Publication VI

Teppo Häyrynen, Jani Oksanen, and Jukka Tulkki. 2011. Dynamics of cavity fields with dissipative and amplifying couplings through multiple quantum two-state systems. Physical Review A, volume 83, number 1, 013801, 6 pages.

PHYSICAL REVIEW A **83**, 013801 (2011)

Dynamics of cavity fields with dissipative and amplifying couplings through multiple quantum two-state systems

Teppo Häyrynen,* Jani Oksanen, and Jukka Tulkki

Department of Biomedical Engineering and Computational Science, Aalto University School of Science and Technology, P. O. Box 12200, FI-00076 AALTO, Finland

(Received 20 September 2010; published 3 January 2011)

We consider simultaneous dissipative and amplifying coupling of cavity fields to multiple two-state systems. We derive a master equation for optical field in a leaky cavity coupled to a reservoir through multiple two-state systems. In our previous works we have limited our study to systems where the reservoir either solely absorbs energy (detector setup) or adds energy (amplifying setup) to the cavity through a single two-state system. In this work we allow both interactions simultaneously and derive a reduced dynamic model for the optical field. We also generalize our model to cover the coupling of the field to several two state systems and discuss its connection to macroscopic interaction, e.g., in semiconductors. Our model includes four physical parameters: the field two-state system coupling γ, the excitation and deexcitation couplings of the two-state system by the reservoir λ_A and λ_D, respectively, and the mirror losses of the cavity C. We solve the steady-state fields at different regimes of these physical parameters. Furthermore, we show that, depending on the parameters, our model can describe the operation of a detector, a light emitting diode, or a laser.

DOI: 10.1103/PhysRevA.83.013801 PACS number(s): 42.50.Nn, 42.50.Lc, 42.50.Ar

I. INTRODUCTION

Dynamics of optical fields interacting with quantum devices like single photon detector or active material reveal interesting phenomena of quantum optics. The progress in experiments have given insight into the single photon creation and annihilation, the statistics of collapsed fields [1,2], and the correlation and coherence properties of the fields [3–5].

We have previously studied a model where the optical field interacts with a two-state quantum system coupled to a reservoir [6,7]. The reservoir acted merely as a dissipative energy drain or as an energy source. We obtained a reduced quantum jump superoperators (QJS) for the field by averaging out the two-state system [6]. Taking mirror losses into account and using the QJSs we investigated how the coupling parameters affect the statistics of the field.

In this work we generalize the model to include simultaneously the dissipative and amplifying couplings to the reservoir. In contrast to our previous work the simultaneous dissipation and energy injection leads to finite reservoir temperatures and to steady-state solutions of the cavity field that will not occur if only one of the interactions is considered. The mirror losses of the cavity are also taken into account. Furthermore, we show that our model can be applied in modeling the optical field that interacts with the reservoir through several two-state systems and compare the results given by our model with the emission and absorption rates in semiconductors.

We will first derive the reduced QJSs for the field interacting with the reservoir through a single two-state system. By numerical comparisons with the solution of the full field two-state system reservoir setup we show that our model can be generalized to setups with multiple two-state systems. Finally, we complete the model by adding mirror losses of the cavity and calculate steady-state properties of the field at different parameter regimes. Using the reduced model we show that depending on the parameters our setup can operate as a light-emitting diode (LED), a laser, or a detector. We also discuss the relation of the coupling constants to the reservoir temperature and speculate about extending our model to semiconductor devices.

*teppo.hayrynen@hut.fi

II. REDUCED MASTER EQUATION OF A CAVITY FIELD COUPLED TO TWO-STATE SYSTEMS

Let γ describe the coupling of the field to a two-state system, λ_D describe the relaxation rate of the excited state $|e\rangle$ of the two-state system into the reservoir, and λ_A describe excitation rate of the ground state $|g\rangle$ of the two-state system by the reservoir. The density operator $\hat{\rho}_{\text{tot}}$ describing both the field and N_s two-state systems evolves according to the Lindblad master equation [8]

$$\frac{d\hat{\rho}_{\text{tot}}(t)}{dt} = -\frac{i}{\hbar}[\hat{H}\hat{\rho}_{\text{tot}}(t) - \hat{\rho}_{\text{tot}}(t)\hat{H}^\dagger] + \sum_{i=1}^{N_s}[2\lambda_D\sigma_-^{(i)}\hat{\rho}_{\text{tot}}(t)\sigma_+^{(i)} + 2\lambda_A\sigma_+^{(i)}\hat{\rho}_{\text{tot}}(t)\sigma_-^{(i)}], \tag{1}$$

where the Hamiltonian is the Jaynes-Cummings Hamiltonian

$$\hat{H} = \frac{1}{2}\hbar\omega_0\sigma_0 + \hbar\omega\hat{a}^\dagger\hat{a} + \hbar\gamma\sum_{i=1}^{N_s}[\hat{a}\sigma_+^{(i)} + \hat{a}^\dagger\sigma_-^{(i)}] - i\hbar\sum_{i=1}^{N_s}[\lambda_D\sigma_+^{(i)}\sigma_-^{(i)} + \lambda_A\sigma_-^{(i)}\sigma_+^{(i)}], \tag{2}$$

