O. V. Lounasmaa Laboratory

Nanocircuits with Superconductivity

Nonequilibrium studies

Juha Voutilainen

Aalto University

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Nanocircuits with Superconductivity: Nonequilibrium studies

Juha Voutilainen

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Abstract

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Abstract

This thesis concerns low-temperature nanoelectronics and, in particular, nonequilibrium phenomena which come about when the concept of temperature loses its significance in nanoscale electric circuits. Under more conventional circumstances, these nonequilibrium phenomena are smothered by thermal motion or die away at large scale but they become essential for a field that is looking for applications in structures smaller than one micron, at temperatures lower than one degree above the absolute zero. The appearance of superconductivity — the technologically-interesting phenomenon of dissipationless transport of electric current — precisely at low temperatures further boosts the chance of finding these applications.

With my collaborators, I consider five different types of setups where nanoconductors are coupled to superconducting electrodes in typical low-temperature operating conditions which lead to the formation of a nonequilibrium state. Using the established theoretical methods of the field, I estimate how the nonequilibrium state affects the properties of the conductor, for example, in metallic conductors which have acquired superconducting properties due to superconducting proximity effect and in graphene, a one-atom thick film of carbon.

As a result of our work, it is possible to predict in detail when nonequilibrium phenomena should become observable. These phenomena include rectification of electric current and the enhancement of supercurrent due to incoherent noise in the electromagnetic environment of the conductor. In particular, our studies imply that the effect of the nonequilibrium state in actively-researched radiation detectors based on electron heating has been so far underestimated, whereas in graphene the equilibrium state is preserved under more extreme conditions than would be expected from the behavior of metallic conductors of the same size. Nevertheless, we observe that the the nonequilibrium phenomena bring about signifant effects in all the studied setups which means that these phenomena must be taken into account when designing any applications in the field of low-temperature nanoelectronics.

Keywords nanoelectronics, superconductivity, nonequilibrium

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Tekijä
Juha Voutilainen

Väitöskirjan nimi
Nanoelektroniikka + Suprajohtavuus: tutkielmia Epätasapainosta

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Tiivistelmä

Tämä väitöskirja käsittelee matalien lämpötilojen nanoelektroniikkaa ja erityisesti epätasapainoilmiöitä, jotka tulevat esiin kun lämpötilan käsite menettää nanomittakaavan virtapiireissä merkityksensä. Nämä epätasapainoilmiöt peittyvät tavanomaisemmissa olosuhteissa lämpöliikkeen alle tai vaimenevat suuressa mittakaavassa merkityksettömiksi, mutta ne muodostuvat keskeisiksi alalla, joka etsii sovellusmahdollisuuksia alle mikrometrin kokoluokkaa olevista rakenteista ja matalimmillaan asteen murto-osan absoluuttisen nollapisteen yläpuolella olevista lämpötiloista. Sähkövastuksen katoamiseen johtavan ja siten myös teknologisesti kiinnostavan ilmiön, suprajohtavuuden, esiintyminen juuri matalissa lämpötiloissa lisää entisestään näitä mahdollisuuksia.

Tarkastelen yhteistyökumppaneitteni kanssa viittä erilaista suprajohteisiin kytkettyä nanojohdinta tyypillisissä matalan lämpötilan toimintaolosuhteissa, jotka johtavat epätasapainotilan muodostumiseen. Alalla vakiintuneita teoreettisia menetelmiä käyttämällä arvioin, miten epätasapainotila muuttaa johtimen ominaisuuksia muun muassa suprajohtavan läheisilmiön vaikutuksesta osin suprajohtavia piirteitä sisältävissä metallijohteissa sekä yhden atomin paksuisesta hiilikalvosta muodostuvassa grafeenissa.

Työn tuloksena on mahdollisuus ennustaa yksityiskohtaisesti, milloin eri nanojohtimissa tulisi näkyä epätasapainoilmiöitä kuten sähkövirran tasasuuntausta tai sähkömagneettisen ympäristön satunnaiskohinasta aiheutuvaa supravirran voimistumista. Erityisesti aktiivisena tutkimuskohteena olevien, elektronien lämmittämiseen perustuvien säteilyilmaisimien osalta epätasapainon vaikutus on tutkimukseni mukaan aliarvioitu, kun taas grafeenissa tasapainotilan rikkominen on vaikeampaa kuin samankokoisten metallijohtimien käyttäytymisestä voisi odottaa. Yleisesti ottaen havaitsemme epätasapainoilmiöillä olevan merkittävä vaikutus kaikissa tutkimissamme kohteissa, mikä tarkoittaa, että ne on myös otettava huomioon mahdollisia matalan lämpötilan nanoelektroniikan sovelluksia suunniteltaessa.

