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Analysis and modeling of piano sustain-pedal effects[a)]

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This paper describes the main features of the sustain-pedal effect in the piano through signal analysis and presents an algorithm for simulating the effect. The sustain pedal is found to increase the decay time of partials in the middle range of the keyboard, but this effect is not observed in the case of the bass and treble tones. The amplitude beating characteristics of piano tones are measured with and without the sustain pedal engaged, and amplitude envelopes of partial overtone decay are estimated and displayed. It is found that the usage of the sustain pedal introduces interesting distortions of the two-stage decay. The string register response was investigated by removing partials from recorded tones; it was observed that as the string register is free to vibrate, the amount of sympathetic vibrations is increased. The synthesis algorithm, which simulates the string register, is based on 12 string models that correspond to the lowest tones of the piano. The algorithm has been tested with recorded piano tones without the sustain pedal. The objective and subjective results show that the algorithm is able to approximately reproduce the main features of the sustain-pedal effect. © *2007 Acoustical Society of America.* [DOI: 10.1121/1.2756172]

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I. INTRODUCTION

Modern pianos have either two or three pedals. The right pedal is always the sustain pedal (or the resonance pedal), which lifts all the dampers and allows the strings to vibrate due to sympathetic coupling. The left pedal is called the *una corda* pedal, and its purpose is to make the output sound softer and give it a “lyrical” quality.[1] The purpose of the possible middle pedal can vary depending on the instrument; it can be the bass sustain pedal or the practice pedal, which softens the sound of the instrument. The sustain pedal, however, is the most important and widely used. It has primarily two purposes. First, it acts as “extra fingers” in situations where *legato* playing is not possible with any fingering. Second, since all the strings are free to vibrate, enrichment of the tone is obtained. The usage of the sustain pedal needs synchronization between the hands and foot, and Repp[2,3] has found that its timing is influenced by the tempo of the performance.

The purpose of this study is to provide information on the sustain-pedal effect through signal analysis and to present an algorithm for modeling the effect. A preliminary version of this work was presented as a part of a Master’s thesis.[4] Now, some modifications and improvements to the algorithm are suggested. In addition, the signal analysis given is in more depth.

The acoustics and the sound of the piano have been widely addressed in the literature. Excellent reviews of the topic are given, for example, in the text book edited by Askenfelt[5] and in the article series by Conklin.[6–8] Recently, research has focused on specific topics, such as the dispersion phenomenon,[9–11] the longitudinal vibrations,[12,13] and the grand piano action.[14] In addition, the physics-based sound synthesis of the piano has gained much popularity during the last two decades. The digital waveguide approach[15–17] is an efficient way to produce realistic piano tones in real time. Another popular approach is the finite difference method; the modeling of the hammer-string interaction[18,19] and the simulation of the piano string vibration[20] have been interesting research topics. Recently, Giordano and Jiang[21] presented a complete finite difference piano model, which consisted of several interacting submodels, such as the hammers, strings, and the soundboard.

Despite the importance of the sustain pedal, only few studies considering the modeling of the effect have been published. De Poli *et al.*[22] presented a model, which is based on the string register simulation with 18 string models of fixed length and ten string models of variable length. The fixed-length strings correspond to the 18 lowest tones of the piano, and the usage of the variable-length strings depends on the situation. For example, some of them can be used to increase the amount of string models in the set of fixed-length strings. In this case, their lengths are set to correspond to the notes subsequent to those of the fixed-length strings. The output from the two junctions is first lowpass filtered and then multiplied with a coefficient that determines the degree of the effect of the sustain pedal. With this system De Poli *et al.* were able to produce a resonance pedal effect for digital electronic pianos.

The problem can be approached also from a theoretical point of view. Carrou *et al.*[23] presented a study that discussed sympathetic vibrations of several strings. They constructed an analytical model of a simplified generic string instrument, in which the instrument body is modeled as a beam to which

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the strings are attached. With this model, Carrou *et al.* obtained results that are applicable to real instruments with many strings, such as the harp. Basically, the situation is similar in the case of the piano, where sympathetic coupling between the strings is a known phenomenon.[1,24,25] However, in the case of the sustain pedal, the number of vibrating strings is nearly 250, and an analytical approach would lead to an extremely complex mathematical problem as well as a heavy computational modeling. In contrast, in this study, the goal is to obtain an efficient model-based synthesis algorithm with a low computational cost, and the approach taken here is rather phenomenological than physical.

Van Duyne and Smith[26] proposed that the sustain-pedal effect can be synthesized by commuting the impulse response of the soundboard and open strings to the excitation point. This approach provides an efficient and accurate way to simulate the sustain-pedal effect.

Jaffe and Smith[27] proposed that sympathetic vibrations can be simulated with a bank of sympathetic strings parallel to a plucked string. Later, some authors have suggested that the sustain pedal could be simulated with a reverberation algorithm,[16] which is often used in room simulations[28,29] and instrument body modeling.[30–32]

The approach taken in this paper follows the aforementioned ideas; the freely vibrating string register is simulated with a set of string models, which are designed to correspond to the lowest tones in the piano. On the other hand, the proposed system can be interpreted as a reverberation algorithm, since the number of modes in the system is very high.

This paper is organized as follows. In Sec. II the recording setup and equipment are described. Section III reports the results of the signal analysis and in Sec. IV the synthesis algorithm with examples and perceptual evaluation of the proposed model are presented. Finally, Sec. V concludes the paper.

II. RECORDINGS

In order to study features of the sustain-pedal effect, recorded piano tones with and without the sustain pedal were analyzed. Obviously, the best choice for the recording place would be an anechoic chamber, but unfortunately this option was out of the question. The next best choice is to carry out the recordings in a maximally absorbent recording studio. Single tones from the bass, middle, and treble range of the piano were played both with and without the sustain pedal.

A. Recording setup and equipment

The recordings were carried out at the Finnvox recording studio in Helsinki, Finland, in February 2006. The instrument was a Yamaha concert grand piano (serial number 3070800) with 88 keys, and it was tuned just before the recording session. The signals that were used in the analysis were recorded on two channels, and the microphones (Danish Pro Audio 4041 with Danish Pro Audio HMA 4000 preamplifier) were positioned 32 cm above the string register. The microphones corresponding to the left and right channels were located 80 cm from the keyboard end and 71 cm from the other end, respectively, and the positions were kept constant throughout the session. Another option for the microphone locations would have been a few meters away from the instrument. This choice would have provided information about the phenomenon from the listener's point of view. On the other hand, our goal was to analyze the effect and gather accurate information for synthesis. Since a better signal-to-noise ratio is obtained when the effects of the transmission path are minimized, it was decided to place the microphones close to the strings.

