographic case are drawn in Figure 5.11 and Figure 5.30. The yearly profits show that the WiMAX technology seems to have lower operation costs and the ADSL technology to have smaller investment costs. Note that the ADSL is modeled to use wires already installed in houses, i.e., the access line is rented from the incumbent operator. The NPV of the ADSL solution is better in the study period, but WiMAX could reach it if the study period were extended and the expansion option to summer cottages is exercised.

The intelligent railway transportation system using Flash-OFDM technology seems like a profitable business case for railway operator. Moreover, the network could be used by operator inside users and cargo companies, i.e., there is a growth option, and the architecture of the system includes a set of technology options.

6.4 Discussion on Models and Further Research

The theoretical model simplifies the investor's problem. Partial models are not having correlations in our probabilistic model though some of them could have impacts on each other. This is one important improvement in the whole model. The valuation of real options is still under rapid developments. Methods with fuzzy logic should be applied and study their usability in telecommunication cases. New option valuation methods for network investments could also be sought.

The simulation model used is modular and could be easily applied to different scenarios. The most relevant further research topic is to analyze different technologies in a similar way as WiMAX and ADSL were analyzed in this study.

The network traffic and capacity calculations in the model used are simplistic. The users are assumed to need a certain throughput via the network. The statistical multiplexing is not considered in the capacity calculations and the backbones of the network may be oversized. The simulation model could be developed further to manage these concerns. Moreover, the effects of network upgradings in the study period should be modeled, since it would give more information on the profitabilities of different technologies.

The forecasting models could also be developed in several ways. Especially the development of broadband access penetration in municipalities or in other small subscriber groups should be studied. In addition to the theoretical reasoning of the social networks affecting in the home broadband subscriber decision making, there is need for practical data and statistical proof on the issue.

The coverage planning ought to estimate the need of different network components more accurately by modeling the geographics and the communications infrastructure already working in the study area. For local broadband markets the price of the connections may be affected by only a few companies and thus the discrete price model might be relevant model and the simulation of adaptation models could be developed further, too.

The competition between operators was not considered in the study. Instead, the invested network was assumed to reach 100% market share in the service area. The competition between operators and different non-profit operator possibilities should be considered in the further research. For example, networks have been built with co-operative voluntary work or by municipalities.

The initial model simulates the taxation of the network investment as if the investment is the only operation of the company. Usually, the operators are bigger and they have many network projects in different regions. In this case the taxation is not playing that significant role in one investment project.

The parameter values of the network components are based on other research publications. The precise levels of the component prices as well as the service tariffs are outside the scope of this analysis as long as they are at approximately right level. The validation of operational and administration costs and rising maintenance cost models in network investments should be done.

This study concentrated on the use of real options in the valuation of a one stage access network investment. The real options could be used in different ways in telecommunications, as was described in Section 4.5. The implementation and analysis of different options and their combinations would deepen the understanding of real options and the important role they play in telecommunications planning.