Avainsanat nanoelektroniikka, suprajohtavuus, epätasapaino

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Preface

I would like to begin by saying that this thesis is a work of love. However, it is not. This thesis is a work of perseverance and resilience on a long road of confusion, doubts, occasional moments of despair, and, to outweigh it all, thrill of accomplishment and learning.

It all started when I stumbled into the Low Temperature Laboratory of then Helsinki University of Technology as a young undergraduate student with little knowledge, high expectations, and the goal, all the way from the start, of one day obtaining a PhD. I was assigned to do theory under the guidance of Nikolai Kopnin and Tero Heikkilä both of whom have greatly impressed me with their dedication to what they do and who I have to thank for their invaluable impact on this particular work. I was not the easiest student as in the realm of theoretical quantum physics with elevated level of abstraction, my most acute ambitions resided on tangible results, concrete real-life applications, and understanding projects in the simplest possible terms. This led me to work with the experimentalists headed by the current director of the laboratory, Pertti Hakonen, and to teach on graduate-level courses given by the university. I have gained a lot from both experiences and their fruits can be traced all the way to the work you hold in your hands.

During my studies, I have had the pleasure to learn from various distinguished individuals. When I started, my supervisor Risto Nieminen advised me to seize the responsibility of funding my own work by applying for scholarships and grants. This advise has been well heeded and I am thankful to Magnus Ehrnrooth Foundation, Foundation of Technology Promotion in Finland, and Finnish Academy of Science and Letters for their ample support which has allowed me to both work independently and broaden my horizon by participating in scientific gatherings around the world. In the open-minded society of scientists I have not only spent

countless occasions on discussing about the latest applications of nanotechnology but also enjoyed numerous enlightening conversations on life in itself — and learned downhill skiing in the process.

Even with all the excitement provided by the international scientific community, this long road would have been a bore if not for a joyful and dependable home base. A few names deserve an extra mention, starting from Pirjo Muukkonen, whose professional can-do attitude was the cure for my miscellaneous problems in the early part of my stay in the laboratory, and Teija Halme, whose ever-available support allowed me to enjoy, without care, the adventure of returning across Europe from Capri island after the volcanic ash incident of 2010.

The long list of people whose professional and social collaboration I highly appreciate includes Khattiya Chalapat, Timothé Faivre, Aurelién Fay, Simone Gasparinetti, Petri Heikkinen, Jaakko Hosio, Pasi Häkkinen, Risto Hänninen, Raphaël Khan, Matti Laakso, Jian Li, Matti Manninen, Teemu Ojanen, Antti Paila, Joonas Peltonen, Antti Puska, Juho Rysti, Anssi Salmela, Karthikeyan Sampath Kumar, Jayanta Sarkar, Xuefeng Song, Jaakko Sulkko, Matti Tomi, Janne Viljas, Pauli Virtanen, and many others with whom my encounters have been briefer. As the departing captain of the laboratory's floorball team, I also wish well for the Pyromaniac Penguins and look forward to hearing them top last year's success.

It is impossible to acknowledge the support I have received during this work without bringing forth my family and friends, including the culture club Ota1. By never asking *anything* about my work, they have provided a perfect counterbalance to the occasionally engulfing world of research. While you cannot choose your relatives, Tarja, Pentti, Arto, and Aino are about the best people you can hope to get stuck with for life. And when it comes to people you can choose, I thank my spouse Maija who I met just after starting my PhD studies, and married just before finishing them. Her support has increased continuously during this process and it culminates in a concrete way on the cover of this work.

I briefly return to reflect the time I have spent in the Low Temperature Laboratory. Even if our research unit has changed its name and location with numerous wonderful people coming and going during the years, for me the fundamental elements have stayed the same as they were on the day I arrived here. To think that now, almost 8 years later, I have finished exactly what I set out to do is perplexing — a sensation underscored by the fact that the future, even for the next 8 *weeks*, is in large parts open

to me. It has been a period of stability and endeavor toward a set goal, combined. Now it is time to see what the foundation built during this time is good for. And to continue learning.

Espoo, April 24, 2012,

Juha Voutilainen

Contents

List of Publications

This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.

I Juha Voutilainen, Tero T. Heikkilä, and Nikolai B. Kopnin,
Nonequilibrium phenomena in multiple normal-superconducting tunnel heterostructures,
Physical Review B, **72**, 054505 (2005).

II Nikolai B. Kopnin and Juha Voutilainen,
Nonequilibrium charge transport in quantum SINIS structures,
Physical Review B, **75**, 174509 (2007).

III Juha Voutilainen, Matti A. Laakso, and Tero T. Heikkilä,
Physics of proximity Josephson sensor,
Journal of Applied Physics, **107**, 064508 (2010).

