

ANALYSIS, PERCEPTION, AND SYNTHESIS OF THE PIANO SOUND

Heidi-Maria Lehtonen



**Aalto University
School of Science
and Technology**

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Heidi-Maria Lehtonen

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<p>This thesis deals with the signal analysis, auditory perception, and physics-based synthesis of the piano sound. Contributions of this thesis can be grouped into four main categories: Analysis and modeling of the sustain pedal effect, analysis of harmonic and inharmonic musical tones by means of an inverse comb filter, loss filter design for waveguide piano synthesis, and perception of longitudinal vibrations in piano tones. The sustain pedal effect is studied through signal analysis of recorded tones, and the results show that the use of the sustain pedal increases the decay times of middle range tones, increases beating, and makes the sounds more reverberant. Based on the results, an algorithm is designed for simulating the sustain pedal effect. Objective and subjective studies show that the algorithm is capable of producing the main effects of the sustain pedal. The signal analysis of tones played with a partial sustain pedal reveals that the tone decay can be divided into three distinct time intervals, namely the initial decay, the damper-string interaction, and the final free vibration. Additionally, the nonlinear amplitude limitation during the damper-string interaction can excite missing modes in the lowest piano tones. Decomposition of harmonic and inharmonic musical instrument tones to tonal and noise components and selecting single partials with an inverse comb filter structure is discussed. The filters are designed based on the fundamental frequency and the inharmonicity coefficient, and they are found to provide a simple and efficient analysis tool for musical signals. A multi-stage ripple filter structure for modeling the complicated decay process of the piano tones is presented. The filter is capable of accurately matching a desired number of partial decay times or, alternatively, modeling the overall decay characteristics of a piano tone. Finally, the threshold of audibility is sought for perception of longitudinal components in fortissimo piano tones through formal listening tests. The results suggest that the longitudinal components are audible up to note C_5 (fundamental frequency 523 Hz), but based on the listeners' opinions modeling the longitudinal components in a piano synthesizer up to note A_3 (fundamental frequency 220 Hz) only is sufficient.</p>			
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<p>Tiivistelmä</p> <p>Tämä väitöskirja käsittelee pianon signaaliäänen analyysia, havaitsemista ja fysiikkaan perustuvaa mallinnusta. Työssä on neljä keskeistä osa-aluetta: kaikupedaalin vaikutusten tutkiminen ja mallintaminen, harmonisten ja epäharmonisten soitinäänten analyysi käänteiskampasuotimella, häviösuotimen kehittäminen ja pitkittäismoodien havaitseminen pianon äänessä. Kaikupedaalin vaikutusta pianon ääneen on tutkittu analysoimalla äänitettyjä yksittäisiä ääniä, ja tulokset osoittavat, että kaikupedaalia käytettäessä keskialueen äänten vaimenemisajat ovat pidempiä verrattuna tilanteeseen, jossa kaikupedaalia ei käytetä. Lisäksi pedaalia käytettäessä huojunta ja tilantuntu lisääntyvät. Kaikupedaalin simuloimista varten on kehitetty algoritmi, ja sekä objektiivisten että subjektiivisten tulosten perusteella voidaan sanoa, että algoritmin avulla voidaan mallintaa kaikupedaalin tärkeimmät piirteet. Osittaista kaikupedaalia on myös tutkittu analysoimalla äänitettyjä ääniä. Tulosten perusteella on todettu, että osittaista kaikupedaalia käytettäessä äänen vaimenemisprosessi voidaan jakaa kolmeen vaiheeseen, jotka ovat alkuvaiheen vaimeneminen, kielen ja vaimentimen interaktio ja vapaa vaimeneminen. Lisäksi epälineaarinen värähtelyamplitudin rajoittaminen saattaa herättää puuttuvia osäänneksiä pianon matalimmissa äänissä. Työssä käsitellään myös harmonisten ja epäharmonisten soitinäänten tonaalisen osan ja jäännösignaalin erottamista ja yksittäisten osäännesten erottelua käänteiskampasuodinrakenteella. Suotimet voidaan suunnitella analysoitavan äänen perustajuuden ja epäharmonisuuskertoimen perusteella. Pianon äänen monimutkaisen vaimenemisen mallintamiseksi on kehitetty moniväresuodin, jonka avulla pystytään tarkasti mallintamaan haluttu määrä osäännesten vaimenemisaikoja. Vaihtoehtoisesti voidaan mallintaa vain vaimenemisen yleistrendi, jolloin suotimen asteluku voidaan valita pienemmäksi. Lopuksi esitetään kuulokynnys pitkittäismoodien havaitsemiselle voimakkaissa pianon äänissä. Kuuntelukokeiden tulos osoittaa, että pitkittäismoodit havaitaan bassoalueella aina nuottiin C_5 (perustajuus 523 Hz) asti, mutta koehenkilöiden mielipiteiden mukaan riittää, että pitkittäismoodit mallinnetaan pianosyntetisaattorissa nuottiin A_3 (perustajuus 220 Hz) asti.</p>			
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Preface

During the past five years the doctoral thesis and everything that comes with it have been my dream. Now that my dream came true I feel this is just the beginning. At this very important moment there are so many things to say.

First of all, I want to express my deepest gratitude to Prof. Vesa Välimäki, who has truly been the best supervisor I could imagine. He has been inspiring, supportive and patient, which has meant a lot to me. I wish to thank my co-authors Prof. Anders Askenfelt, Dr. Balázs Bank, Dr. Timo I. Laakso, Dr. Henri Penttinen, and Dr. Jukka Rauhala who have all contributed significantly to my thesis. I would like to thank my pre-examiners Dr. Anssi Klapuri and Prof. Davide Rocchesso, who both gave valuable and encouraging comments. I am grateful to Luis Costa for carefully proofreading my manuscripts.

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Otaniemi, October 22, 2010

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Contents

Preface	7
Contents	9
List of Publications	11
Author’s contribution	13
List of Abbreviations	15
List of Symbols	17
1 Introduction	19
2 Acoustics of the piano	21
2.1 Keyboard and action	21
2.2 Strings	24
2.3 Soundboard and frame	26
2.4 Pedals	27
3 Analysis of musical instrument tones	31
3.1 Estimation of the fundamental frequency and the inharmonicity coefficient	31
3.2 Separation of tonal and broadband components	33
3.3 Decay time analysis	35
4 Physically informed synthesis of the piano sound	37
4.1 Overview of physics-based synthesis methods	37
4.2 Digital waveguide modeling	39
4.3 Modeling of losses	44
4.4 Modeling the sustain pedal	47

5	Perception of the piano sound	49
6	Main results of the thesis	53
6.1	Publication PI: Analysis and modeling of piano sustain-pedal effects	53
6.2	Publication PII: Analysis of the part-pedaling effect in the piano . .	56
6.3	Publication PIII: Canceling and selecting partials from musical tones using fractional-delay filters	58
6.4	Publication PIV: Analysis of piano tones using an inharmonic inverse comb filter	61
6.5	Publication PV: Sparse multi-stage loss filter design for waveguide piano synthesis	62
6.6	Publication PVI: Perception of longitudinal components in piano string vibrations	65
7	Conclusion and future directions	68

List of Publications

This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.

- I** H.-M. Lehtonen, H. Penttinen, J. Rauhala, and V. Välimäki, “Analysis and modeling of piano sustain-pedal effects,” *J. Acoust. Soc. Am.*, vol. 122, no. 3, pp. 1787-1797, September 2007.
- II** H.-M. Lehtonen, A. Askenfelt, and V. Välimäki, “Analysis of the part-pedaling effect in the piano,” *J. Acoust. Soc. Am. Express Lett.*, vol. 126, no. 2, pp. EL49-EL54, August 2009.
- III** H.-M. Lehtonen, V. Välimäki, and T. I. Laakso, “Canceling and selecting partials from musical tones using fractional-delay filters,” *Comput. Music J.*, vol. 32, no. 2, pp. 43-56, Summer 2008.
- IV** H.-M. Lehtonen, “Analysis of piano tones using an inharmonic inverse comb filter,” In *Proc. Int. Conf. on Digital Audio Effects*, pp. 333-340, Espoo, Finland, September 1-4, 2008.
- V** H.-M. Lehtonen, J. Rauhala, and V. Välimäki, “Sparse multi-stage loss filter design for waveguide piano synthesis,” In *Proc. IEEE Workshop on Applications of Signal Processing to Audio and Acoustics*, pp. 331-334, New Paltz, NY, USA, October 16-19, 2005.
- VI** B. Bank and H.-M. Lehtonen, “Perception of longitudinal components in piano string vibrations,” *J. Acoust. Soc. Am. Express Lett.*, vol. 128, no. 3, pp. EL117-EL123, September 2010.

Author's contribution

Publication I: “Analysis and modeling of piano sustain-pedal effects”

The author of this doctoral thesis designed the recordings in collaboration with the second author, carried out the recordings, analyzed the data and presented the results, developed the algorithm in collaboration with the co-authors, conducted the listening test, and wrote the article excluding Secs. IV.A.2 and IV.A.3.

Publication II: “Analysis of the part-pedaling effect in the piano”

The present author participated in the recordings that were carried out in Stockholm, Sweden, using the recording setup designed by the second author, and designed and carried out the analysis procedure in collaboration with the co-authors, presented the results, and wrote the article.

Publication III: “Canceling and selecting partials from musical tones using fractional-delay filters”

The original idea for the article was suggested by the second author of the paper. The present author designed all the parameter values for the inverse comb filter (ICF) structure and designed the method for setting and refining the pole radius and frequency of the resonator in order to accurately cancel out one of the zeros of the ICF. She also compared the proposed method to other methods and coded all the case studies. The sections which describe the theory of the fractional delay ICFs and the harmonic extraction filter were written in collaboration with the co-authors and especially the derivations of equations are based on the work of the third author. The section describing the case studies was written by the present author.

Publication IV: “Analysis of piano tones using an inharmonic inverse comb filter”

The author is solely responsible for designing the inharmonic inverse comb filter (IICF) structure and the FIR filter, making the comparisons to the reference IIR filter, coding all the examples, and writing the article.

Publication V: “Sparse multi-stage loss filter desing for waveguide piano synthesis”

The present author designed the filter structure in collaboration with the co-authors, designed all the parameter values, made the comparisons to the reference filters, and coded all the examples excluding the reference filter described in Ref. [5] of the article, which was done by the second author. The present author was responsible for writing the article.

Publication VI: “Perception of longitudinal components in piano string vibrations”

The present author designed the listening test in collaboration with the first author, coded the test program, selected the subjects, and carried out the listening test. In addition, she produced the tables and the figure that present the results of the listening test and wrote Section 3 in the article.

List of Abbreviations

DWG	digital waveguide
FIR	finite impulse response
HEF	harmonic extraction filter
ICF	inverse comb filter
IFIR	interpolated finite impulse response
IICF	inharmonic inverse comb filter
IIR	infinite impulse response
LTI	linear and time-invariant
STFT	short-time Fourier transform

List of Symbols

B	inharmonic coefficient value
c	speed of sound
d	diameter of the string
g_k	filter gain at partial k
f_0	nominal fundamental frequency
f_k	frequency of partial k
f_s	sampling frequency
$H(z)$	transfer function of an inharmonic comb filter
$H_{fd}(z)$	transfer function of a fractional delay filter
k	partial index
l	length of the string
L	delay line length
Q	Young's modulus
t	time variable
T	tension of the string
T_{60}	decay time in seconds
τ_k	decay time constant of partial k
x	coordinate along the string
y	displacement of the string

1 Introduction

The piano is one of the most popular instruments in Western music. It has a complex structure, which has evolved from its predecessor, the harpsichord, at the end of the 18th century (Fletcher and Rossing, 1991). Actually, the piano has two forms, the upright and the grand piano, the first being popular especially among common households, while the latter mainly in professional use, for example in concert halls. This work deals mainly with the grand piano, but the methods and results are, in general, applicable to the upright piano.

During the past decades digital pianos have become popular, since they are usually affordable, easily movable, and they do not produce structural noise, which is convenient especially in apartment houses. Traditional digital pianos use the sampling technique, which provides excellent sound quality since recorded, real piano tones are played back from the memory every time the key is pressed down. Another synthesis approach that can be applied in digital instruments is physically informed sound synthesis, or physics-based sound synthesis. In this approach, the sound is computed from scratch with algorithms that are designed based on the physics of the instrument. This offers great flexibility in sound synthesis, since the parameters and algorithms can be modified, depending on how the instrument is played. The algorithms can also offer scalability, which takes the properties of human hearing into account; a trade-off can be made between computational complexity and sound quality.

In practice, physics-based sound synthesis covers two main synthesis approaches. The first one is “true” physical modeling, in which the physical principles of the instrument are applied very strictly, while the other approach takes advantage of various signal processing tricks in order to simplify the algorithms and to ease the computational load. These two techniques are often mixed, however, when designing

a physics-based synthesizer. In this thesis, the approach taken is based mainly on signal processing, while the acoustics of the instrument still give the basis for the process.

Developing a physics-based piano synthesizer has several important steps. In order to be able to create an algorithm that follows the sound production mechanism of an instrument one has to study its acoustics and working principles. Parameters for the algorithms can be extracted from recorded tones with various signal analysis tools. In order to design efficient algorithms that model the most essential features of the sound from the human hearing point of view, listening tests can be conducted to study how (accurately) humans perceive the piano sound. Synthesis algorithm design forms, of course, the most important part of the development process.

This thesis presents new results that can be used to develop a physics-based piano synthesizer. The thesis consists of the introductory part and six articles that have been published either in international, peer-reviewed journals or scientific conferences. The introductory part starts with an overview of the acoustics of the piano in Section 2. Special emphasis is put on the sustain pedal, which is the topic of articles [PI] and [PII]. Next, in Section 3, the signal analysis of musical instrument tones is dealt with. Especially the separation of the harmonic and broadband components is discussed, which is also the topic of articles [PIII] and [PIV]. In addition, the estimation process of important parameters for the synthesis algorithm design is presented. Section 4 gives an overview of the physics-based synthesis of the piano sound with special attention on the loss filter design, which is the subject of article [PV], and the simulation of the sustain pedal, which is discussed in article [PI]. In Section 5, the perception of various features of the piano sound, such as the longitudinal components of vibration, the subject of article [PVI], is discussed. Finally, the main results of the articles [PI]-[PVI] are presented in Section 6 and conclusions are drawn in Section 7.

2 Acoustics of the piano

This section gives a brief overview of the acoustics of the piano, a topic that has been widely researched for several decades. Excellent studies covering the topic include the book edited by Askenfelt (1990), the article by Suzuki and Nakamura (1990), and the article series by Conklin (1996a,b,c), which approach the subject especially from the instrument designer point of view. Traditionally the structure of the piano has been divided into five functional parts: the keyboard, the action, the strings, the soundboard, and the frame (Fletcher and Rossing, 1991). In addition to this, the pedals form yet another important part in the acoustics of the piano.

Figure 2.1 gives a simplified schematic view of the structure of the piano. When a key is pressed down the action sets the hammer into motion toward the string and the corresponding damper is lifted. When the hammer hits the string, the kinetic energy is stored in the normal modes of the string until it is transferred to the soundboard via the bridge. The speaking length of the string is measured from the pin block to the bridge, while the final termination of the string is in the hitch pin. The dampers are controlled by the key and the sustain pedal through the damper mechanism. The sustain pedal rod nut can be used to adjust the distance between the dampers and the strings.

