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Game-Theoretic Validation and Analysis of Air Combat Simulation Models

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***Abstract*—This paper presents a new game-theoretic approach toward the validation of discrete-event air combat (AC) simulation models and simulation-based optimization. In this approach, statistical techniques are applied for estimating games based on data produced by a simulation model. The estimation procedure is presented in cases involving games with both discrete and continuous decision variables. The validity of the simulation model is assessed by comparing the properties of the estimated games to actual practices in AC. These games are also applied for simulation-based optimization in a two-sided setting in which the action of the opponent is taken into account. In optimization, the estimated games enable the study of effectiveness of AC tactics as well as aircraft, weapons, and avionics configurations. The game-theoretic approach enhances existing methods for the validation of discrete-event simulation models and techniques for simulation-based optimization by incorporating the inherent game setting of AC into the analysis. It also provides a novel game-theoretic perspective to simulation metamodeling which is used to facilitate simulation analysis. The utilization of the game-theoretic approach is illustrated by analyzing simulation data obtained with an existing AC simulation model.**

***Index Terms*—Air combat (AC), discrete-event simulation, game theory, military decision making, simulation-based optimization, validation.**

I. Introduction

THE application of constructive simulation is often the most convenient as well as the least money- and time-consuming way to obtain information about the performance of systems used in air combat (AC) or the value of new ways for conducting AC missions [1], [2]. A realistic AC simulation model requires components representing aircraft, weapons, radars, and other apparatus. The simulation model has to also adequately represent the decision making [3]–[5] and situational awareness [6] of pilots. Furthermore, uncertainties affecting AC must be taken into account. A suitable way for modeling the aforementioned features as well as the dynamic nature of AC is offered by the discrete-event simulation methodology (e.g., [7]), and thus, there exist several AC simulation models based on this methodology (e.g., [3] and [8]–[10]).

A discrete-event AC simulation model is controlled with input parameters and variables that affect the components describing the pilots' decision making as well as the properties of aircraft and other hardware. Uncertainty related to AC is represented by random factors in the simulation model whose effect on simulation output, e.g., the number of aircraft shot down, is analyzed using the Monte Carlo method (e.g., [7]). In this method, each AC scenario is replicated several times with different realizations of random factors determined by nonoverlapping pseudorandom number streams to produce statistical estimates for the simulation output.

In practice, the nature of a discrete-event AC simulation model may be almost black box due to its high complexity. Therefore, establishing that it performs as intended, i.e., the validation of the simulation model, is a challenging task (e.g., [7]). It should also be noted that the model is never completely validated, but various methods and techniques can be used to test it in order to increase one's confidence in its validity [11], [12]. Once the satisfactory level of confidence has been reached, the simulation model can be utilized in simulation-based optimization that offers a powerful tool for comparing available tactics or hardware configurations. The optimization analysis, called *strategy analysis* in this paper, gives valuable insight into AC scenarios under consideration although it may not provide the optimal solution of time-dependent courses of action for entire AC campaigns. However, the obtained information can be utilized in planning AC operations or purchases of aircraft and weapon systems. It is also acknowledged in instruction and training of pilots.

In both validation and strategy analysis, the action of the opponent must be taken into account in a rational and realistic manner. In this paper, this issue is tackled by presenting a novel approach to AC simulation analysis that utilizes game theory. The approach consists of statistical procedures for the estimation of games from simulation data as well as the utilization of the estimated games in validation and in simulation-based optimization. Analyses of AC have been widely supported by the application of either game theory or simulation (e.g., [1]). However, the approach presented in this paper is the first application of game theory that explicitly includes the action of the opponent into the simulation analysis. All the simulation analyses presented earlier in the open literature have been one sided as they have ignored this inherent feature of AC.

Games estimated from simulation data can also be seen as a new type of simulation metamodel [12]–[14]. In the simulation literature, metamodels refer to simpler analytical models auxiliary to simulation models [14]. Existing metamodels [15]–[17] are used for various purposes such as understanding and

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validating of a simulation model as well as optimizing the output of the model [18]. Clearly, the estimated games are also applicable for all these purposes.

Game theory (e.g., [19]–[21]) gives a systematic way for analyzing decision problems with several players pursuing their own objectives. A game consists of players, their decision variables, and payoffs that depend on the decision variables and evaluate the attainment of players' objectives. Decision alternatives available to the players are presented by the ranges of the decision variables. The outcome of the game, i.e., the values of the players' payoffs, is determined based on the decision alternatives selected by the players. Using the game, one can identify the players' best responses to the opponents' decisions, i.e., the game optimal value of the player's decision variable when the action of the opponent is fixed. Together, the players' best responses are used to find the equilibrium solution of the game.

The inherent game setting of AC is raised by conflicting objectives of the two sides. Therefore, several types of game formulations for AC and air operations have been constructed. Such formulations are, e.g., matrix games [22], [23], discrete-time dynamic games [24]–[30], differential games [31]–[36], two-target games [37], [38], and influence diagram games [39]. The dynamic and differential games enable the solution of game-optimal time-dependent controls or decision sequences for the players. In order to be tractable, such game formulations have to be limited in detail and realism. For example, aircraft and missiles are described using 3-DOF point-mass models [4], [5], [22], [32], [34]–[36], [39], [40] or even more simplified equations of motion [24], [25], [28]–[30], [33], [41] that result in unrealistic flight paths that cannot be implemented in practice. In addition, many decision-making problems related to AC do not necessitate a dynamic or differential game formulation. Such problems are, e.g., the selection of flight tactics or hardware configurations that can be analyzed effectively using AC simulation models. Compared with dynamic or differential games, simulation models allow for more detailed modeling of flight mechanics as well as of sensory and avionics systems. Furthermore, by using simulation, multiple actors representing individual pilots and their situational awareness that depends on uncertain information as well as coordinated actions within flights can be included in the analysis. However, the game setting of AC has been ignored in the earlier simulation analyses.

The game-theoretic approach introduced in this paper consists of four phases. First, the AC scenario is determined. In the scenario, the sides of AC are assumed to have a set of tactical alternatives related to available tactics or hardware configurations. The objectives of the sides are represented by measures of effectiveness (MOEs). Second, the scenario is simulated with the combinations of the tactical alternatives that determine the input of the simulation model. Then, MOE estimates are calculated based on the simulation output. Third, a suitable game is estimated to capture the dependence between the tactical alternatives and MOEs. In the game, the tactical alternatives are represented by decision variables and the MOE estimate is used as the payoff. This paper presents the estimation procedures for games involving continuous or discrete decision variables. In the discrete case, one obtains a matrix game in which the MOE estimates are classified by applying analysis of variance (e.g., [42]). With continuous variables, a multivariate regression model (e.g., [43] and [44]) is fitted to the simulation data. Finally, the estimated game is used for validating the simulation model or conducting strategy analysis.

In the simulation literature, there exists a versatile set of validation methods for discrete-event simulation models [7], [45]. Commonly used methods include comparing the simulation results with actual data. Alternatively, a subject-matter expert can assess the validity of the model output. To aid the assessment, several techniques can be used to describe the model output such as calculating descriptive statistics and presenting the results graphically. One can also perform a sensitivity analysis with respect to the model input to see how it affects the simulation output. While such methods are also suitable for the validation of an AC simulation model, they omit the game setting of AC which is taken as an integral part of the approach presented in this paper.

In validation, the structures and solutions of estimated games are compared with real-world AC scenarios. This comparison focuses on the following properties of the estimated games. First, symmetric scenarios should result in symmetric games. Second, the payoff of the game should depend on the decision variables in a manner that is consistent with the tactical alternatives and MOE in actual AC. The analysis of these two properties is straightforward and can be carried out even without a subject-matter expert. If an expert is available, also the following properties can be analyzed. The best responses of the games should be justifiable based on the corresponding tactical alternatives of the scenario. Furthermore, the effect of initiatives in the games should reflect the actual AC scenario. If these properties of the games are considered plausible, this is taken as positive evidence on the validity of the simulation model. Thus, the game-theoretic approach gives insight that includes information about the optimal decisions and behavior of the players that could not be obtained by simply studying the output of the simulation model.

