Errata

Publication I

The proof of Proposition 5.5 is incorrect, and thus the proof of Proposition 5.6 is flawed as regards the parts based on using Proposition 5.5. Accordingly, the proof for the efficiency estimate of Theorem 5.8 is incomplete for boundary edges with a non-vanishing ε. However, there is strong numerical evidence that the estimator proposed is also efficient as shown in the numerical results in Section 7. We can show the following suboptimal estimate for the boundary edges with $\varepsilon \neq 0$,

$$\eta_E^2 \leq (\varepsilon + h_E)\|(\sigma_h - \sigma)\cdot \boldsymbol{n}\|_{0,E}^2 + \frac{1}{\varepsilon + h_E}\|u_h^* - u\|_{0,E}^2 + (\varepsilon + h_E)\|g - g_h\|_{0,E}^2.$$

The above can be shown by directly inserting the exact boundary condition into the boundary edge estimator η_E yielding

$$\begin{aligned}\varepsilon(\sigma_h\cdot \boldsymbol{n} - g_h) + u_h^* - u_0 &= \varepsilon(\sigma_h\cdot \boldsymbol{n} - g_h) + u_h^* - \varepsilon(\sigma\cdot \boldsymbol{n} - g) - u \\ &= \varepsilon(\sigma_h - \sigma)\cdot \boldsymbol{n} + (u_h^* - u) + \varepsilon(g - g_h).\end{aligned}$$

Using the triangle inequality and the relation $\varepsilon/\sqrt{\varepsilon + h_E} \leq \sqrt{\varepsilon} \leq \sqrt{\varepsilon + h_E}$ gives the desired result. Note, that the above estimate is suboptimal in the sense that given an irregular boundary load g the contribution from the boundary load error can be substantial and grows as the root of the ε parameter. Furthermore, the flux estimate is in the $\|\cdot\|_{\varepsilon,h}$ norm in contrast to the reliability and convergence estimates given in the L^2 norm. However, assuming a certain degree of regularity for the solution the estimator can be shown to converge with the same ratio as the error.

The authors are working towards presenting a corrected proof for the original results.