of two-state systems with eigenenergies $\pm\hbar\omega_0/2$ coupled to a photon mode having frequency ω with additional terms $i\hbar\lambda_D\sigma_+\sigma_-$ and $i\hbar\lambda_A\sigma_-\sigma_+$ describing the reservoir couplings. Furthermore, $\hat{a}$ and $\hat{a}^\dagger$ are the photon annihilation and creation operators, $\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|$, $\sigma_0 = |e\rangle\langle e| - |g\rangle\langle g|$, $\omega = \omega_0$ so exact resonance is assumed, and $\sigma_\pm^{(i)}$ operates on the i^{th} two-state system. In this model the energy adding event is the excitation of the two-state system by the reservoir. Correspondingly, the dissipation event is the relaxation of the two-state system releasing its energy into the reservoir. We have assumed that each two-state system has the same coupling constants λ_A, λ_D, and γ. First we set $N_s = 1$ and calculate the dissipation and amplification rates and QJSs. Then we generalize the rates and the QJSs for the cases with $N_s > 1$. In Ref. [6] we have calculated cases where the reservoir only absorbs energy (i.e., $\lambda_A = 0$ and $\lambda_D > 0$) and where the reservoir only injects energy (i.e., $\lambda_D = 0$ and $\lambda_A > 0$) into the system. In this manuscript we will consider a case where both $\lambda_A > 0$ and $\lambda_D > 0$. This corresponds to a reservoir at finite temperature as will be shown below.

A. Single two-state system

We will first calculate the reduced amplification and dissipation rates and then form the corresponding QJSs. As in the standard Jaynes-Cummings model [9] we can divide the density operator in blocks $\hat{\rho}_{n+1}$ describing the evolution of the states $|g,n+1\rangle$ and $|e,n\rangle$ (i.e., the ground state with $n+1$ photons and the excited state with n photons). The evolution of the density operator $\hat{\rho}_{n+1}$ for the case of a single two-state system is obtained from Eq. (1) by setting $N_s = 1$ as

$$\frac{d}{dt}\hat{\rho}_{n+1} = \begin{bmatrix} -2\lambda_D\rho_{ee}^{n+1} + 2\lambda_A\rho_{gg}^{n} - \mathrm{i}\gamma\sqrt{n+1}\left(\rho_{ge}^{n+1} - \rho_{eg}^{n+1}\right) & -(\lambda_D+\lambda_A)\rho_{eg}^{n+1} - \mathrm{i}\gamma\sqrt{n+1}\left(\rho_{gg}^{n+1} - \rho_{ee}^{n+1}\right) \\ -(\lambda_D+\lambda_A)\rho_{ge}^{n+1} - \mathrm{i}\gamma\sqrt{n+1}\left(\rho_{ee}^{n+1} - \rho_{gg}^{n+1}\right) & 2\lambda_D\rho_{ee}^{n+2} - 2\lambda_A\rho_{gg}^{n+1} - \mathrm{i}\gamma\sqrt{n+1}\left(\rho_{eg}^{n+1} - \rho_{ge}^{n+1}\right) \end{bmatrix}, \quad (3)$$

where ρ_{ee}^{n+1} and ρ_{gg}^{n+1} (the diagonal elements of $\hat{\rho}_{n+1}$) are the probabilities of states $|e,n\rangle$ and $|g,n+1\rangle$, respectively, and ρ_{eg}^{n+1} and ρ_{ge}^{n+1} (the off-diagonal elements of $\hat{\rho}_{n+1}$) describe the interactions of these states. Note that $\hat{\rho}_{n+1}$ interacts with $\hat{\rho}_n$ and $\hat{\rho}_{n+2}$ and that $\frac{d}{dt}\rho_{gg}^0 = -2\lambda_A\rho_{gg}^0 + 2\lambda_D\rho_{ee}^1$.

Next we assume that the two-state system achieves equilibrium with the instantaneous photon distribution. With the equilibrium excited-state and ground-state probabilities, solved from the steady-state equations of motion, we determine the photon creation and annihilation rates. In the steady-state we obtain the off-diagonal elements as

$$\rho_{eg}^{n+1} = -\rho_{ge}^{n+1} = \frac{\mathrm{i}\gamma\sqrt{n+1}}{\lambda_D+\lambda_A}\left(\rho_{ee}^{n+1} - \rho_{gg}^{n+1}\right). \quad (4)$$

Substitution of Eq. (4) into the two remaining steady-state equations obtained from Eq. (3) gives

$$0 = -2\lambda_D\rho_{ee}^{n+1} + 2\lambda_A\rho_{gg}^{n} - \frac{2\gamma^2(n+1)}{\lambda_D+\lambda_A}\left(\rho_{ee}^{n+1} - \rho_{gg}^{n+1}\right) \quad (5)$$

$$0 = 2\lambda_D\rho_{ee}^{n+2} - 2\lambda_A\rho_{gg}^{n+1} + \frac{2\gamma^2(n+1)}{\lambda_D+\lambda_A}\left(\rho_{ee}^{n+1} - \rho_{gg}^{n+1}\right). \quad (6)$$