FIG. 1. Illustration of the microphone positioning above the string register. The letters L and R indicate the locations of the microphones corresponding to the left and right channel, respectively.

Figure 1 illustrates the recording setup. The microphone locations for the left and right channels are indicated with the letters L and R, respectively. The signals were recorded with the sampling rate of 44.1 kHz and 16 bits. Finally, only the right channel was chosen for the analysis, since it provided a slightly better signal-to-noise ratio.

B. Recorded tones

An extensive set of bass, middle, and treble tones were played several times without the sustain pedal and then with the sustain pedal, and finally the features of five of them were selected to be shown in the present study. The dynamics of the tones were determined to be *mezzo forte* in all cases. It is assumed that the soundboard and the sympathetically resonating string register behave linearly, and thus the model that is calibrated based on signal analysis performed to *mezzo forte* tones is also suitable for playing with different dynamics.

In order to keep the dynamics as constant as possible during the whole session, a weight of 0.5 kg was used to press down the keys in the low and middle range of the piano. A pile of small coins attached to each other made up the weight used. For the highest range, that is, the 17 uppermost keys of the piano, the method proved to produce a pronounced sound effect while depressing the key, when the key touched the front rail and the key frame under the keyboard. Thus, the highest keys were played manually in the conventional way while keeping the dynamic level as constant as possible.

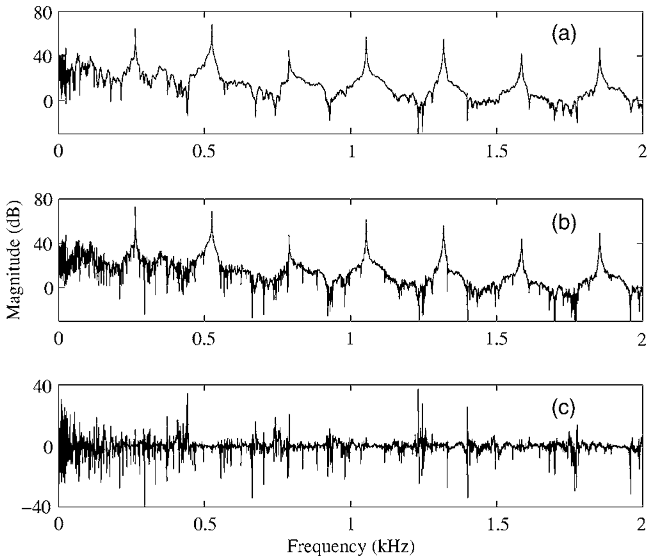

FIG. 2. Tone C4 (a) without and (b) with the sustain pedal in frequency domain. (c) shows the difference between the magnitude spectra in (a) and (b).

In addition, the proportion of the energy of sympathetic vibrations was investigated by first playing the tone with the sustain pedal and then damping the string group that corresponds to the key that was pressed down. After damping the string group, the string register still keeps ringing due to sympathetic coupling, and it is possible to obtain information on the relation between the energy of the direct sound and sympathetic vibrations. This experiment was carried out for several keys from the bass, middle, and treble tones, and each case was repeated a couple of times.

A visualization of the effect of the sustain pedal on the piano tone is shown in Fig. 2. Figures 2(a) and 2(b) illustrate the spectrum of the tone C4 (key index 40, f_0=262.7 Hz) when the sustain pedal is not used and when it is used, respectively. It can be seen that when the tone is played with the sustain pedal, the spectrum contains additional components between the partials. In order to better see these additional components, the difference between the magnitude spectra of Figs. 2(a) and 2(b) is presented in Fig. 2(c). The spectra are computed with a 2^{20} point fast Fourier transform (FFT) using a rectangular window.

III. SIGNAL ANALYSIS

In this section, the signal analysis procedure is described. First, the sustain-pedal effect is analyzed by extracting single partials from recorded tones played with and without the sustain pedal and their features are compared in Sec. III A. Second, in Sec. III B, the residual signals are investigated by canceling all partials from tones that were played with and without the sustain pedal. The residual signal energies were measured in critical bands[33] in order to study in which frequency ranges there are differences. Additionally, possible physical explanations for the observed features are discussed.

TABLE I. The fundamental frequencies and the inharmonicity coefficients of the example tones.

Tone	f_0 (Hz)	B
C2	65.6	3.8×10^{-5}
C3	131.2	1.1×10^{-4}
C4	262.7	3.3×10^{-4}
D5	589.1	1.2×10^{-3}
C6	1052.8	2.3×10^{-3}

A. Extracted partials

Recorded signals were analyzed by extracting single partials by bandpass filtering tones with and without the sustain pedal. In general, three features of the partials were of interest: the possible differences in initial levels, decay times, and amplitude beating characteristics. These features were investigated in the case of the five example tones listed in Table I with their fundamental frequencies and inharmonicity coefficients.[34] These parameters can be estimated from tones manually, or by using some specific algorithm, e.g., the inharmonic comb filter method presented by Galembo and Askenfelt.[35] In the present study, the fundamental frequencies and the inharmonicity coefficients of the example tones were estimated using a simple algorithm based on a peak picking technique. In this algorithm, local maxima of the whitened spectrum of a piano tone are searched and the corresponding frequencies are used as estimates for the harmonic component locations. The theoretical partial frequencies are computed using Eq. (1)[36]

$$f_m = mf_0\sqrt{1+m^2B}, \tag{1}$$

where m is the partial index and f_m is the corresponding frequency. The best B and f_0 estimates are obtained by finding the minimum mean-square error between the partial frequencies and the theoretical partial frequencies obtained from Eq. (1) with different values of f_0 and B. The initial guesses for these parameters are based on the key index information.

In the case of the piano, the harmonics exhibit a complicated decay process because of the two-stage decay and beating.[1] In order to obtain an approximation of the decay times of the tones, a straight line was fitted to the first six seconds of signal log envelopes in a least squares sense.[37,38] After this, estimates for T_{60}-times, that is, the time it takes for a partial to decay 60 dB, were computed based on the slope of the fitted straight line. Additionally, the same procedure was repeated for partials that correspond to the fundamental frequencies of the tones.