2.1 Keyboard and action

The keyboard and pedals form the user interface of the piano. The compass of the instrument is wide: The keyboard consists of 88 keys, and the fundamental frequency of the lowest tone is approximately 27.5 Hz, while that of the highest tone is about 4186 Hz. The action is a complicated system, which has gone through several stages of development during the history of the instrument. The mechanism of the

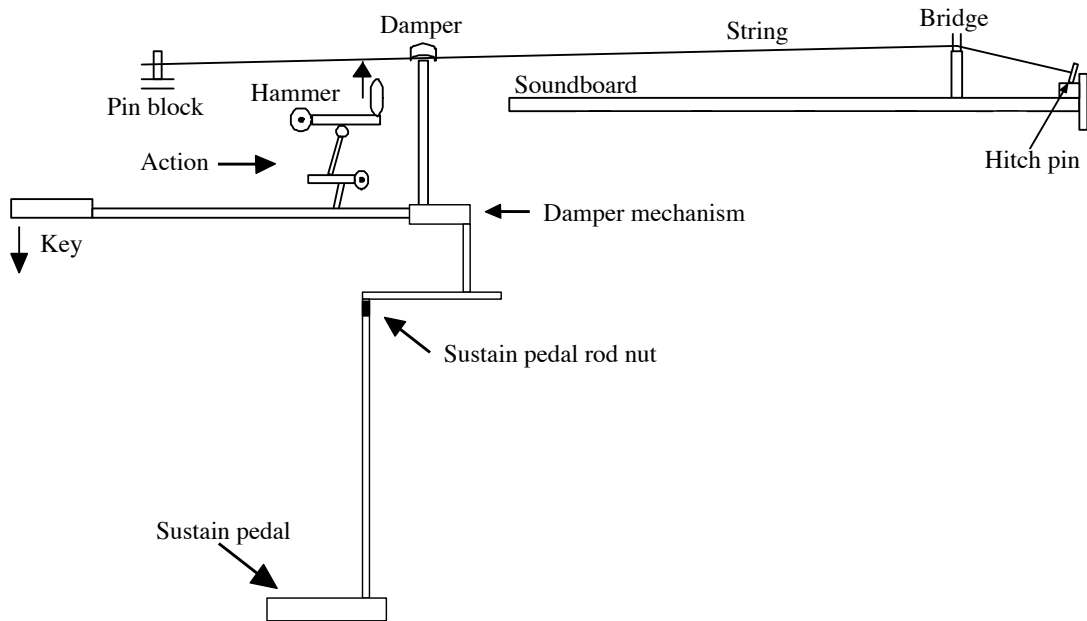


Figure 2.1: Simplified schematic view of the grand piano. When the key is pressed down the hammer starts its journey toward the string. At the same time, the damper is lifted up to allow the string to vibrate freely. The vibrational energy of the string is transmitted to the soundboard via the bridge.

action, and especially the nonlinear behavior of the hammer, have been studied both theoretically and experimentally. Hall (1986) provided a thorough mathematical theory for the interaction between a very light hard hammer and the string. Later he expanded this work to a general solution for a point hammer of any mass that hits a perfectly flexible string (Hall, 1987a) and to the case where the hammer is expected to be a soft point hammer with infinite mass (Hall, 1987b). This somewhat simplified theory, mostly because of the assumption of the linear hammer-string system, was finally studied by Hall and Askenfelt (1988) through measurements of real piano hammer and strings to test how well the theory and practice are in agreement. They concluded that the nonlinear hammer compliance should be included in the theory, which was further developed by Hall (1992).

Askenfelt and Jansson (1990) studied the timing in grand piano action through experiments by mapping the motions of different moving parts of the action onto a timetable. They found, for example, that the timing in the action depends both on regulation¹ and on the dynamic level. In their following work, Askenfelt and Jansson (1991) studied the motion of the key and the hammer. One of the interesting questions was the role of “touch”, which traditionally has divided the opinions of pianists and physicists. The results of the article show, for example, that the hammer motion differs when the key is pressed in *legato* or *staccato* style. Later, Goebel et al. (2005) also reported that the travel times of the hammer are dependent on the type of touch.

Askenfelt and Jansson (1993) presented a work dealing with the wave motion of piano strings and the corresponding spectra in different regions of the piano compass. The results show, among other things, that the hammer mass and compliance have an effect on the spectra, while the effect of voicing is smaller, and that the efficiency of dampers is lower in the low range of the piano compared to the high. The effect of the hammer position on the string mode excitation was presented by Hall and Clark (1987). They concluded that when the hammer hits within a few millimeters of the nodal position of a certain mode, this mode will be weaker or even missing from the spectrum.

In addition to the theoretical and experimental approach, the action mechanism can be studied through a model-based analysis procedure. A model-based analysis of the nonlinear hammer-string interaction was presented by Suzuki (1987) and Boutillon (1988). Stulov (1995) studied the hysteretic behavior of the hammer felt through a mathematical model and Hayashi et al. (1999) presented a simple dynamic mechanical model that was used to analyze the hammer motion before its interaction with the string.

¹Regulation in the context of the piano means careful adjustment of different parts of the action in order to achieve smooth and even-tempered sensation throughout the whole keyboard while playing the instrument.

2.2 Strings

The properties and organization of strings define many features of the piano sound. The string register, that is, the complete set of strings, includes nearly 250 strings that are organized in groups of one to three strings, one string group corresponding to one key. The lowest tones have only one wound string, while in the middle and high ranges the string groups have two and three strings, respectively. The highest strings are solid wire. The strings are stiff, which causes dispersion and makes the tones inharmonic. Inharmonicity means that the partials² of the piano tone are not integral multiples of the fundamental frequency, but are stretched slightly upwards in frequency. In addition to string stiffness, Ortiz-Berenguer et al. (2006) suggested that the soundboard impedance also contributes to the inharmonicity.

Several works dealing with the dispersion phenomenon in the piano have been published, the oldest probably being the work by Schuck and Young (1943) and Young (1952). Fletcher and his colleagues were the first to derive the formula which defines the relation between the partial frequencies f_k and the inharmonicity coefficient B (Fletcher et al., 1962):

$$f_k = kf_0\sqrt{1 + Bk^2}, \quad (2.1)$$

where f_0 is the nominal fundamental frequency of an ideal string (and does not exist in the spectrum), k is the index of the partial, and the B coefficient can be written as

$$B = \frac{\pi^3 Q d^4}{64 l^2 T}, \quad (2.2)$$

²In this context, the harmonics of the piano tones are called partials, since the overtones are not in harmonic relation to the fundamental frequency. Harmonics, in contrast, refer to overtones of such musical instrument tones that do not express inharmonicity.

with Q being Young's modulus and d , l , and T the diameter, length, and the tension of the string, respectively. Inharmonicity needs to be taken into account also in the tuning process by stretching the frequency ratios of the lowest and highest octaves to be more than a 2:1 frequency ratio (Martin and Ward, 1961; Fletcher and Rossing, 1991). The effect of inharmonicity on the tuning of the piano has been studied, for example, by Lattard (1993), who also developed a mathematical simulation of tuning. Recently, the effect of inharmonicity in piano tuning raised some interest in the scientific community when Bryner (2009) published a letter that the celebrated theoretical physicist Richard Feynman wrote to his piano tuner in 1961. In his letter, Feynman discusses whether it is better to tune the piano by ear or by absolute frequencies.

Another important feature is the complicated decay process of the piano tone. The decay times of bass and treble tones vary significantly (Martin, 1947). Moreover, the partials of a piano tone decay at different rates, depending mainly on how rapidly the vibrational energy leaks from the string group to the soundboard at different frequencies (Fletcher et al., 1962). In addition, the tone decay has two phases, the rapid initial decay and the longer final decay. This comes from the fact that the predominant direction of vibration changes from perpendicular to parallel due to the characteristics of the impedance mismatch between the strings and the soundboard and that the strings within one string group are coupled (Weinreich, 1977). In addition, it has been shown that beating can occur in sounds produced by single strings because of the anisotropy in the bridge admittance (Capleton, 2003). Cartling (2005) showed that the piano tone can be affected by weak coexcitation of adjacent tones through the beat frequency and amplitude modulation.

In addition to the transverse vibration, the strings vibrate also in the longitudinal direction. Giordano and Korty (1996) suggested that the longitudinal modes are generated by the stretching of the string during its transverse vibration in a nonlinear fashion. Conklin (1996c) reported that the frequencies of the longitudinal modes

in relation to those of the transversal ones affect the quality of piano tones, for example, by causing confusion in pitch or undesirable changes in tone color. Nakamura and Naganuma (1993) found another series of modes in piano sound spectra, which had a lower inharmonicity compared to the series of transverse modes. This series of modes was consequently named “phantom partials” by Conklin (1997), who later explained that the phantom partials are generated due to nonlinear mixing (Conklin, 1999): an “even phantom” occurs at the frequency that equals double the frequency of a transverse mode, while an “odd phantom” appears at the sum or difference frequencies of two (usually adjacent) transverse modes. Bank and Sujbert (2003, 2005) were the first to explain the mathematical theory behind the origin of the longitudinal string vibrations. They also suggested that the longitudinal modes could be dealt with together, since fundamentally they represent the same phenomenon: longitudinal modes are the free vibration of the string and the phantom partials arise from the forced vibration caused by the transversal displacement of the string.

2.3 Soundboard and frame

The frame of a piano is usually made of cast iron, and its task is to keep the instrument together and withstand the high tension of the strings but, since it is not a good radiator, string energy dissipation within should be prevented. The wooden soundboard, in turn, is the main radiating part of the instrument, and its shape and properties largely determines the directivity and radiation patterns of the piano sound. The frequency-dependent impedance of the bridge and the soundboard is the property that defines how the vibrational energy of the strings is transferred to the soundboard. The ribs, which are glued to the soundboard at right angles to the grain of the wood, are used to add stiffness in the cross-grain direction. Usually the soundboard properties, such as the modal frequencies, are studied experimentally rather than theoretically, since the structure and the shape of the soundboard are complex. During the past decade, however, efficient computational methods,

such as finite-element modeling, have provided a tool to analyze the soundboards numerically. This approach is found in the work of Berthaut et al. (2003).

Probably the first reported soundboard measurements were conducted by Billhuber and Johnson (1940). They reported, among other things, that the properties of the soundboard have an impact on the duration of tones, which is a major factor in the “liveliness” of the instrument. Suzuki (1986) measured the behavior of a grand piano with modal analysis and surface-intensity methods and concluded that the radiation efficiency of the soundboard is very low below 80 Hz, while the sound radiates more efficiently in the frequency range 100 Hz – 1 kHz and most efficiently above 1.4 kHz. Giordano (1998a) studied the mechanical impedance of the soundboard of an upright piano and reported that the ribs contribute to the soundboard stiffness significantly and that the impedance depends strongly on frequency through the spacing of the ribs. In a following work, Giordano (1998b) studied the sound generation of a piano soundboard by measuring the relation of the sound pressure and the soundboard velocity by applying a force at several locations on the bridge. The results indicate that the relation is largest around 1 kHz, while it decreases below 100 Hz and above 5 kHz. Moore and Zietlow (2006) applied the electronic speckle pattern interferometer to study the deflection shapes of an upright piano soundboard. They concluded that the mode shapes and resonant frequencies are highly dependent on the pressure exerted by the strings.

2.4 Pedals

The pedals are probably the least studied topic in the acoustics of the piano. Even so, from the piano performance point of view, they are one of the most important parts of the instrument. The pedals provide another way for the pianist to control the instrument in addition to the keyboard, since the pedals control the dampers. Especially the sustain pedal (also called the resonance pedal, damper pedal, or

“forte” pedal), which is the rightmost and without a doubt the most important pedal, is an essential part of most piano performances. When pressed down, it causes the dampers to lift off the strings allowing them to vibrate freely.

The organization of other pedals depends on the instrument. In grand pianos the left pedal is usually the *una corda* pedal, which slightly shifts the keyboard and the action rightward so that the hammer hits two of the three strings within the string group. In upright pianos the *una corda* pedal is realized by moving the hammers a bit closer to the strings when the pedal is depressed. Weinreich (1977) showed that the usage of the *una corda* pedal reduces the decay rates of piano tones and, thus, affects the timbre of the tones rather than makes the sound of the instrument more quiet. The third pedal, if present, is usually the *sostenuto* pedal, which sustains only those notes that are played before pressing the pedal down. In some upright pianos the middle pedal is the practice pedal, which when depressed sets a piece of felt between the hammers and the strings when the left pedal is used and thus makes the sound of the instrument more quiet.

While the acoustics of the sustain pedal is less studied, pianists’ ways to use the sustain pedal has been addressed in the literature, especially in the field of psychology (Heinlein, 1929, 1930; Taguti et al., 1993; Repp, 1996, 1997). Indeed, this is an interesting topic, since the usage of the sustain pedal in piano playing requires temporal coordination of hands and feet, and it is considered a complex, perceptually and often subconsciously guided motor behavior.

A few studies dealing with the acoustical effects of the sustain pedal exist, though. Brauss (2006) showed through recordings that different pedal motions result in different sound decay patterns. He reported, for example, that the sustain pedal increases beating and that the decay time of the tone is increased when the sustain pedal is used compared to the case where the tone is played without the sustain pedal. These phenomena are due to the fact that the energy of the vibrating string

group corresponding to the struck key excites modes from other freely-vibrating strings. The analysis is, however, performed only for one tone, which implies that the results cannot necessarily be generalized to apply for all tones. Schutz et al. (2008) analyzed the effect of the sustain pedal in order to use this information in a detection system. In particular, they were interested in the increase of partial decay times and the level of the noise floor, which appears when the sustain pedal is used, as detection features. Article [PI] presents the main effects of the sustain pedal on the piano tone through signal analysis performed on several piano tones. In addition, the physical explanations behind the effects are discussed in more detail.

Sustain pedaling is rarely a simple on-off-process. Different pedal applications include the accentuation pedal, which means fast pedaling together with key stroke followed by either fast or slow release of the pedal, pulsation pedal in which the pedal position follows wave-like patterns, and the vibrato pedal with fast pedal oscillation. These and other pedaling techniques are described in more detail by Brauss (2006), Banowetz (1985), and Parncutt and Troup (2002). The partial sustain pedal, which is addressed in [PII], is an often used pedaling technique in which the pedal is not pressed down completely but left somewhere between the two extremes. Brauss (2006) studied the effect of the pedal depression depth on the decay times of the tones and found that the range in which the pedal depression depth has an effect on the tone is relatively small, only about 30% – 50% of the total range, 0% being the no-depression condition and 100% being the full-depression condition. This suggests that the mapping from the pedal movement to the damper movement is nonlinear. From the synthesizer design view-point the interesting part is the interaction between the dampers and the strings. When the dampers touch the strings only partially, the amplitude of the string vibration is limited and the string register is not totally free to vibrate. In this case, the decay pattern of the tone can be divided into three distinct intervals: the initial free vibration, the damper-string interaction, and the final free vibration. With low tones, the nonlinear amplitude limitation affects the string motion and may excite missing partials. These effects

of the partial sustain pedal are reported and discussed in more detail in article [PII].

3 Analysis of musical instrument tones

This section describes the process of analyzing musical instrument sounds using digital signal processing algorithms. Special emphasis is placed on stringed instrument sounds, especially the piano. In general, signal analysis is a wide topic that covers many applications in audio. In this thesis, the concept signal analysis refers to the process where the goal is to estimate parameters, such as the fundamental frequency f_0 , the dispersion coefficient B , and the partial decay times T_{60} (the time that it takes for a partial to decay 60 dB), which can be used in synthesis algorithm and filter design processes. In addition to parameters, other kind of information, such as the partial beating characteristics, soundboard frequency-response properties, or contents of the spectrum in different playing situations may be of interest.

3.1 Estimation of the fundamental frequency and the inharmonicity coefficient

Usually the first step in process of analyzing musical instrument sounds is the estimation of the fundamental frequency³. Traditionally, autocorrelation-based methods have been used widely, especially in speech processing (Rabiner, 1977). This method performs well for static harmonic musical instrument tones, but fast changes and a very large frequency range can cause problems in the estimation. de Cheveigné and Kawahara (2002) presented an autocorrelation-based technique, the YIN method, which includes several modifications and improvements of the traditional autocorrelation method.

³The fundamental frequency refers to a physical, measured quantity. It is often called misleadingly the pitch, which refers to a subjective attribute, the perceived fundamental frequency. Especially in the case of the piano, this mixing in terminology is somewhat risky, since the inharmonicity affects also the perceived pitch even if the fundamental frequency is kept constant (Järveläinen et al., 2000, 2002; Anderson and Strong, 2005).

In the case of the piano, however, the estimation of the fundamental frequency is more complicated due to the dispersion phenomenon. The partial frequencies depend on the inharmonicity coefficient B according to Eq. (2.1). Since there are two unknown parameters in Eq. (2.1), these parameters are often estimated jointly. Galembo and Askenfelt (1999) presented an estimation method based on the inharmonic comb filter. The idea is to construct the filter by defining a set of nonequidistant filter bands in the frequency domain and to compute and sum up the energy of the spectrum of a piano tone in each band. Starting with rough initial f_0 and B parameter values, the comb filter output is computed and the parameters are refined iteratively in a few iteration steps. The parameter values that give the largest output are chosen to be the estimates.

Another estimation method, developed by Askenfelt and Galembo (2000), is based on the computation of the cepstrum and the harmonic product spectrum, which are common algorithms for estimating the fundamental frequency. Basically, the idea is to present the signal in a different domain and to graphically visualize the degree of inharmonicity. In addition, formulas for computing the B coefficient are given based on the characteristic parameters of the cepstrum and the harmonic product spectrum. The estimation algorithms presented by Galembo and Askenfelt (1999) and Askenfelt and Galembo (2000) are capable of producing accurate estimates, but the drawback is the computational complexity of the algorithms.

Rauhala et al. (2007b) proposed a method called the partial frequency deviation method in which the difference of the expected spectral components compared to the amplitude peaks in the spectrum is minimized. The initial guess for the B and f_0 values are refined based on the deviation trend. The process evolves iteratively until the stop conditions are satisfied. In comparison to the inharmonic comb filter method, the partial frequency deviation method proved to give better results at a smaller computational cost.