Next we change the notation from the elements of density matrix to probabilities and use the fact that the probability of any n photon state is given as a sum of the probabilities of the states $|g,n\rangle$ and $|e,n\rangle$. Thus, by substituting $\rho_{gg}^{n+1} = p_g^{n+1}, \rho_{ee}^{n+1} = p_e^n, p_g^{n+1} = p_{n+1} - p_e^{n+1}$ and $p_e^n = p_n - p_g^n$ we obtain

$$0 = -2\lambda_D\left(p_n - p_g^n\right) + 2\lambda_A p_g^n - \frac{2\gamma^2(n+1)}{\lambda_D+\lambda_A}\left(p_e^n - p_g^{n+1}\right) \quad (7)$$

$$0 = 2\lambda_D p_e^{n+1} - 2\lambda_A\left(p_{n+1} - p_e^{n+1}\right) + \frac{2\gamma^2(n+1)}{\lambda_D+\lambda_A}\left(p_e^n - p_g^{n+1}\right) \quad (8)$$

giving

$$p_e^{n+1} = \frac{\lambda_A(\lambda_D+\lambda_A)p_{n+1} + \gamma^2(n+1)p_g^{n+1} - \gamma^2(n+1)p_e^n}{(\lambda_D+\lambda_A)^2} \quad (9)$$

$$p_g^n = \frac{\lambda_D(\lambda_D+\lambda_A)p_n - \gamma^2(n+1)p_g^{n+1} + \gamma^2(n+1)p_e^n}{(\lambda_D+\lambda_A)^2}. \quad (10)$$

Next in Eq. (9) we approximate $p_e^n \approx p_e^{n+1}$and in Eq. (10) we approximate $p_g^{n+1} \approx p_g^n$. These approximations can be made since the steady-state photon distributions are usually smooth, except in a case of high dissipation $\lambda_A \ll \lambda_D$ where a vacuum field is obtained as a steady-state solution. However, in that case also the reduced model gives a vacuum field as a steady-state solution. In addition to the approximations we also substitute $p_g^{n+1} = p_{n+1} - p_e^{n+1}$ into Eq. (9) and $p_e^n = p_n - p_g^n$ into Eq. (10) giving

$$p_e^{n+1} \approx \frac{\lambda_A(\lambda_D+\lambda_A) + \gamma^2(n+1)}{(\lambda_D+\lambda_A)^2 + 2\gamma^2(n+1)}p_{n+1} \quad (11)$$

$$p_g^n \approx \frac{\lambda_D(\lambda_D+\lambda_A) + \gamma^2(n+1)}{(\lambda_D+\lambda_A)^2 + 2\gamma^2(n+1)}p_n. \quad (12)$$

Furthermore, the total approximate ground-state and excited-state probabilities are $p_g = \sum_{n=0}^{\infty} p_g^n$ and $p_e = \sum_{n=0}^{\infty} p_e^n$.

The dissipative and energy adding terms $2\lambda_D\sigma_-\hat{\rho}_{\text{tot}}(t)\sigma_+$ and $2\lambda_A\sigma_+\hat{\rho}_{\text{tot}}(t)\sigma_-$ in the master equation (1) give, by direct calculation, that the dissipation and amplification rates are $2\lambda_D p_e(t)$ and $2\lambda_A p_g(t)$. Therefore, using the approximate ground-state and excited-state probabilities we obtain the following dissipation and amplification rates

$$r_D = 2\lambda_D \sum_{n=0}^{\infty} \frac{\frac{\lambda_A}{\lambda_D+\lambda_A} + \left(\frac{\gamma}{\lambda_D+\lambda_A}\right)^2 n}{1 + 2\left(\frac{\gamma}{\lambda_D+\lambda_A}\right)^2 n} p_n \tag{13}$$

$$r_A = 2\lambda_A \sum_{n=0}^{\infty} \frac{\frac{\lambda_D}{\lambda_D+\lambda_A} + \left(\frac{\gamma}{\lambda_D+\lambda_A}\right)^2 (n+1)}{1 + 2\left(\frac{\gamma}{\lambda_D+\lambda_A}\right)^2 (n+1)} p_n. \tag{14}$$

Setting $\lambda_A = 0$ in Eqs. (11) and (13) reproduces the excited-state probability and the dissipation rate obtained in Ref. [6] for the dissipative (detector) setup. Similarly, setting $\lambda_D = 0$ in Eqs. (12) and (14) gives the ground-state probability and the amplification rate obtained in Ref. [6] for the amplifier setup.