A. SIMULATED ADAPTATION

A.1 HOMOGENEOUS DECISION MODEL WITH NORMAL THRESHOLD

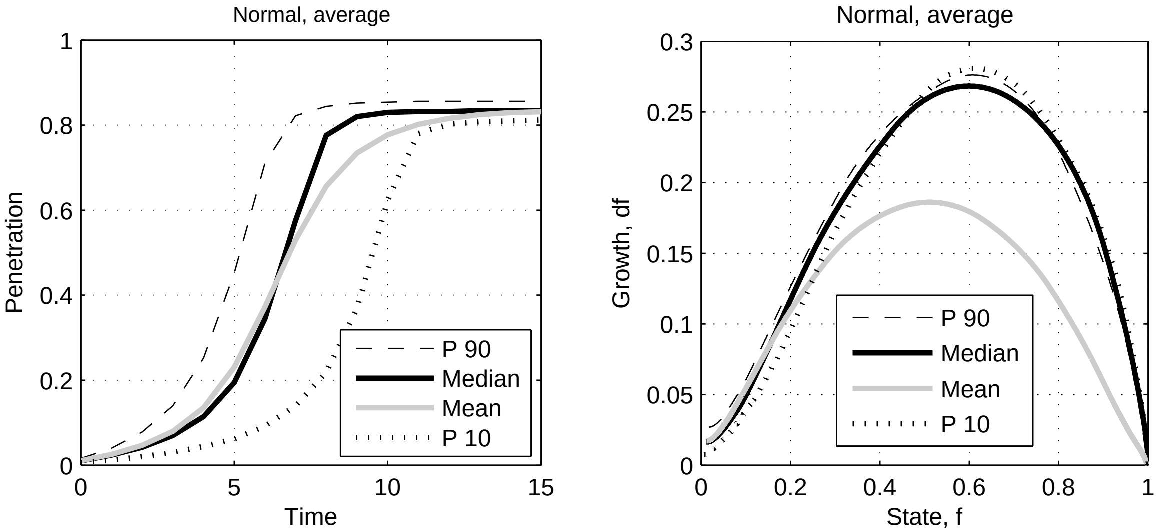

Figure A.1: Simulated adaptation with homogeneous model and normal, average thresholds

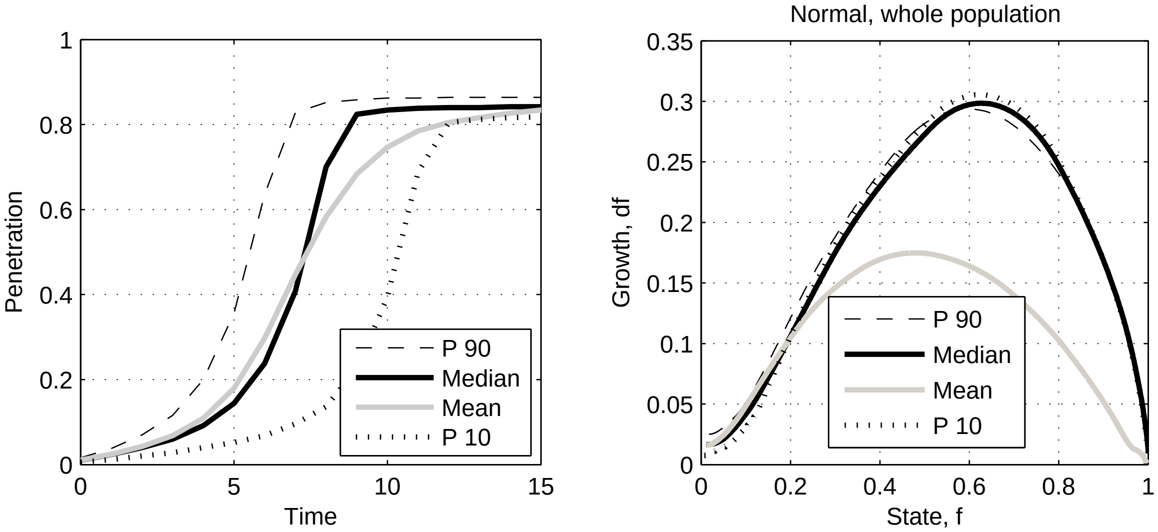

Figure A.2: Simulated adaptation with homogeneous model on total subscribers and normal thresholds