3.2 Separation of tonal and broadband components

A basic operation in the analysis of musical instrument tones is the division of the signal into tonal and broadband components. The tonal component includes the fundamental frequency and partial components and the broadband component (sometimes called the noise component, the stochastic component, or the residual component) is the remaining part of sound. For piano tones, this kind of decomposition is useful, for example, when the estimation of the partial decay times, the beating patterns, or the properties of the soundboard with various signal analysis methods are of interest.

One of the most powerful tools in musical analysis is the sinusoidal modeling technique (McAulay and Quatieri, 1986; Serra, 1989; Serra and Smith, 1990), which can be used also for separation of single partials. The analysis starts with the computation of the windowed short-time Fourier transform (STFT). Then, frequency, amplitude, and phase trajectories are formed based on the STFT computation by connecting the data points in adjacent analysis frames. As a result, the partials and, thus, the tone without the noise component can be reconstructed based on the trajectories. Sinusoidal modeling is especially useful in the analysis of very complex, time-varying signals. On the other hand, implementing the algorithm requires many choices in parameters, such as the length and type of the window used in the STFT computation, which may affect, for example, spectral or temporal smearing in the synthesis stage. Ono et al. (2008) have recently presented a method for real-time separation of harmonic and percussive components from popular music songs. The idea is to utilize the fact that these two kinds of components have different structures in the spectrogram, since harmonic components are essentially horizontal, while percussive components are broadband and thus vertical. The advantage of the method is that it requires no *a priori* knowledge of the signal. The technique is useful for example in the preprocessing stage of various music information retrieval related tasks, and it would be interesting to test how this technique performs with single musical instru-

ment tones when the target is to extract parameters for synthesis. Alternatively, the harmonic content of musical signals can be analyzed with wavelets (Guillemain and Kronland-Martinet, 1996; Evangelista, 1993; Wang and Tan, 2008), frequency-zooming ARMA modeling (Karjalainen et al., 2002; Esquef et al., 2003), or other high-resolution tracking methods (David et al., 2006; Badeau et al., 2006, 2008). For the analysis of isolated harmonic musical instrument tones, however, these kind of generic algorithms may be overly thorough and heavy to implement.

Keane (2007) suggested a method for separating the piano tones into tonal and broadband components by removing the partial peaks from the spectrum. Basically, the idea is to first compute the spectrum of the tone and create an approximate stochastic component spectrum that is used to replace the partial peaks in the original spectrum. This is done by combining the two points around each peak using linear interpolation. Finally, the tonal component can be separated from the piano tone by subtracting the broadband component from the original tone. Although this method proved to give good results, the author points out that it is applicable only for tones in the middle range of the piano. A similar approach was taken by Lee et al. (2007) in the excitation extraction for sound synthesis of the guitar. The authors obtained the residual component of a recorded guitar tone by removing the spectral peaks and reconstructing the missing gaps using statistical interpolation.

The tonal and broadband components can be separated also by using digital filters. The advantage of such filter-based methods is often their simplicity, especially if the filter can be designed in closed-form with a small number of parameters. Välimäki et al. (2004a) used this approach in order to decompose harmonic musical instrument tones into tonal and broadband components. Their idea is to replace the delay line in an inverse comb filter (ICF) with a large allpass filter in order to accurately implement the fractional delay, which is generally needed in order to match the notch locations to corresponding partial locations in the tone. The allpass filter is either the Thiran filter (Thiran, 1971) or the truncated Thiran filter (Välimäki,

2000). Single harmonics can be separated by cascading a resonator with the ICF. The pole of the resonator is used to cancel one of the zeros of the ICF in order to retain the desired partial when the signal is processed. Article [PIII] augments this idea of decomposing the tonal and broadband components of harmonic tones using an ICF with a fractional delay filter. Various fractional delay filter design methods are tested with the ICF, and especially the harmonic extraction procedure is improved so that the pole radius and frequency of the resonator can be set more accurately compared to the previous work (Välimäki et al., 2004a). Article [PIV] extends this idea further to the analysis of inharmonic piano tones by replacing the fractional delay filter with a filter that has a frequency-dependent phase delay.

3.3 Decay time analysis

After separating the tonal and broadband components from isolated musical instrument tones, the next step is usually to analyze the temporal evolution of the partial components. For the work done in this thesis, the most important parameters are the T_{60} times for each partial. The estimated decay times can be converted to target loss filter gain values. The loss filter design process is discussed in Sec. 4.3.

Various methods for estimating the partial decay times have been presented. Laroche and Meillier (1994) suggested a method in which a 3D power spectrum of the signal is first obtained by accumulating the power spectra that are computed at different time instants. Finally the damping factors, which can be converted to decay times, are estimated from the slope of the 3D power spectrum after detecting the trajectories of sinusoidal components and their frequencies using peak picking. The idea of this method is based on the articles of Schroeder (1965) and Jot (1992). The work of Schroeder (1965) deals with estimating the reverberation in concert halls by smoothing the decay curves using a backward integration method. Later, Jot (1992) generalized this work to a time-frequency representation. This approach was

also taken by Välimäki and Tolonen (1998): Once the smoothed amplitude envelope curves are obtained, the decay times can be computed based on the slope of the straight line that can be fitted to amplitude envelope of each partial on a logarithmic dB scale.

Välimäki et al. (1996) suggested that the damping factors can be measured from musical instrument tones by first computing the STFT of the signal and then tracking the amplitude and frequency of each partial. The partial locations are found by searching first the local minima around the assumed maximum. The largest amplitude value between the minima is expected to be the partial peak. The trajectories of partial amplitude and frequency pairs form the envelope curve to which a straight line can be fitted. This analysis approach is also taken in the work of Erkut et al. (2000).

In this thesis and especially in articles [PI] and [PII], the decay times are measured from the amplitude envelopes after extracting partials with the method described in articles [PIII] and [PIV]. A straight line is fitted to the amplitude envelopes in a logarithmic dB scale in a least-squares sense. The T_{60} time for each partial can be then computed from the slope of the fitted line.

4 Physically informed synthesis of the piano sound

The idea behind physical modeling of musical instruments is to build a set of computational rules that imitate the sound production mechanism of the instrument to produce authentic sound. In the case of the piano, this approach is challenging, since the piano is a very complex instrument. It has nearly 250 strings whose length and diameter, and thus acoustical properties, vary significantly throughout the compass of the instrument. The materials, size, and shape of the instrument, as well as the condition of the action affect the sound significantly. Usually the synthesis models are somehow simplified, for example in order to reduce the computational load. It is also common to facilitate the modeling process by using the physical properties of the instrument just as the basis of the synthesis process and improve computational efficiency with signal processing tricks. This approach is usually called physically informed or physics-based sound synthesis.

The first commercial, truly modeled pianos have come to the market only recently, although the idea of physics-based sound synthesis is much older. The first commercial physics-based piano synthesizer software, Pianoteq, was announced by Modartt (2006). Recently, Roland brought their V-Piano to the market, which is the first hardware digital piano without any samples (Roland, 2008). For the moment it seems that physics-based modeling has come to stay in the world of piano synthesizers.

4.1 Overview of physics-based synthesis methods

The physics-based discrete-time synthesis methods can be divided into six classes: finite-difference models, digital waveguides (DWG), mass-spring networks, modal decomposition methods, wave-digital filters, and source-filter models (Välimäki et al.,

2006). In the finite-difference modeling approach, partial differential equations are solved numerically by replacing the differentials with finite-difference approximations in order to discretize the time and position of a single point of a waveform (Hiller and Ruiz, 1971a,b). The theory of finite-difference models and their applications to sound synthesis are presented in the book by Bilbao (2009). This method has been used also in piano synthesis, for example by Chaigne and Askenfelt (1994a,b) and Giordano and Jiang (2004).

In physics-based piano synthesis, however, the most popular synthesis approach is DWG modeling (Smith, 1983; Jaffe and Smith, 1983; Smith, 1992). The first DWG piano model was presented by Garnett (1987). Other DWG piano models have been presented, for example, by Smith and Van Duyne (1995a,b), Van Duyne and Smith (1995), Borin et al. (1997), Aramaki et al. (2001), Bensa et al. (2002, 2003b), Bensa (2003), Bank (2000a, 2006), Bank et al. (2003), Rauhala (2007a), and Rauhala et al. (2008). The model presented by Borin et al. (1997) was remarkable, since it was completely physical and it was playable in real time with decent polyphony. Since DWG is the modeling approach chosen in this thesis, a more detailed overview of the technique is given in Sec. 4.2.

The mass-spring network approach aims at building up vibrating structures with masses, springs, and damping systems. The elements are finally discretized using finite differences. This method was first presented by Cadoz et al. (1983).

The idea of modal decomposition methods (or modal synthesis) is to simulate the modes of vibration with resonant filters (Adrien, 1991). Recently, Bank et al. (2010) proposed a real-time piano synthesizer based on the modal synthesis technique. Trautmann and Rabenstein have presented the functional transformation method (Trautmann and Rabenstein, 1999; Rabenstein and Trautmann, 2003) in which the impulse response of a vibrating system is computed using the Laplace and the Sturm-Liouville transforms. The resulting set of sinusoids are then modeled with resonators.

Wave-digital filters were originally developed for digitizing lumped analog electrical circuits (Fettweis, 1986). They can be applied also in the acoustical domain for synthesis purposes, like in the case of the wave-digital piano hammer model (Van Duyne and Smith, 1994; Pedersini et al., 1999; Bensa et al., 2003a). Source-filter models consist of a wide class of different techniques, a common feature being that all of them have an excitation and a subsystem that somehow processes the excitation (Roads, 1995). For example, some DWG models can be interpreted as source-filter systems.

4.2 Digital waveguide modeling

The origins of the DWG modeling are in the synthesis algorithm presented by Karplus and Strong (1983). This simple method was developed further toward more realistic sound synthesis of musical instruments by Smith (1983) and Jaffe and Smith (1983). The idea behind the DWG technique is to model the discretized version of the traveling-wave solution of the wave equation, which can be written as a superposition of two waves,

$$y(x, t) = f^+(ct - x) + f^-(ct + x), \quad (4.1)$$

where f^+ and f^- are the two waves traveling to the right and to the left, respectively, c is the sound velocity, t is time, x is the location on the string, and y is the displacement of the string.

After discretization, the system can be modeled with two delay lines of length $L/2$. In the case of an ideal string the terminations are rigid, corresponding to multiplication with the reflection coefficient -1 . In practice, however, simulation of losses of a real string needs to be included by distributing loss blocks equally along the

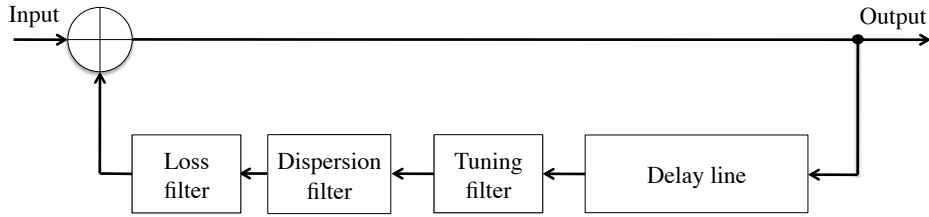


Figure 4.1: Block diagram of the DGW string model.

string. Since the system is linear and time invariant (LTI), the delay lines can be combined and the loss elements can be lumped to a single point. The total length of the delay line L is defined as follows:

$$L = \frac{f_s}{f_0}, \quad (4.2)$$

where f_s is the sampling frequency and f_0 is the desired nominal fundamental frequency. In addition, the terminations of the original string structure with two delay lines can be placed one after the other because of the LTI property. This results in the two reflecting coefficients canceling each other out. In the case of the piano, dispersion blocks can be treated in the same way as the losses. A more thorough description of DWG modeling is presented by Smith (1992, 2010) and Karjalainen et al. (1998).

In practice, the losses and dispersion are modeled with digital filters. In addition, a tuning filter is usually needed to accurately adjust the desired fundamental frequency. Since the delay line length must be an integer and it may happen that the result of Eq. (4.2) is not an integer, the fractional part of the division needs to be modeled separately. Usually this is done with a low-order fractional-delay filter (Jaffe and Smith, 1983; Laakso et al., 1996). Figure 4.1 illustrates the DWG string model.

The dispersion filter is usually an allpass filter whose goal is to simulate the frequency-

dependent propagation velocity of the wave in a stiff piano string with a proper phase delay. Several design approaches have been proposed. Paladin and Rocchesso (1992) presented a real-time implementation for modeling dispersive strings and air columns. The dispersion effect is realized with an allpass structure that is designed based on the group delay specification. Van Duyne and Smith (1994) simulated the dispersion phenomenon with cascaded equal first-order allpass sections and Rocchesso and Scalcon (1996) presented a filter which is designed based on the least-squares equation error criteria (Lang and Laakso, 1994). With this design method, the authors were able to match the phase response for the first several tens of partials. Other dispersion-filter design techniques based on optimization algorithms have been presented by Bensa et al. (2003b, 2005). Rauhala and Välimäki (2006a,c) presented a dispersion-filter design technique based on the Thiran allpass filter design (Thiran, 1971), which provides closed-form formulas for the filter coefficients. Abel and Smith (2006) presented a method in which the target group delay is divided into bands covering the area of 2π which are modeled with complex first-order allpass sections. Recently, Abel et al. (2010) proposed a closed-form design for this design method. In [PIV], a very high-order FIR dispersion filter is presented for offline analysis of piano tones.

Other important parts of a DWG piano synthesizer include the modeling of excitation, beating and two-stage decay, longitudinal modes, the soundboard, and the sustain pedal. A schematic structure of a DWG piano model including these features is presented in Fig. 4.2. This model follows the DWG piano model presented in Rauhala et al. (2008), and it includes the string model, the soundboard and the sustain pedal blocks, which are usually implemented as serial blocks, and the parallel and additional serial blocks which can consist of the simulation blocks for the beating and two-stage decay, and/or the blocks for modeling the longitudinal string vibrations, depending on the synthesis approach. The blocks can be LTI, time-varying, or nonlinear, and these properties should be taken into account when designing the other parts of the synthesizer.

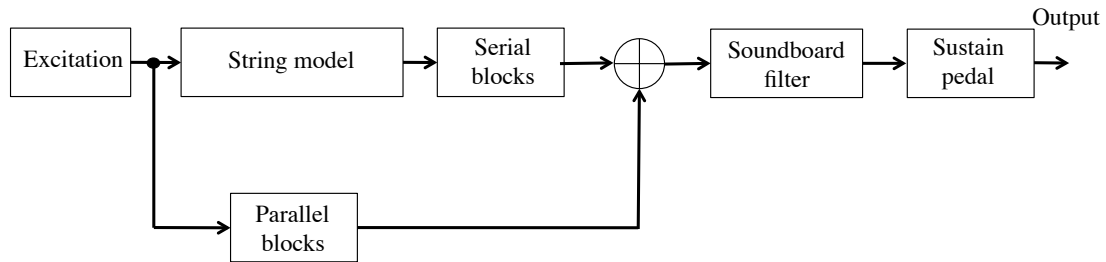


Figure 4.2: Possible (simplified) block diagram of the DGW piano model. Parallel and serial blocks can include modeling of the longitudinal modes and/or the beating effect.

Excitation modeling for DWG synthesis of the piano has been considered somewhat problematic because of the nonlinear behavior of the hammer. Usually, the hammer is considered as a mass that is connected to a nonlinear spring. This system can be described with a differential equation. One option for simulating the hammer is to discretize the differential equation of the mass (Chaigne and Askenfelt, 1994a). Borin and De Poli (1996) suggested that the solutions for the hammer equation can be computed in advance and stored in a lookup table in order to avoid stability problems. Van Duyne et al. (1994) presented a wave digital hammer, which is based on the traveling-wave decomposition of the mass-spring system. Smith and Van Duyne (1995a) proposed that the effect of the hammer can be linearized with a lowpass filter whose input is an impulse and output simulates the hammer-string force pulse. This idea was further developed by Van Duyne and Smith (1995). Bank (2000b) introduced a hammer model that operates at twice the sampling rate of the string model. This way stability, which is often problem in the hammer models, can be ensured. Bensa et al. (2004) suggested a hybrid model for hammer-string interaction, in which the excitation is generated with a subtractive signal model. Rauhala and Välimäki (2006b) presented a model in which the excitation signal is generated using additive synthesis and an equalizing filter that is controlled by the hammer velocity.

Models for beating and two-stage decay can be implemented with two waveguide models simulating the horizontal and vertical polarizations that are connected to each other through two coupling filters, as suggested by Daudet et al. (1999). This work was later expanded to the case of two strings by Aramaki et al. (2001). The authors developed a mathematical theory which is finally realized with digital coupling filters having parameters that can be determined from the analysis of real sounds. Bank et al. (2000) proposed that beating and two-stage decay characteristics can be modeled with time-invariant resonators that are run parallel to the string model. In order to save computational costs, Bank (2001) suggested that the resonators operate at a lower sampling frequency. Karjalainen et al. (2000) proposed that beating can be added to electric guitar tones by first extracting the desired partial and then modulating it with a low-frequency oscillator. A similar approach was used by Rauhala et al. (2007a) for modeling the beating in a DWG piano synthesizer. Later, Rauhala (2007b) suggested that beating and two-stage decay can be simulated in selected partials with equalizers that are connected in series with the string model by modulating the peak gains of equalizers.