In addition to validation, the game-theoretic approach allows the use of simulation-based optimization in strategy analysis. In such an analysis, the combination of simulation and game theory provides an AC analyst with a thorough understanding over decision problems in AC scenarios. The utilization of the estimated games in optimization enhances the existing techniques for simulation-based optimization (e.g., [12], [46]–[49]) into a two-sided setting as the joint effects of the sides' decisions are now taken into account—instead of unilateral optimization. Most importantly, the game-theoretic analysis offers a systematic framework for both sifting through the available tactical alternatives and managing various objectives of the combatants.

In practice, strategy analysis is based on the following aspects of the games. Several games with different payoffs are analyzed to assess the performance of tactical alternatives with respect to different objectives, which provides a comprehensive understanding over the AC decision problem. On the other hand, by varying the opponent's payoff, one can see how this affects the opponent's decisions and the outcome of AC. To identify the effective and ineffective tactical alternatives, one can solve the best responses to all opponent's decisions. Finally, in worst case analysis [19], decision alternatives are compared based on the most disadvantageous payoff values in which they can result. This comparison provides the alternatives that guarantee the best available outcome in case the opponent acts in the worst possible manner and aims at minimizing the respective payoff.

In this paper, the game-theoretic approach is illustrated by representing the analysis of a discrete-event AC simulation model, X-Brawler [3], [8]. In X-Brawler, aircraft, weapons, and other systems as well as pilots' decision making are modeled at a high level of detail which should provide a good representation of actual AC. Validation examples are presented in three scenarios that study the effects of pilot aggressiveness, commit maneuvers executed at the beginning of AC, and the action taking place after the launch of a medium-range air-to-air missile. Strategy analysis is illustrated by analyzing a decision problem regarding the launch range of an air-to-air missile.

This paper is organized as follows. The necessary terminology of game theory is briefly summarized in Section II. In Section III, the game-theoretic approach is introduced by detailing the estimation of games for both discrete and continuous decision variables as well as by discussing the use of estimated games in validation and strategy analysis. Examples of validation and strategy analysis are given in Sections IV and V, respectively. Finally, conclusions are given in Section VI.

II. Required Game-Theoretic Concepts

Game theory is a branch of science that uses mathematical models to study decision-making problems with multiple actors [19]–[21]. Due to the nature of AC, here, the discussion is limited to noncooperative game theory [19] where several players make decisions in order to attain their own, possibly conflicting, objectives. Additionally, the discussion is limited to games with two players that represent the sides of AC. The players are called the blue and red flights, or simply *blue* and *red*.

In games, the players' decision alternatives are represented by decision variables having a discrete or continuous range of values. Due to technical and practical reasons, the players' decisions are now limited to selecting exact values of the decision variables. In other words, the players are allowed to use only pure strategies, and mixed strategies are excluded, i.e., the players are not allowed to present decisions as probability profiles over available decision alternatives. Additionally, in this paper, the term "strategy" is excluded to avoid possible confusion with strategy analysis.

The objectives of the players are presented using payoffs whose values depend on the decision variables. The players are assumed to be rational, i.e., their sole goal is to maximize the value of their own payoff. Once the players have chosen their decision alternatives, the selected alternatives determine the outcome of the game, i.e., the values of players' payoffs. For continuous decision variables, the dependence between decision variables and payoffs is modeled using payoff functions.

A game can be either zero sum or non-zero sum. In a zero sum game, there is only one payoff that is maximized by one player and minimized by the other, i.e., the players' objectives are completely opposite. A zero sum game is presented either by a single payoff matrix in the case of discrete decision variables or by a payoff function for continuous decision variables. If the game is non-zero sum, the players have separate payoffs so that an increase in the payoff of one player does not necessarily lead to a decrease in the payoff of the opponent. In other words, the objectives of the players are not directly opposite. In a non-zero sum game formulation, there are two separate payoff matrices, i.e., a bimatrix, or two payoff functions.

The game also describes the information available to the players when making their decisions. The players can act simultaneously without knowing the opponent's decision or in a predetermined order. Furthermore, the players may or may not be aware of the opponent's payoff. In a Stackelberg game setting [19], the players have the roles of a leader and a follower. The leader makes a decision first, knowing the follower's payoff, i.e., the follower's response to the leader's decision. The follower simply observes the leader's decision and acts accordingly being unaware of the leader's payoff.

In this paper, Stackelberg games are used to model the effect of initiative in AC. In a zero sum setting, the Stackelberg solution equals a maximin solution [19] where the follower's response is always the most disadvantageous for the leader, i.e., the minimum of the leader's potential payoff values. The leader then maximizes its payoff by selecting the appropriate decision alternative. In this paper, maximin solutions are used to perform worst case analysis. This term is adapted from the field of controller design in which game formulations are applied for controlling a dynamic system under uncontrollable disturbances [19]. Here, the action of the opponent is treated as such disturbance.

One way to analyze games is to study the best responses of a player against the decisions of the opponent. The best responses are solved by selecting one decision alternative of the opponent at a time and optimizing one's payoff against that alternative. If a decision alternative is the best response to all of the opponent's decisions, it is called a dominating alternative. Similarly, a decision alternative is dominated if there exists another alternative that performs at least as well against all and better against at least one of the decision alternatives of the opponent. In the case of continuous decision variables, the player's best responses generate the best response curve of the player which gives the player's game optimal decision as a function of the opponent's decision.

The equilibrium solutions of games are obtained by studying the players' best responses. If two decision alternatives are best responses to each other, they form a Nash equilibrium (e.g., [19]–[21]). In such a case, neither player is willing to deviate from the equilibrium, and the players cannot gain anything by unilaterally choosing a different decision alternative. Thus, a Nash equilibrium gives a stable solution for the game, and rational players are assumed to play the game in a way that results in a Nash equilibrium. The Nash equilibrium is not necessarily unique, and when the players are not allowed to randomize their decision making, it may not exist. The Stackelberg equilibrium of the Stackelberg game and the maximin solution of a zero sum game are determined using similar inference based on the players' best responses.

TABLE I
CORRESPONDENCES BETWEEN QUANTITIES IN AC SCENARIOS AS WELL AS IN SIMULATION MODELS AND GAMES

AC scenario	Simulation model	Game
tactical alternative	input variable	decision variable
MOE	MOE estimate	payoff

III. GAME-THEORETIC APPROACH

The game-theoretic approach to AC simulation proceeds as follows. First, the AC scenario of interest is defined at a suitable precision so that the definition contains all the necessary information for performing the simulation. This includes the number and types of aircraft, weapons, sensory, and other systems as well as the initial geometry of the scenario. The definition of the scenario also includes the objectives of the flights, the measure of the attainment of these objectives which is called a MOE, and the flights' means for achieving these objectives, i.e., their available tactics or hardware configurations. Here, these means are called the flights' tactical alternatives.

The MOE represents the outcome of the combat as perceived by the flights, and it can be selected in many ways depending on the aim of the analysis and the objectives of the flights. In general, AC-related decision problems contain multiple objectives, but now, it is assumed that the objectives can be presented with a scalar-valued MOE. It can be, for example, the mean of kills, the mean of losses, or a linear combination of the former. One can also study probabilities of scenario-specific AC events. In defensive scenarios, success can be measured by, e.g., the probability of taking down some prespecified aircraft or destroying the entire attacking bomber fleet. On the other hand, in offensive scenarios, the MOE can be, for example, the probability of reaching a given route point unharmed or destroying an important ground target.

To study how tactical alternatives affect a MOE, the AC scenario is simulated with suitable combinations of tactical alternatives that are selected using design of experiments [12], [42]. The alternatives are entered to the simulation model using input variables, and other scenario information is presented by input parameters. For each combination of input variable values, a MOE estimate is obtained from the simulation output. Once the simulation data are collected, a suitable game is estimated using the statistical techniques presented in Section III-A. In the estimated game, the players' decision variables and payoffs are associated with the simulation input variables and the MOE estimates (Table I). The nature of tactical alternatives determines whether the decision variables of the game are discrete or continuous.