A.2 Homogeneous Decision Model with Exponential Threshold

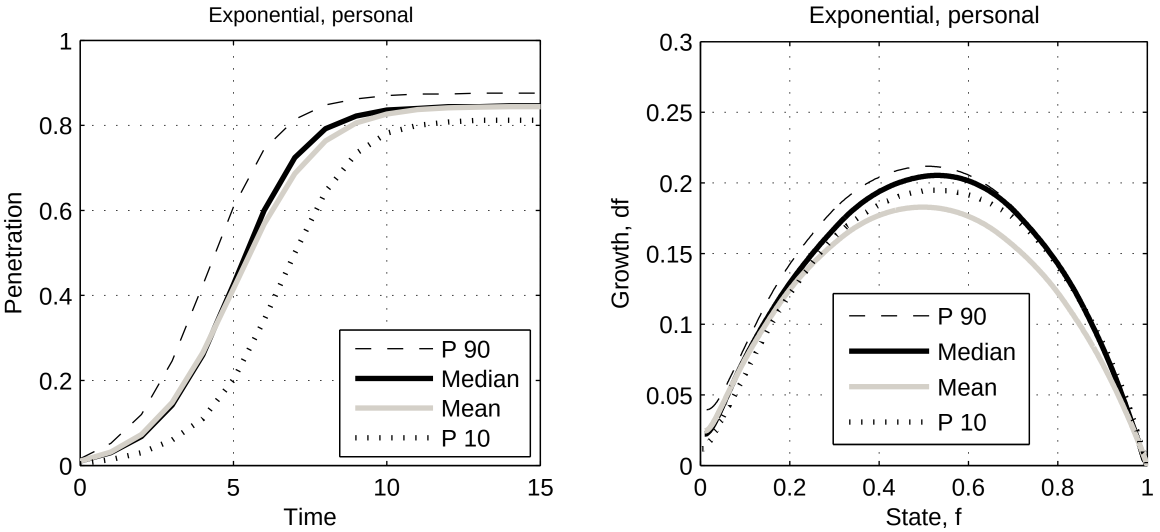

Figure A.3: Simulated adaptation with homogeneous model and exponential thresholds

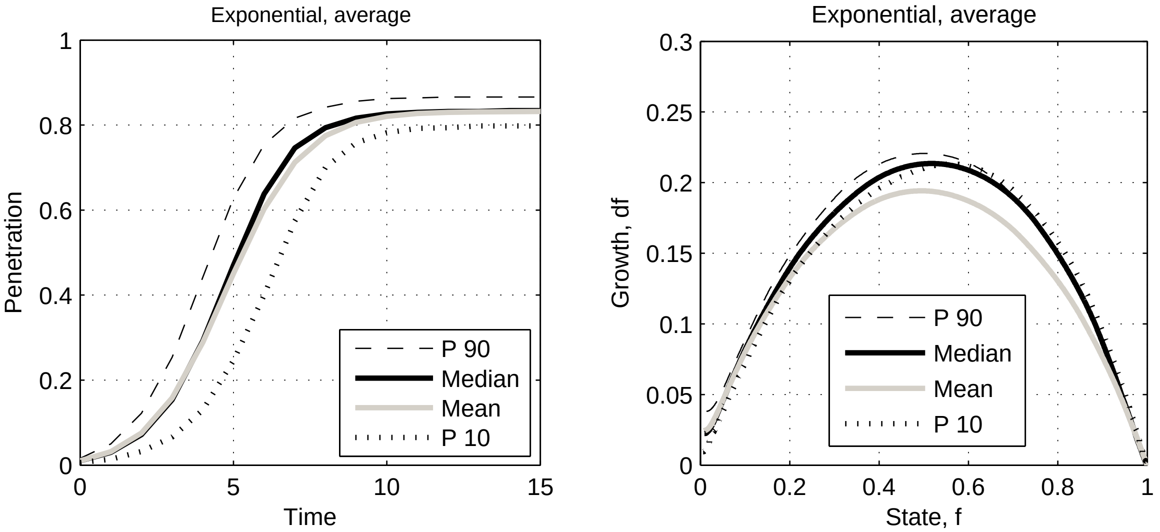

Figure A.4: Simulated adaptation with homogeneous model and exponential, average thresholds