Synthesizing of longitudinal vibrations in piano strings is a relatively new topic. Bank and Sujbert (2003) suggested that the longitudinal modes and phantom partials can be modeled with two additional string models with proper parameters in the DWG synthesis scheme. Bensa and Daudet (2004) proposed that the phantom partials can be modeled by multiplying the original signal with a filtered version of itself. Mixing the two signals produces additional components that correspond to the phantom partials. The advantage of this method is that it is applicable with any physics-based sound synthesis approach. Bank and Sujbert (2005) presented two ways of simulating the longitudinal vibrations. In the first method, the transversal string vibrations are computed with a finite-difference string model that gives the inputs for a set of resonators that simulate the longitudinal vibrations. The second method models both the transverse and the longitudinal vibrations with a set of second-order resonators, thus providing a computationally more efficient approach

compared to the finite-difference scheme.

The effect of the soundboard can be implemented with a filter with a high number of resonances. One possible solution for the design is to model the measured impulse response with an FIR filter. With this method, it is easy to obtain good sound quality, but this comes at the expense of very high-order filters, which are not applicable in the DWG context (Bank, 2006). Borin et al. (1997) suggested that the soundboard can be modeled with a feedback delay network (Jot and Chaigne, 1991; Rocchesso and Smith, 1997). This kind of reverberation-based way of modeling is computationally efficient, but the parameter design for the algorithm is not always a simple task, and it may happen that the resonance density is not high enough.

Commutated waveguide synthesis (Smith, 1993; Karjalainen et al., 1993) is an advantageous method that can be applied in the context of DWG modeling, especially in modeling the body of the instrument, or, in the case of the piano, the soundboard (Smith and Van Duyne, 1995a). The idea is to commute the effect of the instrument body in the excitation signal to avoid the heavy computational load of the soundboard filter. This can be done because of the LTI properties of the effect of the soundboard, provided that all the other parts of the model are LTI as well. Another approach is multirate modeling (Bank, 2000a; Bank et al., 2002), in which the soundboard filter is divided into two frequency bands, where the lower band operates at a reduced sampling rate. This way it is possible to retain the beneficial features of the FIR soundboard filters, while the computational load is acceptable.

4.3 Modeling of losses

The loss filter in a DWG piano model is usually a low-order FIR or IIR filter. The target gain for the loss filter is obtained from the partial decay times that are estimated from recorded tones, as discussed in Sec. 3.3. The filter gain value g_k at

the frequency corresponding to the partial k can be written as a closed-form formula (Bank, 2000a; Bank and Välimäki, 2003):

$$g_k = e^{\frac{-1}{f_0 \tau_k}}, \quad (4.3)$$

where f_0 is the fundamental frequency and τ_k is the decay time constant of the partial k . Eq. (4.3) assumes that the loss filter is applied once per fundamental period. The decay time constant is the time it takes for the signal to decay by $1/e$, where e is the base of the natural logarithm. The decay time constant can be computed from the corresponding $T_{60,k}$ value as (Välimäki, 1995):

$$\tau_k = \frac{T_{60,k}}{\ln(1000)}. \quad (4.4)$$

In practice, fitting a frequency response to the target data is not a trivial task mainly for two reasons. First, a small error in amplitude fitting results in a large error in the perceptually more important T_{60} domain, since the relation between g_k and $T_{60,k}$ is nonlinear. Second, the important data is concentrated approximately in the lowest 10% of the audio range, especially in the case of the lowest piano tones. Large deviations in accuracy can also lead to stability problems if the target gain value is very close to 1, since the loss filter is placed in the feedback loop of the string model. In addition, the decay times of even adjacent partials vary significantly making the modeling task even harder. This problem can be facilitated, however, with data smoothing (Bank and Välimäki, 2003; Rauhala et al., 2005). Even if the small details in data are lost with smoothing, the overall trend of the target decay times is preserved. Moreover, it has been shown that the human ear is quite insensitive to small changes in overall decay characteristics (Järveläinen and Tolonen, 2001). In general, loss filters with linear phase are preferred, since they do not affect the delay the signal is facing in the DWG. This is not a very strict requirement, however, because when the deviation is small it has only a minor effect on the partial locations.

Standard low-order filters, such as a second-order FIR filter designed with the least-squares error criterion (Borin et al., 1997) or a one-pole IIR filter (Välimäki et al., 1996), have been proposed, but these filters are only capable of matching the overall decay characteristics excluding the details. Bank (2000a) proposed that the decay rate estimates can be approximated with an even-order polynomial after which the polynomial parameters can be converted to one-pole loop filter coefficients. This method was extended to higher-order filters by Erkut (2001a). Erkut (2001b) proposed a technique that is based on polynomial regression to fit decay time data. The idea is to obtain the optimum polynomial order that minimizes the mean-square error. The target response for the floss filter is finally obtained from the truncated polynomial.

Bank and Välimäki (2003) presented a weighting function that emphasizes on large decay times and can be used with standard filter design techniques. The method follows the idea that partials that have a long decay time are perceptually more important than those with short decay times. Välimäki et al. (2004b) presented a loss filter design method for harpsichord synthesis that is capable of accurately matching one partial decay time with the overall trend. This design method was later extended by Rauhala et al. (2005) so as to match several partial decay times. Recently, van Walstijn (2010) proposed a physically parametric FIR loss filter design technique in which the filter coefficients are derived from the underlying partial differential equation instead of experimental data.

Article [PV] develops a multi-stage loss filter capable of accurately matching a desired number, say, 50, of partial decay times. The key idea is to design the filter with a reduced sampling rate in order to be able to concentrate on the important frequency range. The filter is finally upsampled for implementation. In addition to highly accurate design, it is possible to model only the overall trend of the decay behavior with a low order filter.

4.4 Modeling the sustain pedal

The key aspect in simulating the sustain pedal is the coupling between the strings in the string register. Garnett (1987) suggested that the effect of the sustain pedal can be taken into account by connecting several strings to the same lumped terminating impedance. This way it is possible to pass vibrational energy from the sounding string to the other strings. In his work, Garnett used six waveguides as a terminating impedance. Avanzini et al. (2001) and Bank et al. (2003) suggested that the terminating impedance can be modeled as a feedback delay network (Jot and Chaigne, 1991; Rocchesso and Smith, 1997), which can be seen as a filter with a high number of peaks in its frequency response. Van Duyne and Smith (1995) suggested that the effect of the sustain pedal can be commuted at the point of excitation. In practice, the impulse response of the soundboard and the freely vibrating string register is used as an excitation signal for the synthesis model. The solution is simple and efficiently simulates the effect of the soundboard and the sustain pedal, but it does not give any possibility to control, for example, the pedal depth unless the impulse response is measured and stored for several pedaling conditions. Even then, only a static response is obtained and varying the pedaling condition in real time, which is often the case in piano performances, is impossible.

De Poli et al. (1998) simulated the freely-vibrating string register with 18 fixed-length and 10 variable-length string models. The 18 fixed-length strings correspond to the 18 lowest tones of the piano, while the 10 variable-length strings can be used in several ways. Their lengths can be set, for example, to correspond to the next 10 piano tones subsequent to the 18 lowest tones. The outputs of the two sets of strings are first lowpass filtered and then summed up. The depth of the sustain pedal can be controlled with a specific parameter that multiplies the output sound. The sustain pedal algorithm presented in article [PI] extends the idea of the method presented by De Poli et al. (1998). In the algorithm of [PI], which consists of 12 string models, two filters are included in each strings model in order

to obtain more realistic behavior of the string register. The first filter is responsible for spreading the harmonic components randomly in the spectrum, whereas the second filter implements frequency-dependent decay.

Zambon et al. (2008) presented a sustain pedal algorithm, which is based on a set of resonators that can be considered to simulate string models. This approach has several advantages over previously presented algorithms. The effect of the sustain pedal can be split as a function of the keyboard in order to control the spread of energy through the piano compass in a more realistic way. In practice, this is realized by several pedal models that are applied in different key regions. Moreover, parameters for the sustain pedal algorithm can be derived directly from recorded tones. On the other hand, the algorithm is computationally more complex than the one presented in [PI]. It would be interesting to compare the sound quality of these two sustain pedal algorithms in a formal listening test.

5 Perception of the piano sound

Relatively few studies discuss the perception of musical instrument tones in the context of physics-based sound synthesis. Despite the fact that the human hearing system is relatively well known and studies concerning the perception of pure tones and noise from various view-points as well as directional hearing have been published, this information is hard to apply to the perception of relatively complex musical tones. Especially for the signal-based approach, information on how accurately humans perceive different features may be of key importance, since the computational power can be addressed to the synthesis of the most important features. In fact, a more relevant question would be what do we not hear, since modeling these features can be excluded and the synthesis algorithm can be simplified.

Traditionally, the most important perceptual attributes are the pitch, loudness, and timbre. In the case of the piano, the pitch sensation is mainly built up by the fundamental frequency and the degree of inharmonicity. The timbre, in turn, is a much more complex attribute which is affected by several perceptual dimensions, such as the attack transient, spectral content, temporal evolution of amplitudes, and modulation. From the synthesis view-point, interesting questions would be, for instance, how accurately the inharmonicity, longitudinal modes, decay times, or the beating characteristics should be modeled. Moreover, each of these tasks can be approached from several perspectives. In the case of the inharmonicity, for example, valid framings of the question are in which frequency region the inharmonicity is important, how accurately the partial locations should match the theoretical frequencies, and how many partials should be matched accurately without an audible difference compared to the reference tone.

Although studies discussing perception of the piano sound are few, studies of perception of other stringed instrument tones, such as the guitar, have been published,

for example, by Järveläinen (2003). It is probable that these results are applicable also to the piano sound, at least to some extent. In general, it is hard to say how much perceptually important features mask each other in different instruments.

The most intensively studied perceptual feature of the piano tone is probably the inharmonicity. Moore et al. (1985) studied the detection thresholds for one mistuned partial in otherwise harmonic complex tones and reported that the inharmonicity is detected in different ways for high and low partials. In the case of low harmonics, the mistuned partial stands out from the tone, while for the high partials the mistuning is perceived as roughness or beating. Moreover, the perception decreases when the mistuned partial number and the fundamental frequency of the tone increased.

Rocchesso and Scalcon (1999) studied the bandwidth of perceived inharmonicity in piano tones with the aim to achieve a certain frequency threshold above which it is not necessary to model the inharmonicity. They reported that the effect of inharmonicity is more prominent in the lower half of the keyboard. Additionally, the bandwidth of perceived inharmonicity increases with the fundamental frequency, although more partials need to be matched properly for low tones due to the low fundamental frequency. Järveläinen et al. (2001) reported audibility thresholds for inharmonicity for five fundamental frequencies. In this study, the timbral changes were of interest. They found that inharmonicity is more easily detected at low fundamental frequencies compared to high. In addition, they pointed out that for long tones with low fundamental frequency the inharmonicity perception threshold is lower compared to short tones of the same fundamental frequency.

Since the inharmonicity effect elevates the partials in frequency, it may happen that the perceived pitch is also changed. This was studied by Järveläinen et al. (2000, 2002) for five fundamental frequencies ranging from A_1 to A_7 . The authors presented a lower limit for the inharmonicity coefficient below which no significant pitch change was observed. In addition, they suggested that the pitch is often

judged based on some of the partials from the six lowest while the changes in timbre are often detected from the higher partials. Anderson and Strong (2005) presented the results of an experiment where the subjects were asked to match the pitch of synthetic inharmonic tones to those of harmonic tones. The nine test tones covered the three first octaves of the piano keyboard. The results suggest that the perceived fundamental frequency is higher compared to the actual fundamental frequency in the case of the lowest piano tones, while the fundamental frequency of the higher tones is perceived more accurately. This is in line with the practice that the lowest tones in an acoustic piano are tuned a bit lower than the theoretical values in order to retain the sensation of a correctly tuned scale.

Galembo et al. (2001) reported that the relative starting phases of the partials of bass tones with piano-like inharmonicity affect the perceived pitch and timbre. For harmonic tones, changing the initial phase conditions affect the pitch and timbre remarkably, while in the case of the inharmonic tones the phases of the partials are finally randomized due to the inharmonicity regardless of the initial phase conditions.

Another interesting question is how the tone decay process is perceived. This has gained much less attention in the literature, however. Järveläinen and Tolonen (2001) studied the perceptual tolerances for two decay parameters in context of loop filter design for DWG synthesis of plucked string instruments. The first parameter is the overall decay parameter, which controls the general level of the loop filter gain (changes in the parameter value contribute the same absolute gain increase or decrease to all frequencies), and the other parameter is the frequency-dependent decay, which controls the tilt of the loop filter magnitude response. For the overall decay, the lower threshold was 75% and the upper threshold 140% of the reference time constant value, suggesting that variations within this range are not perceived. For the frequency-dependent decay parameter, the reported lower and upper thresholds were 83% and 116% of the reference value, respectively. The results indicate that quite large deviations in decay parameters can be accepted without significant

audible effects.

The situation is somewhat different and often more complex with the piano tones, since the decay times of even adjacent partials vary significantly, and a loop filter with a lowpass character is often considered to result in too simple a decay. On the other hand, based on the work by Järveläinen and Tolonen (2001), it can be reasoned that a strictly accurate design is hardly required thus allowing the use of low-order filters in the DWG modeling approach.

Järveläinen and Karjalainen (2002) have studied the perception of beating and two-stage decay in dual-polarization string models. The test tones were synthetic tones in which the horizontal and vertical polarizations were implemented by mixing the outputs of two basic string models delay lines that were slightly mistuned. The results of the listening test showed that beating is noticed if the level of the vertical component is 7 – 18 dB lower than that of the horizontal component. For the two-stage decay, differences between 30% and 90% in the time constants of the two polarizations are acceptable, depending on the level differences of the two polarizations. Based on the results the authors pointed out that the sensitivity of the human hearing system to dual-polarization effects is relatively weak. This suggests that the human hearing system is relatively insensitive also to beating and two-stage decay in piano tones.

Yet another interesting research topic is the audibility of longitudinal components of vibration. While the physical origins of the longitudinal components are known (Bank and Sujbert, 2005), little is known about their perceptual relevance. It has been suggested that the longitudinal components are best audible in low fortissimo tones (Conklin, 1990) and that they add warmth to the sound (Bensa and Daudet, 2004). In [PVI] perception thresholds for longitudinal components in fortissimo piano tones is presented. The results suggest that the effect of the longitudinal components is perceived even in treble range, but the effect is subtle above the tone A_3 .

6 Main results of the thesis

6.1 Publication PI: Analysis and modeling of piano sustain-pedal effects

Article [PI] presents new information about the effect of the sustain pedal on the piano sound. Piano tones with and without the sustain pedal engaged were recorded in a recording studio and the tones were analyzed with signal-processing tools. The instrument that was recorded was a Yamaha concert grand piano, which was played with a pile of coins glued together in order to use exactly the same force when pressing the keys, as illustrated in Fig. 6.1.

The signal analysis consisted of the analysis of harmonic content and the analysis of the string register response. The yield of the analysis of extracted harmonics was the following observations: the decay times of tones increased in the middle range of the piano when the sustain pedal is used, while in the bass and treble range this phenomenon was not found; the initial amplitudes of the tones were the same regardless of whether the sustain pedal was used or not; and beating increased. These results are in line with observations presented previously (Brauss, 2006). Figure 6.2 shows the logarithm of the envelopes of the partials of the piano tone C_3 ($f_0 = 131.2$ Hz and $B = 0.00011$) (a) without and (b) with the sustain pedal engaged.

The beating and decay-time increase occur mainly because the coupling of the strings through the bridge allows energy transfer to those strings that are attached to the same bridge. The undamped string register is excited, in addition to the energy transfer from the string that has been excited by pressing the key, also by the mechanical impact of the hammer. This effect was observed to be prominent especially



Figure 6.1: Pile of coins used to press down the keys in order to keep the dynamics as constant as possible during the recording session.