The reduced master equation for the field can now be written as

$$\frac{d\hat{\rho}}{dt} = -\frac{A_D}{2}(\hat{O}^\dagger\hat{O}\hat{\rho} - 2\hat{O}\hat{\rho}\hat{O}^\dagger + \hat{\rho}\hat{O}^\dagger\hat{O}) - \frac{A_A}{2}(\hat{O}\hat{O}^\dagger\hat{\rho} - 2\hat{O}^\dagger\hat{\rho}\hat{O} + \hat{\rho}\hat{O}\hat{O}^\dagger). \tag{15}$$

By comparing the rates in Eqs. (13) and (14) with the rates obtained from the reduced Lindblad master equation we can identify the operators in the dissipative term $A_D\hat{O}\hat{\rho}\hat{O}^\dagger$ and in the amplifying term $A_A\hat{O}^\dagger\hat{\rho}\hat{O}$ as

$$\hat{O} = \frac{1}{\sqrt{1 + B\hat{a}\hat{a}^\dagger}}\hat{a} \tag{16}$$

$$B = 2\frac{\gamma^2}{(\lambda_D + \lambda_A)^2} \tag{17}$$

$$A_A = \lambda_A B \text{ and } A_D = \lambda_D B. \tag{18}$$

Note that the terms independent of n in the numerators of the rates in Eqs. (13) and (14) cancel in the master equation. The reduced Lindblad master equation (15) no longer includes the two-state system, but the model still captures the average evolution of the field as shown in Fig. 1, where the numerical solution of the full model is compared with the reduced model. Figure 1 shows that the reduced model accurately gives the expectation value of the number of photons and the photon number distribution. The results in Fig. 1 focus on parameter regime where both type of the reservoir interactions present. By setting $\lambda_A = 0$ or $\lambda_D = 0$ we obtain the dissipative or energy adding scheme, respectively, derived in Ref. [6].

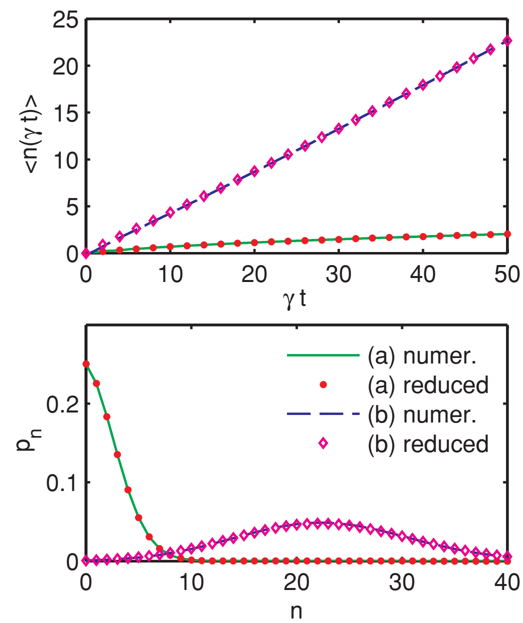

FIG. 1. (Color online) Comparison of the reduced model and the numerical solution of the full system. In case (a) $\lambda_D = 0.1\gamma$ and $\lambda_A = 0.1\gamma$ and in case (b) $\lambda_D = 0.5\gamma$ and $\lambda_A = 1.0\gamma$. The upper figure shows the expectation values of the number of photons while the lower figure shows the photon distribution at $\gamma t = 50$. The full system was initially in the state $|g,0\rangle$ and the reduced system in the state $|0\rangle$. Note that the solution given by the reduced model accurately follows the exact solution.

B. Multiple two-state systems

Next we will consider a single mode optical field coupled to a reservoir through multiple two-state systems by setting $N_s > 1$ in Eqs. (1) and (2). We assume that the two-state systems do not interact with each other directly, but they do interact with the field and with the reservoir. The dipole-dipole interactions between the two-state systems can be neglected by assuming that wave functions of the different two-state systems do not overlap as is done in the standard Dicke model (or Tavis-Cummings model) [10]. The assumption can be made in typical atomic systems since the dipole-dipole interaction is proportional to $1/\text{distance}^3$ [11]. Furthermore, we have assumed that each two-state system has the same coupling constants λ_A, λ_D, and γ. Since the two-state systems do not interact directly with each other, the reduced master equation (15) can be generalized for multiple two-state systems by adding separate dissipative and amplifying terms for each of the two-state systems. The terms are equal due to the equal coupling constants. Therefore, the reduced master equation for multiple two-state systems is obtained by scaling the A parameters with N_S as