The results for the five example tones are listed in Tables II and III, giving the overall decay rates and the decay rates of the partials corresponding to the fundamental frequencies, respectively, along with their initial levels. As can be seen, the decay times in general are larger in those cases where the sustain pedal is used, especially in the middle range of the piano. In the treble range, there are no remarkable differences in the decay times, while in the bass tones the overall decay time seems to be even slightly decreased, but the decay time of the first partial is increased when the sustain

TABLE II. The overall T_{60}-times and the initial levels of the tones without (N) and with (P) the sustain pedal.

Tone	T_{60}/N (s)	T_{60}/P (s)	Level/N (dB)	Level/P (dB)
C2	20.3	18.1	−7.5	−7.0
C3	12.5	14.8	−7.3	−7.3
C4	9.5	15.3	−6.8	−6.6
D5	14.7	18.3	−5.0	−4.5
C6	12.0	12.0	−4.3	−3.7

pedal is used. The changes in initial levels of the partials are minor, within 1 dB, which implies that these changes are, in general, difficult to hear.

There are several possible physical explanations for the observed changes in decay times presented in Tables II and III. The bass strings are attached to a separate bridge, which inhibits the energy from leaking to the middle and treble strings and, thus, the decay times are not altered substantially when the bass tones are played with the sustain pedal compared to the situation where the sustain pedal in not engaged. In the middle region, on the other hand, the number of strings is higher because of the duplets and triplets, and the middle and treble strings are attached to the same bridge, which enables more coupling to surrounding strings. It is likely that some of the surrounding strings have modes near the modes of the strings corresponding to the struck key, and when these modes are not damped by the damper, the decay time of the played tone increases. In the treble region the decay times are not likely to increase, since the harmonic structure is sparse. Even if the frequency of a partial of a high tone is nearly the same as some of the partials of the lower tones, the high tone does not contain much energy for exciting the lower strings. Moreover, the high strings are relatively far away from the more energetic middle strings.

The relevance of the increased decay times from the perceptual point of view is a more complicated question. Järveläinen and Tolonen[39] presented perceptual tolerances for decay parameters and concluded that decay time variations are inaudible if the change is between 75% and 140%. Despite the fact that the reported perceptual tolerances were determined for synthetic tones that resemble guitar plucks, they can be expected to indicate that the human hearing is similarly inaccurate in analyzing the decay process of piano tones. In this light, the decay time difference seems to be audible only in the case of the tone C4, since the T_{60} time is 161% of the original T_{60} time when the sustain pedal is engaged.

TABLE III. The T_{60}-times of the fundamental frequencies with the initial levels for five example tones without (N) and with (P) the sustain pedal.

Tone	T_{60}/N (s)	T_{60}/P (s)	Level/N (dB)	Level/P (dB)
C2	9.3	14.8	−14.1	−13.7
C3	10.0	14.3	−10.3	−9.9
C4	10.3	17.0	−13.8	−14.2
D5	14.2	16.0	−5.2	−4.5
C6	9.0	8.3	−4.4	−3.8

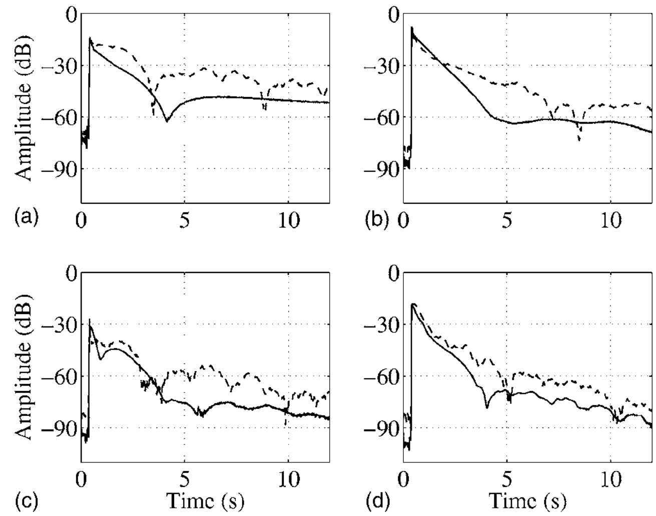

FIG. 3. Envelopes of the partials (a) 1, (b) 2, (c) 3, and (d) 4 of the tone C4. The solid and dashed lines represent the tones without and with the sustain pedal, respectively.

The amplitude beating in the partials is slightly increased, as is shown in Fig. 3, which presents the envelopes of the partials Nos. 1–4 of the tone C4. The solid and dashed lines represent the tones with and without the sustain pedal, respectively. The differences in the noise floors, which are visible before the onset of the tones, are due to the contact sound between the dampers and the strings when the sustain pedal is pressed down before pressing down the key. The amplitude beating characteristics are similar also in the bass and treble tones.

The increased beating results from the energy transfer from the excited string group to the string register via the bridge. As all the dampers are lifted and the string register is allowed to vibrate freely, the modes near those of the excited string group gain energy.

Another observation is that the two-stage decay structure of the partials becomes less obvious when the tone is played with the sustain pedal compared to the situation when the sustain pedal is not engaged. This feature is also due to the energy leakage from the string register, and it depends on the admittance of the bridge.

B. String register hammer response

When the hammer hits a string group, the freely vibrating string register and the soundboard are excited by the impulse of the hammer. The behavior of the undamped string register was studied by removing the partials from recorded tones corresponding to the struck key. The aim was to compare the residual signals of the tones with and without the sustain pedal in the frequency domain.

The frequency analysis was done with a 2048 point FFT applying a Hanning window having 512 samples with a hop size of 256 samples. Figures 4(a) and 4(b) illustrate the time-frequency plot of the residual signals obtained from the tone C4 without and with the sustain pedal, respectively. It can be seen that when the tone is played with the sustain pedal the level of the residual signal during the time interval 1–5 s is approximately 10 dB higher than in the reference case. The