IV Juha Voutilainen, Aurelién Fay, Pasi Häkkinen, Janne K. Viljas, Tero T. Heikkilä, and Pertti J. Hakonen,
Energy relaxation in graphene and its measurement with supercurrent,
Physical Review B, **84**, 045419 (2011).

V Juha Voutilainen, Pauli Virtanen, and Tero T. Heikkilä,
Absorption of heat into a superconductor-normal metal-superconductor junction from a fluctuating environment,
eprint arXiv:1202.3358, 5 pages + 3 pages of supplementary information (2012).

Author's Contribution

Publication I: "Nonequilibrium phenomena in multiple normal-superconducting tunnel heterostructures"

Author did the numerics and participated in writing the paper.

Publication II: "Nonequilibrium charge transport in quantum SINIS structures"

Author participated in solving the problem and writing the paper.

Publication III: "Physics of proximity Josephson sensor"

Author was responsible for writing the paper and producing the content, apart from Eq. (8) derived by Matti Laakso.

Publication IV: "Energy relaxation in graphene and its measurement with supercurrent"

Author was responsible for writing the paper and producing the content, apart from the experimental Chapter II primarily done by Aurelién Fay.

Publication V: "Absorption of heat into a superconductor-normal metal-superconductor junction from a fluctuating environment"

Author did the numerics and participated in writing the paper.

1. Introduction

This thesis concerns *nonequilibrium transport* in *nanoscale electronic circuits* with *superconducting components*. In the big picture, nanotechnology and the discovery of superconductivity are two major breakthroughs of 20th century physics. The first makes it possible to manipulate objects down to the atomic scale (and further...) whereas the latter is a fascinating phenomenon with the hallmark of zero electrical resistance below a material-dependent critical temperature. They both promise a plethora of applications to improve the efficiency of our work and the quality of our lives, some of which have already turned to reality. However, despite the heavy research effort invested in both fields during the latter part of the 20th century, we can safely say that in terms of technological advancement both fields are still at most in their adolescence — and this is even more evident when we consider the opportunities in merging the two fields. This means that what is known is not enough: From the perspective of fundamental research there is still a lot of work to be done to understand the possibilities emerging from our newly-acquired ability to shape matter at the sub-microscopic level. According to nobelist Herbert Kroemer in his account on the prospects of the emerging field: [1]

> *"Nanotechnology should be expected to have a rich future with new applications, but the details of that future are resistant to prediction."*

The goal of our research then is to shed light on these details.

Nanotechnology itself is a broad concept so here we apply nanotechnology specifically to electronic components operated at temperatures of the order of at most 1 Kelvin. However, the resulting field of *low-temperature nanoelectronics* is still incredibly diverse. This is because of the appearance of quantum-mechanical effects which markedly alter the properties

of nanoelectronic components at small length scales, typically below 1μm, and sub-Kelvin temperatures which, due to advances in experimental techniques, are nowadays realized in research laboratories in a workmanlike manner. As such, nanoelectronics prepares the way for a completely new kind of engineering. While some research groups exploit these newfound skills for conventional purposes, such as producing different types of detectors with vastly improved sensitivity, there are also others who have embarked on visionary quests such as searching new exotic particles [2] or building a *quantum computer* [3]. Whichever the goal, the possibilities of nanoelectronics engineering are only beginning to emerge. And to make matters even more interesting, superconductivity is based on quantum phenomena as well, making it an ideal addition to the repertoire of building blocks for tomorrow's nano circuits. There is a long way to go to understand not only the intrinsic properties of these blocks but, most importantly, their interplay with one another in practicable nanoelectronics realizations.

The choice of building blocks, while important, is only the first part of devising a nanoelectronic circuit. Equally important is the question of how the blocks are manipulated by external stimuli such as electric voltages, radiation fields or thermal fluctuations in the environment. These stimuli drive the system out of thermodynamic equilibrium, so that temperature is no longer well defined, and this can lead to exceptional outcomes for three reasons:

1. In nanoscopic conductors even a slight input of external energy may change the dynamics of the system
2. In sub-Kelvin temperatures thermal noise and the usual energy relaxation processes are no longer dominant
3. We can include superconductors as nonlinear circuit elements

The result is observable in transport of both energy and electric charge.

In this thesis, I present our studies on a choice of nanoelectronics systems, where we can use external driving to erect a nonequilibrium state which substantially alters the properties of the system so that peculiar outcomes unfold. The systems are composed of superconductors connected to each other by different types of *nanoconductors* and they can be manipulated to show a range of nonequilibrium effects from current rectification to enhancement of the dissipationless supercurrent. This is theoretical work but the experimental realizations are not far away as I show below.