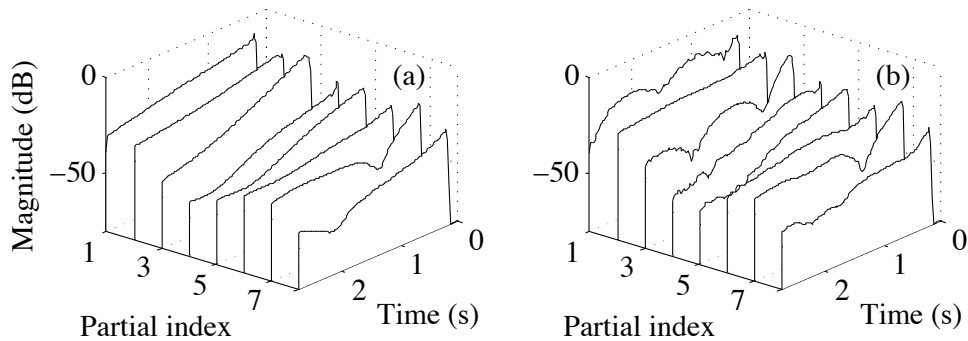


Figure 6.2: Envelopes of the first eight partials of the tone C_3 (a) without and (b) with the sustain pedal engaged.

in the beginning of the tone. Later, similar results were presented also by Schutz et al. (2008). Figure 6.3 illustrates the string register responses obtained by removing the partials from the tones played (a) without and (b) with the sustain pedal engaged.

The basis of the algorithm design for the simulation of the sustain pedal effect is a

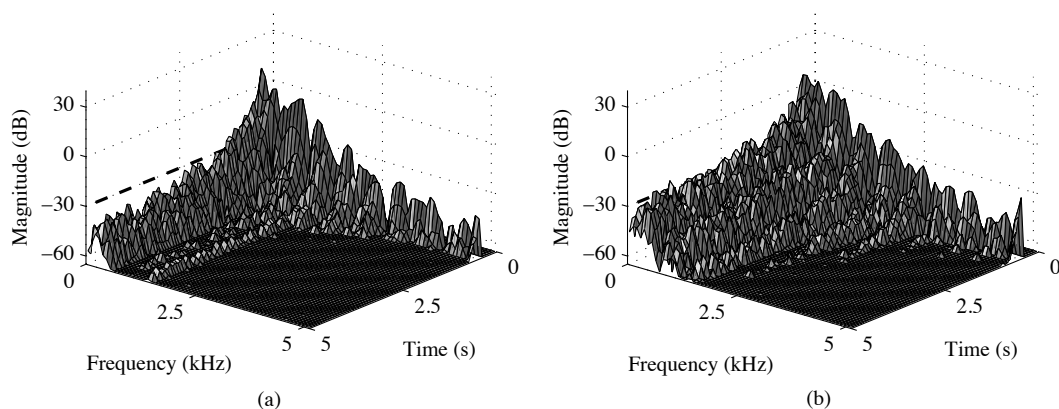


Figure 6.3: Time-frequency plot of the residual signal of the tone D_5 (a) without and (b) with the sustain pedal engaged. The dashed line in (a) and (b) shows the residual signal magnitude 5 seconds after the excitation when the sustain pedal is engaged. (After Lehtonen et al. (2007))

reverberation algorithm with 12 string models corresponding to the 12 lowest tones of the piano. The idea behind the design is that the fundamental frequencies and the overtones of the string models together model the modes of the whole string register. The string models consist of delay lines, dispersion filters, and lowpass filters. The purpose of the dispersion filters in this context is not to model the inharmonicity accurately, but rather to spread the modes of the reverberation algorithm randomly in the sympathetic spectrum. The basis for the dispersion-filter design follows the work of Rauhala and Välimäki (2006a,c). The lowpass filter is used to model the frequency-dependent losses in the strings. For this, a first-order IIR filter is used (Välimäki et al., 1996). In addition to the string block, the algorithm consists of a tone corrector and a mixing coefficient. The tone corrector is basically a bandpass filter, the purpose of which is to target the energy of the simulated string register to the right frequency band. The mixing coefficient is used to control the proportion of the effect of the string register in the output sound, and it varies depending on the tone that is processed by the algorithm. Low tones have a larger mixing coefficient than the high tones.

Finally, the sound quality of the proposed algorithm was compared to an earlier sustain pedal algorithm presented by De Poli et al. (1998) through a listening test. The results showed that the algorithm presented in article [PI] was able to produce a more realistic sustain pedal effect compared to the reference algorithm. Example tones are available online at <http://www.acoustics.hut.fi/publications/papers/jasa-piano-pedal/>.

6.2 Publication PII: Analysis of the part-pedaling effect in the piano

Article [PII] supplements the results of article [PI] by presenting the effects of the part-pedaling through signal analysis that has been carried out on piano tones that were played with different amounts of sustain pedal. In addition to near-field recordings, the damper acceleration signal was recorded in order to obtain temporal information on the hammer-string interaction under different playing conditions. In the recording session, the depth of the sustain pedal, i.e. the distance between the dampers and the strings, was controlled by turning the sustain pedal rod nut (see Fig. 2.1) in steps of 60 degrees, which was verified to correspond to 0.08 ± 0.01 mm change in damper height. Figure 6.4 illustrates the turning process.

Two types of recordings were carried out. In the first phase, all strings but those corresponding to the selected notes were damped in order to minimize the effect of the string register in the analysis of the temporal evolution of the partials. The distance between the damper and the strings was first adjusted by turning the sustain pedal rod nut and after this the key was pressed down. The keys were played in the traditional way, unlike in the previous work in which a pile of coins was used. The results of signal analysis show that when part-pedaling is used the piano tone decay can be divided into three distinct intervals: the initial free vibration,



Figure 6.4: Prof. Askenfelt turning the sustain pedal rod nut that controls the distance between the strings and the dampers.

the damper-string interaction, and the final free vibration. During the final free vibration, the decay times of tones increase as a function of the sustain pedal depth. When the depth of the sustain pedal reaches its maximum the three-stage decay is no longer found, which is in line with the results presented in [PI].

In the second recording phase, the damper adjustment was returned to normal, i.e. turning the sustain pedal rod nut affected all the dampers in the same way. The signal analysis revealed that in the case of low tones nonlinear amplitude limitation caused by the interaction of the damper and the string affects the tone by exciting missing modes when energy is transferred from one mode to another. This kind of nonlinear effect can occur in vibrating strings due to various reasons, for example, through tension modulation (Legge and Fletcher, 1984) or the slapped bass effect, where the fret or the fingerboard limits the amplitude of the string vibration (Rank and Kubin, 1997). However, the excitation of missing modes was not observed in mid-range and high-range tones. The timbre of all tones is changed, since the

damper-string interaction has an effect on the temporal evolution of partials and their relation. Sound examples are available online at <http://www.acoustics.hut.fi/publications/papers/jasael-part-pedaling/>.

6.3 Publication PIII: Canceling and selecting partials from musical tones using fractional-delay filters

Article [PIII] deals with the analysis of harmonic musical instrument tones. An efficient filter-based analysis method for separating the harmonics from the residual signal, selecting single harmonics, and the separation of odd and even harmonics is presented. The article extends the work by Välimäki et al. (2004a, 2007). The filter is an inverse comb filter (ICF) in which the delay line is replaced with an allpass filter to produce a fractional delay that is needed to accurately match the notches of the ICF to the partial frequencies of the tone. The transfer function of an ICF can be written as

$$H(z) = \frac{1}{2}[1 - H_{\text{fd}}(z)], \quad (6.1)$$

where $H_{\text{fd}}(z)$ is the transfer function of the fractional delay filter. Four different fractional delay filter design methods were tested with the ICF structure: the standard Lagrange filter (Laakso et al., 1996), the truncated Lagrange filter (Välimäki and Haghparast, 2007), the standard Thiran filter (Thiran, 1971), and the truncated Thiran filter (Välimäki, 2000). All the filter coefficients can be designed in closed form, based on the desired fractional part of the delay, which is determined by the fundamental frequency and the sampling frequency (see Eq. 4.2). The truncated Thiran filter and the truncated Lagrange filter were found to give the best accuracy in the analysis of both synthetic test and real musical instrument tones.

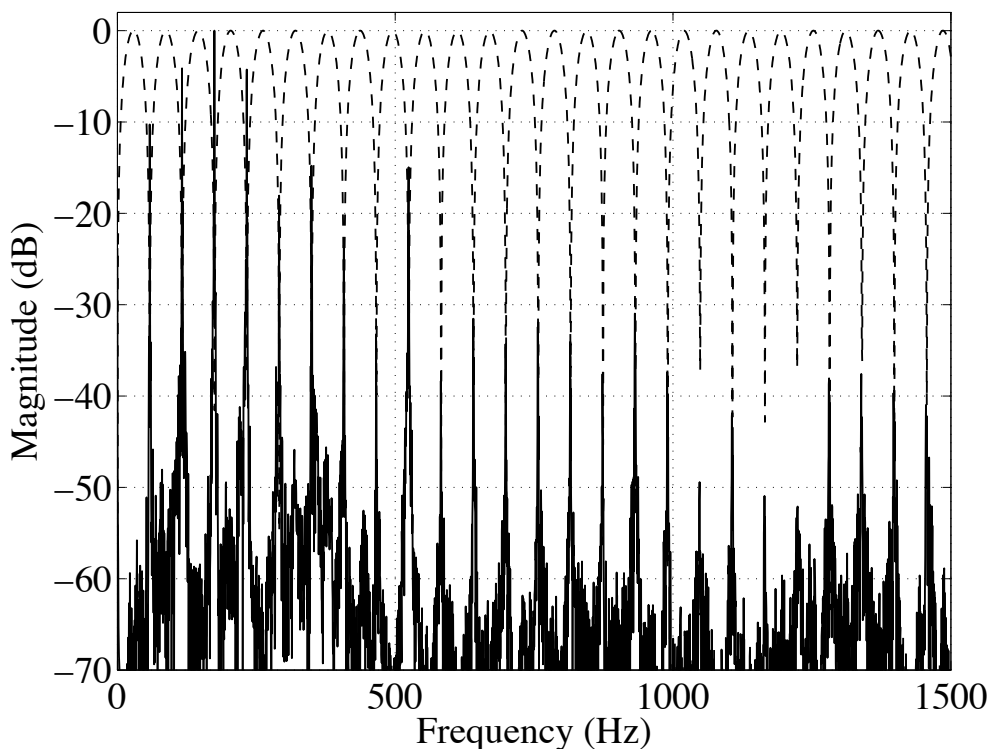


Figure 6.5: Canceling harmonics from a double bass tone $A_1^\#$ with the ICF. The allpass filter is the truncated Thiran filter of order 80 and prototype order 720. The solid and dashed lines represent the magnitudes of the tone spectrum and the ICF frequency responses, respectively.

Figure 6.5 shows the magnitude responses of a bowed double bass tone $A_1^\#$, $f_0 = 58.27$ Hz (solid line) and an ICF filter (dashed line) which is used to cancel the harmonics from the tone. The allpass filter is a truncated Thiran filter of order 80 and prototype order 720. The desired attenuation at the bottom of the notches is 120 dB. As can be seen, the notches of the filter accurately match the harmonic locations. The result of the filtering operation is shown in Fig. 6.6.

Single harmonics can be separated with a harmonic extraction filter (HEF), which consists of an ICF and a second-order resonator. The pole of the resonator cancels one of the zeros of the ICF and, thus, one harmonic is preserved while the others are filtered out. Furthermore, even and odd harmonics can be separated by setting the

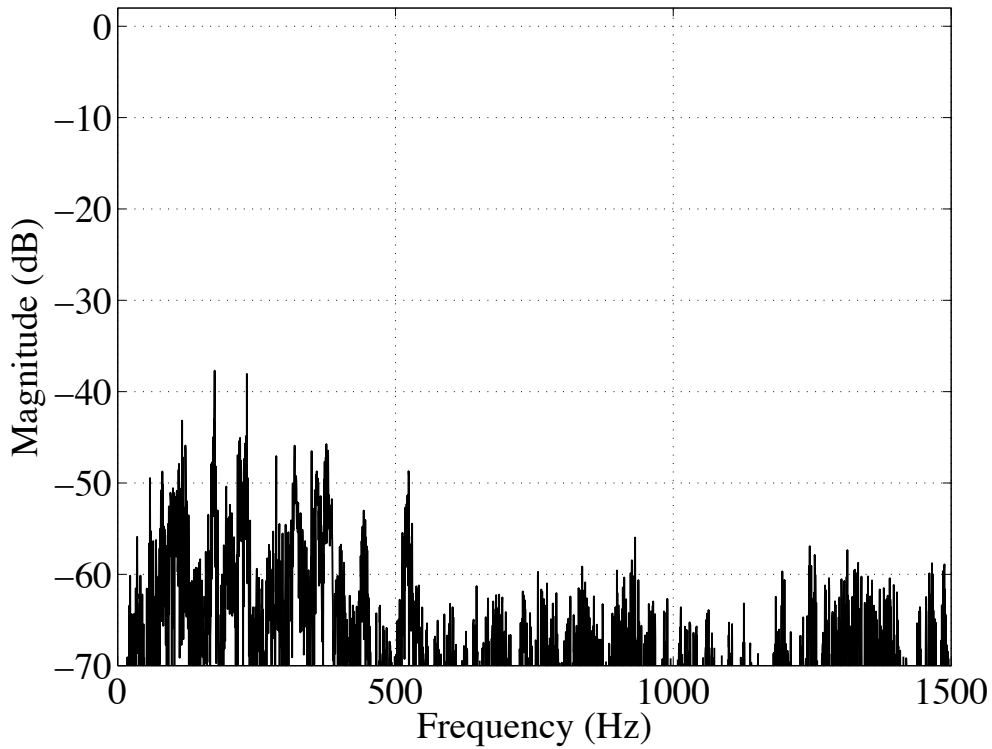


Figure 6.6: Spectrum of the residual of the double bass tone $A_1^\#$.

delay of the ICF to be half of the original desired delay. This way the notches appear at the multiples of the second harmonic and the odd harmonics are preserved. The even harmonics can be obtained by subtracting the odd harmonics from the original tone.

The analysis process using the ICF is simple and no further synthesis stage is required like in many other analysis methods. In addition, comparison against sinusoidal modeling and to other filter design methods (the windowed-sinc interpolation and the truncated Lagrange interpolation) shows that the ICF with a truncated Thiran filter performs best.

6.4 Publication PIV: Analysis of piano tones using an inharmonic inverse comb filter

In this article, the idea in article [PIII] is extended to the analysis of inharmonic piano tones. The fractional delay filter is replaced with a filter that has a frequency-dependent phase delay in order to set the notches of the inharmonic inverse comb filter (IICF) to match the partial locations of a piano tone. Two filters, a new FIR and a reference IIR filter were considered and their performance was compared. The FIR filter was designed with frequency sampling (Parks and Burrus, 1987). The phase response of the FIR filter is defined based on the fact that the phase is a multiple of 2π radians at partial locations and the magnitude response imitates a lowpass filter with a passband ranging from 0 Hz to 20 kHz. Once the frequency response is defined, the inverse discrete Fourier transform is applied in order to obtain the impulse response and thus the FIR filter coefficients. The length of the impulse response can be truncated with a rectangular window.

The IIR filter design method was suggested by Abel and Smith (2006). The method divides the desired phase delay characteristics into 2π -wide segments and assigns a pole-zero pair to each section, which yields to complex allpass sections that can be combined into biquads with real coefficients. After designing the dispersion filters they can be inserted to the ICF structure.

Single partials can be extracted by cascading a resonator with the ICF, as was done in article [PIII]. This is illustrated in Fig. 6.7, where the tone $A_2, f_0 = 110.0$ Hz, is analyzed with the inharmonic HEF structure with an FIR filter of length 1500. The desired attenuation at the bottom of the notches is 120 dB. The solid and dashed lines represent the spectrum of the tone and the frequency response of the filter, respectively. Figure 6.7 shows how the notches of the HEF match the partial locations, except for partial no. 10, which is preserved, as seen in Fig. 6.8.

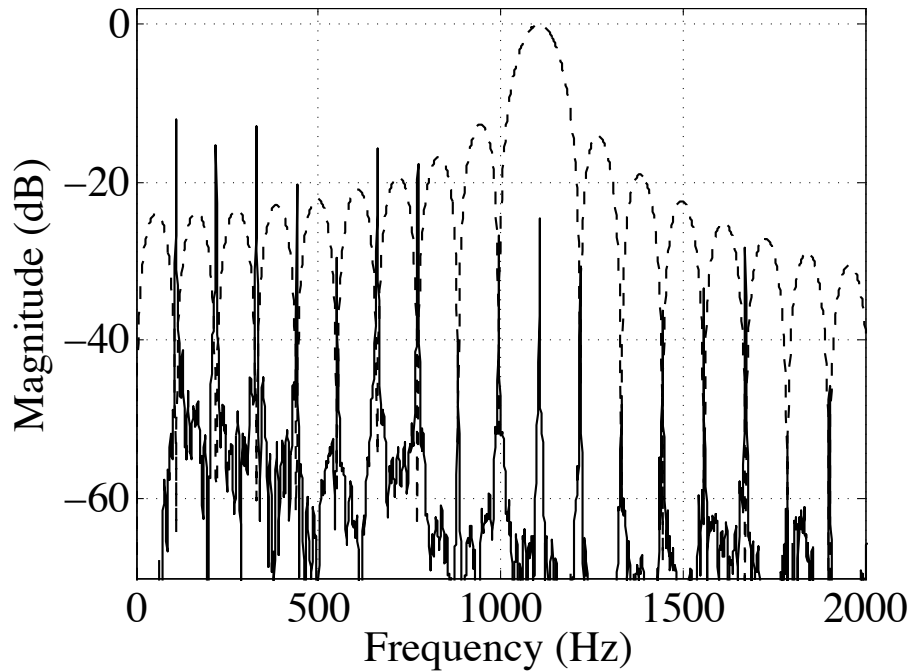


Figure 6.7: Illustration of the extraction of 10th partial from the piano tone A_2 with the HEF structure. The solid and dashed lines represent the magnitudes of the tone spectrum and the frequency response of the filter, respectively.