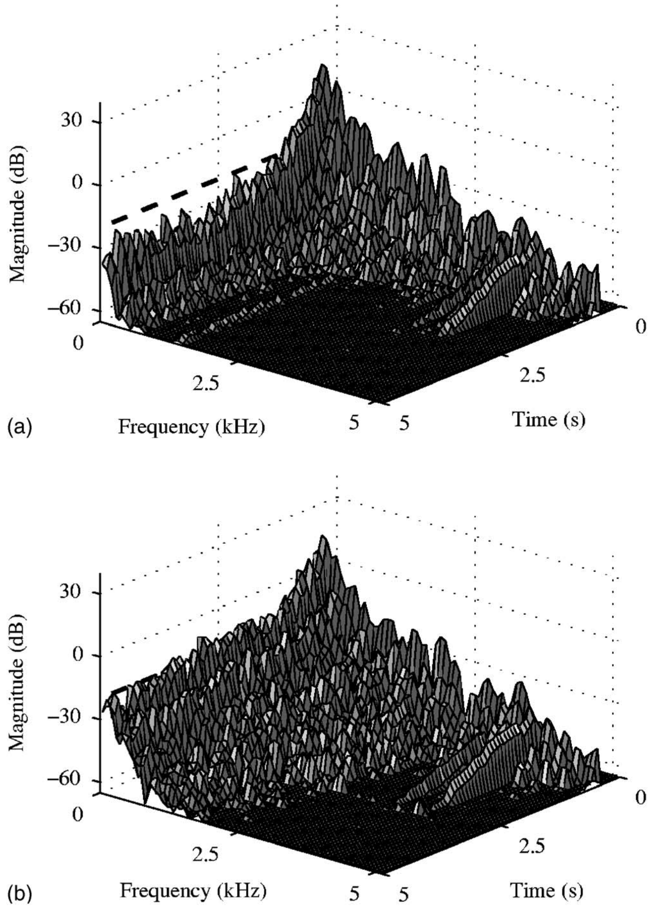

FIG. 4. Time-frequency plot of the residual signal obtained from tone C4 (a) without and (b) with the sustain pedal. The dashed line in (a) indicates the corresponding residual signal magnitude 5 s after the excitation when the tone is played with the sustain pedal (b).

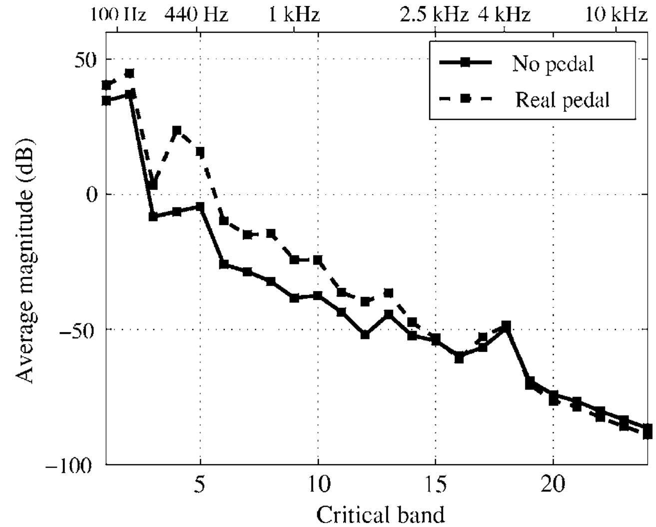

FIG. 5. Energies of the residual signals as a function of frequency on the Bark scale. The solid and dashed lines represent the cases where the tone C4 is played without and with the sustain pedal, respectively.

dashed line in Fig. 4(a) illustrates the corresponding level of the residual signal when the sustain pedal is used [see Fig. 4(b)]. The trend is similar when the analysis is performed on lower and higher tones.

The energies of the residual signals were measured on the Bark scale.[33] The Bark scale was chosen because it provides information from the psychoacoustical point of view, which is important for perceptually meaningful synthesis of the sustain pedal effect. Figure 5 illustrates the energies of the residual signals for the tone C4 with and without the sustain pedal. The energies were computed from a 1 s excerpt 110 ms after the excitation. Figure 6 presents the difference of the two curves of Fig. 5 as well as the corresponding differences for the tones C2 and C6. As a conclusion, the residual signal energy is increased 5–30 dB on Bark bands 1–14, which correspond to the frequency range 0–2.5 kHz.

The aforementioned frequency range affected by the sustain-pedal effect seems reasonable from the physical point of view. It can be seen in Fig. 5 that the differences between the residual signal energies start to decrease around 1.5 kHz, and finally roll off towards 0 dB around 2.5 kHz. This frequency range is approximately the same where the strings do not have dampers anymore. As a result, the highest strings are more or less excited always, regardless of the usage of the sustain pedal.

IV. SYNTHESIS ALGORITHM DESIGN

The main idea of the proposed reverberation algorithm is to approximate the string register with 12 simplified digital waveguide string models[40] that correspond to the 12 lowest strings of the piano. The proposed structure is basically an extension to the algorithm presented by De Poli *et al.*,[22] where 18 fixed length and ten variable length strings were used to simulate the string register.

In addition, in the present study a dispersion filter $A_k(z)$ and a lowpass filter $H_k(z)$, where k represents the key index, are included in every string model. The design processes of these filters are described in Secs. IV A 1 and IV A 2, respectively. The input to the string models is filtered with a tone corrector filter $R(z)$ and their summed output is multiplied with a mixing coefficient, which is then added to the input tone. The tone corrector filter and the mixing coefficient are discussed in Secs. IV A 4 and IV A 5, respectively. A sche-

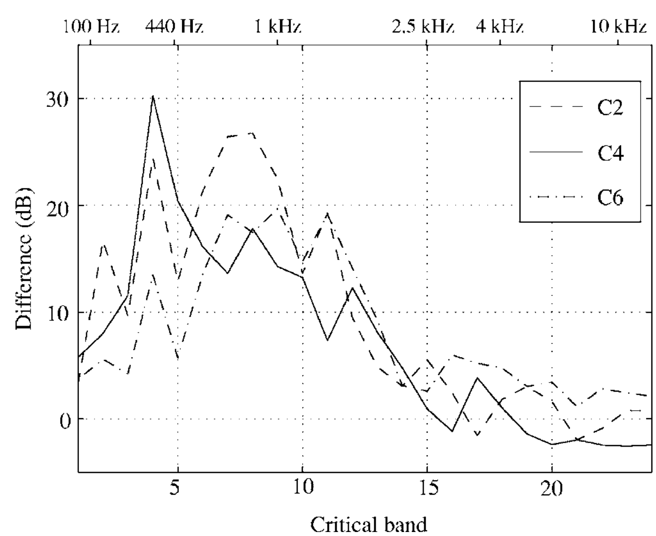

FIG. 6. Differences of the residual signal energies in the case of tones C2, C4, and C6 as a function of frequency on the Bark scale.