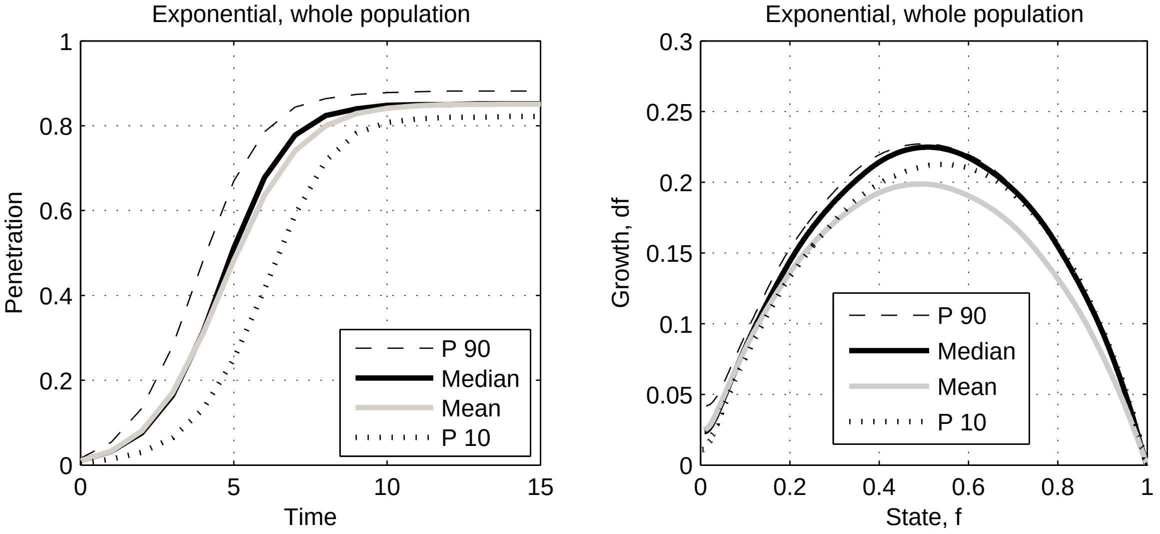

Figure A.5: Simulated adaptation with homogeneous model on total subscribers and exponential thresholds

B. NORMAL DISTRIBUTION AND STATISTICS

A *random variable* X is a variable whose numerical value cannot be determined exactly. However, the *instance* of X is exact after the occurrence of *random event*. The different possible outcomes e of X — i.e., e is in the *event space* $\mathcal{E}(X)$ of X — have different probabilities to occur.

Consider a random variable X with event space $\mathcal{E}(X) = \mathcal{R}$. The *(cumulative) distribution function* (cdf) $F(x) = P\{e \in \mathcal{E}(X) | e \leq x\}$ of X defines the probabilities of the events [220]. The random number is *(absolutely) continuous* if the distribution function is continuously differentiable, i.e., $F(x) = \int_{-\infty}^{x} f(x)dx$. In other words, the probability $P(e)$ of any specific value e of a continuous random variable X is zero, i.e., $\forall e \in \mathcal{E}(X) : P(e) = 0$, and the probability sum is one, i.e., $P[\mathcal{E}(X)] = 1$. In summary, the *probability density function* (pdf) $f(x)$ of a continuous random variable X has the following properties

- $f(x) \geq 0$ for all $x \in \mathcal{E}(X)$,
- $P[\mathcal{E}(X)] = \int_{-\infty}^{\infty} f(x)dx = 1$,
- $P[a \leq x \leq b] = F(b) - F(a) = \int_{a}^{b} f(x)dx$.

The *expectation* (or *mean*) of a continuous random variable X is

$$E[X] = \int_{-\infty}^{\infty} x f(x) dx.$$

Furthermore, the expectation $E[X^i] = \int_{-\infty}^{\infty} x^i f(x)dx$ is called ith moment α_i of the distribution and $\mu_i = E[(X - E[X])^i] = \int_{-\infty}^{\infty} (x - E[X])^i f(x)dx$ the ith moment about the mean. The *variance* of X is defined as $var[X] = \sigma^2 = \mu_2$. The square root of variance, i.e., σ, is called the *standard deviation*. More information on probability theory and statistics can be found in [188] or more theoretically in [67, 220].