After the estimation of the game, it is used in the validation of the simulation model or in strategy analysis. To validate the simulation model, the game is evaluated in the sense of how well it reflects the actual AC scenario. On the other hand, the estimated game can be used as a part of the strategy analysis to give insight into a decision problem arising from the scenario. The validation and strategy analyses are further discussed in Sections III-B and C, respectively.

To summarize, the game-theoretic approach consists of the following phases.

1) Define the AC scenario.
2) Simulate the scenario according to a suitable experimental design.
3) Estimate games from the simulation data using suitable statistical techniques.
4) Use the games in validation and/or strategy analysis.

A. Estimation of Games

1) Simulation Data: To estimate games, one needs AC simulation data that consist of values of simulation input variables and resulting MOE estimates. If the tactical alternatives of the AC scenario are discrete, e.g., different maneuvers or missile types, they are presented by discrete-valued input variables for blue and red, which are denoted by $x_1, \ldots, x_n$ and $y_1, \ldots, y_m$, respectively. Then, the scenario is simulated using the input variable values (x_i, y_j) to gather the needed data.

Continuous tactical alternatives are entered into a simulation model using input variables that have a continuous range of values and are denoted by x for blue and y for red. Examples of such alternatives include, for example, missile launch distances or missile support times. For continuous input variables, games can be constructed in two ways. One approach is to estimate games containing continuous decision variables, even though it is not possible to simulate the AC scenario using all feasible values of input variables. In such case, a discrete set of values (x_i, y_j) is chosen according to a suitable experimental design (e.g., [42]) to produce the necessary simulation data.

On the other hand, the continuous input variables can also be discretized and treated as discrete-valued input variables. In discretization, there exists a tradeoff between the accuracy of estimated games and the number of necessary simulation replications. In general, games obtained by discretization necessitate a lesser number of simulation replications, whereas games containing continuous decision variables provide more precise results.

Because of random factors in a simulation model, a MOE can be regarded as a random variable with an unknown probability distribution. Similarly, a given AC event associated with the MOE takes place during simulation with an unknown probability. When the AC scenario is simulated with input variable values (x_i, y_j), the simulation output gives a random sample from the MOE distribution or observations of the occurrence of the AC event. These are used to produce MOE estimates denoted by $\hat{U}(x_i, y_j)$. In practice, the MOE estimate is the mean of the sample estimating the expectation of the MOE distribution or the relative frequency of the occurrence of the AC event estimating the probability of the event. One can also estimate other descriptive statistics, such as quantiles, but now, such an analysis is omitted. The values of the input variables and the MOE estimates form the simulation data $(x_i, y_j, \hat{U}(x_i, y_j))$.

2) Discrete Decision Variables: When tactical alternatives are discrete, the AC scenario is converted into a matrix game where the decision alternatives of the players coincide with the input variable values (x_i, y_j) and the payoffs are based on the MOE estimates $\hat{U}(x_i, y_j)$. Due to the random factors of the simulation model, the MOE estimates may not be entirely accurate and have to be classified statistically using variance analysis methods (e.g., [42] and [43]). In these methods, the estimates

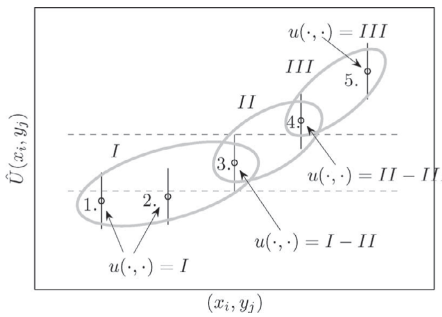


Fig. 1. Classification of MOE estimates for a matrix game. Simulation with input variable values (x_i, y_j) produces MOE estimates $\hat{U}(x_i, y_j)$ that are grouped into classes I, II, and III. The payoff has four values, i.e., $u(x_i, y_j) \in \{I, I\text{–}II, II\text{–}III, III\}$.

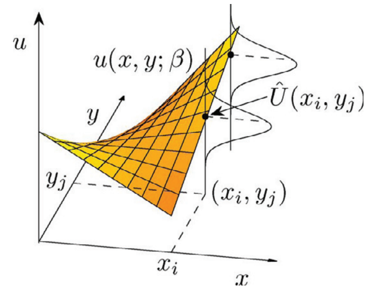


Fig. 2. Construction of the payoff function. Simulation with the input variable values (x_i, y_j) produces the MOE estimate $\hat{U}(x_i, y_j)$. The payoff function $u(x, y; \boldsymbol{\beta})$ is fitted to the data in order to approximate the MOE for all values of the variables x and y.

are compared pairwise to find out which pairs are statistically significantly different. In this paper, the MOE estimates are classified using the Tukey–Kramer procedure [42] that carries out all the comparisons simultaneously to avoid the multiple-comparison problem. The Tukey–Kramer procedure also allows unequal sample sizes and variances for the MOE estimates. This is practical as some of the simulation replications may crash, resulting in unequal sample sizes. Furthermore, there is no guarantee that different values of input variables lead to equal variances.

In the Tukey–Kramer procedure, the MOE estimates are grouped into classes. Two estimates belong to the same class if they do not differ from each other statistically significantly. Now, the classes are indexed in ascending order so that the lowest estimates belong to class I, the second lowest belong to class II, and so on. The classification gives the payoff, denoted by $u(x_i, y_j)$, that maps the combinations of decision alternatives (x_i, y_j) to the payoff values associated with classes I, II, III, etc. Then, the player maximizing the payoff prefers the outcome of the game with the higher payoff value. Note that a MOE estimate can simultaneously belong in two classes. Thus, the payoff value can be, for example, I–II. When comparing two payoff values, they are considered as equal if they share a common class, e.g., I–II and II–III. It should also be noted that the classification gives only ordinal information about the ranking of the payoff values, i.e, the payoff value I is considered smaller than II but the magnitude of the difference between the payoff values is unknown.

Fig. 1 shows an example of the classification of five MOE estimates $\hat{U}(x_i, y_j)$ into three classes I, II, and III. The MOE estimates are marked with circles, and each estimate is associated with a vertical line that corresponds to the simultaneous 95% confidence interval used in the comparison of the estimates. If two vertical lines overlap, the corresponding estimates do not differ from each other statistically significantly. For example, in Fig. 1, the horizontal dashed lines represent the comparison of the third estimate with the others. The first three estimates do not significantly differ from each other. Thus, they are regarded as equal and belong to class I. The third and fourth estimates are also deemed equal and belong to class II. Finally, the fourth and fifth estimates form class III.

In Fig. 1, the payoff $u(x_i, y_j)$ has four values I, I–II, II–III, and III. Again, two combinations of decision alternatives are considered to provide equal payoff values if the corresponding values overlap, i.e., they share a common class. For example, the third payoff value is I–II, and therefore, it is considered equal to the first payoff value I and the fourth payoff value II–III. On the other hand, the fifth payoff value III is statistically significantly greater than the third I–II.

3) Continuous Decision Variables: In the case of continuous tactical alternatives, one needs a payoff function that approximates the dependence between the tactical alternatives and the MOE. There exists random variation in the MOE estimates that has to be accounted for in the approximation. Therefore, the payoff function $u(x, y; \boldsymbol{\beta})$ is constructed as a regression model [42], [43] based on the simulation data $(x_i, y_j, \hat{U}(x_i, y_j))$, as shown in Fig. 2. Now, the simulation input variables (x, y) are taken as the decision variables of blue and red. In order to achieve the best possible approximation, the payoff function is fitted to simulation data by selecting an appropriate type of regression model and estimating its parameter vector $\boldsymbol{\beta}$ with the method of least squares. The correctly constructed regression model then describes the dependence between the decision variables and the payoff of the game as well as approximates the payoff values for all values of the decision variables.

