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Comparison of various sources of uncertainty in stand-level net present value estimates

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ABSTRACT

The objective of this study was to compare the relative importance of various sources of uncertainties in determining the net present value of forest stands and forested property. This was achieved by performing stand-level simulations that took into account: i) input data errors (airborne laser-scanning data vs. ocular standwise field inventory data), ii) stochastic future development of timber assortment prices and iii) errors in stand-level growth projection models. The starting point of the study was a simulated forest estate comprising 40 stands of various types sufficiently represented (e.g. with respect to species composition, development class distribution, and site quality). Stochastic timber price models were formulated, employing geometric mean-reverting principles. The results showed that sources of uncertainty all had significant effects on the probability distribution of the net present value of the stand. The relative standard deviations of stand net present values averaged 8% for stochastic timber price, 29% for errors in standwise field inventory data, 26% for errors in airborne laser-scanning data and 33% for errors in growth projection models when applying a 3% discount rate. When all three sources of uncertainty were analysed simultaneously, the highest average standard deviation was 47.4%. Interestingly, errors in the growth projections and the quality of inventory data contributed more to the variation in stand net present value than fluctuation in timber price did, although this result was based on the assumption that the forestry industry maintains its competitiveness in the long run. Our modeling approach made it possible to compare various sources of uncertainty and to set confidence intervals for net present value estimates. This approach can also result in information on which sources of uncertainty are focused.

1. Introduction

The economical value of forests is crucial information for landowners and various forestry organizations. Estimates of the value of forest estates are needed for many purposes, e.g. in real estate business, land divisions and exchanges and for considering forestry investment. The need for determining the value and the value development of forests has become more important since forests are increasingly considered as one possible investment outlet amongst other real or financial assets. International Financial Reporting Standards (IFRS) requires that forest enterprises present systematically computed estimates of the value of their forested land annually. The methodologies used for assessing forest value vary among organizations. Probably, the most common method currently used for assessing forest estate value is computing the net present value (NPV) of forests based on predicted future cash transactions. The sales comparison approach, in which the value of a forest estate is determined using historical data covering market prices or realized forest estate sales in the same region, is also used to some extent.

The market price of forested land is highly dependent on its characteristics, e.g. the amount of timber, fertility of the soil etc. Thus, it may not always be possible to obtain good estimates on the value of individual estates by using information on past sales in the region. In such cases, computing the NPV of forests based on predicted flows of future revenues and costs gives more accurate and better justified estimates of land value and its probability distribution.

The NPV of forested land is subject to various uncertainties. Some of these uncertainties are economic (e.g. price and interest rate developments), some are related to the quality of inventory data, while others are related to forest growth and occurrence of natural hazards. The economic literature on forest stand management has focused on studying the effects of stochastic timber prices (e.g. Brazee and Mendelsohn, 1988; Thomson, 1992; Insley and Rollins, 2005) on optimal harvesting. Most economic studies typically include only one source of uncertainty at a time. As exceptions, Valsta (1992) accounted for both stochastic tree growth and natural hazards, and Reed and Haight (1996) for both price risk and tree growth uncertainty when

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predicting the NPV distributions of forest stands. Alvarez and Koskela (2007) investigated optimal timber harvesting under both stock and price uncertainty, using a theoretical model.

The starting point for evaluating forested property must always be forest inventory data describing the property's timber resources, sites and possible values other than as related to timber production. If the computational input data are inaccurate, it will significantly affect the functionality of the simulation models, resulting in unrealistic simulation outputs and correspondingly suboptimal optimization solutions. Various sources of error may, of course, also have countereffects. The accuracy of this information is strongly dependent on the inventory methodology used. In practice, ocular standwise field inventories carried out either for forest-planning purposes or separately have been used. Future information obtained by laser scanning and mensuration of aerial photography will be utilized increasingly.

The mean errors of traditional ocular standwise field inventory used in operational forest management planning vary for mean volume from 16% to 38% in Finland (Poso, 1983; Haara and Korhonen, 2004; Saari and Kangas, 2005). One problem in standwise field inventory is the poor accuracy of species-specific estimates. For example Haara and Korhonen (2004) obtained relative root-mean-squared errors (RMSEs) of 29%, 43% and 65%, for volumes of Scots pine (*Pinus sylvestris* L.), Norway spruce (*Picea abies* L.H Karst.) and birch (*Betula* L. spp.), respectively. In addition, standwise field inventory often contains significant bias (0–25%), that varies with the person doing the inventory, due to the subjective nature of the method.

Airborne laser scanning (ALS) is the most accurate remote-sensing technique for standwise forest inventory providing accuracies ranging between 10% and 27% for the mean volume at stand-level (e.g. Næsset, 1997, 2002; Lim et al., 2003; Holmgren, 2003; Packalén and Maltamo, 2006). Current data acquisition costs are comparable to those of standwise field inventory. ALS devices providing small-footprint diameters (10–30 cm) allow accurate height determination of the forest canopy. The two main approaches to deriving forest information from small-footprint ALS data have been those based on laser canopy height distribution (area-based method, Næsset, 1997) and individual tree detection (Hyyppä and Inkinen, 1999; Persson et al., 2002; Popescu et al., 2003).

Some of the causes of uncertainty in evaluating forest value include growth and yield projection models. The errors in forest growth projections can be caused by model misspecification, random estimation errors of the model coefficients and residual variation of the models, i.e. the random variation inherent in most natural processes (Kangas, 1999). A number of studies focusing on the effects of various uncertainty components in forest growth projection systems exist (Gertner and Dzialowy, 1984; Mowrer, 1991; Kangas, 1997).

Sources of uncertainty in forest NPV predictions can be taken into account using various forest-planning scenarios (e.g. Pukkala, 2005, 2006). One approach to calculating a forest property's NPV is to determine it as a sum of individual stand or logging site NPVs. Pukkala (2005) adopted this approach and derived models for mineral soil Scots pine, Norway spruce and silver birch (*Betula pendula* Roth) stands by which the stand NPVs can be calculated as functions of temperature sum, interest rate, timber assortment prices, site quality and stock structure. The models derived are based on thousands of stand-level optimization runs in which stand treatment was optimized by maximizing NPV, using varying assortment prices and interest rates.

Our hypothesis is that uncertainties related to errors in input data, errors in growth projections and variations in timber assortment prices have a decisive influence on a stand's NPV and on the economic value of a forest estate. However, to our knowledge there are no studies that simultaneously incorporate economic risk, quality of inventory data and growth and yield projection errors in the valuation of forest estate. There is also very little discussion in the existing literature on the relative importance of different sources of uncertainty on forest valuations.

Our main objective was to assess the relative importance of different sources of uncertainty in forest NPV computations. The term uncertainty here refers to the variation in estimated forest NPVs caused by random variations in timber assortment prices, errors in input data and random errors in growth and yield projections. The uncertainty caused by input data errors was assessed, using two alternative inventory methods: ocular standwise field inventory and ALS-based estimation. The study was carried out as a simulation study, applying the Monte Carlo method and using stand-level growth and yield projection models. An essential objective was to produce quantitative information on the uncertainty in forest planning and forest NPV computations, which so far has been lacking.