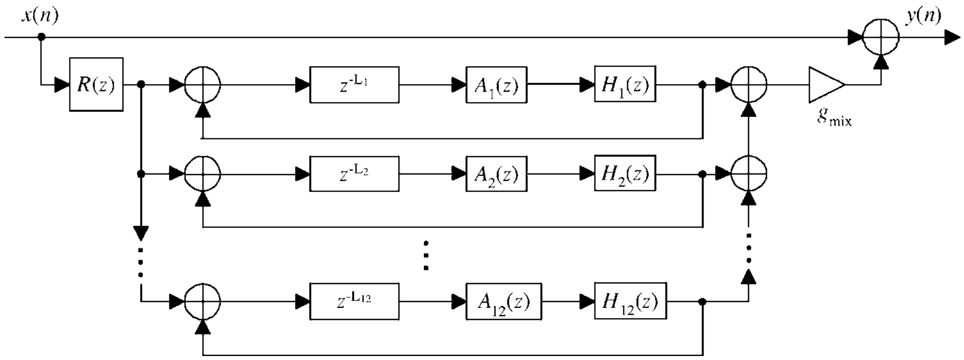

FIG. 7. Block diagram that shows the structure of the sustain-pedal algorithm.

matic view of the algorithm is presented in Fig. 7. In the following, the choice of the algorithm parameters is discussed. The synthesis algorithm is designed with the MATLAB software using a 44.1 kHz sampling rate and a double-precision floating-point arithmetic.

A. Algorithm parameters

1. Delay line lengths

In theory, nearly 250 string models are needed in order to accurately imitate the string register. This choice is, however, not feasible for real-time processing. Thus, the number of string models must be reduced. Since the lowest tones contain more partials than the highest tones, it is advantageous to choose the delay line lengths according to the N lowest tones. When the value of the parameter N is chosen to be 12, the string register is simulated with the first octave of the piano, and the partials of these tones roughly approximate the higher octaves.

The values of the delay line lengths $L_1, \ldots, L_{12}$ are defined by the frequencies indicated in Table IV with the following rounding operation:

$$L_k = \text{floor}\left(\frac{f_s}{f_{0,k}} + \frac{1}{2}\right), \tag{2}$$

where floor is the greatest integer function, f_s is the sampling frequency, and $f_{0,k}$ refers to the fundamental frequency of the tone that corresponds to the key index k. The rounding operation is performed, because the ratio of the sampling frequency and the fundamental frequency is usually not an integer. This makes a slight error to the delay line length, which is ordinarily compensated with an allpass fractional delay filter.[27,41,42] In this case, however, this compensation is not necessary, since only the lowest tones with long delay lines are dealt with. The fractional part is about 0.05% of the whole delay line length, and thus this minor error is inaudible.

TABLE IV. Parameters for the sustain-pedal algorithm.

Key index k	f_0 (Hz)	L	B	a_1	g
1	27.5	1602	3.0×10^{-4}	−0.974	0.9918
2	29.2	1511	2.9×10^{-4}	−0.972	0.9903
3	30.9	1428	2.7×10^{-4}	−0.971	0.9942
4	32.7	1349	2.6×10^{-4}	−0.969	0.9928
5	34.6	1273	2.5×10^{-4}	−0.966	0.9929
6	36.8	1199	2.4×10^{-4}	−0.964	0.9941
7	38.9	1134	2.3×10^{-4}	−0.961	0.9941
8	41.2	1070	2.2×10^{-4}	−0.959	0.9954
9	43.7	1009	2.1×10^{-4}	−0.956	0.9947
10	46.3	952	2.0×10^{-4}	−0.953	0.9958
11	49.1	899	1.9×10^{-4}	−0.949	0.9938
12	52.1	847	1.8×10^{-4}	−0.946	0.9929

2. Dispersion filter design

The stiffness of the piano strings, making the strings dispersive, produces inharmonic tones. It is known that the inharmonicity is a perceptually important phenomenon in the bass range of the piano as it adds "warmth" to the sound.[36] In the proposed sustain-pedal algorithm, however, the purpose of the dispersion filters is not to produce inharmonicity, but to spread the harmonic components of the string models more randomly in the sympathetic spectrum. This approach follows the idea of Väänänen *et al.*,[43] where the authors used comb-allpass filters cascaded with delay lines in the reverberator algorithm in order to obtain a dense response for room acoustics modeling. In the present work, the dispersion filters have an effect especially on the tonal quality of the sound decay. When the dispersion filters are excluded from the model, the decay process may sound somewhat too regular or metallic.

In this model, the dispersion filters are designed with the tunable dispersion filter method.[44,45] The original tunable dispersion filter method offers closed-form formula to design a second-order dispersion filter. This method has been extended to an arbitrary number of first-order filters in cascade,[45] which is used in this work to design a single first-order dispersion filter. The transfer function of the dispersion filter can be denoted as

$$A_k(z) = \frac{a_{1,k} + z^{-1}}{1 + a_{1,k}z^{-1}}, \tag{3}$$

where k is the index of the string model and $a_{1,k}$ is the filter coefficient, which can be calculated as[27,42]

$$a_{1,k} = \frac{1 - D_k}{D_k + 1}, \tag{4}$$

where D_k is the phase delay value at dc. Roughly speaking, increase in the D_k value corresponds to increase in the inharmonicity coefficient value. This is applied in the tunable dispersion filter method, which provides a closed-form formula to determine the D_k value based on the desired fundamental frequency and the desired inharmonicity coefficient value.[44,45]

The parameter values used in this work are shown in Table IV. The inharmonicity coefficient values used in the dispersion filter design are realistic values for the first 12 keys of the piano.[35,46] The delay at the $f_{0,k}$, produced by the dispersion filter, can be approximated with[45] D_k.

3. Lowpass filter design

As the purpose of the algorithm is to simulate real strings, also the frequency-dependent losses in the string are approximated by digital means. A first-order lowpass filter is used for modeling the losses. This filter is often applied in physical modeling of string instruments, because it is easy to design, efficient to implement, and it sufficiently brings about the slow decay of low-frequency partials and fast decay of high-frequency partials at the same time.[38] The transfer function of $H_k(z)$ is given as[38]

$$H_k(z) = \frac{b_k}{1 + cz^{-1}}, \tag{5}$$

where $b_k = g_k(1+c)$. The losses of the string are simulated so that the parameters g_k and c control the overall decay and the frequency-dependent decay, respectively. In this work, the parameters g_k are calibrated based on the 12 lowest tones played without the sustain pedal, and they are listed in Table IV. The c parameter, which controls the frequency-dependent decay, is designed as an approximation based on several tones for the middle, and treble tones. This procedure was found suitable, since especially the treble tones decay fast compared to bass tones. If the parameter was calibrated for bass tones, the decay process of the middle and treble tones with synthetic sustain pedal would be too long. Thus, the c parameter is set to -0.197 for all the strings.