In practice, there are no apparent limitations for the functional form of the regression model, as it can be, for example, a linear, logistic, or some nonlinear regression model [42]. The model needs to be complex enough to accurately capture the dependence between tactical alternatives and a MOE. The goodness of fit of the regression model is studied using residuals, coefficient of determination, and/or deviance of the model [42]–[44]. Simultaneously, the model needs to be as parsimonious as possible. Thus, statistically insignificant parameters are excluded from the model in order not to fit the model into random variation of the simulation data. The relevance of the parameters is analyzed with p-values [42]–[44]. The model selection is case dependent, and the suitability of alternative models is also affected by the type of MOE. For example, logistic regression models [44], [50] are ideal for modeling probabilities of AC events. Furthermore, several regression models can be combined to define more complex payoff functions, e.g., the difference between probabilities of two AC events.

B. Validation Analysis

The purpose of validation analysis is to ensure that the simulation model gives a satisfactory representation for AC. A widely used validation approach is to study the simulation data with statistical methods [11] such as regression analysis and analysis of variance. The methods are used to analyze how different factors and variables affect the outcome of the simulated AC. Traditionally, this one-sided analysis is performed separately for each flight. If the simulation model is proper and functional, the statistical models should be consistent with the AC scenario under consideration.

Games estimated from simulation data are utilized in validation in the same way as the one-sided statistical models. Now, validation is conducted in a two-sided setting from the viewpoint of both flights by studying whether the games are consistent with the actual AC scenario. If the games are found unsatisfactory, this indicates a need for improvement in the simulation model or its settings.

In this paper, validation analysis concentrates on the following properties of games: MOE estimates, symmetry, dependences, best responses of players, equilibrium solutions, and effect of initiative. To ease the comparison of the games and their properties with actual AC practices, the players are not allowed to randomize their decision making. It should also be noted that the symmetry and, up to certain extent, the dependences can be analyzed even without subject-matter expertise, whereas the analysis of best responses and the effect of initiative requires a more profound familiarity with AC and its practices.

1) Symmetry: Symmetric AC scenarios are an integral part of the validation analysis. An AC scenario is said to be symmetric if the initial geometry is symmetric and the flights have similar aircraft and other hardware as well as similar tactical alternatives. In such a case, the estimated games are supposed to reflect this symmetry. Asymmetric games, on the other hand, point toward problems in the simulation model or in the execution of simulation.

In a symmetric game, the payoff values are the same for each player under comparable circumstances, i.e., the effect of a decision on the payoffs is independent of the player making the decision. The estimation of separate games with the same payoff for blue and red should result in similar games that have same payoff values. For example, in a symmetric setting, the games with payoffs equaling the number of blue and red kills should be alike. Furthermore, in the case of continuous decision variables, the symmetric payoff functions are expected to depend on the decision variables in a concurrent manner and the parameters of the payoff functions are supposed to mirror each other. In symmetric games, best responses and equilibrium solutions should also be symmetric. For example, if there is a Nash equilibrium where the players make certain decisions, the decision combination where the decisions of the players are reversed should also be a Nash equilibrium.

Validation analysis can also be extended to asymmetric scenarios to see how differences between the flights affect the simulation results. For example, one can start with a perfectly symmetric AC scenario and gradually make it more uneven, e.g., by enhancing the performance capability of one flight or by changing the initial geometry. By comparing corresponding games, it is possible to study the effect of increased asymmetry on the outcome of AC. Clearly, the outcome should favor the flight having superior aircraft or advantageous initial position.

2) Dependence Between Decision Variables and Payoff: The games show how the players' decisions affect the players' payoff values and the corresponding MOE estimates, reflecting the outcome of the simulated AC scenario. Therefore, the payoffs and their dependence on the decision variables can be studied to see if they are reasonable, e.g., do more effective weapon systems result in a better outcome or do more defensive tactics reduce losses. Furthermore, the joint effects of players' decisions can be analyzed, e.g., to see what happens to the number of kills when both flights behave very aggressively.

3) Best Responses and Equilibrium Solutions: Best responses are a special case of the joint effects of the players' decisions which give the players' game optimal decisions when the opponent's decision is fixed. The best responses can be solved for all decision alternatives of the opponent, and the logic behind these responses should concur with the respective AC scenario. If the estimated games have dominating alternatives, this should also be justifiable on the basis of the scenario.

For example, if the AC scenario involves maximizing kills, one may want to engage in a direct confrontation with the opponent. Therefore, the best responses in this situation should represent the aggressive behavior of the player, e.g., choosing a shorter missile launch range or maneuvering directly toward the opponent. In such a scenario, it would also be plausible to have a dominating alternative that performs best regardless of the opponent's decision.

Nash equilibria of the games are also used in the validation analysis. If an estimated game has one or more such equilibria, there should be an explanation based on the AC scenario to show why the players would behave in the given manner. This is compared with actual AC by considering whether similar decisions would be made by pilots.

4) Initiative: There are AC situations where holding initiative, i.e., making one's decision first, is advantageous compared with making the decision after the opponent or simultaneously. For the estimated games, the effects of initiative can be studied by solving the equilibrium solutions of the game in a Stackelberg setting. When the players alternate as the leader and the follower, the payoff values they receive in the equilibria determine whether it is advantageous to make one's decision before or after the opponent. This property of the estimated game can then be compared with the AC scenario to validate the simulation model.

C. Strategy Analysis

The aim of strategy analysis is to increase the understanding of an AC scenario and decision-making problems associated with it by comparing the effectiveness of tactical alternatives. For comparison of the tactical alternatives, the most straightforward approach would be simulation-based optimization in which the scenario is simulated for all tactical alternatives that are ranked according to their performance [49]. Unfortunately, such an analysis ignores the action of the opponent. Therefore, a two-sided setting is introduced to simulation-based optimization by analyzing games estimated from simulation data.

In strategy analysis, four properties of games are studied. Multiple games are estimated for various payoffs to examine the scenario from several points of view. For each individual game, players' best responses as well as equilibrium solutions are solved and the dominance between decision alternatives is studied. By studying maximin solutions, the decision alternatives can also be compared based on the worst possible outcome in which they may result. In general, these properties of the games are to be studied by an analyst who is familiar with AC practices.

1) Best Responses and Equilibrium Solutions: For all estimated games, the best responses are solved for both players to study the interaction of the players' decision alternatives. Most importantly, the best responses determine the optimal decision alternatives when the opponent's decision is known and present the effects of the opponent's action in an explicit manner. The best responses of the players define the equilibrium solutions of the game which provide the potential courses of AC and the resulting payoff values. With this knowledge, one can compare tactical alternatives according to the opponent's best responses in order to achieve the most desirable outcome of AC.

2) Dominance Between Alternatives: An important feature of strategy analysis is the recognition of dominated decision alternatives. By solving the player's best responses against all decisions of the opponent, one may find out that some decision alternatives are never among the best responses. Such information simplifies the decision-making problem by pruning out tactical alternatives and directing the focus of further analysis toward the more relevant ones.

3) Different Objectives: To gain a thorough understanding of the AC scenario and the decision-making problem at hand, games with different payoffs should be estimated. By analyzing these games together, one obtains a multidimensional picture of the AC and understands how different objectives are best achieved. The analysis of several games also implies how the opponent might act in different situations. Overall, the games with different payoffs yield information on how AC should be conducted against an opponent with given objectives as well as on what courses of AC may take place in these situations.

4) Maximin Solution: In a game setting, the effectiveness of a decision alternative may depend critically on the decision of the opponent. In some situations, a player may be unaware of the objectives of the opponent. One way to handle this uncertainty is to assume that the opponent's objectives are completely opposite to the objectives of the first player. Furthermore, it may be assumed that the opponent is able to respond to the decisions of the first player in the most disadvantageous manner. These are the assumptions of worst case analysis [19] where the decision alternatives are compared based on the worst outcome in which they may result.

In practice, worst case analysis is performed by studying zero sum maximin solutions where the opponent observes the first player's decision and responds with the alternative that minimizes the payoff of the first player. Then, the first player selects the alternative that maximizes the payoff given the response of the opponent. Worst case analysis yields the decision alternatives that guarantee the highest payoff value when the opponent tries to minimize the first player's payoff. In other words, the application of tactical alternatives obtained with worst case analysis ensures that one succeeds always, at least, as well as planned or better.