2. Material and methods

2.1. Standwise field inventory and ALS estimation datasets

We used two datasets, collected with two alternative inventory methods, for modeling the input data errors. These two datasets included an ocular standwise field inventory dataset and a stand-level ALS estimated dataset. The standwise field estimation data (D_{FIELD}) consisted of partly visual estimates by experienced forest planners for 1158 stands and the control data from reference sample plots located inside the stands (for details see Haara and Korhonen (2004)).

The area-level ALS data (D_{ALS}) were collected from a study area in northeastern Finland and included 89 stands. The values for the attributes were estimated and measured at the tree species stratum-level and the estimates were based on a k-most similar neighbour (k-MSN) procedure (for details see Packalén and Maltamo (2006)). The D_{ALS} dataset also included both the ALS estimates and reference measurements from sample plots located inside the stands.

In both datasets, the reference sample plot measurements were presumed to represent the true values of the forest properties, although the reference sample plot measurements contain at least some amount of sampling error. However, the reference data errors were not taken into account as they were insignificant compared to the estimation errors. The attributes that were estimated visually in the field inventory and by ALS estimation and measured from sample plots were mean diameter (D_{gM}), mean height (H_{gM}), basal area (G), number of stems per hectare (N) and total volume (V). Descriptive statistics, such as mean and standard deviation (*sd*), aggregated to stand-level, are in Table 1.

Table 1
Descriptive statistics on datasets D_{FIELD}, D_{ALS} and D_{SIM}.

	D_{FIELD}					D_{ALS}					D_{SIM}				
	n	Min	Mean	Max	*sd*	*n*	Min	Mean	Max	*sd*	*n*	Min	Mean	Max	*sd*
D_{gM} [cm]	1158	7.4	19.1	36.4	5.8	89	8.9	18.6	32.5	6.1	40	0.0	12.6	29.0	9.4
H_{gM} [m]	1158	4.7	15.3	47.5	5.2	89	6.5	13.6	21.9	4.2	40	0.7	12.2	26.8	8.4
G [m^2/ha]	1158	1.0	21.0	51.6	6.5	89	8.0	17.9	33.3	6.4	40	0.0	11.0	27.5	9.0
N [trees/ha]	1158	45	1074	3776	554	89	306	1340	2884	528	40	360	1223	2200	578
V [m^3/ha]	1158	7.9	161.0	495.3	81.8	89	32.4	126.3	350.3	69.5	40	0.0	102.2	341.6	104.8

2.2. NPV simulation input data

The input data for the NPV simulations (D_{SIM}) consisted of a synthetic dataset of 40 simulated forest stands having an even distribution of development classes, from young thinning stands to mature stands. First, 40 different development class, site class, and tree species-combinations were generated. Then, typical attribute values were looked up for these 40 stands from growth and yield tables by Vuokila and Väliaho (1980) and Oikarinen (1983). The data covered all site classes and all main tree species (Scots pine, Norway spruce and birch) in Finland and the temperature sum was 1100, which is typical for central Finland. Attributes depicting the environment and site properties were on the stand-level and attributes depicting growing stock properties were on the tree species stratum-level. Descriptive statistics on the growing stock properties of the D_{SIM} dataset is in Table 1.

2.3. NPV simulations

The relative importance of the three sources of uncertainty in forest NPV computations was studied by simulating each stand repeatedly with the Monte Carlo method. Another approach for assessing the uncertainty would have been to use some analytical error propagation technique, such as Taylor series approximation (e.g. Gertner, 1987; Mowrer, 1991). However, implementing the analytical approach becomes increasingly complex as more sources of uncertainty are introduced to the system and the complexity of the model system itself increases. The uncertainty caused by random variation in future timber assortment prices is referred to as U_{PRICE}. The uncertainty caused by input data errors is referred to as U_{ALS} and U_{FIELD} for the datasets D_{ALS} and D_{FIELD}, respectively. The uncertainty caused by random errors in growth projections is referred to as U_{GROWTH}. The sources of uncertainty were included in the simulations in various combinations: separately, in pairs and all three simultaneously (Table 2), enabling us to determine how the different sources of uncertainty and their combinations affect the NPV distributions. In addition, we simulated each combination with interest rates of 3%, 4% and 5%. Note that U_{ALS} and U_{FIELD} were alternatives and were not included in any of the combinations simultaneously.

The actual simulation was done with a stand-level growth and yield simulator implemented in SIMO (SIMulation and optimization), a flexible and extendable open-source forest-planning framework (http://www.simo-project.org, Rasinmäki et al., 2009). The stand-level growth equations in the simulator were by Vuokila and Väliaho (1980) and Huuskonen (2008) for Scots pine and Norway spruce, and by Oikarinen (1983), Saramäki (1977) and Mielikäinen (1985) for birches. The dependent variables in the growth equations of established, i.e. non-seedling stands were increment of G and increment of dominant height H_{dom} for Scots pine and Norway spruce and increments of G, V and H_{dom} for birches. For other than pine seedling stands, the dependent variable in the growth equations was the percentual increment of attributes D_{gM}, G, H_{dom} and V (Table 3).

The increments predicted with the growth equations were added to the values of the stand-level attributes in question, except for pine seedling stands, where the models by Huuskonen (2008) predicted the actual attribute values in given age. The dependent and independent variables of the growth equations for different tree species, for seedling stands and established stands, are in Table 3.

Table 2
Uncertainty source combinations used in the NPV simulations.

	1	2	3	4	5	6	7	8	9	10	11
U_{PRICE}	o				o	o	o			o	o
U_{ALS}		o			o			o		o	
U_{FIELD}			o			o			o		o
U_{GROWTH}				o			o	o	o	o	o

Table 3
Dependent and independent variables of the stand-level growth equations by stand type and tree species.