Organization of this thesis

This thesis is an article dissertation composed of an overview, where I aspire to give the reader a broad understanding of what we have done and why, and a bundle of our articles published in peer-reviewed scientific journals which provide the actual details of the work. The main matter of the overview is divided into three parts: Chapters 2, 3, and 4, which can also be summarized as “Phenomena”, “Methods”, and “Results”. In the first part I explain what is meant by nonequilibrium in context of nanoelectronics, how the nonequilibrium state is formed in a nanoconductor and how superconductivity affects it. In the second part, I present the theoretical techniques we have used to describe practical nanosystems. In the third part I apply this knowledge to a group of select systems and show how the state of the system affects observables such as the (super)current in various up-and-coming nanodevices and how this can sometimes be exploited. In addition, in Chap. 5 I reflect on the prospects of this field of research.

I have also included a selection of references in *Bibliography* to works I **a)** have found personally beneficial when putting this thesis together and **b)** consider topical to its content at the time of its publishing. The field of nanotechnology is ever-changing and new results keep pouring in with commendable speed. I consciously aim to refer to any usable reviews where this progress has been analyzed and, when none exist, to the latest advances on a particular well-defined topic. For an interested reader, the latter generally include a useful review-like summary of prior works (usually as early as in the introductory part) and I warmly recommend reading up on them for better understanding of the background of each particular topic. Finally, I include the heart of the thesis, my five publications, in unamended form.

2. The concepts: Nanoconductors + Nonequilibrium + Superconductivity

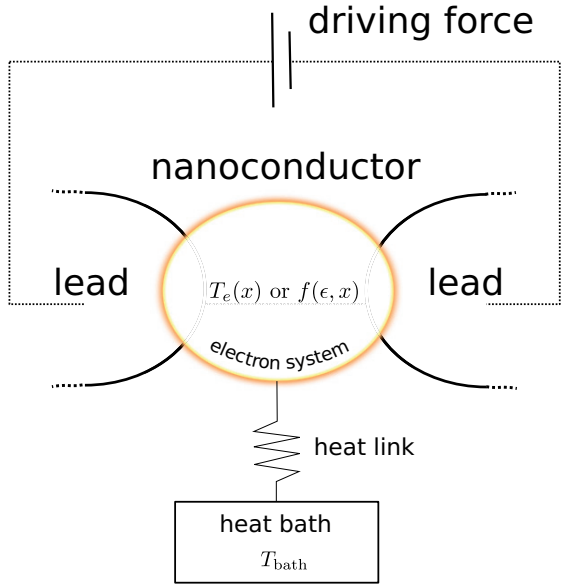

Figure 2.1. Basic outline of a nanoelectronics system

Thermodynamical equilibrium is defined as a state where there are no gradients in the essential state variables [4]. This means that systems composed of discrete blocks where an electric or thermal current flows from one block to another due to a difference in chemical potential, temperature, etc., such as the one in the center of Fig. 2.1, should as a whole be considered to be in a nonequilibrium state. However, such systems can typically attain a time-invariant steady-state, where the properties of the system stay constant when averaged over a time scale longer than the short intrinsic relaxation time of the system (in practice, significantly smaller than for example 1 ms). While the behavior of the system is thus constant in time, its properties can still be greatly affected by the balance between the driving force(s) and the relaxation processes which work to equilibrate the system.

2.1 Driven nanosystems out of equilibrium

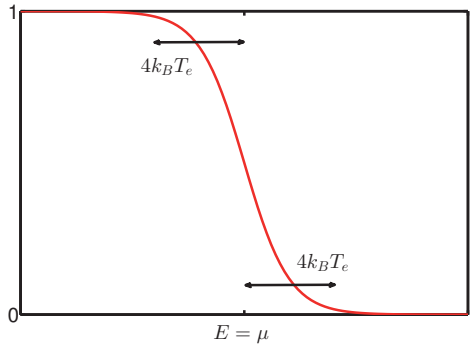

Figure 2.2. (Quasi)equilibrium energy distribution at temperature T_e

When we study how the properties of nanoelectronics systems change under external driving, we use the word *equilibrium* not for the whole system but for the equilibrium established between the *electron system* in the nanoconductor[1] (in the center of Fig. 2.1) and its own local *heat bath*, i.e., the phonon system at temperature $T_{\rm bath}$. As such, the electron system is in equilibrium when it can be described by the Fermi–Dirac distribution

$$f_0(\epsilon, x) = \frac{1}{\exp\left[(E - \mu(x))/k_B T_e(x)\right] + 1}, \tag{2.1}$$

defined by only two parameters: the local chemical potential $\mu(x)$[2] and the local electron temperature $T_e(x) = T_{\rm bath}(x)$ at position x. The former determines the zero point of the distribution with excitations of energy $\epsilon = E - \mu(x)$, whereas the latter defines its slope around this point, as visualized in Fig. 2.2. The nanoconductor can be driven out of this equilibrium state by feeding energy into the system, for example, from a voltage or current source, high-temperature environment or an external radiation source. The state of the system is then determined by what happens to this excess energy after it is first deposited in the system, either as a new high-energy electron or as an excitation of one of the pre-existing electrons of the nanoconductor.