IV. Example of Validation Analysis

In the following, the use of the game-theoretic approach in validation is demonstrated by analyzing three scenarios that are based on simulations conducted with the AC simulation model X-Brawler [3], [8]. The aim of the analysis is to explore the decision-making model of the simulated pilots and to ensure that settings and hardware models used in simulation are proper and functional. In the scenarios, flights engage in AC with identical aircraft and weapon systems. Furthermore, the initial geometry of the combat is symmetric. Such a situation should result in symmetric games that can be analyzed without a subject-matter expert.

The scenarios are selected in order to describe AC at different levels of resolution. The first scenario presents an analysis that spans the entire engagement and compares three types of pilot behavior ranging from low to high aggression level. The second scenario studies the effect of a single maneuvering decision during the commit phase of AC. In the third scenario, a support time game [40] is studied. The game of this type takes place in AC when pilots have launched their medium range air-to-air missiles.

A. Scenario 1: Level of Aggression

In this scenario, the effect of pilot aggressiveness is studied in two-versus-two AC where both flights perform a maneuver called *end run* (see Section IV-B) during the commit phase of AC. The flights' tactical alternatives represent aggression levels for the simulated pilots. They are briefly summarized as follows.

1) Low aggression level. The flights engage the opponent and launch a medium-range air-to-air missile toward it. After the launch, the pilots support their missiles, i.e., relay state information about the opponent to the missile in order to increase the likelihood of a hit but not at the cost of being easy targets for the opponent. Finally, the flights retreat from the opponent. The distance to the opponent is maintained at beyond visual range during the entire engagement.
2) Medium aggression level. Similar to the low aggression level, but now, having retreated from the opponent, the flights reenter the combat. Should a pilot get caught within the visual range area, the engagement is continued as a dogfight.
3) High aggression level. The flights engage the opponent and launch and support their missiles. Then, the aircraft are flown toward the opponent, and a dogfight is engaged within visual range. During the engagement, the pilots do not perform any defensive maneuvers, such as retreating.

		RED (*min*)		
		low	*medium*	*high*
BLUE (*max*)	*low*	I (0.179)	III (1.200)	III (1.205)
	medium	$\boxed{II}$ (0.344)	IV (1.501)	IV (1.498)
	high	$\boxed{II}$ (0.330)	IV (1.504)	IV (1.489)

Fig. 3. Matrix game with the payoff representing the number of blue kills. The Nash equilibria are enclosed in squares.

		RED (*max*)		
		low	*medium*	*high*
BLUE (*min*)	*low*	I (0.257)	$\boxed{II}$ (0.346)	$\boxed{II}$ (0.320)
	medium	III (1.156)	IV (1.488)	IV (1.475)
	high	III (1.162)	IV (1.468)	IV (1.485)

Fig. 4. Matrix game with the payoff representing the number of red kills. The Nash equilibria are enclosed in squares.

		RED (*min*)		
		low	*medium*	*high*
BLUE (*max*)	*low*	$\boxed{II}$ (−0.077)	IV (0.855)	IV (0.885)
	medium	I (−0.811)	III (0.013)	III (0.023)
	high	I (−0.833)	III (0.036)	III (0.004)

Fig. 5. Matrix game with the payoff representing the difference of blue and red kills. The Nash equilibrium is enclosed in a square.

The MOEs analyzed in the scenario are the number of blue and red kills as well as the difference between kills. Here, the number of kills means the number of opposing aircraft shot down. To produce necessary MOE estimates, the scenario is simulated 2400 times for each of the nine combinations of input variable values representing the levels of pilot aggressiveness. The MOE estimates are classified using the procedure presented in Section III-A2 which results in matrix games.

The estimated games with the MOE estimates are shown in Figs. 3–5. Blue maximizes its kills in Fig. 3 and the difference of kills in Fig. 5. In Fig. 4, blue minimizes red kills. All the games are zero sum, and thus, the objective of red is always opposite to blue. It should also be noted that I corresponds to the payoff value with the smallest MOE estimates and that the estimates ascend according to the indexing.

To validate the simulation model, the symmetry of the games is considered first. The games for blue and red kills in Figs. 3 and 4 reflect the symmetry of the scenario as the game for blue kills is essentially the same as the game for red kills because the former game matrix can be obtained by transposing the latter. The game for the difference of kills in Fig. 5 is also perfectly symmetric. Note that payoff value I is the most advantageous for red while IV is preferred by blue. Additionally, when the players' decisions coincide, the MOE estimates do not differ statistically significantly from zero. All the games discussed previously represent the symmetric AC scenario, and there is no reason to challenge the validity of the simulation model.

Next, the best responses of the players as well as the dependence between the decision variables and the payoff are analyzed. In Figs. 3 and 4, an increase in the level of aggression leads to higher casualty rates for both flights. The best response for the player minimizing losses, i.e., blue in Fig. 4 and red in Fig. 3, is always the low aggression level, whereas for the player maximizing kills, i.e., blue in Fig. 3 and red in Fig. 4, the best response is the medium or high aggression level. The game for the difference of kills (Fig. 5) implies that the low aggression level is the dominating alternative for both players as it gives always the most desirable payoff value. Thus, according to this game, it is most effective to launch the missile and disengage. It should also be noted that the medium and high aggression levels produce identical payoff values. This could be explained, e.g., by an ineffective implementation of evasive maneuvers or overtly effective missile models that render the maneuvering after the missile launch irrelevant.

		RED (*min*)		
		end run	*split*	*cross*
BLUE (*max*)	*end run*	I (1.423)	II (1.545)	$\boxed{I}$ (1.418)
	split	II (1.553)	II (1.585)	$\boxed{I}$ (1.424)
	cross	I (1.433)	II (1.547)	$\boxed{I}$ (1.455)

Fig. 6. Matrix game with the payoff representing the number of blue kills. The Nash equilibria are enclosed in squares.

To summarize, the symmetric AC scenario under consideration produces symmetric games. In addition, the games are reasonable as an increase of aggression increases the casualty rates for both flights which is compatible with the actual AC. These observations point toward the validity of the simulation model. However, the analysis also implies some shortcomings in the simulation results. The dominance of the low aggression level in Fig. 5 and the identical payoff values for the alternatives *medium* and *high* may not be entirely realistic. Therefore, further analysis of the decision-making model of the simulated pilots as well as of the aircraft and missile models is recommended.

B. Scenario 2: Commit Maneuver

In the second scenario, the effect of the commit maneuver on the outcome of a two-versus-two AC is studied. The overall pilot behavior is similar to the medium aggression level in Scenario 1, but now, the flights can approach their opponent using three different maneuvers during the commit phase of AC. In other words, the tactical alternatives of the scenario present the maneuvering of flights at the beginning of the engagement. Three commit maneuvers are compared.

1) End Run. The flight approaches the opponent from left or right, depending on which side it finds the most advantageous.
2) Split. The flight splits, and the aircraft approach the opponent simultaneously from both sides.
3) Cross. The flights head toward the opponent, and the aircraft zigzag.

As in Scenario 1, the MOEs are the number of blue and red kills as well as the difference of kills. To produce MOE estimates, the scenario is simulated 2400 times for each of the nine combinations of input variable values that represent the commit maneuvers. The classification of the estimates gives the matrix games presented with the MOE estimates in Figs. 6–8. The players maximize their kills and minimize their losses. In Fig. 8, the difference of kills is defined so that it is maximized by blue and minimized by red.

		RED (*max*)		
		end run	*split*	*cross*
BLUE (*min*)	*end run*	*I–II* (1.444)	*IV–V* (1.541)	*II–III* (1.483)
	split	*III–IV* (1.526)	*V* (1.577)	*V* (1.578)
	cross	$\boxed{\textit{II–III}}$ (1.483)	*I* (1.411)	$\boxed{\textit{I–II}}$ (1.448)

Fig. 7. Matrix game with the payoff representing the number of red kills. The Nash equilibria are enclosed in squares.