B.1 Normal and Log-Normal Distributions

A random variable X has *normal distribution* $X \sim \mathcal{N}(\mu, \sigma)$ with parameters $\mu \in \mathcal{R}$ and $\sigma > 0$ if the probability density function is [188]

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

The expectation and standard deviation of the normal distribution are μ and σ, respectively. The distribution $\mathcal{N}(0, 1)$ is called the *standard normal distribution*. The shapes of all normal distributions are symmetric, see Figure B.1.

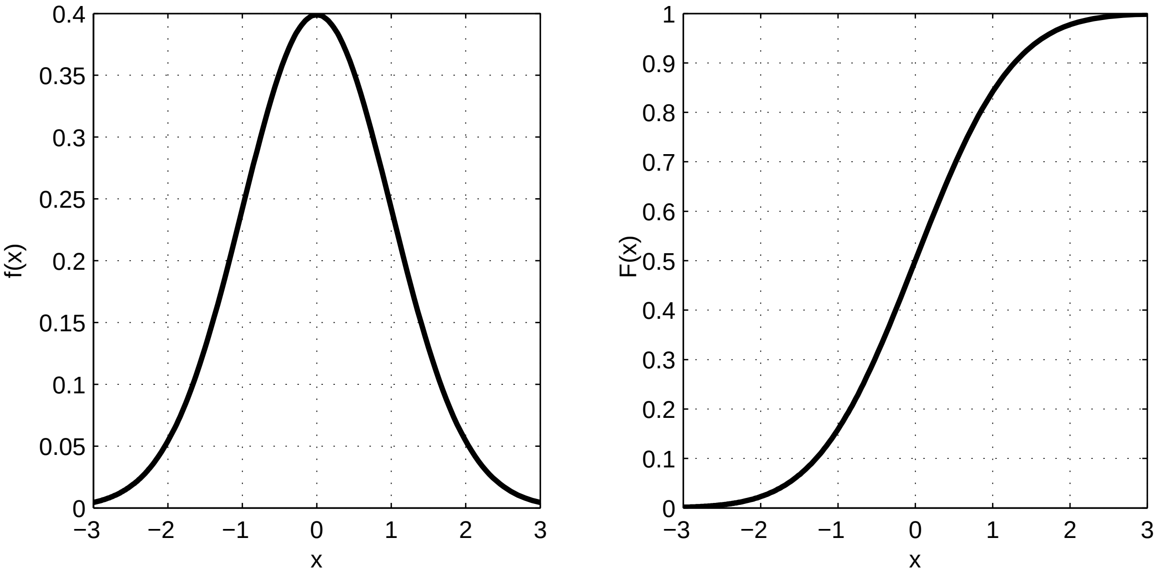

Figure B.1: The probability density and the cumulative standard normal distribution