[1] In superconductors the independent *quasiparticles* are not strictly speaking electrons but rather combinations of electrons and holes. In this overview, we however talk simply about electrons.

[2] Generally, $\mu(x)$ is determined by the driving processes but it can also be controlled by applying an additional *gate voltage* to the nanoconductor through a capacitive connection. This *gating* is usually performed evenly across the length of the nanoconductor so that only the overall level of μ changes: $\mu(x) \to \mu(x) + \delta\mu$.

The energy of the first excitation can be transferred forward in the system in the following ways:

1. to the leads by the escape of the electron,
2. to the heat bath, i.e., the lattice phonons,
3. to "the sky", through electromagnetic radiation,
4. to other electrons in the nanoconductor.

The first three alternatives result in the energy escaping the electron system altogether whereas the inelastic *electron-electron scattering* only redistributes the energy inside the system. The significance of each mechanism is, however, strongly dependent on the operating conditions.

Typically, inelastic *electron-phonon scattering* is the strongest relaxation mechanism and in the practical situations of this thesis work, it always dominates the radiative *electron-photon relaxation*, which I briefly justify in Sec. 3.2.1. It becomes weak only at very low temperatures and the equilibrium state of Eq. (2.1) is thus established whenever electron-phonon scattering dominates, i.e., the corresponding relaxation time is smaller than the time scale with which energy is deposited in the system, $\tau_{\mathrm{e-ph}} \ll \tau_{\mathrm{drive}}$. For electrical driving with current I this driving time scale is simply the inverse rate with which individual electrons enter the conductor: $\tau_{\mathrm{drive}} = (I/e)^{-1}$.

When the operating temperature is lowered so that $\tau_{\mathrm{e-ph}} > \tau_{\mathrm{drive}}$, the electron system is driven out of equilibrium and other relevant time scales come about. Assuming the electron-electron relaxation is strong enough, $\tau_{\mathrm{e-e}} \ll \tau_{\mathrm{drive}}$, the electron system can still be described by Eq. (2.1) but with a distinct electron temperature $T_e \neq T_{\mathrm{bath}}$. In this case, T_e is determined by the balance of heat currents flowing to and from between the leads and the nanoconductor and we call such a state *quasiequilibrium*. When the electron system is cut off from its direct link to the heat bath in this way, it becomes possible to heat the electrons independent of the lattice (see Ref. [5] from 1985) or even cool them as demonstrated first in 1994 in Ref. [6].

In the previous case, we still assume that the electron-electron interaction is strong compared to the driving forces. However, in structures of the order of microns and smaller this is not necessarily true anymore (I give a quantitative measure also for this in Sec. 3.2.1). When the electron-electron relaxation time $\tau_{\mathrm{e-e}} > \tau_{\mathrm{drive}}$, the system is in full nonequilibrium.

In this case, the noninteracting electrons have no means to thermalize even amongst themselves and the distribution function $f(\epsilon, x)$ may assume a form drastically different from that of Eq. (2.1), demonstrated experimentally in 1997 [7]. This can, in turn, result in strongly nonlinear features in the electric or heat current through the system.

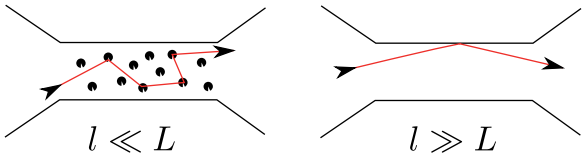

Figure 2.3. Diffusive and ballistic conductivity for a nanoconductor with length L and elastic relaxation length l which is roughly of the same order as the distance between individual elastic scattering events.

The final important and typically the shortest relaxation time scale for nanoconductors is the elastic relaxation time τ which describes relaxation from collisions between electrons and lattice defects. While these collisions no longer affect the relaxation of energy, they have a strong influence on how fast the electrons escape the nanoconductor and consequently on the dynamics of the system. Apart from Publication II, we assume that $\tau \ll \tau_{\mathrm{e-ph}}, \tau_{\mathrm{e-e}}, \tau_{\mathrm{drive}}$ and that the corresponding elastic relaxation length l is much smaller than the system dimensions relevant to transport. As the result, the electron loses memory of its initial direction and the state of the system as described by Eq. (2.1) is independent of the direction of electron momentum. Such nanoconductor is *diffusive* and this is typical for metallic conductors which have $l \gtrsim 10$ nm. However, there exists exotic nanoconductors such as semiconductor nanowires and carbon nanotubes where $l > 1$ μm is viable and, with the other relaxation processes also suppressed, they can function as *ballistic* conductors in which there is no scattering except at the boundaries of the conductor (see Ref. [8]). We consider nonequilibrium effects in such a conductor in Publication II.