		RED (*min*)		
		end run	*split*	*cross*
BLUE (*max*)	*end run*	$\boxed{\textit{II–III}}$ (−0.021)	*II–III* (0.004)	$\boxed{\textit{II}}$ (−0.065)
	split	*III* (0.027)	*II–III* (0.008)	*I* (−0.155)
	cross	$\boxed{\textit{II–III}}$ (−0.048)	*IV* (0.136)	$\boxed{\textit{II–III}}$ (0.007)

Fig. 8. Matrix game with the payoff representing difference of kills. The Nash equilibria are enclosed in squares.

First, the symmetry of the games is considered. The payoff values do not concur in Figs. 6 and 7. This results in disparate best responses for the players and asymmetric Nash equilibria. For example, in Fig. 6, the combination *split* for blue and cross for red is an equilibrium, but in Fig. 7, the combination cross and *split* is not. Altogether, the games do not mirror each other in a way that is suggested by the scenario at hand. On the other hand, the game for the difference of kills (Fig. 8) as well as its best responses and Nash equilibria are symmetric. Furthermore, if the flights perform the same maneuver, the MOE estimates do not differ statistically significantly from zero as one could expect based on the symmetric AC scenario.

Next, the players' best responses and dominating alternatives are studied. In the game for blue kills (Fig. 6), *split* leads to the highest payoff value regardless of the decision of red, and therefore, it is the dominating alternative for blue. In the case of red kills (Fig. 7), *split* is not the dominating alternative for red, and thus, the games are inconsistent. In Figs. 6 and 7, *cross* is found to be the dominating alternative for the player who minimizes losses. When the payoff is the difference of kills (Fig. 8), both players find cross to be the dominating alternative. This may not be entirely realistic, and the implementation of the commit maneuvers should be studied further.

To study the effect of initiative, the games are analyzed in the Stackelberg setting. The players are set alternately as the leader and the follower to see how the order of action affects the equilibrium solution of the game. In the first two games, the existence of the dominating alternatives makes initiative meaningless as the players select the dominating alternative regardless of the sequence of decisions. In the game for the difference of kills, the Stackelberg setting results in the same equilibria as the game with simultaneous action. Thus, initiative and information on the opponent's decision have no influence on the solutions of the games which is not entirely in accordance with the actual AC.

Overall, the findings do not concur with the AC scenario at hand. The games are not perfectly symmetric, and initiative does not affect any of the games. Most importantly, *cross* should not be the dominating alternative in the presented scenario. Therefore, the validation analysis suggests that the implementation of the commit maneuvers is flawed and that there exists a need for further development in the simulation settings.

TABLE II
PARAMETER VECTORS $\boldsymbol{\beta}_B$ AND $\boldsymbol{\beta}_R$ OF THE LOGISTIC REGRESSION MODELS REPRESENTING THE PROBABILITIES OF BLUE AND RED KILLS

		Prob. of blue kill	Prob. of red kill
variable	parameter	$\boldsymbol{\beta}_B$	$\boldsymbol{\beta}_R$
constant	β_0	-3.444 (±0.165)	-3.529 (±0.169)
x	β_1	0.289 (±0.027)	-0.126 (±0.022)
y	β_2	-0.131 (±0.021)	0.300 (±0.028)
x^2	β_3	-0.009 (±0.001)	0.011 (±0.001)
y^2	β_4	0.012 (±0.001)	-0.009 (±0.001)
xy	β_5	0.003 (±0.001)	0.003 (±0.001)
model deviance		1331	1310
degrees of freedom		44991	44991
statistical significance		1.000	1.000

C. Scenario 3: Support Time of a Missile

The third scenario concentrates on one-versus-one AC where the pilots face each other at the limit of the missile launch range. The pilots launch their missiles and support them, i.e., relay radar information about the opponent to their missiles to increase the likelihood of a hit. The tactical alternatives of the scenario are the support times of blue and red, denoted by $x, y \in [0, 15]$ (in seconds). The main idea of the scenario is that supporting one's missile increases the hitting probability. On the other hand, longer support times take the pilot to a more disadvantageous position with regard to evading the opponent's missile and increase the probability of being hit. The ranges of the support are dictated by the properties of the missile. In the scenario, there is no reason to support the missiles for longer than 15 s because earlier simulations have proved this to be the upper limit of the missiles' flight time.

Unlike the previous scenarios, now, MOEs are the probability of blue kill, the probability of red kill, and the weighted sums of the probabilities. The tactical alternatives are presented by the continuous input variables of the simulation model. The scenario is simulated for a set of input variable values that are selected according to a central composite design [42]. The used experimental design includes 12 combinations of the input variable values that are simulated 3000 times and a central combination that is simulated 12 000 times. Because the MOEs are probabilities, a logistic regression model [44] is fitted to the simulation data which gives regression models of the form

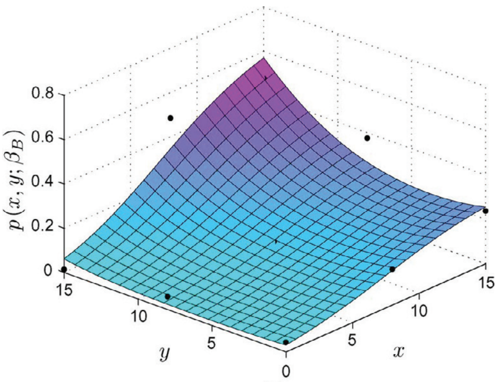
$$p(x, y; \boldsymbol{\beta}) = \frac{\exp\left(q(x, y; \boldsymbol{\beta})\right)}{1 + \exp\left(q(x, y; \boldsymbol{\beta})\right)} \tag{1}$$

where $q(x, y; \boldsymbol{\beta})$ is a quadratic function of the decision variables x and y, i.e.,

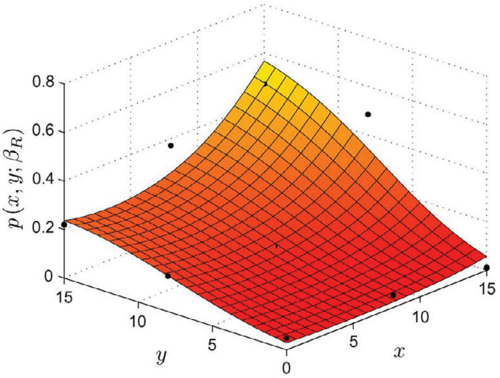
$$q(x, y; \boldsymbol{\beta}) = \beta_0 + \beta_1 x + \beta_2 y + \beta_3 x^2 + \beta_4 y^2 + \beta_5 xy. \tag{2}$$

The functional form of $q(x, y; \boldsymbol{\beta})$ is selected to allow for the curvature of the regression model. Most importantly, the term xy links the players' decision variables together and turns the model into a game instead of two independent optimization problems.

Games for the probabilities of blue and red kill estimated from simulation data and the resulting parameter vectors $\boldsymbol{\beta}_B$ for blue and $\boldsymbol{\beta}_R$ for red are presented in Table II. The 95% confidence intervals of the parameter estimates are given in



(a)



(b)

Fig. 9. Logistic regression models used in defining the payoffs of the support time game. (a) Probability of blue kill $p(x, y; \boldsymbol{\beta}_B)$ as the function of the blue support time x and the red support time y. (b) Probability of red kill $p(x, y; \boldsymbol{\beta}_R)$ as the function of the blue support time x and the red support time y.

parentheses. All the parameters are found to be statistically significant as their p-values are of magnitude 10^{-4} or smaller. The values of the model deviances are small compared with the respective degrees of freedom, implying that they have no statistical significance and are probably results of random variation (see Table II). Thus, the models fit the data well, and there is no sign of lack of fit or need for the addition of higher order variables. The MOE estimates and the regression models are shown in Fig. 9.