Type of stand	Tree species	Dependent variable	Independent variables	Model adopted from
Seedling	Scots pine	G, H_{dom}, D_{gM}, V	G, H_{dom}, H_{gM}, N, Age, site class, regeneration method	Huuskonen (2008)
	Norway spruce, *Betula* spp. and other deciduous	Increment of G, H_{dom}, D_{gM}, V	Age, site class, tree species	
Established	Scots pine and Norway spruce	Increment of G	G, Age, H_{dom}, site index	Vuokila and Väliaho (1980)
		Increment of H_{dom}	Age, H_{dom}, site class	Vuokila and Väliaho (1980)
	White birch	Increment of G	G, Age, proportion of birch of total G	Mielikäinen (1985)
		Increment of H_{dom}	Age, H_{dom}, site class	Oikarinen (1983)
		Increment of V	Age, V, site class	Oikarinen (1983)
	Pubescent birch and other deciduous	Increment of G	G, Age, site class	Mielikäinen (1985)
		Increment of H_{dom}	V, Age, site class, site index	Mielikäinen (1985)
		Increment of V	Age, H_{dom}, site class	Saramäki (1977)

The development of each stand was simulated until the next regeneration harvest, or a maximum of 100 years, using one-year *timestep*, and repeating the whole simulation process 100 times for each source of uncertainty and interest rate combination. The thinning schedules were based on silvicultural recommendations of the forestry extension organization Tapio in Finland (Hyvän metsänhoidon suositukset, 2006). Regeneration harvests were done as soon as the 5-year moving average of value growth percentage, or so-called v-value, of the stand was less than the chosen interest rate. In order to estimate the timber assortment volumes and incomes from the harvests, tree diameter distributions were constructed for each stand before the harvests, using distribution models by Kilkki (1989), Siipilehto (1999) and Maltamo and Kangas (2000). The value of each diameter class was then predicted with the taper curve functions of Laasasenaho (1982) and optimal stem bucking algorithm of Näsberg (1985). The basic functions and simulation logic of the used stand-level growth and harvest simulation system is described in Fig. 1. Simulated NPV npv_{il} for each simulation iteration l ($l = 1\ldots100$) of each stand i ($i = 1\ldots40$) in dataset D_{SIM}, was calculated as the sum of discounted net cash flows C_{ilt} from thinnings and regeneration harvests during next 100 years, or until the next regeneration harvest, so that

$$npv_{il} = \sum_{t=1}^{100}\left(\frac{C_{ilt}}{(1+r)^t}\right) \quad (1)$$

where t is the simulation year and the r had values $r = 0.03, 0.04, 0.05$ for the interest rates 3%, 4% and 5%, respectively. The costs of silvicultural treatments and regenerations were not included in the analysis. The above simulation was repeated separately for each source of uncertainty and interest rate combination. In addition, a reference NPV npv_i^{REF} was simulated for each stand i and interest rate, so that all three sources of uncertainty were disabled.

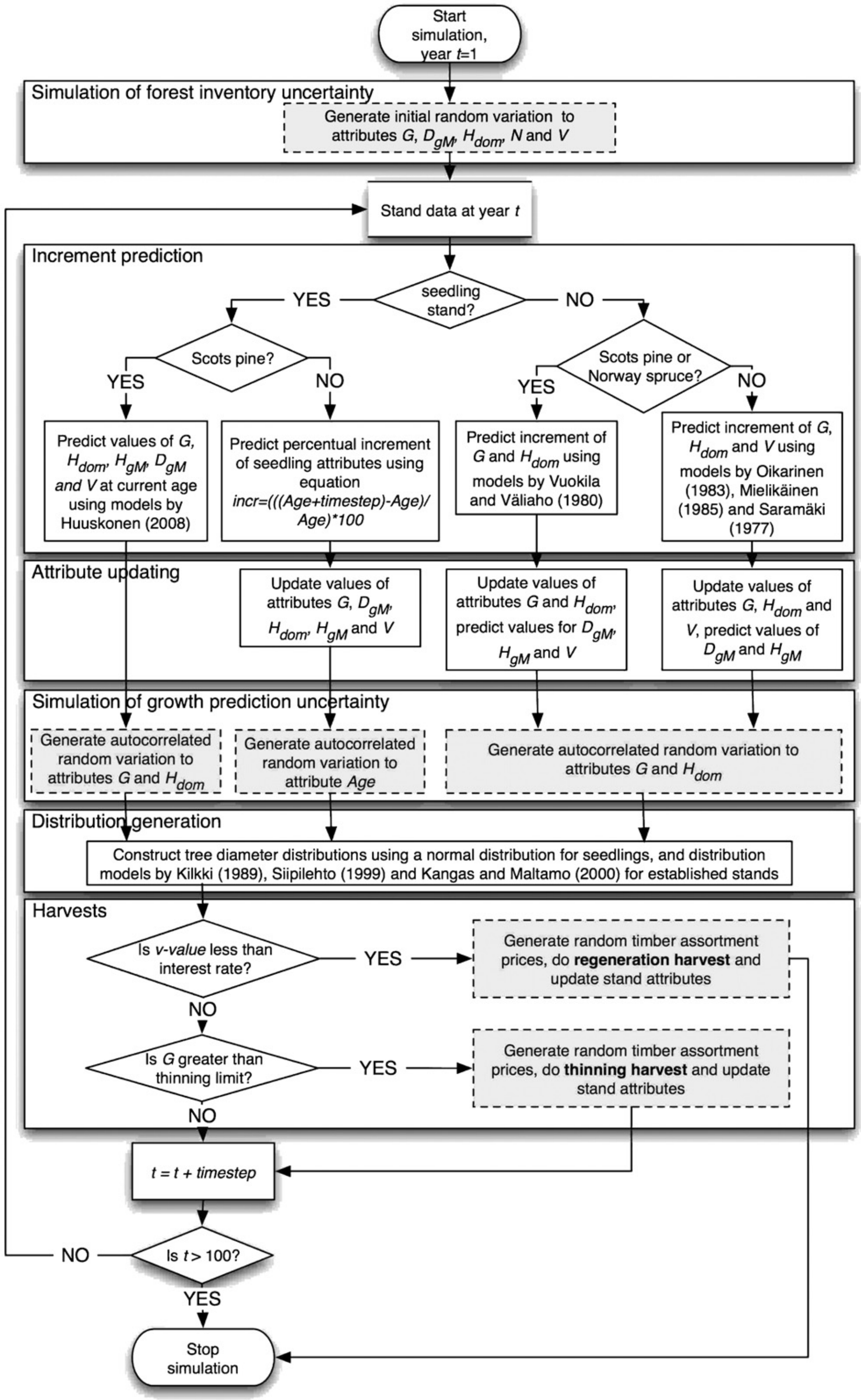

Fig. 1. A flow chart of the simulation logic in the stand-level growth and harvest simulator used for the NPV simulations. Grey boxes with dotted line are associated with simulation of the various sources of uncertainty. Thinning limits in the harvests are based on Hyvän metsänhoidon suositukset (2006). Variable t is the simulation year and *timestep* is one year.

2.4. Simulation of the sources of uncertainty

2.4.1. Simulation of stochastic timber assortment prices

Monthly statistics on real stumpages in Finland between January 1986 and August 2008 (see Fig. 1) were used for estimating the equation for future development of timber assortment prices (Finnish Statistical Yearbook of Forestry, 2008). The nominal prices were deflated, using wholesale price index for domestic goods, and the base year for deflation was August 2005 (Fig. 2). The price statistics are given separately for saw logs and pulpwood for the three main commercial timber species in Finland: Scots pine, Norway spruce, and birch.