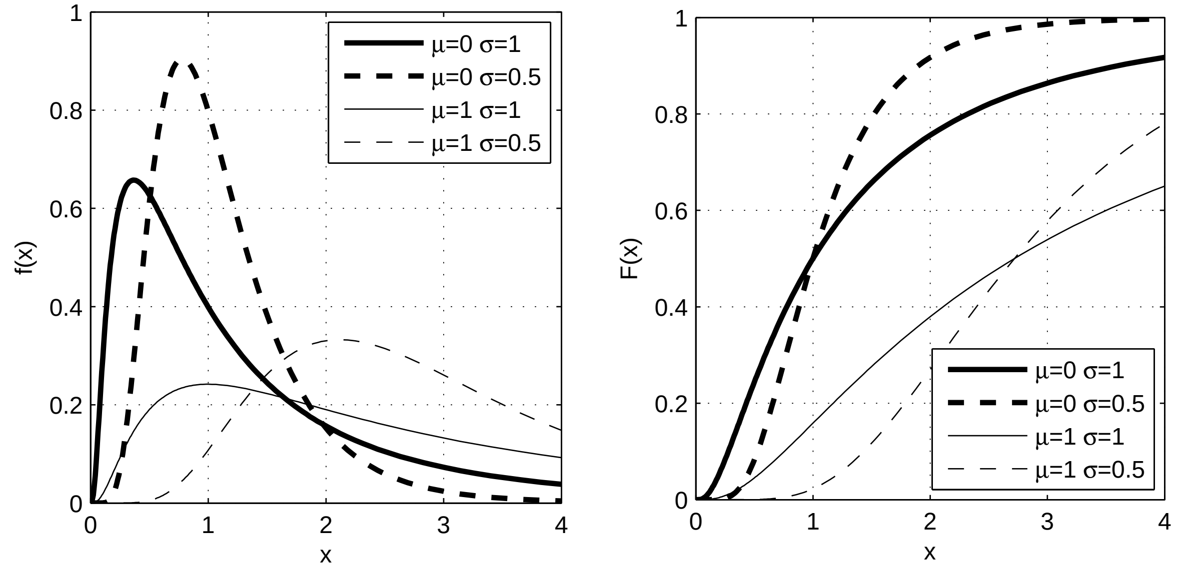

Figure B.2: The probability density and the cumulative functions for log-normal distributions

Consider a distribution $Y = e^X$ such that X is normally distributed, i.e., $X \sim \mathcal{N}(\mu, \sigma)$. Now, $Y \sim \mathcal{LN}$ is called *log-normal distribution* with parameters $\mu \in \mathcal{R}$ and $\sigma > 0$ [188]. The density function of the log-normal distribution is thus

$$g(y) = \frac{1}{\sqrt{2\pi}\sigma y} e^{-\frac{(ln(y)-\mu)^2}{2\sigma^2}}.$$

Note that the expectation and the variance of the log-normal distribution are not equal to parameters. Instead, $E[Y] = e^{\mu+\sigma^2/2}$ and $var[Y] = (e^{\sigma^2} - 1)e^{2\mu+\sigma^2}$. The event space of the log-normal distribution is the positive real axis and the distribution is not symmetric, see Figure B.2.

B.2 SKEWNESS

The *skewness* is the measure how asymmetric the distribution is. The concept is used mainly in determining whether the statistical data may be modeled as a normal distribution or not. The skewness of distribution X is [67]

$$\gamma_1 = \frac{\mu_3}{\sigma^3},$$

where μ_3 is the third moment about the mean and σ is the standard deviation. If γ_1 is positive, the distribution is positively skewed and the probability density function have tail on positive axis, see Figure B.3. If the skewness is negative, the tail is on left and the distribution is negatively skewed. For symmetric distributions, e.g., normal distribution, the skewness is zero.

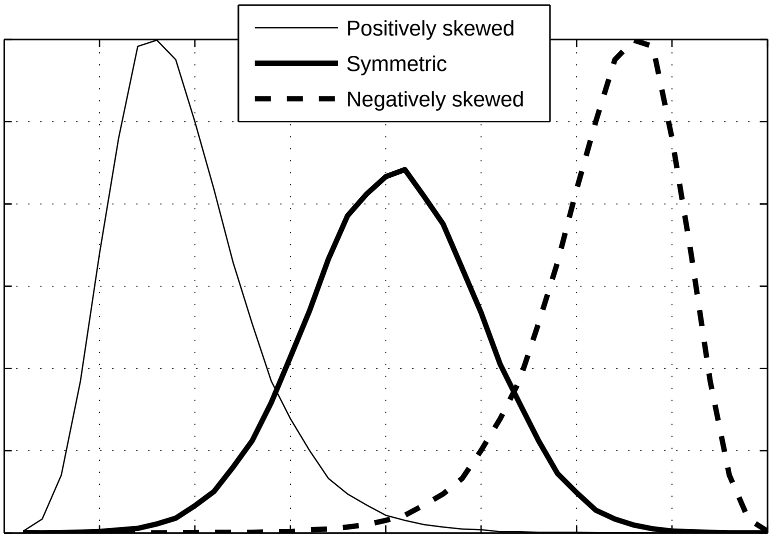

Figure B.3: Probability density functions for symmetric and asymmetric distributions

B.3 KURTOSIS

The *kurtosis* measures the peak of the distribution. The lower the kurtosis the more flatten the distribution is. The commonly used definition for kurtosis (or *kurtosis excess*) is [67, 244]

$$\gamma_2 = \frac{\mu_4}{\sigma^4} - 3,$$

where μ_4 is the fourth moment about the mean and σ the standard deviation. The normal distribution has the kurtosis (excess) zero and it is called *mesokurtic*. Flatten distributions have negative kurtosis and are called *platykurtic* and peaked distributions are *leptokurtic* with positive kurtosis, see Figure B.4.