In comparison, the FIR filter was found to perform better, since it provided a better attenuation at partial frequencies when all partials were canceled to obtain the residual signal. This was also found when single partials were extracted from synthetic tones.

6.5 Publication PV: Sparse multi-stage loss filter design for waveguide piano synthesis

Article [PV] presents a multi-stage ripple-filter design technique which consists of three subfilters that are designed in subbands with a reduced sampling rate and finally up-sampled for implementation. The basis for the filter design is in frequency sampling (Parks and Burrus, 1987) and the IFIR technique (Neuvo et al., 1984).

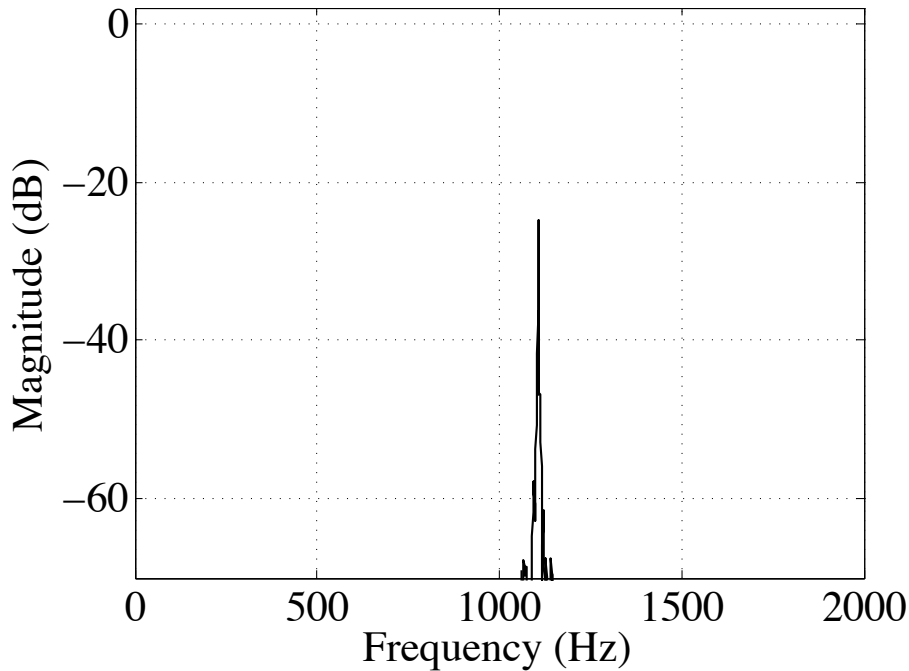


Figure 6.8: Spectrum of the extracted partial no. 10 of the piano tone A_2 .

The advantage is that it is possible to concentrate on the important frequency range, which, in the case of the lowest piano tones, is only about 10% of the audio range. Moreover, it is possible to accurately match many, say 50, partials. The three subfilters are the equalizer, the anti-imaging filter, and the multi-ripple filter. The equalizer is a first-order FIR filter and it sets the general trend of the piano tone decay. It is designed by matching two data points, such as the fundamental frequency f_0 and the Nyquist frequency. The anti-imaging filter is also a first-order FIR filter, and it defines the decay trend of the piano tone below 5 kHz. It is designed in the same way as the equalizer, but on a reduced frequency band.

The multi-ripple filter is designed by first reducing the frequency range with critical down-sampling and then forming the target magnitude response based on the measured T_{60} times, as defined by Eq. 4.3. The target impulse response is computed with the inverse discrete Fourier transform and is truncated by choosing the

N largest values and setting the other values to zero. When N is chosen to equal the number of partials that are to be matched, it is possible to model the target frequency response accurately. When the multi-ripple filter is up-sampled, the anti-imaging filter is used to suppress the image frequency responses that result from the upsampling operation. For lower-order filters, the fit can be improved with the weighting function proposed by Bank and Välimäki (2003).

Figure 6.9 illustrates how each of the subfilters contributes to the matching of decay times of the 50 lowest-order partials of the tone B_1 ($f_0 = 30.87$ Hz). The measured data is depicted with diamonds in each subfigure; in (a), (c), and (e) as filter gain values and in (b), (d), and (f) as decay times. From subfigures (a), (c), and (e) it can be noticed that the important frequency range is a small fraction of the whole audio range when the sampling frequency of 44.1 kHz is used. The dash, dash-dotted, and the solid line represent the frequency responses of the equalizer of order 1, anti-imaging filter of order 1, and the multi-ripple filter that has a sparse structure with 50 non-zero coefficients, respectively. Subfigures (b), (d), and (f) show how the combination of the corresponding filters on the left fit the target data in the T_{60} domain. Figure 6.9 (f) shows that the combination of the three filters is capable of accurately matching the partial decay times. At higher frequencies the filter's response decays gently with frequency.

The advantage of the design technique is that it allows a tradeoff between the filter order and the number of partial decay times that are matched to the target response. In comparison with previous loss filters, the proposed multi-stage filter was found to perform better with the same or lower computational load.

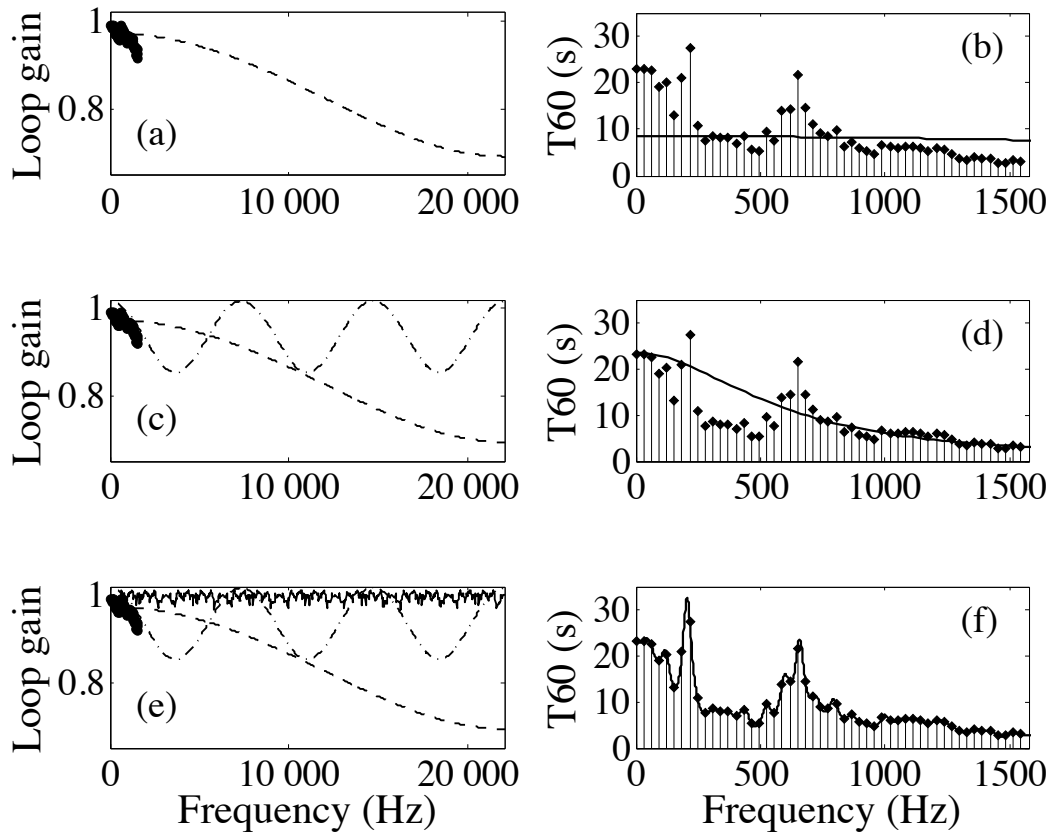


Figure 6.9: Illustration of the three stages of the loss-filter design process. The target data is shown as diamonds in every subfigure; in (a), (c), and (e) as loop gain values and in (b), (d), and (f) as T_{60} times. The dashed, dash-dotted, and the solid lines in (a), (c), and (e) represent the frequency responses of the equalizer, anti-imaging filter, and the multi-ripple filter, respectively. The solid line in (b), (d), and (f) show the combination of the corresponding frequency responses on the left in the T_{60} domain.

6.6 Publication PVI: Perception of longitudinal components in piano string vibrations

Publication [PVI] presents the threshold of audibility for the longitudinal components (longitudinal modes and the phantom partials) in piano tones through an ABX listening test. Recorded fortissimo tones from two grand pianos and one upright piano were resynthesized with and without longitudinal components in order

to accurately control the spectral content of the test tones. In the ABX test (Clark, 1982), X is the unknown reference tone and A and B are the tones without and with the longitudinal components, respectively. The subjects' task is to judge whether the reference tone X is the same tone as A or B. Figure 6.10 presents the user interface of the test program, which was coded with Matlab.

Eight subjects took part in the test, which was divided into two parts. In the first part, the first six tone pairs from a Steinway grand piano were presented to the subjects 16 times each in random order, and then six tones pairs from a Yamaha upright piano followed in the same manner. In the second part, six tone pairs from a Yamaha grand piano were judged and, additionally, subjects took a preference test in which they were asked whether they think the difference between the tones with and without the longitudinal components is substantial enough to be modeled in a piano synthesizer. This was a "yes" or "no" type of question without the possibility to answer "I don't know".

The results show that the longitudinal components are audible up to note C_5 , but based on the listeners' opinions it is important to model the longitudinal components up to note A_3 only. In the audibility test, it was concluded that the subject is able to hear the difference if 75% of the trials (12 out of 16 repetitions) are judged correctly. Interestingly, more than half of the listeners could discriminate the difference for the tone C_5 , which is in contrast to the general belief that the longitudinal components would be audible only in the bass range. In the preference test, the difference was considered to be significant and worth modeling in a piano synthesizer if more than 25% of the listeners thought so.

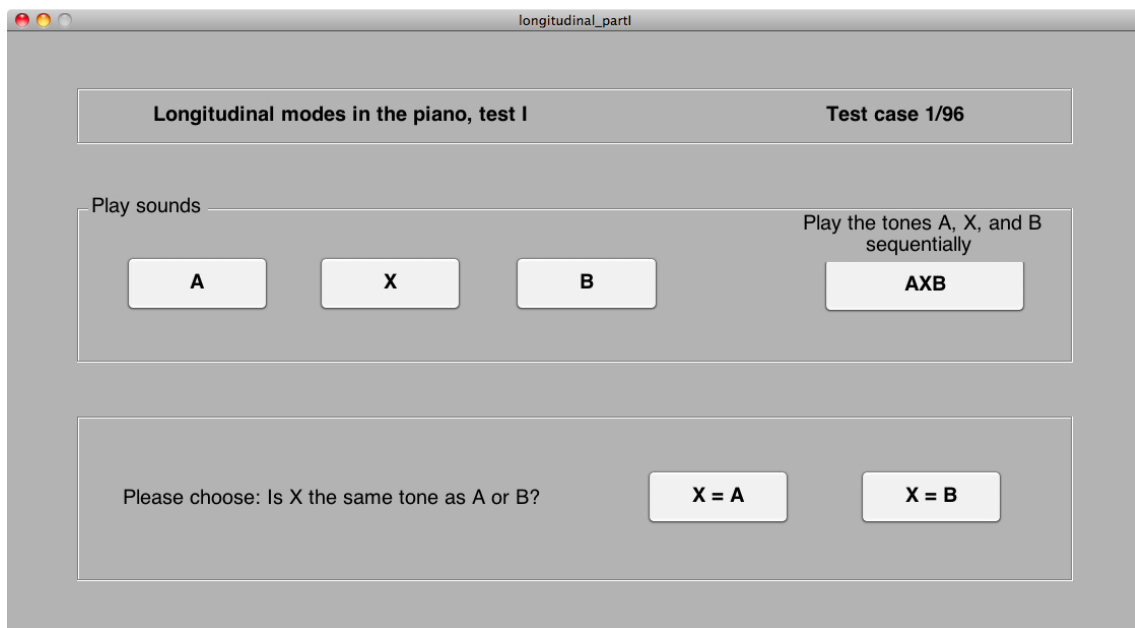


Figure 6.10: User interface of the ABX listening test.

7 Conclusion and future directions

This thesis discusses different steps in the development of a physics-based piano synthesizer. Topics include research on acoustics, analysis, perception, and algorithm design. The sustain pedal effect has been studied through signal analysis of recorded piano tones played with the full and partial sustain pedal. The analysis results show that when the sustain pedal is engaged, the beating increases, the two-stage decay characteristics of the piano tone are distorted, and the freely-vibrating string register adds reverberation to the sound. Moreover, when the partial sustain pedal is used, the nonlinear amplitude limitation can excite the missing modes of the lowest piano tones. A reverberation algorithm for simulating the main features of the full sustain pedal effect has been developed.

The analysis of harmonic and inharmonic musical instrument tones by means of an ICF is discussed. The partials can be efficiently cancelled or selected by replacing the delay line of the ICF with a suitable fractional delay filter. Different fractional delay filters have been tested for harmonic and inharmonic tones.

A sparse multi-stage loss filter for modeling the complicated decay of the piano tones is presented. This filter consists of three subfilters that are designed with different sampling rates in order to concentrate on the most important frequency regions. Finally, the filter is up-sampled for implementation. With this filter design, it is possible to accurately match several partial decay times. Alternatively, the computational load can be eased by modeling only the overall decay characteristics.

The perception of longitudinal vibrations is studied through a formal listening test. The goal was to find a threshold of audibility for fortissimo piano tones. The results suggest that the longitudinal vibrations are audible up to note C_5 , but, based on the listeners' opinion, it is necessary to model the longitudinal vibrations up to note

A_3 only in a piano synthesizer.

At the present moment it seems that physics-based sound synthesis will most likely form the trend in the development of future digital pianos. In addition to the efficient synthesis algorithm design, two main research areas need to be taken into account. The first is perceptual studies. It is clear that humans do not hear every single detail that is visible in the results of signal analysis of piano tones, which means that it is not reasonable to model every feature of the sound. This topic is challenging, though, because of the complexity of both the instrument and our hearing system. On the other hand, the results of listening tests would possibly reveal such information that is of crucial importance in the design of efficient algorithms.

The second important issue is collaboration with musicians and piano technicians. We engineers tend to stick to the analysis of clean, single tones instead of exploring the wider field of piano performances. After all, we hope that the piano synthesizers are played by professionals, whose wishes and high standards we should be able to satisfy. The analysis of single tones is justified, of course, since it is obvious that the underlying acoustical properties are best revealed from simple cases, but discussion with professional pianists opens new vistas, in particular for the properties that the synthesizer should be capable of reproducing. Examples of such properties are the variety of pedaling techniques. It is not yet clear what happens between the dampers and the strings in the different ways of pedaling, but as becomes evident from the literature targeted for professional pianists and students of piano playing (Banowetz, 1985; Parncutt and Troup, 2002; Brauss, 2006), these different techniques are common practice in almost every piano performance.

Despite of the diligent work that has been done in the field of piano acoustics and synthesis, the instrument still seems to preserve some of its secrets. However, this is part of the beauty of the piano: it has always something to offer to those who are enthusiastic and open-minded.