4. Tone corrector

The tone corrector $R(z)$ is designed based on the information obtained from residual signal analysis. In Sec. III B it was concluded that the energy increases mainly in the frequency band 0–2.5 kHz, and in the highest frequency range there is no significant increase in the energy of the residual signal due to strings without the dampers. This is visible also in Fig. 6. In order to design the tone corrector filter $R(z)$, differences of the residual signal energies were computed for several bass, middle, and treble tones. Then, a second-order filter consisting of a dc blocking filter and a lowpass filter was fitted manually to the average of the energy differences. The transfer function can be written as

$$R(z) = \frac{0.1164(1 - z^{-1})}{1 - 1.880z^{-1} + 0.8836z^{-2}}. \tag{6}$$

The average energy difference and the magnitude spectrum of the tone corrector are shown in Fig. 8 with a dashed and solid lines, respectively. The maximum error in the fit is about 5 dB.

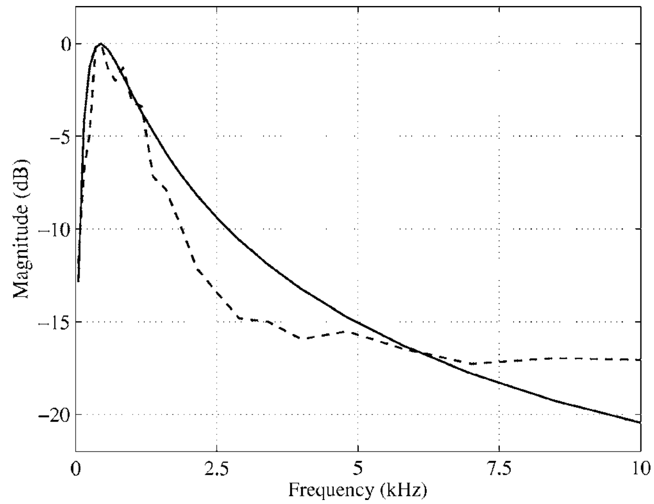

FIG. 8. The average energy difference (dashed line) and the magnitude spectrum of the tone corrector $R(z)$ (solid line).

5. Mixing coefficient

In order to determine the value of the mixing coefficient g_{mix}, the proportion of the energy of sympathetic vibrations has been investigated from the recordings. The tone was played with the sustain pedal, but after the sound had decayed 1–2 s, the string group corresponding to the tone was damped while the rest of the string register still kept ringing. The energies of two 500 ms excerpts were calculated 200 and 3000 ms after the onset of the tone, and the relation of the energies was computed. The first and the second excerpt corresponds to the tone before and after the damping, respectively. It was found that the energy difference before and after the damping is approximately −30 dB for the lowest tones and −45 dB for the highest tones.

In order to find an appropriate mixing coefficient value that produces a similar relation between the direct and artificially reverberated sound, the algorithm has been tested with input tones that are truncated after 2 s from the onset. This approximately simulates the situation, in which the strings are damped about 2 s after the onset of the tone. Based on this analysis, we have found that appropriate mixing coefficient values g_{mix} that yield similar relations between energies of the direct and reverberated sound are 0.015 for the lowest range and 0.005 for the highest range of the piano. Table V lists the used mixing coefficient values as a function of the key index.

It should be pointed out that the exact mixing coefficient value g_{mix} is not a critical issue; small changes in the value do not produce drastic changes to the output sound, provided that the value is chosen from the aforementioned range.

TABLE V. Mixing coefficient values used in the sustain-pedal algorithm.

Key index k	g_{mix}
1–16	0.015
17–28	0.01
29–40	0.008
41–54	0.006
55–88	0.005

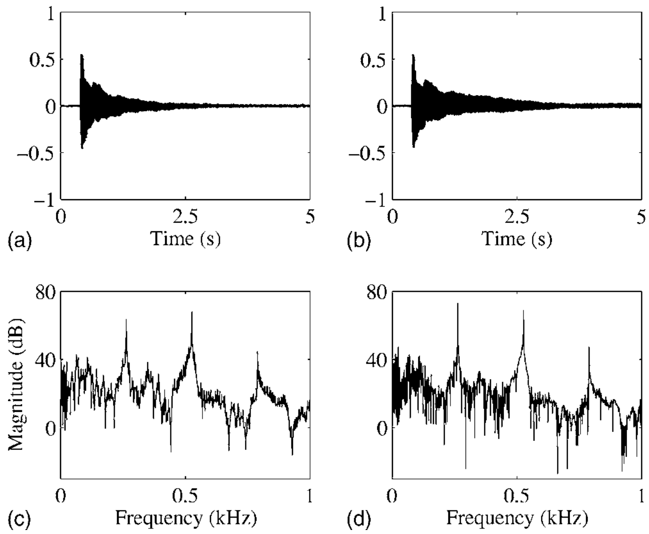

FIG. 9. Tone C4 (a) with the synthetic sustain pedal and (b) with the real sustain pedal. In (c) and (d) the magnitude spectra of the same signals are presented.

B. Results

The algorithm has been tested with recorded piano tones without the sustain pedal, and the processed tones have been compared to recorded tones with the real sustain pedal, Figures 9(a) and 9(b) illustrate the tone C4 with the synthetic and real sustain pedals in the time domain, and (c) and (d) depict the same signal in the frequency domain, respectively. The spectrum is computed with a 2^{20} point FFT using a rectangular window. As can be seen, the algorithm is able to approximately reproduce the additional spectral content between the partials. This effect was noted in Fig. 2, where the spectra of the tones with and without the sustain pedal were compared. The results are similar also in the bass and treble regions.

Additionally, the residual signals of the tones with the synthetic sustain pedal were computed and analyzed. Figure 10(a) presents the time-frequency plot of the residual signal obtained from the tone C4 with the synthetic sustain pedal. For comparison, the residual signal of the tone played with the real sustain pedal is illustrated in Fig. 10(b). The differences between the synthetic and real sympathetic vibration, the T_{60} times were measured in frequency bands 0–2000 Hz, 2000–4000 Hz, and 4000–6000 Hz. These results are listed in Table VI for three example tones: C2, C4, and C6. The T_{60} times are computed in the same way as described in Sec. III A, except that the straight line was fitted to the first 2 s of the signals. For the two highest frequency bands of the tone C6, the straight line was fitted to the first second of the signal, however, because of the fast decay.