Timber prices fluctuate heavily in a short run. However, Dixit and Pindyck (1994, p. 77) suggested that if data were available for only

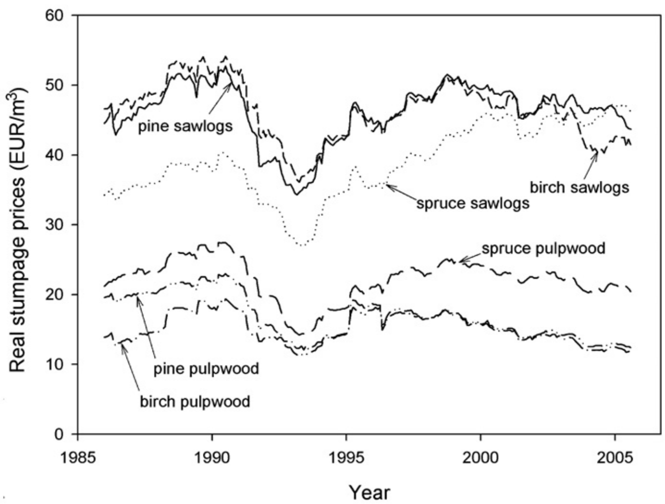

Fig. 2. Historical data on real timber prices as EUR/m^3 (price level of August 2005).

30–40 years (in our case 23 years), it would be difficult to distinguish statistically between random walk and mean-reverting processes. As a result, the decision on price model should be based more on common understanding of the nature of the price process rather than outcomes of the statistical tests. In the short run, the prices of raw commodities tend to fluctuate but in the longer run they will draw back towards long-run marginal cost of their production. We assume that there will also be future demand for timber products, and we model the timber price as a geometric mean-reverting (hereafter GMR) process. The future development of the stumpage price for each timber assortment is given by

$$dp = \eta\left(\overline{p} - p\right)dt + \sigma p dz \tag{2}$$

where $\overline{p}$ is the long-run average price. The parameters η and σ denote the speed of reversion and the level of annual variation and dz represent the increment of the wiener process (Dixit and Pindyck 1994, p. 77). In forest economics literature, geometric Brownian motion (hereafter GBM) has been widely applied because of its tractability. Clarke and Reed (1989), Thomson (1992), Yoshimoto and Shoji (1998) supported the use of GBM. On the other hand, Insley (2002), Insley and Rollins (2005) and Yoshimoto (2009) chose the GMR process. Although there is no analytical solution for the GMR process, they chose it based on economic reasoning.

Timber assortment prices have correlated strongly in the past. We assume that the relative changes in timber assortment prices will continue to correlate similarly also in the future. This is accounted for in price predictions by multiplying the Cholesky decomposition of the variance–covariance matrix (see Table 4) of the timber assortment price variations by the matrix of normally distributed random variables with zero mean and *sd* of one (for detailed description of simulating correlated random vectors applying the Cholesky decomposition, see Rubinstein, 1981).

Table 5
Parameters for timber assortment price forecasts.

	Sawlogs			Pulpwood		
	Pine	Spruce	Birch	Pine	Spruce	Birch
η	0.2045	0.0402	0.2615	0.1544	0.1035	0.3146
$\overline{p}$	56.37	68.59	53.72	17.28	24.97	17.61
σ	0.05145	0.06727	0.05312	0.08753	0.04942	0.05352

The parameter values were estimated by a local linearization method introduced by Ozaki (1985) and applied in forest economics by Yoshimoto and Shoji (1998, 2002). The basic idea of the method is that an original nonlinear stochastic differential equation (price process Eq. (1)) is first converted to a stochastic differential equation with a constant diffusion term and then the nonlinear drift of the derived equation is locally approximated by a linear function of state. The resulting stochastic differential equation is analytically solvable and the corresponding likelihood function for parameter estimation can be achieved (for mathematical proofs see Yoshimoto and Shoji (2002)). The parameter values of the estimation are given in Table 5.

2.4.2. Simulation of random variation in forest inventory data

The effect of random variation, i.e. measurement and sampling errors, in forest inventory data was taken into account by generating true values from the estimates in simulation input dataset D_{SIM}, using so-called true value models. In order to generate realistic true values into the simulation input data D_{SIM}, we first modeled the relation between the estimates and the true values, separately in the two datasets D_{ALS} and D_{FIELD}.

The estimates for attribute k ($k = D_{gM}, H_{gM}, G, N, V$) are represented by vector $\hat{x}_k$ and the true values by vector $\mathbf{x}_k$, where element x_{kij} was the value for attribute k in tree species stratum j of stand i. Each stand i contained one or more tree species stratums j, depending on the forest structure, and the range of i was $i = [1\ldots89]$ for dataset D_{ALS} and $i = [1\ldots1158]$ for dataset D_{FIELD}.

The true value models were constructed so that trends, distribution shapes and correlations between the various attributes were taken into account, as this is encouraged in earlier studies (e.g. Duvemo and Lämås, 2006). First we calculated the difference vector between the true values and the estimates $\mathbf{d}_k = \mathbf{x}_k - \hat{x}_k$. In most of the attributes, there was a trend in $\mathbf{d}_k$, which was modeled with linear regression so that $\mathbf{d}_k = \beta_{k0} + \beta_{k1}\hat{x}_k + \boldsymbol{\varepsilon}_k$ for D_{ALS} and $\mathbf{d}_k = \beta_{k0} + \beta_{k1}\hat{x}_k + \beta k_2\hat{x}_k^2 + \boldsymbol{\varepsilon}_k$ for D_{FIELD}, where $\boldsymbol{\varepsilon}_k$ is the random error component vector for variable k. The parameters of the models are in Table 6. In many cases there were no strong trends, especially in attributes D_{gM} and H_{gM} in dataset D_{ALS}, which together with wide residual variation led to the model parameters being insignificant. Dataset D_{ALS} attributes G, N and V had distinct positive trends indicating that the true values in stands with large trees were higher than the estimates.

Table 4
Variance–covariance matrix of timber assortment prices. The matrix is computed from normalised historical timber prices.

		Sawlogs			Pulpwood			Average price
		Pine	Spruce	Birch	Pine	Spruce	Birch	
Sawlogs	Pine	1.003861						
	Spruce	0.934673	1.003861					
	Birch	0.786532	0.781244	1.003861				
Pulpwood	Pine	0.832771	0.789512	0.874596	1.003861			
	Spruce	0.780352	0.822987	0.905541	0.906665	1.003861		
	Birch	0.765291	0.644382	0.793820	0.880564	0.741503	1.003861	
Average price		0.969309	0.966317	0.868148	0.913052	0.899018	0.802218	1.003861

Table 6
Parameters of the true-value trend models.

	D_{FIELD}			D_{ALS}	
	β_0	β_1	β_2	β_0	β_1
D_{gM}	5.2753 ***	−0.3933 ***	0.0061 ***	−0.5542	0.0748
H_{gM}	6.0388 ***	−0.6710 ***	0.0161 ***	0.0946	0.0444
G	1.4222 ***	−0.0781 *	−0.0001	−0.7899 ***	0.1178 ***
N	5.5538 ***	0.0137	−0.0003 *	14.2113	0.1937 *
V	99.4900 ***	0.1059 ***	−0.0001 ***	−6.1998 ***	0.1439 ***

Table 8
Correlation matrices $cor(\mathbf{d}_{.ij})$ for the datasets D_{FIELD} and D_{ALS}.