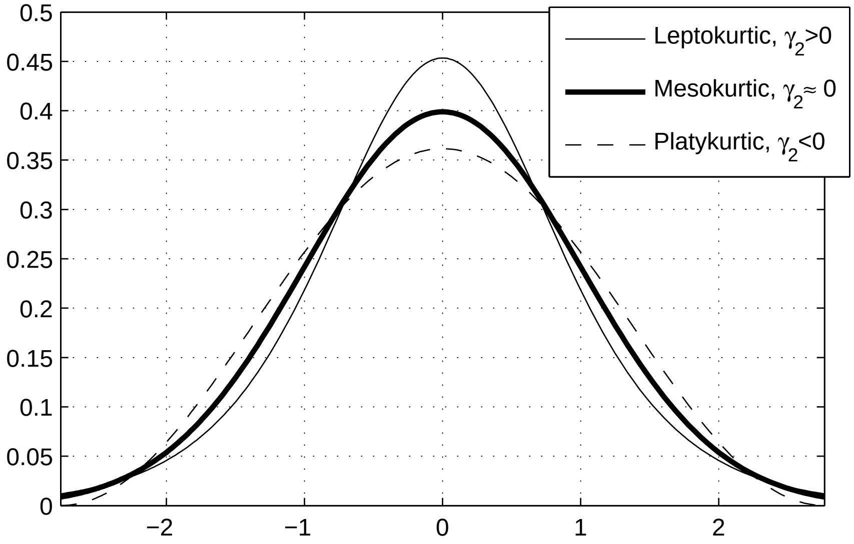

Figure B.4: Probability density functions for some distributions with mean zero and variance one

B.4 STATISTICS AND ESTIMATORS

Research is often based on *samples* and the theory behind the phenomena are surveyed using statistics. The samples are taken from *population*, which is to be analyzed. Consider a statistical sample S such that there are n different values e_i. The statistical analysis determines the characteristics of the underlying population and tests different research hypotheses.

To estimate the parameters and other properties of a real random variable X, the sample properties are analyzed. The *estimator* of a characteristic θ is a function $T(e_1, e_2, ..., e_n)$. The mean $\overline{S}$ and the moments m_k of the sample are

$$\overline{S} = \frac{\sum_{i=1}^{n} e_i}{n}, \tag{B.1}$$

$$m_k = \frac{\sum_{i=1}^{n} (e_i - \overline{S})^k}{n}. \tag{B.2}$$

The mean $\overline{S}$ and the second moment m_2 are used as estimators for expectation and variance of the population, respectively.

The estimator is asked to give right answer. Moreover, it should be efficient, i.e., the answer should not vary too much. In statistics, these two criteria can be expressed as [67, 188, 220]

- $E[T] = \theta$,
- minimize $Var[T]$.

If the first criteria is satisfied, the estimator is *unbiased*. The second criteria stands for *minimum variance*. It has been proved that the mean $\overline{S}$ (B.1) of the sample is unbiased minimum variance estimator for the expectation of the population [188, 220]. However, the variance m_2 (B.2) of the sample is biased estimator because [188, 220]

$$E[m_2] = \frac{n-1}{n}\sigma^2 \neq \sigma^2.$$

The unbiased estimator for the population variance is [67, 188, 220]

$$s^2 = \frac{n}{n-1}m_2 = \frac{\sum_{i=1}^{n}(e_i - \overline{S})^2}{n-1}.$$

Moreover, s^2 is the minimum variance unbiased estimator [220].