References

- J. S. Abel and J. O. Smith. Robust design of very high-order allpass dispersion filters. In *Proc. Int. Conf. on Digital Audio Effects (DAFx-06)*, pages 13–18, Montreal, Quebec, Canada, Sept. 18-20 2006. http://www.dafx.ca/proceedings/papers/p_013.pdf.
- J. S. Abel, V. Välimäki, and J. O. Smith. Robust, efficient design of allpass filters for dispersive string sound synthesis. *IEEE Signal Process. Lett.*, 17(4):406–409, 2010.
- J. M. Adrien. The missing link: Modal synthesis. In G. De Poli, A. Piccialli, and C. Roads, editors, *Representations of Musical Signals*, pages 269–297. The MIT Press, Cambridge, Massachusetts, USA, 1991.
- B. E. Anderson and W. J. Strong. The effect of inharmonic partials on pitch of piano tones. *J. Acoust. Soc. Am.*, 117:3268–3272, 2005.
- M. Aramaki, J. Bensa, L. Daudet, P. Guillemain, and R. Kronland-Martinet. Resynthesis of coupled piano string vibrations based on physical modeling. *J. New Music Res.*, 30(3):213–226, 2001.
- A. Askenfelt, editor. *Five Lectures on the Acoustics of the Piano*. Royal Swedish Academy of Music, 1990. Available online at http://www.speech.kth.se/music/5_lectures/.
- A. Askenfelt and A. S. Galembo. Study of the spectral inharmonicity of musical sound by the algorithms of pitch extraction. *Acoust. Phys.*, 46(2):121–132, 2000.
- A. Askenfelt and E. V. Jansson. From touch to string vibrations. I: Timing in the grand piano action. *J. Acoust. Soc. Am.*, 88(1):52–63, 1990.
- A. Askenfelt and E. V. Jansson. From touch to string vibrations. II: The motion of the key and hammer. *J. Acoust. Soc. Am.*, 90(5):2383–2393, 1991.

- A. Askenfelt and E. V. Jansson. From touch to string vibrations. III: String motion and spectra. *J. Acoust. Soc. Am.*, 93(4):2181–2196, 1993.
- F. Avanzini, B. Bank, G. Borin, G. De Poli, F. Fontana, and D. Rocchesso. Musical instrument modeling: the case of the piano. In *Proc. of the Workshop on Current Research Directions in Computer Music*, pages 124–133, Barcelona, Spain, 2001.
- R. Badeau, B. David, and G. Richard. High resolution spectral analysis of mixtures of complex exponentials modulated by polynomials. *IEEE Trans. Signal Process.*, 54(4):1341–1350, 2006.
- R. Badeau, G. Richard, and B. David. Performance of ESPRIT for estimating mixtures of complex exponentials modulated by polynomials. *IEEE Trans. Signal Process.*, 56(2):492–504, 2008.
- B. Bank. *Physics-based Sound Synthesis of the Piano*. Master’s thesis, Budapest University of Technology and Economics, Budapest, Hungary, 2000a. Published as Report 54 of TTK Laboratory of Acoustics and Audio Signal Processing, available online at <http://www.acoustics.hut.fi/~bbank/thesis.html>.
- B. Bank. Nonlinear interaction in the digital waveguide with the application to piano sound synthesis. In *Proc. Int. Computer Music Conf.*, pages 54–57, Berlin, Germany, September 2000b.
- B. Bank. Accurate and efficient modeling of beating and two-stage decay for string instrument synthesis. In *Proc. MOSART Workshop on Current Research Directions in Computer Music*, pages 134–137, Barcelona, Spain, 2001.
- B. Bank. *Physics-based Sound Synthesis of String Instruments Including Geometric Nonlinearities*. PhD thesis, Budapest University of Technology and Economics, Budapest, Hungary, 2006. Available online at <http://home.mit.bme.hu/~bank/phd/index.html>.
- B. Bank and L. Sujbert. Modeling the longitudinal vibration of piano strings. In *Proc. Stockholm Music Acoust. Conf*, pages 143–146, Stockholm, Sweden, 2003.

- B. Bank and L. Sujbert. Generation of longitudinal vibrations in piano strings: From physics to sound synthesis. *J. Acoust. Soc. Am.*, 117(4):2268–2278, 2005.
- B. Bank and V. Välimäki. Robust loss filter design for digital waveguide synthesis. *IEEE Signal Process. Lett.*, 10(1):18–20, 2003.
- B. Bank, V. Välimäki, L. Sujbert, and M. Karjalainen. Efficient physics-based sound synthesis of the piano using DSP methods. In *Proc. Eur. Signal Process. Conf.*, pages 2225–2228, Tampere, Finland, 2000.
- B. Bank, G. De Poli, and L. Sujbert. A multi-rate approach to instrument body modeling for real-time sound synthesis applications. In *Proc. 112th AES Convention*, Munich, Germany, May 2002.
- B. Bank, F. Avanzini, G. Borin, G. De Poli, F. Fontana, and D. Rocchesso. Physically informed signal processing methods for piano sound synthesis: A research overview. *EURASIP J. Appl. Signal Process.*, 2003(10):941–952, 2003.
- B. Bank, S. Zambon, and F. Fontana. A modal-based real-time piano synthesizer. *IEEE Trans. Audio, Speech, Language Process.*, 18(4):809–821, 2010.
- J. Banowetz. *The Pianist’s Guide to Pedaling*. Indiana University Press, Bloomington, 1985.
- J. Bensa. *Analysis and Synthesis of Piano Sounds Using Physical and Signal Models*. PhD thesis, Université de la Méditerranée, Marseille, France, 2003. Available online at <http://www.lma.cnrs-mrs.fr/~bensa/>.
- J. Bensa and L. Daudet. Efficient modeling of “phantom” partials in piano tones. In *Proc. Int. Symp. on Musical Acoustics*, pages 207–210, Nara, Japan, 2004.
- J. Bensa, S. Bilbao, R. Kronland-Martinet, and J. O. Smith. From the physics of piano strings to digital waveguides. In *Proc. Int. Computer Music Conf.*, pages 45–48, Göteborg, Sweden, 2002.

- J. Bensa, S. Bilbao, R. Kronland-Martinet, and J. O. Smith. A power normalized non-linear lossy piano hammer. In *Proc. Stockholm Music Acoust. Conf.*, pages 365–368, Stockholm, Sweden, 2003a.
- J. Bensa, S. Bilbao, R. Kronland-Martinet, and J. O. Smith. The simulation of piano string vibration: From physical models to finite difference schemes and digital waveguides. *J. Acoust. Soc. Am.*, 114(2):1095–1107, 2003b.
- J. Bensa, K. Jensen, and R. Kronland-Martinet. A hybrid resynthesis model for hammer-string interaction of piano tones. *EURASIP J. Appl. Signal Process.*, 2004(7):1021–1035, 2004.
- J. Bensa, S. Bilbao, R. Kronland-Martinet, J. O. Smith, and T. Voinier. Computational modeling of stiff piano strings using digital waveguides and finite differences. *Acta Acust. united Acust.*, 91:289–298, 2005.
- J. Berthaut, M. N. Ichchou, and L. Jézéquel. Piano soundboard: Structural behaviour, numerical and experimental study in the modal range. *Appl. Acoust.*, 64(11):1113–1136, 2003.
- S. Bilbao. *Numerical Sound Synthesis: Finite Difference Schemes and Simulation in Musical Acoustics*. John Wiley and Sons, Ltd, UK, 2009.
- P. H. Billhuber and C. A. Johnson. The influence of the soundboard on piano tone quality. *J. Acoust. Soc. Am.*, 11(3):311–320, 1940.
- G. Borin and G. De Poli. A hysteretic hammer-string interaction model for physical model synthesis. In *Proc. Nordic Acoustical Meeting*, pages 399–406, Helsinki, Finland, 1996.
- G. Borin, D. Rocchesso, and F. Scalcon. A physical model for music performance. In *Proc. Int. Computer Music Conf.*, pages 350–353, Thessaloniki, Greece, 1997.
- X. Boutillon. Model for piano hammers: Experimental determination and digital simulation. *J. Acoust. Soc. Am.*, 83(2):746–754, 1988.

- H. Brauss. *The Singing Piano Tone or the Artistry of Pedaling*. Trafford Publishing, Victoria, Canada, 2006.
- J. C. Bryner. Stiff-string theory: Richard Feynman on piano tuning. *Physics Today*, 62(12):46–49, 2009.
- C. Cadoz, A. Luciani, and J. Florens. Responsive input devices and sound synthesis by simulation of instrumental mechanisms: The CORDIS system. *Comput. Music J.*, 8(3):60–73, 1983.
- B. Capleton. False beats in coupled piano string unisons. *J. Acoust. Soc. Am.*, 115(2):885–892, 2003.
- B. Cartling. Beating frequency and amplitude modulation of the piano tone due to coupling of tones. *J. Acoust. Soc. Am.*, 117(4):2259–2267, 2005.
- A. Chaigne and A. Askenfelt. Numerical simulations of piano strings. I. A physical model for a struck string using finite difference methods. *J. Acoust. Soc. Am.*, 95(2):1112–1118, 1994a.
- A. Chaigne and A. Askenfelt. Numerical simulations of piano strings. II. Comparisons with measurements and systematic exploration of some hammer-string parameters. *J. Acoust. Soc. Am.*, 95(3):1631–1640, 1994b.
- D. Clark. High-resolution subjective testing using a double-blind comparator. *J. Audio Eng. Soc.*, 30(5):330–338, 1982.
- H. A. Conklin. Piano design factors — their influence on tone and acoustical performance. In A. Askenfelt, editor, *Five Lectures on the Acoustics of the Piano*, chapter 1. Royal Swedish Academy of Music, 1990. Available online at http://www.speech.kth.se/music/5_lectures/conklin/conklin.html.
- H. A. Conklin. Design and tone in the mechanoacoustic piano. Part I. Piano hammers and tonal effects. *J. Acoust. Soc. Am.*, 99(6):3286–3296, 1996a.

- H. A. Conklin. Design and tone in the mechanoacoustic piano. Part II. Piano structure. *J. Acoust. Soc. Am.*, 100(2):695–708, 1996b.
- H. A. Conklin. Design and tone in the mechanoacoustic piano. Part III. Piano strings and scale desing. *J. Acoust. Soc. Am.*, 100(3):1286–1298, 1996c.
- H. A. Conklin. Piano strings and “phantom” partials. *J. Acoust. Soc. Am.*, 102(1):659, 1997.
- H. A. Conklin. Generation of partials due to nonlinear mixing in a stringed instrument. *J. Acoust. Soc. Am.*, 105(1):536–545, 1999.
- L. Daudet, P. Guillemain, and R. Kronland-Martinet. Resynthesis of piano strings vibrations based on physical modeling. In *Proc. Int. Computer Music Conf.*, pages 48–51, Beijing, China, 1999.
- B. David, R. Badeau, and G. Richard. HRHATRAC algorithm for spectral line tracking of musical signals. In *Proc. Int. Conf. Acoustics, Speech, and Signal Process.*, pages 45–48, Toulouse, France, May 15-19 2006.
- A. de Cheveigné and H. Kawahara. YIN, a fundamental frequency estimator for speech and music. *J. Acoust. Soc. Am.*, 111(4):1917–1930, 2002.
- G. De Poli, F. Campetella, and G. Borin. Pedal resonance effect simulation device for digital pianos. United States Patent 5,744,743, April 1998. Appl. No. 618379, filed: March 1996.
- C. Erkut. Loop filter design techniques for virtual string instruments. In *Proc. Int. Symp. on Musical Acoustics*, pages 259–262, Perugia, Italy, 2001a.
- C. Erkut. Model order selection techniques for the loop filter design of virtual string instruments. In *Proc. of the 5th World Multi-Conference on Systemics, Cybernetics and Informatics*, pages 529–534, Orlando, FL, USA, 2001b. Available online at <http://lib.tkk.fi/Diss/2002/isbn9512261901/article8.pdf>.

- C. Erkut, V. Välimäki, M. Karjalainen, and M. Laurson. Extraction of physical and expressive parameters for model-based sound synthesis of the classical guitar. In *Proc. 108th AES Convention*, Paris, France, 2000. Available online at <http://lib.tkk.fi/Diss/2002/isbn9512261901/article5.pdf>.
- P. A. A. Esquef, M. Karjalainen, and V. Välimäki. Frequency-zooming ARMA modeling for analysis of noisy string instrument tones. *EURASIP J. Appl. Signal Process.*, 2003(10):953–967, 2003.
- G. Evangelista. Pitch synchronous wavelet representations of speech and musical signals. *IEEE Trans. Signal Process.*, 41(12):3313–3330, 1993.
- A. Fettweis. Wave digital filters: Theory and practice. *Proc. IEEE*, 74(2):270–327, 1986.
- H. Fletcher, E. D. Blackham, and R. Stratton. Quality of piano tones. *J. Acoust. Soc. Am.*, 34(6):749–761, 1962.
- N. H. Fletcher and T. D. Rossing. *The Physics of Musical Instruments*. Springer-Verlag, New York, USA, 1991.
- A. Galembo and A. Askenfelt. Signal representation and estimation of spectral parameters by inharmonic comb filters with application to the piano. *IEEE Trans. Speech Audio Process.*, 7(2):197–203, March 1999.
- A. Galembo, A. Askenfelt, L. L. Cuddy, and F. A. Russo. Effects of relative phases on pitch and timbre in the piano bass range. *J. Acoust. Soc. Am.*, 110(3):1649–1666, 2001.
- G. E. Garnett. Modeling piano sound using waveguide digital filtering techniques. In *Proc. Int. Computer Music Conf.*, pages 89–95, Urbana, Illinois, USA, 1987.
- N. Giordano. Mechanical impedance of a piano soundboard. *J. Acoust. Soc. Am.*, 103(4):2128–2133, 1998a.

- N. Giordano. Sound production by a vibrating piano soundboard: Experiment. *J. Acoust. Soc. Am.*, 104(3):1648–1653, 1998b.
- N. Giordano and M. Jiang. Physical modeling of the piano. *EURASIP J. Appl. Signal Process.*, 2004(7):926–933, 2004.
- N. Giordano and A. J. Korty. Motion of a piano string: Longitudinal vibrations and the role of the bridge. *J. Acoust. Soc. Am.*, 100(6):3899–3908, 1996.
- W. Goebel, R. Bresin, and A. Galembo. Touch and temporal behavior of grand piano actions. *J. Acoust. Soc. Am.*, 118(2):1154–1165, 2005.
- P. Guillemain and R. Kronland-Martinet. Characterization of acoustic signals through continuous linear time-frequency representations. *Proc. IEEE*, 84(4):561–585, 1996.
- D. E. Hall. Piano string excitation in the case of small hammer mass. *J. Acoust. Soc. Am.*, 79(1):141–147, 1986.
- D. E. Hall. Piano string excitation II: General solution for a hard narrow hammer. *J. Acoust. Soc. Am.*, 81(2):535–546, 1987a.
- D. E. Hall. Piano string excitation III: General solution for a soft narrow hammer. *J. Acoust. Soc. Am.*, 81(2):547–555, 1987b.
- D. E. Hall. Piano string excitation. VI: Nonlinear modeling. *J. Acoust. Soc. Am.*, 92(1):95–105, 1992.
- D. E. Hall and A. Askenfelt. Piano string excitation V: Spectra for real hammers and strings. *J. Acoust. Soc. Am.*, 83(6):1627–1638, 1988.
- D. E. Hall and P. Clark. Piano string excitation IV: The question of missing modes. *J. Acoust. Soc. Am.*, 82(6):1913–1918, 1987.
- E. Hayashi, M. Yamane, and H. Mori. Behavior of piano-action in a grand piano. I. Analysis of the motion of the hammer prior to string contact. *J. Acoust. Soc. Am.*, 105(6):3534–3544, 1999.

- C. P. Heinlein. A discussion of the nature of pianoforte damper-peddalling together with an experimental study of some individual differences in pedal performance. *J. Gen. Psychol.*, 2:489–508, 1929.
- C. P. Heinlein. Pianoforte damper-peddalling under ten different experimental conditions. *J. Gen. Psychol.*, 3:511–528, 1930.
- L. Hiller and P. Ruiz. Synthesizing musical sounds by solving the wave equation for vibrating objects: Part 1. *J. Audio Eng. Soc.*, 19(6):462–470, 1971a.
- L. Hiller and P. Ruiz. Synthesizing musical sounds by solving the wave equation for vibrating objects: Part 2. *J. Audio Eng. Soc.*, 19(7):542–551, 1971b.
- D. A. Jaffe and J. O. Smith. Extensions of the Karplus-Strong plucked-string algorithm. *Comput. Music J.*, 7(2):56–69, 1983.
- H. Järveläinen. *Perception of Attributes in Real and Synthetic String Instrument Sounds*. PhD thesis, Helsinki University of Technology, Espoo, Finland, 2003. Published as Report 68 of TKK Laboratory of Acoustics and Audio Signal Processing, available online at <http://lib.tkk.fi/Diss/2003/isbn9512263149/>.
- H. Järveläinen and M. Karjalainen. Perception of beating and two-stage decay in dual-polarization string models. In *Proc. Int. Symp. on Musical Acoustics*, pages 1–10, Mexico City, Mexico, 2002.
- H. Järveläinen and T. Tolonen. Perceptual tolerances for decay parameters in plucked string synthesis. *J. Audio Eng. Soc.*, 49(11):1049–1059, 2001.
- H. Järveläinen, T. Verma, and V. Välimäki. The effect of inharmonicity on pitch in string instrument sounds. In *Proc. Int. Computer Music Conf.*, pages 237–240, 2000.
- H. Järveläinen, V. Välimäki, and M. Karjalainen. Audibility of the timbral effects of inharmonicity in stringed musical instrument tones. *Acoust. Res. Lett. Online*, 2(3):79–84, 2001.