		D_{gM}	H_{gM}	G	V	N
D_{FIELD}	D_{gM}	1				
	H_{gM}	0.47	1			
	G	0.16	0.18	1		
	V	0.13	0.48	0.72	1	
	N	−0.21	−0.04	0.59	0.34	1
D_{ALS}	D_{gM}	1				
	H_{gM}	0.85	1			
	G	0.32	0.28	1		
	V	0.33	0.33	0.96	1	
	N	−0.28	−0.25	0.11	0.09	1

In dataset D_{FIELD}, attributes D_{gM} and H_{gM} had also positive trends, but the trends of attributes G, N and V were negative.

After we had fitted the true-value trend models, we examined the distributions of the model residuals $\hat{\varepsilon}_k$, that were the differences $\mathbf{d}_k$ without the trend. For most of the attributes, the distributions were probably not generated from a normal distribution, according to a Shapiro–Wilk test of normality with a 95% confidence level. Since the distributions were skewed and had high kurtosis, we used logit-logistic distribution for the true-value simulations, which fitted the distributions rather well. Logit-logistic distribution is a four-parameter distribution flexible enough to cover a large number of different distribution shapes (Wang and Rennolls, 2005). We fitted the logit-logistic distributions into the residuals $\hat{\varepsilon}_k$, using the maximum likelihood (ML) method. The estimated parameters for the logit-logistic distributions are in Table 7. The cumulative distribution function and the probability density function of the logit-logistic distribution are given by Eqs. (3) and (4), respectively.

$$F(y\,|\,\psi,\lambda,\phi,\tau) = \frac{1}{1+\exp\left(\frac{\phi}{\tau}\right)\left(\frac{x-\psi}{\lambda-x}\right)^{-\frac{1}{\tau}}} \tag{3}$$

$$f(y\,|\,\psi,\lambda,\phi,\tau) = \frac{\lambda-\psi}{\tau(x-\psi)(\lambda-x)}\frac{1}{\exp\left(-\frac{\phi}{\tau}\right)\left(\frac{x-\psi}{\lambda-x}\right)^{\frac{1}{\tau}}+\exp\left(\frac{\phi}{\tau}\right)\left(\frac{x-\psi}{\lambda-x}\right)^{-\frac{1}{\tau}}+2}. \tag{4}$$

Since it was possible that the $\mathbf{d}_k$ values between the different attributes were correlated, we computed correlation matrices from the residual vectors $\hat{\varepsilon}_{.ij}$, where $\hat{\varepsilon}_{.il} = (\hat{\varepsilon}_{DgM,\,il}, \hat{\varepsilon}_{HgM,\,il}, \hat{\varepsilon}_{G,\,il}, \hat{\varepsilon}_{N,\,il}, \hat{\varepsilon}_{V,\,il})$, of each stand and tree species stratum i and j and found out that the correlations between the residuals of attributes D_{gM} and H_{gM} and G and V were strong, especially in the D_{ALS} dataset (Table 8).

After we had modeled the trends, distributions and correlations of the differences, we could simulate realistic random true values into the simulation input data D_{SIM}. For generating the correlated non-normal multivariate random attributes, we used the copula approach (Kolev et al., 2006; Mehtätalo et al., 2008). We transformed the residual vectors $\hat{\varepsilon}_k$ into normally distributed random variable vectors $\mathbf{Y}_k$ through transformation $Y_{kij} = \Phi^{-1}(F(\hat{\varepsilon}_{kij}|\hat{\psi}_k, \hat{\lambda}_k, \hat{\varphi}_k, \hat{\tau}_k))$. From the transformed residuals, we computed a variance–covariance matrix cov ($\mathbf{Y}_{.ij}$), where $\mathbf{Y}_{.ij} = (Y_{D_{gMij}}, Y_{H_{gMij}}, Y_{Gij}, Y_{Nij}, Y_{Vij})$, and multiplied it's Cholesky decomposition with a normal random variable vector for each stand and stratum i and j in order to get correlated multinormal variable vectors $\tilde{\mathbf{Y}}_{.ij}$ (Rubinstein, 1981). These variable vectors were again transformed into logit-logistic variable vectors, using the transformation $\tilde{\varepsilon}_{kij} = F\ (\Phi^{-1}(\tilde{Y}_{kij}|\hat{\psi}_k,\ \hat{\lambda}_k,\ \hat{\varphi}_k,\ \hat{\tau}_k))$. To obtain the actual simulated differences $\tilde{d}_k$, we added the modeled trends to the simulated residuals $\tilde{\varepsilon}_k$. The simulated true values were thus $\tilde{\mathbf{x}}_k = \hat{x}_k + \tilde{d}_k$. For more detailed description on simulating forest inventory errors using similar approach as in here, see Mäkinen et al. (in press).

2.4.3. Simulation of random growth prediction errors

The uncertainty caused by the stand-level growth models, U_{GROWTH}, was taken into account by including a random variation component in the growth projections. We divided the random errors caused by this variation into inter stand error u and intra stand error e so that the total random error at time t was $\mu_t = u + e_t$. The random error caused by intra stand variation was modeled as an autoregressive process in which e_{t+1} is dependent on e_t so that $e_{t+1} = \alpha \times e_t + b$, where α is the autocorrelation coefficient and b is a random coefficient.

The normally distributed random error u was generated once at the beginning of the growth simulation for each tree species stratum of each stand and again after regeneration of each stand. The value for the random coefficient b was generated again at each time step t of the growth and yield simulation. The total variance of μ was based on a study by Haara (2005). This variance was divided again into inter stands variance and intra stand variance, applying the results of Kangas (1999).

The random error coefficients u and e were generated separately for attributes H_{dom} and G, except in seedling stands, since we used stand-level dominant height (H_{dom}) and basal area (G) growth models in this study. The total random error μ_t was then added to the growths of H_{dom} and G at each time step t to simulate the random variation in the growth and yield projections. In pine seedling stands, the growth projection was similar to the non-seedling stands. In spruce and birch stands, the growth was projected using a simple model that predicts the number of years required for the stand to reach breast height (1.3 m), using stand age, site class and tree species as the independent variables After that, the growth of seedling stands

Table 7
Parameters of the logit-logistic distributions, fitted (ML) to the $\hat{\varepsilon}_k$ values of the datasets D_{ALS} and D_{FIELD}.