The sample moments, m_k are unbiased estimators [67]. However, the skewness and kurtosis are often estimated using these and other biased estimators [139]. The skewness and kurtosis excess of the sample are

$$\begin{aligned} g_1 &= \frac{m_3}{m_2^{\frac{3}{2}}}, &\text{(B.3)}\\ g_2 &= \frac{m_4}{m_2^2} - 3. &\text{(B.4)} \end{aligned}$$

To reduce the bias, the estimators used in this study are [139]

$$\begin{aligned} G_1 &= \frac{\sqrt{n(n-1)}}{n-2}g_1, \\ G_2 &= \frac{n-1}{(n-2)(n-3)}((n+1)g_2 + 6), \end{aligned}$$

where sample variables g_1 and g_2 are as in (B.3) and (B.4).

B.5 TESTS FOR NORMALITY

Many theoretical models use the normal distribution as basis for calculations. This is because many real phenomena seem to be normally or nearly normally distributed. Thus there are many techniques developed for normal distributions, too. However, all the distributions cannot be modeled using normal distributions. To help decide whether or not to use normal models, different *tests for normality* are used.

The most simple, and perhaps the most reliable and efficient too, normality tests are graphical comparisons [208]. From the statistical data the cumulative distribution function of the sample can be drawn. Furthermore, using mean and variance estimators, the corresponding cumulative distribution function of normal distribution can be plotted in the same figure. The resulting figure with two curves tells whether the distribution can be normal or not. The graphical comparison helps comparing the shapes of distributions, but the significance of the differences may be hard to specify from figure. To estimate the significance, some normality tests are developed. In this study, normality tests by Kolmogorov–Smirnov and Lilliefors and Jarque–Bera are considered.

Consider a *sample distribution* for sample $S = \{e_1, e_2, ...e_N\}$ from population P

$$S_N(x) = \begin{cases} 0, & x < e_1^* \\ k/N, & e_k^* \leq x < e_{k+1}^* \\ 1, & x \geq e_N^* \end{cases}$$

where e_1^* is the smallest sample in S and e_2^* the second smallest and so on. In normality test there are two hypotheses:

- *Null hypothesis* H_0: P is normally distributed,
- *Alternative hypothesis* H_1: P is not normally distributed.

The hypotheses are tested using test statistics and the aim is to reject H_0 if the difference between S_N and the corresponding normal distribution is significant.

Kolmogorov–Smirnov test statistics is defined as [181]

$$d = \max_x |F_0(x) - S_N(x)|, \tag{B.5}$$

where F_0 is the distribution to which the sample distribution S_N is tested. The test assumes H_0 and rejects it if the probability of the test statistics to be as high as or greater than d is low. The *P-value* of the test is the calculated probability value. Commonly used significance level is 0.05 which means that 5% of the samples from normal distributions can have test statistics d greater than or equal to sample. In this study, 2% significant level is used with rather large sample size 1000. The limiting distribution of the test value d was introduced by Kolmogorov and developed further by Smirnov. Moreover, the distribution can be approximated by Monte-Carlo and the significance of the difference between distributions F_0 and S_N measured [181]. However, the test does not give reliable results if the parameters of the distribution are not known, i.e., if the mean and variance must be estimated. In such cases the same test statistics d (B.5) follows Lilliefors distribution [175, 188]. This modification of the Kolmogorov–Smirnov test is called *Lilliefors test*.

Jarque–Bera test for normality is based on the skewness and kurtosis. The distribution of the sample skewness is proved to follow $N(0, \sqrt{6N})$ and the distribution of the sample kurtosis $N(0, \sqrt{24N})$ [42]. Thus the test statistics

$$T = N\left(\frac{G_1^2}{6} + \frac{G_2^2}{24}\right)$$

is Chi-squared distributed with two degrees of freedom [42, 135]. If the skewness and kurtosis of the sample are far from the zero, the test statistics get values much above zero and the P-value of the H_0 hypothesis is small. Jarque–Bera test should not be used for small samples.

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