The payoff functions of the game are formulated by combining the probabilities of blue and red kills as weighted sums

$$u_B(x, y) = w_B p(x, y; \boldsymbol{\beta}_B) + (1 - w_B)\,(1 - p(x, y; \boldsymbol{\beta}_R)) \quad (3)$$

$$u_R(x, y) = w_R p(x, y; \boldsymbol{\beta}_R) + (1 - w_R)\,(1 - p(x, y; \boldsymbol{\beta}_B)) \quad (4)$$

where the weights $0 \leq w_B$ and $w_R \leq 1$. The payoff of blue $u_B(x, y)$ consists of the probability of blue kill $p(x, y; \boldsymbol{\beta}_B)$ and the probability of avoiding blue loss $(1 - p(x, y; \boldsymbol{\beta}_R))$. The larger the weight w_B, the more willing blue is to sustain losses,

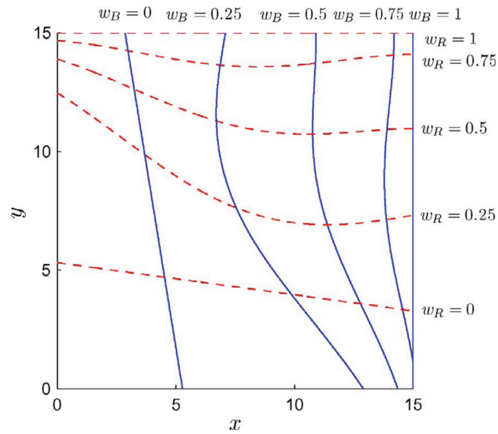


Fig. 10. Players' best response curves with different weights w_B and w_R. Blue's best response curves are marked with solid lines and red's with dashed lines.

e.g., by setting $w_B = 1$, the payoff reduces to the probability of blue kill. Similarly, when $w_B = 0$, the payoff equals the probability of avoiding blue loss. Thus, the weight w_B can be interpreted as a measure of aggressiveness for blue. The payoff of red $u_R(x, y)$ is constructed in a similar manner.

Players' best response curves, i.e., the optimal support times against a given support time of the opponent, are solved by maximizing the payoffs (3) and (4) while holding the opponent's support time constant. The responses are shown in Fig. 10 for a set of weights w_B and w_R. For instance, if blue is only interested in the number of blue kills, i.e., $w_B = 1$, the best response of blue to all red's support times is to support for as long as possible, i.e., $x = 15$. As mentioned in the definition of the scenario, there is no reason for supporting the missile any longer due to its limitations. If blue wants to minimize its losses, i.e., $w_B = 0$, it is optimal for blue to support its missile for approximately 5 s, and the optimal support time decreases linearly as a function of the red support time.

The payoff functions and the best response curves presented previously are used in validation analysis by considering their symmetry and the dependences implied by them. The scenario is symmetric, and therefore, also, the game should be symmetric, which holds for the presented game. The parameters in $\boldsymbol{\beta}_B$ and $\boldsymbol{\beta}_R$ are symmetric (Table II). For example, the parameter of x in the model for the probability of blue kill is 0.289 ± 0.027 and the parameter of y in the model for the probability of red kill is 0.300 ± 0.028. These estimates do not differ statistically significantly. Hence, the decision variables affect the probabilities of kills in a similar manner. This is also pointed out by the regression models in Fig. 9 which are perfect mirror images. Therefore, the payoff functions derived from the probabilities are also symmetric. Furthermore, the response curves in Fig. 10 are symmetric with respect to the line $x = y$ as expected.

The prolonging of the support times x and y generally results in higher kill probabilities (Fig. 9). Therefore, the estimated game seems to be mostly reasonable as it shows that supporting one's missile increases both probabilities of kill. However, if the launcher of the missile does not support the missile at all, the probability of kill is very small regardless of the support time chosen by the opponent. One could argue that the zero probability of a hit for an unsupported missile is not entirely realistic. The missile has also its own radar, and the performance of the missile should not be entirely dependent upon the launching aircraft's radar. Therefore, the performance and behavior of the missiles in the simulation model should be further studied and confirmed.

The best response curves are realistic as an increase in the weight assigned to the probability of kill leads to longer support times (Fig. 10). However, the best responses corresponding to the minimization of losses, i.e., the weights $w_B = 0$ or $w_R = 0$, deviate from the expected. Based on the actual scenario, the pilots should not support their missiles at all in order to minimize the probability of being hit. In the game, the players' best response is to support their missiles for approximately 5 s before heading away from the opponent. This is an inconsistency compared with the actual scenario that warrants further analysis of the implementation of the evasive maneuvers as well as of the range of the missiles and the detection range of their radars. Nevertheless, in general, the estimated game and the best response curves are in concordance with the actual AC scenario, and the analysis supports the validity of the simulation model.

V. Example of Strategy Analysis

In this section, the utilization of strategy analysis is illustrated with an example where an AC analyst aims for better understanding of a decision problem related to an AC scenario. In the problem, the analyst is supposed to assess the best launch range of an air-to-air missile in a two-versus-two AC scenario and study the impact of the missile launch range on the progress of AC. For demonstration purposes, the analyst's flight is called blue and the opponent's flight is called red.

At the beginning of the scenario, blue and red flights are flying directly toward each other. The flights have otherwise identical aircraft and weapon systems but the aircraft of red are substantially faster with maximum flight speed of 1.6 Mach compared with 0.85 Mach by blue. On the other hand, the blue flight has an initial altitude advantage as it flies at 10 000 ft compared with the 5000-ft altitude of the red flight. The flights launch their missiles and support them for a short period of time to increase the probability of a hit. After the support phase, the pilots execute a drag maneuver to avoid the opponents' missiles.

The tactical alternatives of the scenario are the launch ranges of the missiles which are, in reality, continuous. Now, to simplify the analysis and to reduce the number of necessary simulation replications, the launch range is discretized into three levels: *short* (10 nm), *medium* (12 nm), and *long* (14 nm). The effect of the missile launch range is studied with regard to the three objectives described by MOEs that are the difference of kills, kills for both sides, and the probability of shooting down all the aircraft of blue. The scenario is simulated using X-Brawler 2400 times for each of the nine combinations of the input variable values in order to produce simulation data necessary for the estimation of the games.

		RED (*min*)		
		short	*medium*	*long*
BLUE (*max*)	*short*	*II* (−1.050)	*I* (−1.176)	*I* (−1.152)
	medium	*III* (−0.848)	*III* (−0.800)	*V* (−0.432)
	long	$\boxed{IV}$ (−0.567)	*V* (−0.495)	*VI* (−0.130)

Fig. 11. Matrix game with the payoff representing the difference of kills. The maximin solution is enclosed in a square.

To thoroughly cover the decision problem, it is analyzed with multiple games that are selected to illustrate the full scope of the strategy analysis presented in Section III-C. In the first game, the maximin solution is determined using the difference of kills as the zero sum payoff. The second game is a non-zero sum game where both players maximize their kills. In the final game, the zero sum payoff is the probability of shooting down all the aircraft of blue. Overall, the decision problem is perceived from several viewpoints of blue, and the effect of different objectives on the outcome of AC is explicitly demonstrated.

A. Difference of Kills

First, the analyst wants to examine both the defensive and offensive aspects of AC from the viewpoint of blue. Thus, the MOE is set as the difference of kills that is maximized by blue. Uncertainty over the behavior of red is taken into account using worst case analysis. It is assumed that red has directly opposite objectives and can always select its best response after the decision is made by blue. As discussed in Section III-C4, worst case analysis is carried out by finding a maximin solution where blue acts first and red reacts to the decision of blue. The estimated game and the MOE estimates are shown in Fig. 11.

The worst case analysis yields the maximin solution of the game that is *long* for blue and *short* for red. Blue selects the decision alternative *long*, and red can choose the short missile launch range without suffering additional losses. Thus, the objective of blue, i.e., maximizing the difference of kills, is best achieved by only avoiding losses. Overall, it should be noted that all the MOE estimates for the difference of kills are negative, implying that red always attains a higher number of kills than blue. Therefore, the discussed AC scenario is disadvantageous for blue, and blue may only attempt to minimize this disadvantage by maximizing the distance to the opponent.

B. Number of Blue and Red Kills

After the worst case analysis, the analyst studies the problem by assuming that both flights act offensively and attempt to shoot down the opponent's aircraft. Then, their MOE is the number of kills. The situation is modeled as a non-zero sum game where the players maximize the payoffs representing their kills. The payoff values are assembled into a bimatrix shown in Fig. 12 where the MOE estimates are presented in parenthesis.