	D_{FIELD}				D_{ALS}			
	λ_k	φ_k	ψ_k	τ_k	λ_k	φ_k	ψ_k	τ_k
D_{gM}	47.174	−2.501	−62.134	0.274	58.961	−2.120	−41.108	−0.362
H_{gM}	81.006	−2.505	−35.303	−0.834	22.061	−1.863	−31.456	0.367
G	61.996	−2.045	−22.051	−1.056	14.548	−1.507	−16.614	0.136
V	753.824	−2.468	−296.026	−0.943	93.974	−1.588	−139.848	0.408
N	2431.233	−1.975	−2232.169	−0.099	1588.913	−0.849	−880.408	−0.670

Table 9
Variance–covariance matrix used for generating correlated inter stand random errors for the attributes H_{dom}, G, T_{spruce} and T_{birch}.

	G	H_{gM}	T_{spruce}	T_{birch}
G	0.00360000	0.00205012	−0.00428519	−0.00462895
H_{gM}	0.00205012	0.00467000	−0.00488065	−0.00527218
T_{spruce}	−0.00428519	−0.00488065	0.00797000	0.00688748
T_{birch}	−0.00462895	−0.00527218	0.00688748	0.00930000

was projected with a very simple growth equation that predicts the percentual increment of the various stand-level attributes, until the stand attributes reached the limits of an established stand. After reaching the limits of an established stand, the growth was projected using the H_{dom}, G and V growth models. The random variation in the development of spruce and birch seedling stands was included by adding a normally distributed random error μ_t to the stand age T. The μ_t values were generated separately for spruce T_{spruce} and birch T_{birch} ages.

We presumed that the inter stand random error coefficients u for the attributes H_{dom}, G, T_{spruce} and T_{birch} were correlated. This was based on the assumption that the inter stand variation is caused by site quality and other site-specific factors that affect the stands' growth. Thus the u values for these attributes were generated from a multinormal distribution, using the Cholesky decomposition and variance–covariance matrix in Table 9.

2.5. Analysis of the uncertainty

The uncertainty in the NPV simulation was analysed by studying the distributions of the NPVs and comparing them to the reference NPVs, separately for each source of uncertainty and interest rate combination. For each stand i, the mean and sd of the NPV distribution, $mean_i^{NPV}$ and sd_i^{NPV}, respectively, were calculated with Eqs. (5) and (6).

$$mean_i^{NPV} = \sum_{l=1}^{100} (npv_{il}) \times \frac{1}{100} \tag{5}$$

$$sd_i^{NPV} = \sqrt{\sum_{l=1}^{100} (npv_{il} - mean_i^{NPV})^2 \times \frac{1}{100}}. \tag{6}$$

The bias, i.e. difference between the reference NPVs and means of the NPV distributions of each stand i, was calculated as $bias_i^{NPV} = mean_i^{NPV} - npv_i^{REF}$ and the relative, or percentual, bias was calculated as $bias\%_i^{NPV} = (mean_i^{NPV} - npv_i^{REF})/npv_i^{REF} \times 100$. We were also interested in the relative variation and thus relative sd (%) was calculated with Eq. (7).

$$sd\%_i^{NPV} = \sqrt{\sum_{l=1}^{100} \left(\left(\frac{npv_{il} - mean_i^{NPV}}{mean_i^{NPV}} \right) \times 100 \right)^2 \times \frac{1}{100}}. \tag{7}$$

3. Results

First, the relative influences of the individual uncertainty sources on the NPVs were investigated, after which the uncertainty sources were analysed in pairs and finally simultaneously. The most essential results of the study are presented in Table 10.

In scrutinizing the results presented in Table 10, it should first be noted that of the individual sources of uncertainty, U_{PRICE} produced the smallest average variation, i.e. average $sd\%^{NPV}$ values (6.9–8.2%), where the input data errors had more considerable influence, with values of 28.8–32.6% and 26.4–28.7% in the cases of U_{FIELD} and U_{ALS}, respectively. Of the individual uncertainty sources, U_{GROWTH} was the most prominent in causing variation, resulting in an average variation of 33.2%–33.4%.

The relative biases, or the differences between the reference NPVs and the simulated distribution means, were mainly negative, except when U_{ALS} is one of the sources of uncertainty in which cases the average NPV biases, i.e. average $bias\%^{NPV}$ values, were mainly positive. The average biases ranged from −12.5% to 10.2%.

Analysis of the different uncertainty source combinations was carried out first in pairs and then by combining all three (U_{PRICE}, U_{ALS}/U_{FIELD}, U_{GROWTH}) uncertainty sources. Of the pairwise analyses, combining U_{PRICE} with U_{ALS} or U_{FIELD}, resulted in a variation of the NPVs that was similar to that achieved with U_{ALS} or U_{FIELD} alone (app. 30%). The combination of U_{PRICE} and U_{GROWTH} resulted in average variation ranging from 46.5% to 50.0%, which is of the same magnitude as the variation achieved by including all three uncertainty sources simultaneously. The largest variation was achieved by combining U_{PRICE}, U_{FIELD} and U_{GROWTH} (47.4% and 51.3% with interest rates of 3% and 5%, respectively). The cumulation of the variation is not straightforward as summing the variances of two or more sources of uncertainty differs from the simulated combined variance. In most cases, the simulated joint variance was higher than the sum of the individual variances.

The effect of interest rate on the variation was relatively small, but the effect on the average NPVs was notable, so that higher interest rates resulted in lower NPVs. Interestingly, the average biases increased along with increasing interest rates.

Table 10
Averages of the relative biases ($bias\%^{NPV}$) and sds ($sd\%^{NPV}$) of the simulated NPV distributions with given source of uncertainty and interest rate combination. The active uncertainty sources in each combination are marked with o.

Active sources of uncertainty				Interest rate					
				3%		4%		5%	
U_{PRICE}	U_{FIELD}	U_{ALS}	U_{GROWTH}	$bias\%^{NPV}$	$sd\%^{NPV}$	$bias\%^{NPV}$	$sd\%^{NPV}$	$bias\%^{NPV}$	$sd\%^{NPV}$
o				−6.1	8.2	−1.5	7.3	−0.9	6.9
	o			−6.8	28.8	−5.4	29.2	−5.7	32.6
		o		1.7	26.5	4.8	26.4	7.3	28.7
			o	−9.5	33.2	−6.7	33.4	−5.8	33.2
o	o			−9.1	29.0	−3.8	32.1	−0.8	33.8
o		o		−1.0	27.4	7.1	28.6	10.2	30.9
o			o	−5.7	34.9	−1.5	35.3	−2.9	34.9
	o		o	−12.5	46.9	−7.9	48.2	−6.4	50.0
		o	o	−2.1	46.5	4.3	46.6	7.0	47.1
o	o		o	−9.2	47.4	−3.6	48.3	−1.8	51.3
o		o	o	0.1	46.5	7.5	46.8	10.0	47.6