$$A_A \to N_s A_A = N_s \lambda_A B \tag{19}$$

$$A_D \to N_s A_D = N_s \lambda_D B. \tag{20}$$

Parameter B is not scaled. In Fig. 2 we show the comparison of reduced and full models for setup with $N_s = 3$. The expectation value of the number of photons and the photon distribution are calculated with three different parameter sets: (a) $\lambda_D = 1.0\gamma$, $\lambda_A = 1.0\gamma$, and $\hat{\rho}(0) = |0\rangle\langle 0|$; (b) $\lambda_D = 2.0\gamma$, $\lambda_A = 3.0\gamma$, and $\hat{\rho}(0) = |0\rangle\langle 0|$; (c) $\lambda_D = 0.5\gamma$, $\lambda_A = 0$, and $\hat{\rho}(0) = |10\rangle\langle 10|$. The two-state systems are initially in the ground state. The reduced model reproduces the results of the full model except for the Rabi-type oscillations [case (c) in Fig. 2] as we also pointed out in the case of purely amplifying or dissipative setup in Refs. [6,7]. However, for $N_s \gg 1$ the phases of the two-state systems in real systems are randomly distributed and the Rabi oscillation are expected to be averaged out naturally. Thus we expect that the reduced model is more accurate for cases $N_S \gg 1$ than for case $N_s = 1$. Results of Fig. 3 support this assumption. In Fig. 3 we show comparison of the reduced and full models with $N_s = 3$ and

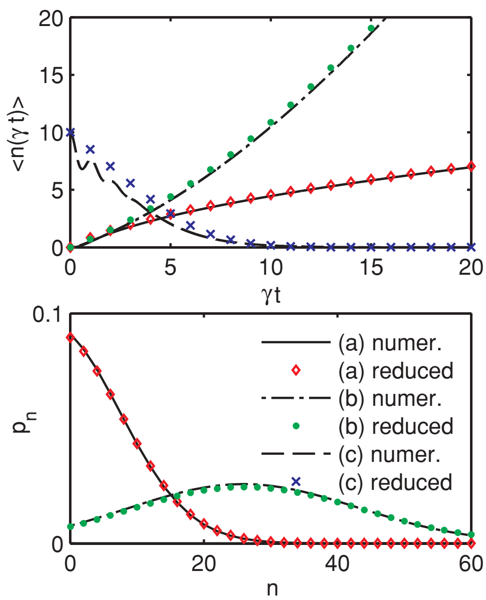

FIG. 2. (Color online) Comparison of the reduced model and the full model with three two-state systems. The upper figure shows the expectation value of the number of photons and the lower figure shows the photon distribution at $\gamma t = 20$. (a) $\lambda_D = 1.0\gamma$, $\lambda_A = 1.0\gamma$, and $\hat{\rho}(0) = |0\rangle\langle 0|$; (b) $\lambda_D = 2.0\gamma$, $\lambda_A = 3.0\gamma$, and $\hat{\rho}(0) = |0\rangle\langle 0|$; and (c) $\lambda_D = 0.5\gamma$, $\lambda_A = 0$, and $\hat{\rho}(0) = |10\rangle\langle 10|$. The two-state systems are initially in the ground state. The probability distributions in case (c) are not shown since $p_0 = 1$.

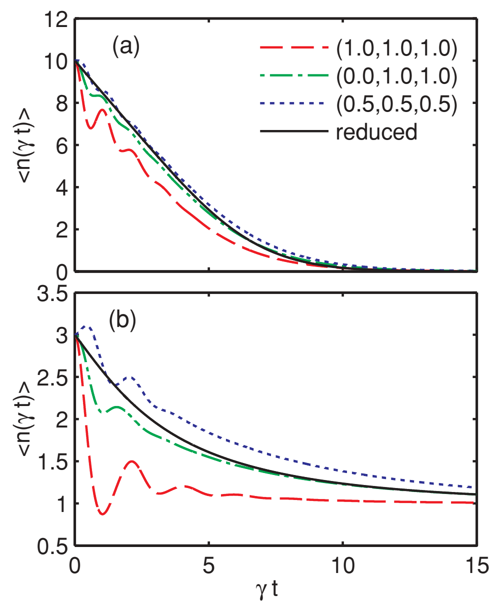

FIG. 3. (Color online) Comparison of the reduced model with the full model having the two-state systems set into varying initial states. (a) $\lambda_A = 0$, $\lambda_D = 0.5\gamma$, $\hat{\rho}(0) = |10\rangle\langle 10|$ [as in Fig. 2 (c)] and (b) $\lambda_A = 0.15\gamma$, $\lambda_D = 0.3\gamma$, $\hat{\rho}(0) = |3\rangle\langle 3|$. The initial states of the two-state systems are given in form (p_g^a, p_g^b, p_g^c). Results suggest that the Rabi oscillations average out for setups with $N_s \gg 1$ two-state systems in random initial states.

the two-state systems set into varying initial states, which corresponds to the randomly distributed phases of the two-state systems.

III. STEADY-STATE SOLUTIONS OF THE REDUCED MODEL WITH MIRROR LOSSES

To make the model more general, we also include a term describing the mirror losses by adding a jump term $C\hat{a}\hat{\rho}\hat{a}^\dagger$ to the reduced master equation with $C = \omega/Q$ (ω is the frequency and Q is the quality factor of the cavity [12]) which is linear with respect to the photon number. The reduced master equation for the field can now be written as

$$\frac{d\hat{\rho}}{dt} = -\frac{A_D}{2}(\hat{O}^\dagger\hat{O}\hat{\rho} - 2\hat{O}\hat{\rho}\hat{O}^\dagger + \hat{\rho}\hat{O}^\dagger\hat{O}) - \frac{A_A}{2}(\hat{O}\hat{O}^\dagger\hat{\rho} - 2\hat{O}^\dagger\hat{\rho}\hat{O} + \hat{\rho}\hat{O}\hat{O}^\dagger) - \frac{C}{2}(\hat{a}^\dagger\hat{a}\hat{\rho} - 2\hat{a}\hat{\rho}\hat{a}^\dagger + \hat{\rho}\hat{a}^\dagger\hat{a}). \quad (21)$$

Depending on parameters γ, λ_D, λ_A, and C, equation (21) can be used to model light-emitting diodes and lasers or detector setups as will be shown below.