The decision alternative *long* is the dominating alternative for blue, and *short* is the dominating alternative for red. The game has three Nash equilibria. The equilibria *short* for blue and *medium*/*long* for red indicate that if red opts for a longer missile launch range, blue has to launch its missiles from the short range to attain the maximal number of kills. The equilibrium *long* for blue and *short* for red differs from the

		RED (*max*)		
		short	*medium*	*long*
BLUE (*max*)	*short*	*II,VI* (0.188, 1.238)	$\boxed{I,VI}$ (0.046, 1.222)	$\boxed{I,VI}$ (0.030, 1.182)
	medium	*II,V* (0.208, 1.056)	*I,IV* (0.048, 0.845)	*I,II* (0.029, 0.461)
	long	$\boxed{III,IV}$ (0.324, 0.891)	*I,III* (0.054, 0.549)	*I,I* (0.039, 0.169)

Fig. 12. Bimatrix game with the payoffs representing the number of blue and red kills. The first value is the payoff for blue, and the second value is the payoff for red. The Nash equilibria are enclosed in squares.

		RED (*max*)		
		short	*medium*	*long*
BLUE (*min*)	*short*	*V* (0.332)	*V* (0.326)	*V* (0.312)
	medium	*IV* (0.224)	*III* (0.136)	*I–II* (0.029)
	long	$\boxed{III}$ (0.151)	*II* (0.058)	*I* (0.003)

Fig. 13. Matrix game with the payoff representing the probability of shooting down all the aircraft of blue. The Nash equilibrium is enclosed in a square.

previous equilibria as, now, blue launches its missiles from the long range and red is forced to select the short launch range to maximize its kills.

Overall, the situation seems to be disadvantageous for blue because red achieves more kills than blue in all the outcomes of the game. In the Nash equilibria, one flight launches its missile from a longer range and the other one closes in before launching its missile. Whichever flight launches its missiles from longer range is at a considerable advantage as the opponent has to enter closer range to achieve maximal number of kills. In effect, the other flight actually flies into the missiles of its opponent and inadvertently maximizes the opponent's payoff.

C. Shooting Down Blue Aircraft

Finally, the analyst takes another perspective to the missile-launching problem by studying the scenario as an air-to-ground operation where blue is attempting a strike against a ground target. To protect the ground target, red maximizes the probability of shooting down all the aircraft of blue before they reach the launch area of air-to-ground missiles. Blue tries to reach the launch area, and thus, the probability in question is minimized by blue. The situation is described using a zero sum game whose payoff is the probability of shooting down all the aircraft of blue.

The game and its Nash equilibrium are shown in Fig. 13. The decision alternative *long* is the dominating alternative for blue, and *short* is the dominating alternative for red. The combination of the dominating alternatives forms the Nash equilibrium. Red has the highest probability of shooting down all the opposing aircraft if it launches the missiles from short range. On the other hand, blue minimizes the probability by launching its missiles from as far as possible.

To conclude, blue has to stay as far as possible from red in order to avoid losing all its aircraft, whereas red tries to launch the missiles from as close as possible. In the earlier phases of the strategy analysis, the scenario is found to be disadvantageous for blue. However, now, one notices that, by launching the missiles from long range, blue has a $1 - 0.151 = 0.849$ chance of preserving at least one aircraft that can reach the launch area of the air-to-ground missiles.

D. Remarks on the Strategy Analysis

The goal of the analyst in the example strategy analysis was to assess the best launch range of air-to-air missiles for the blue flight as well as to study the effect of the launch range on the course of the AC scenario. The analyst's observations from the games estimated during the analysis can be summarized as follows. From the view point of blue, the tactical alternative *long* is dominating in all the cases as the speed advantage of red makes it disadvantageous for blue to enter closer combat. Thus, blue should launch its missiles from the long range. On the other hand, if blue selects the longest launch range, red is able to launch its missiles from closer range due to its speed advantage.

In addition to the information on practical launch ranges and potential courses of AC, the strategy analysis yields a holistic conception of the AC scenario. The constructed games indicate that the scenario is highly disadvantageous for blue. Thus, based on the strategy analysis, the considered scenario should be avoided whenever possible.

The game settings studied in this example have been chosen to illustrate all aspects of strategy analysis introduced in Section III-C. Each game employs different payoffs, providing distinct reasoning about the scenario and its outcome. Even though the payoff of the game presented in Section V-C is closely related to the number of red kills, the example demonstrates the flexibility of strategy analysis as the payoff can be chosen to present almost any AC event within the scope of the simulation model. It should also be noted that all the games presented were estimated from a single simulation batch. Thus, the analysis of multiple objectives of the flights does not necessitate any additional simulation replications compared with the analysis of a single objective, provided that simulation output is collected comprehensively.

VI. Conclusion

This paper has presented a new approach to the analysis of AC combining game theory and discrete-event simulation. In the approach, data obtained by simulating AC scenarios are converted into games using statistical techniques. The payoffs of the games containing discrete or continuous decision variables are estimated using analysis of variance and multivariate regression analysis, respectively. The estimated games are applied for validating simulation models as well as for simulation-based strategy analysis. The game-theoretic approach extends one-sided statistical validation methods and simulation-based optimization approaches by taking into account the game setting that is a critical part of AC. On the other hand, the estimated games can be considered as a new type of simulation metamodel.

The application of games in validation is based on the comparison of their properties with actual AC practices. Such properties include symmetry of games, dependence between decision variables and payoffs, best responses, equilibrium solutions, and the effect of initiative. The utilization of these properties in validation is illustrated with the example analysis of an existing discrete-event simulation model. The analysis both gave positive feedback on the accuracy of the simulation model and revealed some inconsistencies in the simulation data. These observations based on, e.g., best responses and equilibrium solutions, could not have been made with traditional one-sided validation methods. Thus, the use of the game-theoretic approach gave additional insight into the validity of the simulation model. Overall, estimated games offer a way to present simulation data in an informative and easily interpretable form. Therefore, the game-theoretic approach is capable of answering validation needs in a transparent manner.

The use of the game-theoretic approach in strategy analysis allows one to study the effectiveness of tactical alternatives available in AC scenarios with respect to the objectives of both flights. The approach reveals the best response dynamic of the scenario that enables the study of the players' best responses to each of the opponent's tactical alternatives. The approach is flexible as it allows the modeling of the diverse objectives of the flights. Games can also be used in worst case analysis, providing tactical alternatives that secure the best possible outcome of the AC scenario for one flight when the opposing flight acts in the worst possible way from the viewpoint of the first flight. Furthermore, the dominating tactical alternatives can be easily pinpointed, which simplifies the decision-making problem under consideration by reducing the number of potential tactical alternatives.

The utilization of these properties is demonstrated with an example strategy analysis. In order not to obscure the aspects of strategy analysis, the example problem was quite basic. Nevertheless, similar analysis could easily be extended to more complex and refined decision problems. Most importantly, strategy analysis offers systematic and structured means for analyzing decision problems related to AC scenarios from several viewpoints using only a single simulation batch.

The game-theoretic approach can also be applied to simulation analyses in other fields than AC. In addition to military problems, the approach lends itself naturally to all studies, involving multiple actors with conflicting objectives and having a compelling need for simulation. Examples of such fields include problems in economics, computer science, and biology. A potential military application area could be the analysis of effect-based operations [51] which is a modern military planning concept that approaches the planning from a systems perspective and takes into account the multiple objectives of military operations.

A potential direction for future research is to apply the principles of the game-theoretic approach in analyses of discrete-event simulation models and simulation data that do not originally represent a game setting. In such analyses, the input of a simulation model is set as the decision variable of a player, and a pertinent random factor included in the simulation model is treated as the decision variable of a virtual opponent. Then, games estimated from the simulation data could be combined with worst case inference to get an overview of the total impact of uncertainties presented by the random factor. In other words, a game against nature is proposed (e.g., [21]). In this way, the application area of the game-theoretic approach could be extended well beyond situations with an inherent game setting.

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