Additionally, the residual signal energies were measured in critical bands. In Fig. 11, the solid and dashed lines represent the situation that was depicted in Fig. 5 and the dash-dotted line represents the energy of the residual signal of the tone with the synthetic sustain pedal.

In Fig. 12, the differences of the residual signal energies of the tones without and with the synthetic sustain pedal are presented in the cases of the example tones C2 (dashed line), C4 (solid line), and C6 (dash-dotted line).

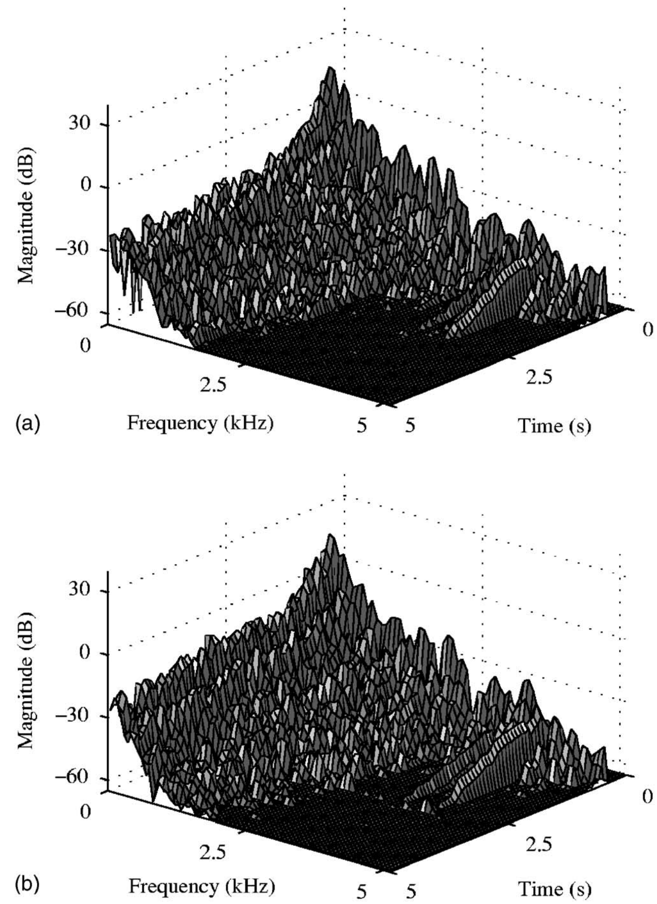

FIG. 10. Time-frequency plot of the residual signal obtained from tone C4 (a) with the synthetic sustain pedal and (b) with the real sustain pedal.

While the algorithm seems to be able to approximate the overall behavior of the real sustain-pedal device, the details of the phenomenon are not modeled perfectly. For example, the partial decay times remain the same in some cases. On the other hand, in Sec. III A it was concluded that the change in the partial decay times is generally inaudible except in the case of the tone C4. If, however, lengthening of partial decay times is of interest, it could be taken into account, for example, with an envelope generator. If the aim is to process synthetic piano sounds, the longer decay times can be taken into account already in the synthesis phase.

Tones with the synthetic sustain pedal and examples of residual signals are available for listening at the web page http://www.acoustics.hut.fi/publications/papers/jasa-piano-pedal/.

TABLE VI. Decay times T_{60} of the synthetic and real sympathetic vibrations in three frequency bands.

Tone	0–2000 Hz	2000–4000 Hz	4000–6000 Hz
C2, synthetic	8.0 s	7.0 s	6.5 s
C2, real	5.8 s	7.3 s	6.4 s
C4, synthetic	5.0 s	4.4 s	5.0 s
C4, real	5.4 s	4.7 s	5.2 s
C6, synthetic	7.4 s	1.7 s	1.8 s
C6, real	5.2 s	1.8 s	1.8 s

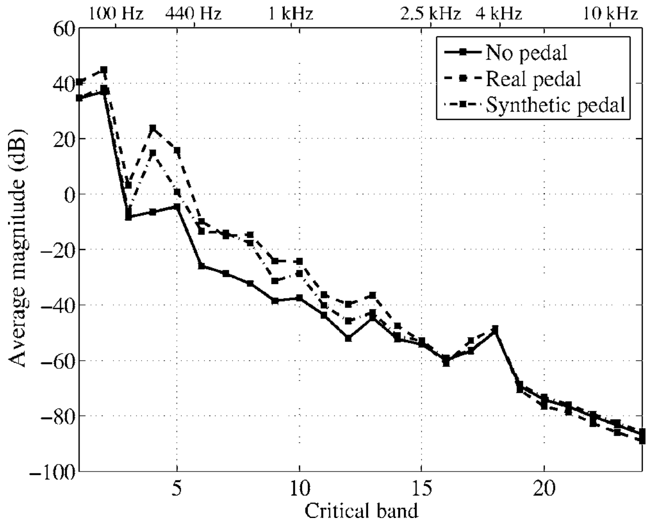

FIG. 11. Energies of the residual signals as a function of frequency on the Bark scale. The solid and dashed lines represent the cases where the tone C4 is played without and with the sustain pedal, respectively. The dash-dotted line represents the residual signal energy of the tone that has been processed with the proposed sustain-pedal algorithm.

C. Perceptual evaluation

In order to evaluate the performance of the proposed algorithm from the perceptual point of view, a listening test was conducted. In addition, the proposed algorithm was compared against a reference algorithm having 28 string models without the dispersion filters. The lowpass filters were replaced with a constant coefficient 0.998, which corresponds to that presented by De Poli *et al.*[22] Also the amount of string models is the same as the maximum amount of string models used by De Poli *et al.*[22] The mixing coefficient was chosen to be half of that used in the proposed model, and the tone corrector was the same in both algorithms.

The main goal of the test was to investigate the indistinguishability between the original tones with the sustain pedal and processed sounds with the synthetic sustain pedal in the two cases. Six subjects all having background in music and audio signal processing took the test. None of the subjects reported a hearing defect, and all had previous experiences from psychoacoustic experiments. The subjects were between 23 and 28 years of age.