2.2 Inclusion of superconductors

We have studied systems where superconductors are used as contacts (or electrodes) sandwiched around a nanoconductor so that they together form a superconductor-nanoconductor-superconductor junction, abbreviated as *SNS junction* where N refers to *normal* conductivity.[3] For the configuration of Fig. 2.1 in the beginning of this chapter, this simply means that the leads are made of superconducting material and the nanoconductor acts as a *weak link* bridging them. Generally, such weak links are governed by the Josephson relations discovered already in 1962 [9]. They state that superconducting junctions can transport tunneling supercurrent I_S when there exists a finite difference $\varphi = \phi_1 - \phi_2$ between the phases $\phi_{1,2}$ of the macroscopic order parameters of the superconductors that form the junction. In the simplest case where the current-biased superconductors are *not* separated by an actual nanoconductor but only by a thin layer of high-resistivity material, implying strong elastic scattering, this phase difference is set by the *dc* Josephson relation

$$I_S(\varphi) = I_c \sin(\varphi). \tag{2.2}$$

The tunneling current has a maximal value, critical current I_c, above which the junction switches to dissipative conduction and a finite voltage V starts building up between the superconductors.

The dissipationless supercurrent in superconductors is carried by correlated pairs of electrons known as the Cooper pairs [10]. While the original work of Josephson predicts tunneling of Cooper pairs, it has long been known that in weak links the pairs may break and reform so that supercurrent transport is actually governed by diverse rules which are set by the properties of the weak link *and* whether the system is in equilibrium or not [11]. As such, $I_S(\varphi)$ can differ markedly from its sinusoidal form depending on the type and state of the junction [12]. However, the dissipative dynamics of φ, determined by the *ac* Josephson relation

$$V = \frac{\hbar}{2e}\frac{d\varphi}{dt}, \tag{2.3}$$

remain independent of these details and are consequently universal — even so that Eq. (2.3) is valid also when the nanoconductor forming the link is out of equilibrium. The total current is composed of both the supercurrent and dissipative-current components and in typical voltage-biased

[3] In this text, I use S to denote a superconducting element and N to stand for a normal (nano)conductor, as is the usual convention within the field.

SNS junctions it changes from the zero-voltage value of Eq. (2.2) to follow the linear Ohm's law $I = V/R_N$ for $V \gg R_N I_c$, where R_N is the resistance of the nanoconductor in the normal-conducting state. In the intermediate regime, the current can be strongly affected by the nonequilibrium effects.

From the circuit perspective, using superconductors makes it possible to run supercurrent through the nanoconductor. However, we are interested in the process from the perspective of the nanoconductor and how the transport process through the conductor is modified when it is contacted to superconductors. There follows four major effects:

1. superconducting proximity effect,
2. modified density of states,
3. Andreev reflection,
4. suppression of heat transport,

which are partly derived from each other. I introduce these below.

2.2.1 Superconducting proximity effect

When superconductors are contacted to normal metals with a good contact, i.e., that of high transparency, the correlations between Cooper pairs do not immediately disappear at the interface but instead decay within a distance of the superconducting coherence length ξ. This *superconducting proximity effect* means that in the presence of superconductors a nanoconductor can also become partly superconducting regardless of its intrinsic type. In intrinsic superconductors, the coherence length is simply of the size of a Cooper pair, $\xi \sim \hbar v_F/\Delta$, where v_F is the material-dependent Fermi velocity and Δ the pairing energy per one electron, whereas in normal conductors, it depends on ϵ of Sect. 2.1, i.e., the single-particle energy of an electron broken away from the pair. In ballistic conductors, we thus have $\xi \sim \hbar v_F/\epsilon$. In diffusive conductors, the collision-hampered electron motion away from the interface is characterized by the sample-dependent diffusion constant D so that $\xi \sim \sqrt{\hbar D/\epsilon}$. Generally, it is useful to define these quantities not for single electrons but for the electron system as a whole in which case ϵ is replaced by the value relevant for the dominant physical processes. For example, in equilibrium the energy content of an electron system is described by temperature and we may use a thermal coherence length by setting $\epsilon \to k_B T_{\rm bath}$ to characterize the system on average.

In addition to the coherent motion of the electron pairs over the distance ξ, i.e., *two-particle coherence*, there exists another type of coherence, namely the quantum mechanical *single-particle phase coherence* of the individual electrons. As a quantum phenomenon, superconductivity also requires single-particle coherence and the induced superconductivity in a nanoconductor does not survive over distances longer than the single-electron phase-coherence length l_ϕ which is of the order of the inelastic relaxation length. Consequently, in SNS structures the proximity effect penetrates the whole nanoconductor only if its length $L < \min(\xi, l_\phi)$.