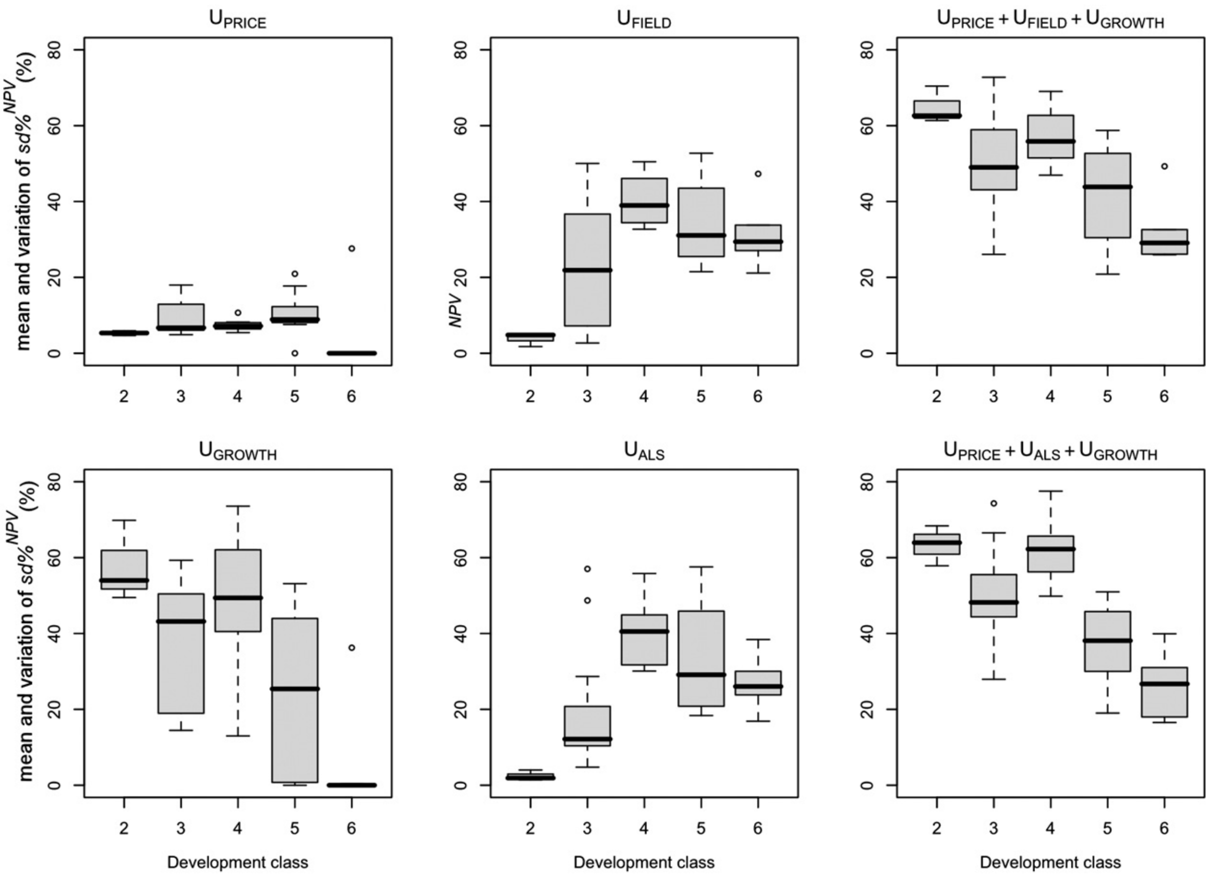

Fig. 3. Mean and variation of $sd\%^{NPV}$ by development class and separately for the various uncertainty sources U_{PRICE}, U_{ALS}/U_{FIELD} and U_{GROWTH}. Development classes: 2—young seedling stand, 3—advanced seedling stand, 4—young thinning stand, 5—advanced thinning stand, 6—mature stand.

The results presented in Table 10 were derived as averages and were calculated using all of the 40 simulated stands. Next, the effects of the uncertainty sources were analysed by development class (Fig. 3).

The effect of the uncertainty sources on the variation by development class is presented in Fig. 3. The variation associated with U_{PRICE} is the highest in young and advanced thinning stands (development classes 4 and 5). The variation due to both U_{ALS} and U_{FIELD} are the highest in young thinning stands, although their effect is considerable also in advanced thinning and mature stands. In respect to U_{GROWTH}, the greatest average variation is found in seedling stands, whereafter the averages decrease in a trendlike fashion towards mature stands. When all three uncertainty sources are considered simultaneously, the highest variation is observed in young stands, due to the strong effect of U_{GROWTH}.

The validity of the NPV computations carried out was determined by comparing the stand-level NPVs produced by the SIMO system to those produced by Pukkala's (2005) models. Fig. 4 indicates that the NPVs produced by the stand-level growth projection system implemented in SIMO are similar to those produced by Pukkala's (2005) models ($R^2 = 0.73$). Complete correspondence cannot be achieved, and it cannot even be assumed, because there are differences between the respective methodologies. The growth models used in the simulations of this research and those behind Pukkala's models are different. In Pukkala's method the timings of both thinning and regeneration harvests are optimized, whereas in the present study only regeneration harvest timings were optimized using the *v*-value. In addition, the NPVs are affected by timber assortment prices, which in this study were different from the prices used by Pukkala.

The distributions of the NPVs caused by different uncertainty source combinations were examined also at the single stand-level. In Fig. 5, we can see the distributions of three stands in different development classes: young seedling stand, young thinning stand and mature stand. The means and variances of the distributions are distinctly different between the three stands representing three development classes. In the mature stand, the means of the two simulated distributions are also rather far apart.

4. Discussion

This study is one of the first attempts to consider the importance of different sources of uncertainty salient to the determination of forest value. The sources of uncertainty included errors in input data, growth projection errors and timber assortment price fluctuations, and were considered simultaneously with variable rates of interest.

The results proved that the study's basic hypothesis is valid, namely that uncertainties related to inventory errors, growth projection errors

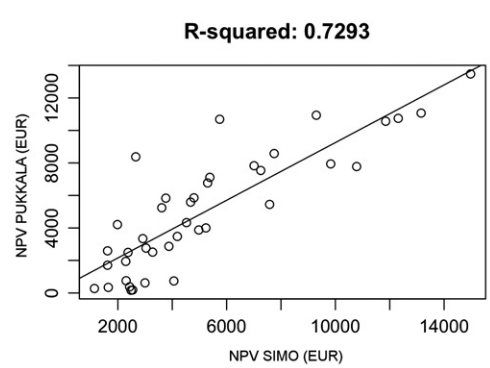

Fig. 4. Stand-level reference NPVs (npv_i^{REF}) in euros. SIMO system vs. Pukkala's (2005) models, using an interest rate of 3%.