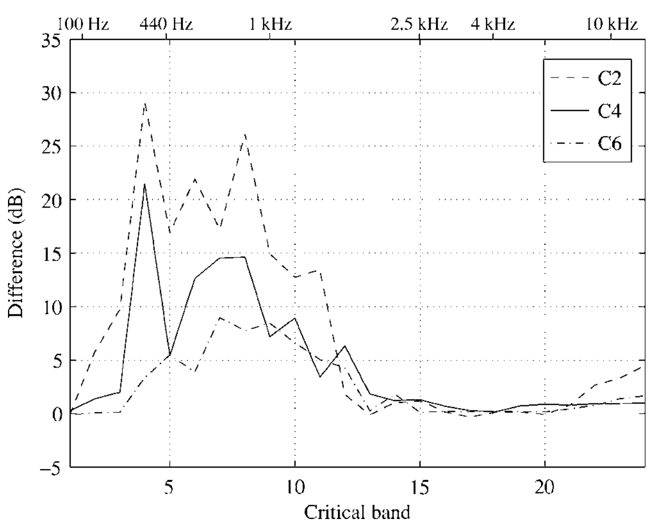

FIG. 12. Differences of the residual signal energies in the case of tones C2, C4, and C6 without the sustain pedal and with the proposed sustain-pedal algorithm as a function of frequency on the Bark scale. These results can be compared to those of Fig. 6, where the same analysis is performed to tones with and without the real sustain pedal.

The test included tones from two instruments. The first instrument (denoted as instrument No. 1) was the same grand piano that was used in the analysis part of this study, and the other one (denoted as instrument No. 2) was another Yamaha grand piano, recorded in a rehearsal room of Espoo Music School. Altogether ten tones were included in the test: tones F2, Gb3, A4, Ab5, and G7 from the instrument No. 1 and tones C1, C2, C4, C5, and G6 from the instrument No. 2. Each tone, with three types per tone (real sustain pedal, synthetic sustain pedal using 12 string models with dispersion filters and lowpass filters, and synthetic sustain pedal using 28 string models with frequency-independent damping coefficients) were repeated five times during the test in a pseudorandom order, making a total of 150 sound samples.

The listening test was conducted in a quiet room and the sound samples were presented to the subjects through Sennheiser HD 580 headphones. In the beginning of the test, the subjects took two rehearsal tests. In the first rehearsal test, they were able to familiarize themselves with five tones with the real sustain pedal and synthetic sustain pedals. The second rehearsal test simulated the actual test, where single tones were played and the subject was asked to determine whether the sustain pedal used in the tones was real or synthetic. The results from the rehearsal tests were not saved. In the actual test, after listening to each sound sample twice, they were asked whether they thought the sustain pedal was real or synthetic.

Following the procedure used by Wun and Horner,[47] the perceived quality of a synthetic sustain pedal was measured with a discrimination factor d defined as

$$d = \frac{P_c - P_f + 1}{2}, \tag{7}$$

where P_c and P_f are the proportions of correctly identified tones with synthetic sustain pedal and tones with real sustain pedal misidentified as tones with synthetic sustain pedal, respectively. The proportions are normalized in the range [0, 1]. Following the convention of Wun and Horner,[47] the tones with synthetic sustain pedal are considered to be nearly indistinguishable from the tones with the real sustain pedal, if the d factor falls below 0.75. Table VII shows the results for the tones used in the test.

From the results it can be seen that the proposed algorithm works well for the low and high range of the piano. In most cases it performs better than the reference algorithm. In the middle range the synthetic sustain pedal is distinguished from the real sustain pedal. In fact, the reference algorithm seems to work slightly better for the tones Gb3 and C4. This is reasonable, since using more string models in the algorithm yields a denser harmonic structure, which is important especially in the middle range, where the amount of relatively energetic strings is high. On the other hand, especially

TABLE VII. Results of the listening test. P and R refer to the results of the proposed and the reference model, respectively. The values that fall below 0.75 are bold for clarity.

Tone	Key index	f_0 (Hz)	Instrument	d factor/P	d factor/R
C1	4	32.8	2	**0.67**	0.82
C2	16	66.1	2	**0.50**	**0.65**
F2	21	87.4	1	**0.53**	**0.53**
Gb3	34	185.7	1	0.80	0.75
C4	40	264.6	2	0.78	0.75
A4	49	441.7	1	0.80	0.90
C5	52	526.4	2	0.83	0.93
Ab5	60	833.8	1	**0.57**	**0.68**
G6	71	1570.5	2	**0.70**	0.83
G7	83	3158.8	1	**0.57**	0.87

in the highest range the decay of the simulated sympathetic vibrations is too long if the frequency-dependent damping is not taken into account. This is probably the reason for the high d factors of the reference algorithm in highest frequency range.

After the listening test most of the subjects reported that the most difficult cases were the bass tones. The middle-range tones were the easiest ones, where the regular and metallic-sounding decay revealed the synthetic cases. In the highest region, the processed tones were judged as synthetic, if the decay process was too long and noisy.

V. CONCLUSIONS

In this paper, the effect of the sustain-pedal device in the grand piano was analyzed. Also, an algorithm for reproducing the effect was proposed. Physically, pressing the sustain pedal lifts dampers which otherwise damp a large bank of sympathetically resonating strings. From the signal analysis it was found that the energy of the residual signal increases when the sustain pedal is used, because the string register is allowed to vibrate freely. Moreover, when the sustain pedal is engaged, the decay times of the tones are longer in the middle range of the piano. This phenomenon was not, however, observed in the case of the bass and treble tones. The initial levels of the harmonics are not substantially changed when the sustain pedal is used; the observed change remains within 1 dB in all example cases. On the other hand, the amplitude beating is slightly increased, and the two-stage decay structure of the tone is not as clear as in the case where the sustain pedal is not engaged.

The proposed algorithm is based on 12 digital waveguide string models corresponding to the 12 lowest tones of the piano, and they consist of delay lines, dispersion filters, and lowpass filters. It was found that the algorithm is able to approximately reproduce the sustain-pedal effect, but the details of the sustain-pedal effect are not modeled perfectly.

A listening test was conducted in order to investigate the naturalness of tones processed with the proposed sustain-pedal algorithm. The result was that the synthetic sustain pedal in low and high tones is perceptually indistinguishable from the real sustain pedal. In the middle range, however, the synthetic sustain pedal is detected from unnaturally regular and metallic decay characteristics. In general, the proposed algorithm performs better than the reference algorithm. The results indicate that by decreasing the amount of string models and including loss filters and dispersion filters the performance of the algorithm is the same or even better than with a larger amount of string models without any filters.

At the moment, the algorithm has been tested only with recorded piano tones. In the future, the aim is to also process synthetic tones. In order to do this, the algorithm needs to be tested and properly calibrated for the synthetic tones.

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