The proximity effect itself has been known for a long time but it returned to the focus of scientific interest in the late 1990's [13]. It leads naturally to intrinsic modifications to the properties of nanoconductors.

2.2.2 Modified density of states

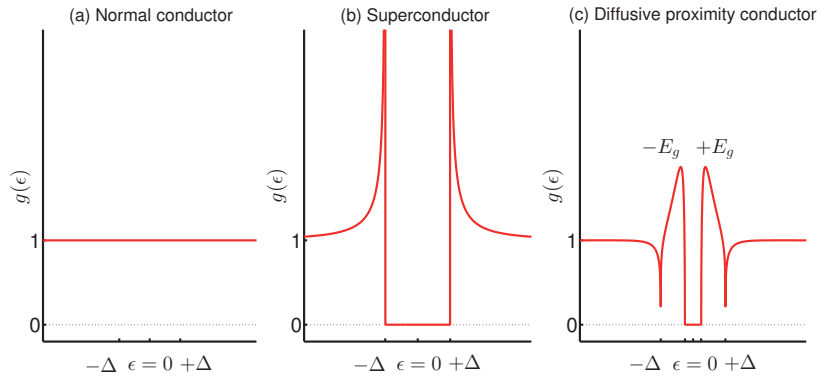

Figure 2.4. Density of states in a normal conductor, in a superconductor and in a specific realization of a diffusive proximity conductor (at the midpoint), all normalized to the density of states of the same system in the normal state so that $g(\epsilon) = N(\epsilon)/N_F$.

While in normal conductors the electron density of states (DOS) is constant N_F around the Fermi level, in superconductors it becomes energy-dependent $N_S(\epsilon)$ and a gap of width 2Δ opens near the Fermi level as visualized in Figs. 2.4(a) and 2.4(b). We can write this superconducting DOS in a simple form [10]

$$g_S(\epsilon) \equiv \frac{N_S(\epsilon)}{N_F} = \begin{cases} \frac{|\epsilon|}{\sqrt{\epsilon^2 - \Delta^2}}, & |\epsilon| > \Delta \\ 0, & |\epsilon| \leq \Delta \end{cases} \tag{2.4}$$

as given by the microscopic theory of superconductivity. Since the proximity effect induces superconductor-like properties into an adjacent nanoconductor, it also modifies the density of states of the conductor.

In a diffusive nanoconductor, the DOS profile of the system depends on the position x inside the conductor and, when placed between superconductors, also on the phase difference φ as measured in Ref. [14]. Near the superconductors, the profile is close to that of Fig. 2.4(b) but at the midpoint of a SNS system, it has a *minigap* like the one shown in Fig. 2.4(c) with the total absence of states restricted to energies $|\epsilon| < E_g < \Delta$. While the DOS profile remains quantitatively position dependent, this feature of a shrunk energy gap is uniform throughout the whole nanoconductor and it even extends a small way into the superconductors, so that their DOS at $\epsilon < \Delta$ becomes finite as well (although very small). The minigap disappears for $\varphi = \pi$, and for $\varphi = 0$ it has a maximum $E_g \approx 3.1E_{\rm Th}$, where $E_{\rm Th}$ is the Thouless energy of the junction corresponding to the inverse of the time τ_D it takes for an electron to diffuse through the nanoconductor. This is the characterizing energy scale for diffusive nanoconductors and for a junction of length L and diffusion constant D it reads

$$E_{\rm Th} = \frac{\hbar}{\tau_D} = \frac{\hbar D}{L^2}. \tag{2.5}$$

Since $E_{\rm Th}$ determines the width of the superconducting-like energy gap in a *diffusive proximity conductor* described here, it also sets the energy scale above which superconducting features begin to disappear in such systems.

In ballistic conductors there is no scattering so the DOS profile is uniform in space. It also follows that when superconductors with transparent contacts are introduced to form a *ballistic proximity conductor*, the resulting DOS profile is in many ways reminiscent to that of a superconductor as in Fig. 2.4(b). However, due to *Andreev* two-particle scattering processes (which we discuss in the next section) at the N-S interfaces, a set of discrete energy states appears at energies $|\epsilon| < \Delta$. The number of these states depends on the flight time $\tau_L = L/v_x$ of the electron traversing the conductor of length L with constant speed v_x so that it is roughly $N \approx 2\Delta\tau_L/\pi\hbar$ with a minimum of two for very short junctions, i.e., point contacts. Because of these levels, the DOS profile for a ballistic proximity conductor at *subgap energies* $|\epsilon| < \Delta$ is composed of sharp peaks at the exact locations of the levels and it is more informative to show the energy spectrum of the junction than to plot the actual DOS profile. This is done in Fig. 2.5 for the subgap energy spectrum, as a function of the phase difference between the contacting superconductors and for both a short ($\Delta\tau_L/\hbar \ll 1$) and a long ($\Delta\tau_L/\hbar \gg 1$) ballistic junction. In both cases, we can see how the spectrum is affected by elastic interface scattering,