From master equation (21) we can obtain the following differential equation for the probability of having n photons in the field

$$\frac{dp_n(t)}{dt} = -\frac{A_D n}{1+Bn}p_n - \frac{A_A(n+1)}{1+B(n+1)}p_n - Cnp_n + \frac{A_D(n+1)}{1+B(n+1)}p_{n+1} + \frac{A_A n}{1+Bn}p_{n-1} + C(n+1)p_{n+1}. \quad (22)$$

The steady-state solution (with the detailed balance condition) is

$$p_n = p_0 \prod_{k=1}^{n} \frac{\frac{A_A}{A_D+C}}{1+\frac{BC}{A_D+C}k} \quad (23)$$

with $p_0 = (\sum_{n=0}^{\infty}\prod_{k=1}^{n}\frac{\frac{A_A}{A_D+C}}{1+\frac{BC}{A_D+C}k})^{-1}$. With the approach used in Ref. [6] the steady-state photon number and the second factorial moment are obtained as

$$\bar{n}_{ss} = \frac{A_A-(A_D+C)}{BC} + \frac{A_D+C}{BC}p_0 \quad (24)$$

$$\overline{n(n-1)}_{ss} = \frac{A_A-(A_D+C)}{BC}\bar{n} + \frac{A_D+C}{BC}(1-p_0). \quad (25)$$

Furthermore, Eqs. (24)–(25) give the second-order coherence degree [12,13] $g^{(2)}(t,t) = \overline{n(n-1)}(t)/\bar{n}^2(t)$ as

$$g_{ss}^{(2)} = \frac{\left[\frac{A_A-(A_D+C)}{BC}\right]\left[\frac{A_A-(A_D+C)}{BC} + \frac{A_D+C}{BC}p_0\right] + \frac{A_D+C}{BC}(1-p_0)}{\left[\frac{A_A-(A_D+C)}{BC} + \frac{A_D+C}{BC}p_0\right]^2}. \quad (26)$$

A. LED and laser operation

If amplification is smaller than losses $A_A < A_D + C$ and $BC \ll A_D + C$ Eq. (23) can be simplified into

$$p_n = \left(1 - \frac{A_A}{A_D + C}\right)\left(\frac{A_A}{A_D + C}\right)^n \tag{27}$$

$$\bar{n} = 1\Big/\left(\frac{A_D + C}{A_A} - 1\right), \tag{28}$$

which corresponds to a thermal field with $\frac{A_D+C}{A_A} = \exp(\frac{\hbar\omega}{k_B T})$, where k_B is the Boltzmann constant and T is the temperature. Thus, under these conditions, the setup operates as an LED. If, on the other hand, amplification is greater than losses $A_A > A_D + C$ and $BC \gg A_D + C$ we obtain

$$p_n = e^{-\lambda_A/C}\frac{(\lambda_A/C)^n}{n!} \tag{29}$$

$$\bar{n} = \lambda_A/C, \tag{30}$$

which is the Poisson distribution and, therefore, a coherent field is obtained. In this regime the setup operates as a laser.

B. Relation of the reservoir temperature to the coupling parameters

The standard model of a single mode cavity field coupled to a thermal reservoir is governed by the following Lindblad master equation [12]

$$\frac{d\hat{\rho}}{dt} = -\frac{\xi}{2}\bar{n}_{th}(\hat{a}\hat{a}^\dagger\hat{\rho} - 2\hat{a}^\dagger\hat{\rho}\hat{a} + \hat{\rho}\hat{a}\hat{a}^\dagger) - \frac{\xi}{2}(\bar{n}_{th} + 1)(\hat{a}^\dagger\hat{a}\hat{\rho} - 2\hat{a}\hat{\rho}\hat{a}^\dagger + \hat{\rho}\hat{a}^\dagger\hat{a}), \tag{31}$$

where ξ is the coupling and $\bar{n}_{th}$ is the mean number of photons in the thermal reservoir. The steady-state solution is, of course, a thermal field with $\bar{n}_{th}$ photons. To compare our model to this result we assume that the cavity mirrors are perfect ($C = 0$) and that the amplification is smaller than dissipation ($A_A < A_D$). We obtain probabilities $p_n = [(\lambda_D - \lambda_A)/\lambda_D](\lambda_A/\lambda_D)^n$ and, furthermore, the steady-state photon number is $\bar{n}_{ss} = (\frac{\lambda_D}{\lambda_A} - 1)^{-1} = \bar{n}_{th} = [\exp(\frac{\hbar\omega}{k_B T}) - 1]^{-1}$. Comparison shows that

$$\frac{\lambda_D}{\lambda_A} = \exp\left(\frac{\hbar\omega}{k_B T}\right), \tag{32}$$

which means that adjusting the excitation and deexcitation rates of the two-state system corresponds to setting the temperature of the reservoir.