- H. Järveläinen, T. Verma, and V. Välimäki. Perception and adjustment of pitch in inharmonic string instrument tones. *J. New Music Res.*, 31(4):311–319, 2002.
- J.-M. Jot. An analysis/synthesis approach to real-time artificial reverberation. In *Proc. Int. Conf. Acoustics, Speech, and Signal Process.*, pages 221–224, San Francisco, CA, USA, 1992.
- J.-M. Jot and A. Chaigne. Digital delay networks for designing artificial reverberators. In *Proc. 90th AES Convention*, Paris, France, February 1991.
- M. Karjalainen, V. Välimäki, and Z. Jánosy. Towards high-quality sound synthesis of the guitar and string instruments. In *Proc. Int. Computer Music Conf.*, pages 56–63, Tokyo, Japan, 1993.
- M. Karjalainen, V. Välimäki, and T. Tolonen. Plucked-string models: From the Karplus-Strong algorithm to digital waveguides and beyond. *Comput. Music J.*, 22(3):17–32, 1998.
- M. Karjalainen, H. Penttinen, and V. Välimäki. Acoustic sound from the electric guitar using DSP techniques. In *Proc. Int. Conf. Acoustics, Speech, and Signal Process.*, pages 773–776, Istanbul, Turkey, 2000.
- M. Karjalainen, P. A. A. Esquef, P. Antsalo, A. Mäkivirta, and V. Välimäki. Frequency-zooming ARMA modeling of resonant and reverberant systems. *J. Audio Eng. Soc.*, 50(12):1012–1029, 2002.
- K. Karplus and A. Strong. Digital synthesis of plucked-string and drum timbres. *Comput. Music J.*, 7(2):43–55, 1983.
- M. Keane. Separation of piano keyboard vibrations into tonal and broadband components. *Appl. Acoust.*, 68(10):1104–1117, 2007.
- T. I. Laakso, V. Välimäki, M. Karjalainen, and U. K. Laine. Splitting the unit delay — tools for fractional delay filter design. *IEEE Signal Process. Mag.*, 13(1):30–60, 1996.

- M. Lang and T. I. Laakso. Simple and robust method for the design of allpass filters using least-squares phase error criterion. *IEEE Trans. Circuits Sys. II: Analog and Digital Signal Process.*, 41(1):40–48, 1994.
- J. Laroche and J.-L. Meillier. Multichannel excitation/filter modeling of percussive sounds with application to the piano. *IEEE Trans. Speech Audio Process.*, 2(2):329–344, 1994.
- J. Lattard. Influence of inharmonicity on the tuning of a piano—measurements and mathematical simulation. *J. Acoust. Soc. Am.*, 94(1):46–53, 1993.
- N. Lee, Z. Duan, and J. O. Smith. Excitation signal extraction for guitar tones. In *Proc. Int. Computer Music Conf.*, Copenhagen, Denmark, August 2007.
- K. A. Legge and N. H. Fletcher. Nonlinear generation of missing modes on a vibrating string. *J. Acoust. Soc. Am.*, 76(1):5–12, 1984.
- H.-M. Lehtonen, H. Penttinen, J. Rauhala, and V. Välimäki. Analysis of the sustain-pedal effect in the grand piano. In *Proc. Int. Congr. Acoust.*, 2007. CD-ROM Proceedings.
- D. W. Martin. Decay rates of piano tones. *J. Acoust. Soc. Am.*, 19(4):535–541, 1947.
- D. W. Martin and W. D. Ward. Subjective evaluation of musical scale temperament in pianos. *J. Acoust. Soc. Am.*, 33(5):582–585, 1961.
- R. J. McAulay and T. F. Quatieri. Speech analysis/synthesis based on a sinusoidal representation. *IEEE Trans. Acoust. Speech, Signal Process.*, 34(4):774–754, 1986.
- Modartt, 2006. The Pianoteq webpage <http://www.pianoteq.com/>.
- B. C. J. Moore, R. W. Peters, and B. R. Glasberg. Thresholds for the detection of inharmonicity in complex tones. *J. Acoust. Soc. Am.*, 77(5):1861–1867, 1985.
- T. H. Moore and S. A. Zietlow. Interferometric studies of a piano soundboard. *J. Acoust. Soc. Am.*, 119(3):1783–1793, 2006.

- I. Nakamura and D. Naganuma. Characteristics of piano sound spectra. In *Proc. Stockholm Music Acoust. Conf.*, pages 325–330, Stockholm, Sweden, 1993.
- Y. Neuvo, C.-Y. Dong, and S. K. Mitra. Interpolated finite impulse response filters. *IEEE Trans. Acoust. Speech, Signal Process.*, 32(3):563–570, 1984.
- N. Ono, K. Miyamoto, H. Kameoka, and S. Sagayama. A real-time equalizer of harmonic and percussive components in music signals. In *Proc. Int. Conf. on Music Information Retrieval*, pages 139–144, Philadelphia, Pennsylvania, USA, 2008.
- L. Ortiz-Berenguer, J. Casajus-Quiros, M. Torres-Guijarro, J. Beracoechea, and J. Perez-Aranda. Simple modeling of piano inharmonicity due to soundboard impedance. In *Proc. 120th AES Convention, Preprint*, volume 6647, pages 1–6, 2006.
- A. Paladin and D. Rocchesso. A dispersive resonator in real time on MARS workstation. In *Proc. Int. Computer Music Conf.*, pages 146–149, San Jose, California, USA, 1992.
- T. W. Parks and C. S. Burrus. *Digital Filter Design*. John Wiley and Sons, Inc., 1987.
- R. Parncutt and M. Troup. Piano. In R. Parncutt and G. E. McPherson, editors, *The Science and Psychology of Music Performance*, chapter 18, pages 285–302. Oxford: University Press, 2002.
- F. Pedersini, A. Sarti, S. Tubaro, and D. E. Inf. Block-wise physical model synthesis for musical acoustics. *Electronics Lett.*, 35(17):1418–1419, 1999.
- R. Rabenstein and L. Trautmann. Digital sound synthesis of string instruments with the functional transformation method. *Signal Process.*, 83(8):1673–1688, 2003.
- L. R. Rabiner. On the use of autocorrelation analysis for pitch detection. *IEEE Trans. Acoust. Speech, Signal Process.*, 25(1):24–33, 1977.

- E. Rank and G. Kubin. A waveguide model for slabbass synthesis. In *Proc. Int. Conf. Acoustics, Speech, and Signal Process.*, pages 443–446, Munich, Germany, April 21–24 1997.
- J. Rauhala. *Physics-based Parametric Synthesis of Inharmonic Piano Tones*. PhD thesis, Helsinki University of Technology, Espoo, Finland, 2007a. Published as Report 85 of TKK Laboratory of Acoustics and Audio Signal Processing, available online at <http://lib.tkk.fi/Diss/2007/isbn9789512290666/>.
- J. Rauhala. The beating equalizer and its application to the synthesis and modification of piano tones. In *Proc. Int. Conf. on Digital Audio Effects (DAFx-07)*, pages 181–187, Bordeaux, France, September 2007b. <http://dafx.labri.fr/main/papers/p181.pdf>.
- J. Rauhala and V. Välimäki. Tunable dispersion filter design for piano synthesis. *IEEE Signal Process. Lett.*, 13(5):253–256, 2006a.
- J. Rauhala and V. Välimäki. Parametric excitation model for waveguide piano synthesis. In *Proc. Int. Conf. Acoustics, Speech, and Signal Process.*, pages 157–160, Toulouse, France, 2006b.
- J. Rauhala and V. Välimäki. Dispersion modeling in waveguide piano synthesis using tunable allpass filters. In *Proc. Int. Conf. on Digital Audio Effects (DAFx-06)*, pages 71–76, Montreal, Canada, 2006c. <http://www.dafx.ca/proceedings/papers/p-071.pdf>.
- J. Rauhala, H.-M. Lehtonen, and V. Välimäki. Multi-ripple loss filter for waveguide piano synthesis. In *Proc. Int. Computer Music Conf.*, pages 729–732, Barcelona, Spain, September 2005.
- J. Rauhala, H.-M. Lehtonen, and V. Välimäki. Toward next-generation digital keyboard instruments. *IEEE Signal Process. Mag.*, 24(2):12–20, 2007a.

- J. Rauhala, H.-M. Lehtonen, and V. Välimäki. Fast automatic inharmonicity estimation algorithm. *J. Acoust. Soc. Am. Express Lett.*, 121(5):EL184–EL189, 2007b.
- J. Rauhala, M. Laurson, V. Välimäki, H.-M. Lehtonen, and V. Norilo. A parametric piano synthesizer. *Comput. Music J.*, 32(4):17–30, 2008.
- B. H. Repp. Pedal timing and tempo in expressive piano performance: A preliminary investigation. *Psychology of Music*, 24(2):199–221, 1996.
- B. H. Repp. The effect of tempo on pedal timing in piano performance. *Psychol. Res.*, 60(3):164–172, 1997.
- C. Roads. *The Computer Music Tutorial*. The MIT Press, Cambridge, Massachusetts, USA, 1995.
- D. Rocchesso and F. Scalcon. Accurate dispersion simulation for piano strings. In *Proc. Nordic Acoustical Meeting*, pages 407–414, Helsinki, Finland, June 1996.
- D. Rocchesso and F. Scalcon. Bandwidth of perceived inharmonicity for physical modeling of dispersive strings. *IEEE Trans. Speech and Audio Process.*, 7(5):597–601, 1999.
- D. Rocchesso and J. O. Smith. Circulant and elliptic feedback delay networks for artificial reverberation. *IEEE Trans. Speech Audio Process.*, 5(1):51–63, 1997.
- Roland, 2008. The V-piano webpage <http://www.roland.com/products/en/V-Piano/>.
- M. R. Schroeder. New method of measuring reverberation time. *J. Acoust. Soc. Am.*, 37(6):1187–1188, 1965.
- O. H. Schuck and R. W. Young. Observations on the vibrations of piano strings. *J. Acoust. Soc. Am.*, 15(1):1–11, 1943.

- A. Schutz, N. Bertin, D. T. M. Slock, B. David, and R. Badeau. Piano “forte pedal” analysis and detection. In *Proc. 124th AES Convention*, Amsterdam, The Netherlands, May 2008.
- X. Serra. *A System for Sound Analysis/Transformation/Synthesis Based on a Deterministic plus Stochastic Decomposition*. PhD thesis, Stanford University, California, USA, 1989. Available online at <http://ccrma.stanford.edu/STANM/stanms/stanm58/>.
- X. Serra and J. O. Smith. Spectral modeling synthesis: A sound analysis/synthesis based on a deterministic plus stochastic decomposition. *Comput. Music J.*, 14(4): 12–24, 1990.
- J. O. Smith. *Physical Audio Signal Processing for Virtual Musical Instruments and Audio Effects*. W3K Publishing, USA, 2010. Available online at <http://ccrma.stanford.edu/~jos/pasp/>.
- J. O. Smith. *Techniques for Digital Filter Design and System Identification with Application to the Violin*. PhD thesis, Stanford University, California, USA, 1983.
- J. O. Smith. Physical modeling using digital waveguides. *Comput. Music J.*, 16(4): 74–91, 1992.
- J. O. Smith. Efficient synthesis of stringed musical instruments. In *Proc. Int. Computer Music Conf.*, pages 64–71, Tokyo, Japan, 1993.
- J. O. Smith and S. A. Van Duyne. Commuted piano synthesis. In *Proc. Int. Computer Music Conf.*, pages 335–342, Banff, Canada, 1995a.
- J. O. Smith and S. A. Van Duyne. Overview of the commuted piano synthesis technique. In *Proc. IEEE Workshop on Applications of Signal Processing to Audio and Acoustics*, pages 226–229, New Paltz, NY, USA, October 1995b.
- A. Stulov. A hysteretic hammer-string interaction model for physical model synthesis. *J. Acoust. Soc. Am.*, 97(4):2577–2585, 1995.

- H. Suzuki. Vibration and sound radiation of a piano soundboard. *J. Acoust. Soc. Am.*, 80(6):1573–1582, 1986.
- H. Suzuki. Model analysis of a hammer-string interaction. *J. Acoust. Soc. Am.*, 82(4):1154–1151, 1987.
- H. Suzuki and I. Nakamura. Acoustics of pianos. *Appl. Acoust.*, 90:147–205, 1990.
- T. Taguti, K. Ohgushi, and T. Sueoka. Individual differences in the pedal work of piano performance. In *Proc. Stockholm Music Acoust. Conf.*, pages 142–145, Stockholm, Sweden, 1993.
- J. P. Thiran. Recursive digital filters with maximally flat group delay. *IEEE Trans. Circuit Theory*, 18(6):659–664, 1971.
- L. Trautmann and R. Rabenstein. Digital sound synthesis based on transfer function models. In *Proc. IEEE Workshop on Applications of Signal Processing to Audio and Acoustics*, pages 83–86, New Paltz, NY, USA, October 1999.
- V. Välimäki. Simple design of fractional delay allpass filters. In *Proc. Eur. Signal Process. Conf.*, pages 1881–1884, 2000.
- V. Välimäki. *Discrete-Time Modeling of Acoustic Tubes Using Fractional Delay Filters*. PhD thesis, Helsinki University of Technology, Espoo, Finland, 1995. Published as Report 37 of TKK Laboratory of Acoustics and Audio Signal Processing, available online at http://www.acoustics.hut.fi/~vpv/publications/vesa_phd.html.
- V. Välimäki and A. Haghparast. Fractional delay filter design based on truncated Lagrange interpolation. *IEEE Signal Process. Lett.*, 14(11):816–819, 2007.
- V. Välimäki and T. Tolonen. Development and calibration of a guitar synthesizer. *J. Audio Eng. Soc.*, 46(9):766–778, 1998.

- V. Välimäki, J. Huopaniemi, M. Karjalainen, and Z. Jánosy. Physical modeling of plucked string instruments with application to real-time sound synthesis. *J. Audio Eng. Soc.*, 44(5):331–353, 1996.
- V. Välimäki, M. Ilmoniemi, and M. Huutilainen. Decomposition and modification of musical instrument sounds using a fractional delay all-pass filter. In *Proc. 6th Nordic Signal Process. Symp.*, pages 208–211, Espoo, Finland, June 2004a.
- V. Välimäki, H. Penttinen, J. Knif, M. Laurson, and C. Erkut. Sound synthesis of the harpsichord using a computationally efficient physical model. *EURASIP J. Appl. Signal Process.*, 2004(7):934–948, 2004b.
- V. Välimäki, J. Pakarinen, C. Erkut, and M. Karjalainen. Discrete-time modelling of musical instruments. *Rep. Progr. Phys.*, 69(1):1–78, 2006.
- V. Välimäki, H.-M. Lehtonen, and T. I. Laakso. Musical signal analysis using the fractional-delay inverse comb filters. In *Proc. Int. Conf. on Digital Audio Effects (DAFx-07)*, pages 261–268, Bordeaux, France, September 2007. <http://dafx.labri.fr/main/papers/p261.pdf>.
- S. A. Van Duyne and J. O. Smith. A simplified approach to modeling dispersion caused by stiffness in strings and plates. In *Proc. Int. Computer Music Conf.*, pages 407–410, Aarhus, Denmark, September 1994.
- S. A. Van Duyne and J. O. Smith. Developments for the commuted piano. In *Proc. Int. Computer Music Conf.*, pages 319–326, Banff, Canada, 1995.
- S. A. Van Duyne, J. R. Pierce, and J. O. Smith. Traveling wave implementation of a lossless mode-coupling filter and the wave digital hammer. In *Proc. Int. Computer Music Conf.*, pages 411–418, Aarhus, Denmark, 1994.
- M. van Walstijn. Parametric FIR design of propagation loss filters in digital waveguide string models. *IEEE Signal Process. Lett.*, 17(9):795–798, 2010.

- E. Wang and B. T. G. Tan. Application of wavelets to onset transients and inharmonicity of piano tones. *J. Audio Eng. Soc.*, 56(5):381–392, 2008.
- G. Weinreich. Coupled piano tones. *J. Acoust. Soc. Am.*, 62(6):1474–1484, 1977.
- R. W. Young. Inharmonicity of plain wire piano strings. *J. Acoust. Soc. Am.*, 24(3):267–273, 1952.
- S. Zambon, H.-M. Lehtonen, and B. Bank. Simulation of piano sustain-pedal effect by parallel second-order filters. In *Proc. Int. Conf. on Digital Audio Effects (DAFx-08)*, pages 199–204, Espoo, Finland, 2008. Available online at http://www.acoustics.hut.fi/dafx08/papers/dafx08_37.pdf.