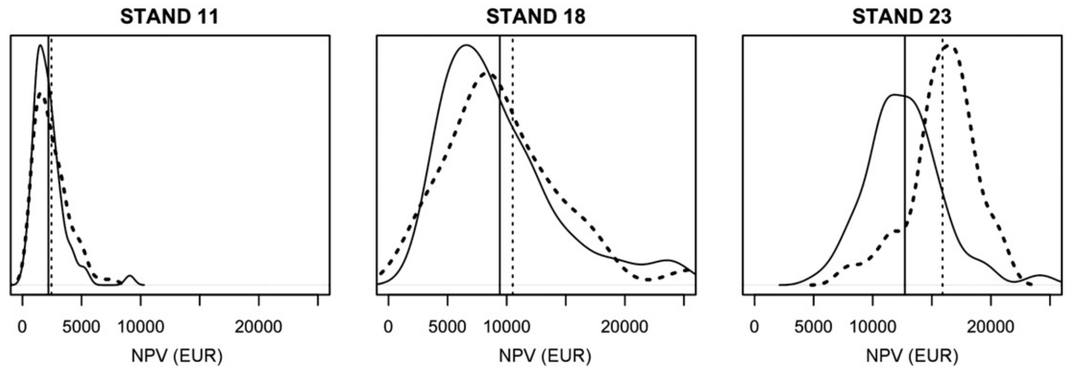

Fig. 5. Probability distribution of NPV as euros for three stands, all situated in medium-rich soil. Two alternative combinations of different sources of uncertainty include U_{PRICE}, U_{FIELD} and U_{GROWTH} (solid line) and U_{PRICE}, U_{ALS} and U_{GROWTH} (dotted line). Interest rate is 3%. The initial development classes were *young seedling stand* for stand 11, *young thinning stand* for stand 18 and *mature stand* for stand 23. The vertical lines are the means of the two uncertainty source combinations.

and timber assortment prices all contribute remarkably to variation in the values of forest stands, and also to the means of the NPVs to some extent. The variation in the NPVs is depicted here with the averages of $sd\%^{NPV}$ values, in which higher values mean wider distribution of the NPV estimates. The effect of individual uncertainty sources, i.e. the average $sd\%^{NPV}$ values, varied with an interest rate of 3% from 8.2% (U_{PRICE}) to 33.2% (U_{GROWTH}). When various uncertainty sources were combined, the highest average variation was 47.4% (U_{PRICE}, U_{FIELD}, U_{GROWTH}) with an interest rate of 3%. The cumulation of the variation was not straightforward. The sum of the variances of the individual sources of uncertainty was less than the variance of the jointly simulated uncertainty. This suggests that this type of simulation study is well-suited for studying the joint effects of multiple sources of uncertainty.

Timber assortment price fluctuations proved to be a somewhat less important source of uncertainty than input data errors and growth projection errors. However, this is largely a consequence of the pricing process applied and the assumption that timber products will also maintain their future competitiveness. If a GBM had been used, the wood price fluctuation would have been considerably greater and the respective uncertainty consequently also greater.

Input data errors result in an uncertainty, the influence of which almost corresponds to the effect of growth projection errors. The uncertainty caused by ALS estimation was slightly smaller than that caused by traditional field inventory. This is due to the higher precision of ALS estimates. The most notable difference between the two inventory methods was in the averages biases, or the average $bias\%^{NPV}$ values. In the case of ALS estimation, the average biases were positive, which means that the true NPV values are in average higher than the references. In the case of traditional standwise field inventory this was opposite. This is explained mostly by the trends in the true-value models used for simulating random variation into the simulation input data. In ALS estimation, and especially with attribute *G*, the true values in stands with large trees were actually higher than the estimates and in traditional field inventory they were smaller. Other sources of uncertainty led systematically to negative average biases.

The interest rates studied (3–5%) had only a minor influence on the average variation. It is clear that interest rate has a decisive influence on the level of NPV in absolute terms. For example, NPVs calculated with an interest rate of 3% are considerably higher than NPVs computed at a 5% rate of interest. However, the variation in NPVs also increased for lower rates of interest. Thus the relative errors remained similar, irrespective of the rate of interest.

The average variations by development class (Fig. 3) are quite logical. The input data errors caused high variation in the NPVs of advanced and mature forests, because revenues from these harvests will soon accumulate. In respect to growth model errors, random variation in the growth of young stands is projected throughout the entire rotation period. The random variation in growth of mature stands does not have sufficient time to make any difference, because the stands will be regenerated before long. This finding is also valid in practical forest planning, i.e. the uncertainty related to the functionality of growth models has more impact in younger stands. On the other hand, both ALS and traditional field inventories produce biased data, especially in mature stands (underestimates) and seedling stands (overestimates), which was taken into account in the true-value models that were used for generating random variation into simulation input data.

In combining the uncertainty sources into pairs (see Table 10) one can examine to which extent the uncertainty can be reduced by acquiring more accurate inventory data. For example, the average relative variation would be reduced from 51% to 35% in the case of traditional field inventory, given that the inventory data would be absolutely accurate. On the other hand, it can be difficult to rid oneself of the uncertainty caused by growth projection error. In addition to the errors in the growth models themselves, uncertainty is also caused by random variation characteristics of the most natural processes, which is difficult to include in the models. In the long term, environmental changes triggered by climatic changes and changes in forest structure caused by new forest treatment practices may also weaken the reliability of empirical growth models. Despite these hindrances, there are suitable approaches for decreasing growth projection errors. One such approach would be calibrating the growth projections with data on past growth of the. However, acquiring this type of data can be difficult and expensive.

Some simplifications had to be made during the course of the study. The effects of stem quality, special timber assortments and fuel wood on logging revenue were not taken into account. It was also assumed that stands were treated according to the traditional low-thinning regimes (Hyvän metsänhoidon suositukset, 2006). All of these factors affect forest NPVs. However, their influence on the importance of the uncertainty sources studied is probably not that significant. The effects of these other sources of uncertainty on forest NPV calculations can be addressed in follow-up studies.

Natural risk factors such as wind, snow, fire, insect and fungi damages were likewise not taken into account. It would, in principle, be possible to also consider forest damage risks as uncertainty sources in forest NPV computations, given that models for predicting the effect of various damage factors on stock growth were available.

Setting a minimum selling price (i.e. reservation price) in advance is one approach in the forest owner's adaptive decision-making (Attfield et al., 1991). This minimum price can be made dependent on the value increment and amount of the growing stock. Therefore it varies on a standwise basis. The application of minimum selling prices considerably improved the net yield of forestry in several studies (Haight and Holmes, 1991; Thomson, 1992; Plantinga, 1998; Brazee and Bulte, 2000). How much does an option of adaptive harvesting decision reduces the levels of various risk factors is a promising theme for future study. More rigorous analysis employing various pricing processes and assumptions on the future use of forest products is also needed.

5. Conclusions

In applying the methodology presented here, the effect of various sources of uncertainty on the outcome of forest NPV computation can be taken into account and confidence intervals can be set for the output results. The methodology also aids in assessing forest property market values during which the current estate market situation must also be taken into account in addition to the computed NPV. The study also resulted in information on which uncertainty sources to focus attention to increase the certainty of the output results. This is currently needed because new inventory methodologies are being used in operative forest planning.

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