C. Comparison to semiconductor devices

In semiconductors the absorption and emission rates are given by [14] $r_{\text{abs}} = W(1 - f_e)(1 - f_h)\bar{n}$ and $r_{\text{em}} = W f_e f_h(\bar{n} + 1)$, respectively, where W is a material-dependent constant and f_e and f_h are the electron and hole occupation probabilities in the conduction and valence bands, respectively. By comparing these rates to the rates given by our reduced model in Eq. (15) we obtain equations

$$\sum_{n=0}^{\infty}\frac{A_D n}{1 + Bn}p_n = W(1 - f_e)(1 - f_h)\bar{n} \tag{33}$$

$$\sum_{n=0}^{\infty}\frac{A_A(n + 1)}{1 + B(n + 1)}p_n = W f_e f_h(\bar{n} + 1). \tag{34}$$

Since we have three parameters in our model we need a third equation to solve them. The steady-state photon number for semiconductor devices can be solved from equation emission = absorption + mirror losses which gives $\bar{n}_{ss} = W f_e f_h/[W(1 - f_e)(1 - f_h) + C - W f_e f_h]$. Setting this solution equal with equation (24) gives the third equation. Using these three relations enables us to solve A_A, A_D, and B as a functions of f_e, f_h, and W. A purely amplifying system ($A_D = 0$) is recovered when $f_e = 1$ or $f_h = 1$ and a purely dissipative system ($A_A = 0$) when $f_h = 0$ or $f_e = 0$. Solving equation (33) and (34) is generally not straightforward analytically. However, for the two limiting cases of a purely spontaneous emission and a laser field, the parameters can be obtained in simple forms as shown below.

From small fields in the regime $B\bar{n} \ll 1$ it is straightforward to approximate the parameters A_D, A_A, and B as

$$A_D = W(1 - f_e)(1 - f_h) \tag{35}$$

$$A_A = W f_e f_h \tag{36}$$

$$B \approx 0. \tag{37}$$

For laser fields we use Eq. (30) to write $B = A_A/(C\bar{n})$ and substitute it into Eqs. (33) and (34) giving

$$\sum_{n=0}^{\infty}\frac{A_D n}{1 + \frac{A_A n}{C\bar{n}}}p_n \approx \frac{A_D}{1 + A_A/C}\bar{n} = W(1 - f_e)(1 - f_h)\bar{n} \tag{38}$$

$$\sum_{n=0}^{\infty}\frac{A_A(n + 1)}{1 + \frac{A_A(n+1)}{C\bar{n}}}p_n \approx \frac{A_A}{1 + A_A/C}(\bar{n} + 1) = W f_e f_h(\bar{n} + 1), \tag{39}$$

where we have assumed $n/\bar{n} \approx 1$ in the denominators. Parameters A_D, A_A, and B can now be evaluated as

$$A_D = \frac{W(1 - f_e)(1 - f_h)}{1 - \frac{W f_e f_h}{C}}, \tag{40}$$

$$A_A = \frac{W f_e f_h}{1 - \frac{W f_e f_h}{C}}, \tag{41}$$

$$B = \frac{A_A}{C\bar{n}}. \tag{42}$$

IV. DETECTOR SCHEME

The evolution of a state $\hat{\rho}(t)$ during an infinitesimal time dt is described by a decomposition of the form

$$\hat{\rho}(t + dt) = \hat{J}\hat{\rho}(t)dt + \hat{S}(dt)\hat{\rho}(t), \tag{43}$$

where the two terms describe the possible quantum trajectories of the system during $[t,t+dt)$. In photodetection these trajectories correspond to the events where a photon is either detected

$$\hat{J}\hat{\rho}(t)dt = A_D\hat{O}\hat{\rho}(t)\hat{O}^\dagger dt \tag{44}$$

or not detected

$$\hat{S}(dt)\hat{\rho}(t) = \exp\left(-\frac{A_D}{2}\hat{O}^\dagger\hat{O}dt\right)\hat{\rho}(t)\exp\left(-\frac{A_D}{2}\hat{O}^\dagger\hat{O}dt\right) \tag{45}$$

during $[t,t+dt)$ [6,7,15]. Our setup can be considered as an ideal single-photon subtraction (detection) scheme if the amplification is set to zero ($\lambda_A = 0$) and, furthermore, we assume a nonleaking cavity ($C = 0$). In this case the one-count operator $A_D\hat{O}\hat{\rho}\hat{O}^\dagger$ corresponding to single-photon detection is

$$\hat{J}\hat{\rho} = \frac{2\gamma^2}{\lambda_D}\left[1+2\left(\frac{\gamma}{\lambda_D}\right)^2\hat{a}\hat{a}^\dagger\right]^{-1/2} \times\hat{a}\hat{\rho}\hat{a}^\dagger\left[1+2\left(\frac{\gamma}{\lambda_D}\right)^2\hat{a}\hat{a}^\dagger\right]^{-1/2}. \tag{46}$$

In the limit $\gamma^2\bar{n} \ll \lambda_D^2$ the one-count operator (46) simplifies to $\hat{J}\hat{\rho} = \frac{2\gamma^2}{\lambda_D}\hat{a}\hat{\rho}\hat{a}^\dagger$ [6,7]. In contrast, in the limit $\gamma^2\bar{n} \gg \lambda_D^2$ the one-count operator (46) simplifies to $\hat{J}\hat{\rho} = \lambda_D\hat{E}\hat{\rho}\hat{E}^\dagger$, where $\hat{E} = (1+\hat{a}^\dagger\hat{a})^{-1/2}\hat{a}$ [6,7]. The first limit coincides with the Srinivas-Davies model [16] describing nonsaturated detectors while the second limit coincides with the model introduced in Ref. [17] describing fully saturated detectors.

V. CONCLUSIONS

We have derived a quantum trajectory based model describing a single-mode optical field in optoelectronic devices. Our model accounts for field-material coupling, the pumping and the losses of the material, and the mirror losses of the cavity. We have shown that the reduced model can be applied not only to fields interacting with a reservoir through a single two-level system but also fields interacting with several two-level systems or macroscopic systems like semiconductors. As examples of the potential applications of the model, we have applied our model to describe the well-known statistical properties of an optical field coupled to a detector and the statistics of active optical components like LEDs and lasers.

We expect our model to be generally applicable to model the photon statistics of different optical components or setups. In particular our model is applicable to describe recent experiments where the cavity field is produced or analyzed using an atomic beam [2,18] and we expect that our model provides a quite useful tool for analyzing and designing new experiments.

ACKNOWLEDGMENTS

This work was partly funded by the Multidisciplinary Institute of Digitalisation and Energy program (MIDE) of Aalto University.

[1] Valentina Parigi, Alessandro Zavatta, Myungshik Kim, and Marco Bellini, Science **317**, 1890 (2007).
[2] Sebastien Gleyzes, Stefan Kuhr, Christine Guerlin, Julien Bernu, Samuel Deleglise, Ulrich Busk Hoff, Michel Brune, Jean-Michel Raimond, and Serge Haroche, Nature (London) **446**, 297 (2007).
[3] M. Assmann, F. Veit, M. Bayer, M. van der Poel, and M. Hvam, Science **325**, 297 (2009).
[4] Yuan Li, Yu-Chi Zhang, Peng-Fei Zhang, Yan-Qiang Guo, Gang Li, Jun-Min Wang, and Tian-Cai Zhang, Chin. Phys. Lett. **26**, 074205 (2009).
[5] Y. Li, G. Li, Y. C. Zhang, X. Y. Wang, J. Zhang, J. M. Wang, and T. C. Zhang, Phys. Rev. A **76**, 013829 (2007).
[6] T. Häyrynen, J. Oksanen, and J. Tulkki, Phys. Rev. A **81**, 063804 (2010).
[7] T. Häyrynen, J. Oksanen, and J. Tulkki, Eur. Phys. J. D **56**, 113 (2010).
[8] C. W. Gardiner and P. Zoller, *Quantum Noise*, 3rd ed. (Springer, Berlin, 2004).
[9] Jin-Sheng Peng and GaoXiang Li, *Introduction to Modern Quantum Optics* (World Scientific, Singapore, 1998).
[10] Andrei B. Klimov and Sergei M. Chumakov, *A Group-Theoretical Approach to Quantum Optics* (Wiley-VCH, New York, 2009).
[11] Gang Chen, Zheng-Yuan Xue, L. F. Wei, and J.-Q. Liang, Europhys. Lett. **86**, 44002 (2009).
[12] M. O. Scully and M. S. Zubairy, *Quntum Optics* (Cambridge University Press, Cambridge, UK, 1997).
[13] Rodney Loudon, *The Quantum Theory of Light* (Oxford University Press, Oxford, UK, 1983).
[14] Jasprit Singh, *Semiconductor Optoelectronics: Physics and Technology* (McGraw-Hill, New York, 1995).
[15] Teppo Häyrynen, Jani Oksanen, and Jukka Tulkki, J. Phys. B **42**, 145506 (2009).
[16] M. D. Srinivas and E. B. Davies, J. Mod. Opt. **28**, 981 (1981).
[17] M. C. de Oliveira, S. S. Mizrahi, and V. V. Dodonov, J. Opt. B **5**, S271 (2003).
[18] G. Nogues, A. Rauschenbeutel, S. Osnaghi, M. Brune, J. M. Raimond, and S. Haroche, Nature (London) **400**, 239 (1999).