assumed negligible in ideal SNS junctions. One may also ask what happens above the gap at energies $|\epsilon| > \Delta$. While the continuum DOS there resembles in part that of Fig. 2.4(b), it may either diverge or disappear altogether at $\epsilon = \pm\Delta$, depending on φ and τ_L [15]. We have however focused specifically on the subgap transport in Publication II.

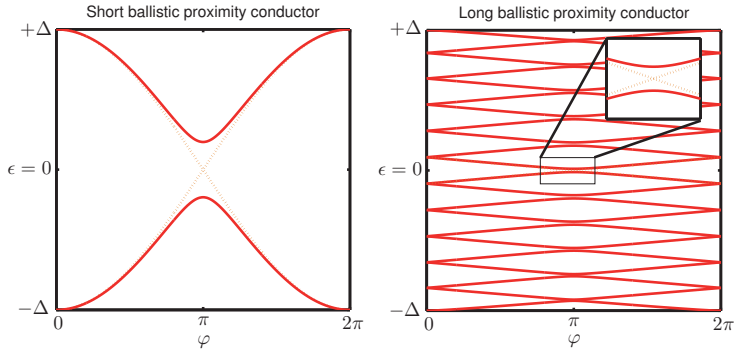

Figure 2.5. Subgap energy spectrum in short and long ballistic proximity conductors as a function of phase difference φ. The dotted line shows the spectrum in the ideal case when the N-S interface is completely transparent and the solid line corresponds to a case where there is some normal scattering at the interface.

Measurable physics: Supercurrent

The density of states changes drastically in superconductors and diffusive/ballistic proximity conductors, as demonstrated above. The question that follows is: What is its significance? This we can answer by taking a look on how supercurrent is transported through the nanoconductor. When the spectrum is expressed as discrete levels as in Fig. 2.5, the supercurrent in an SNS junction is a sum across these levels [12]

$$I_S = \frac{2e}{\hbar} \underbrace{\sum_i \frac{\partial \epsilon_i}{\partial \varphi}}_{\text{spectrum}} \underbrace{[1 - 2f(\epsilon_i)]}_{\text{distribution}} \tag{2.6}$$

and when the spectrum is a continuum expressed with a DOS function as in Fig. 2.4, the supercurrent is an integral over the states

$$I_S = \frac{1}{eR_N} \underbrace{\int d\epsilon j_S(\epsilon, \varphi)}_{\text{spectrum}} \underbrace{[1 - 2f(\epsilon)]}_{\text{distribution}}. \tag{2.7}$$

These two expressions are essentially the same with the small difference that in the latter one the quantum of resistance $h/e^2 \approx 26$ kΩ for a single transport channel is replaced by the total normal-state resistance R_N of the nanoconductor and the spectral part is expressed by a *spectral supercurrent* function j_S. Most importantly, in both cases the expression for

the current can be easily divided into two parts: the spectral component which depends on the configuration of energy levels in the nanoconductor and the state component which depends on the occupation of these energy levels as determined by the distribution function $f(\epsilon)$. While driving the system into a nonequilibrium state can strongly modify the latter as discussed in Sec. 2.1, the total supercurrent is equally dependent on any changes in the spectral part. As seen from Figs. 2.4, such changes can lead to sharp features at energies $\epsilon = \pm E_g, \pm\Delta$ or blocking of processes altogether at small energies $\epsilon < E_g$.

2.2.3 Andreev reflection and suppression of heat transport

Instead of describing the contact between a superconductor and a nanoconductor in terms of the proximity effect, we can alternatively use the picture of *Andreev reflection* proposed as early as in 1964 [16]. It is an interface phenomenon but it turns out that what happens at the N-S interface ultimately results in changes inside the nanoconductor. In this sense, Andreev reflection at the interface generates the proximity effect and is the fundamental cause behind all the modifications to nanoconductor properties in SNS systems [17].

Although Andreev reflection takes place at all energies ϵ, the process is best visualized at subgap energies $|\epsilon| < \Delta$ where the DOS in the superconductor at an N-S interface vanishes but electron- (and hole-) type excitations may still appear on the normal-conducting side. Since there are no states in the superconductor, it is impossible for such an excitation to enter the superconducting side. Instead, it reflects as a hole (electron) with the same spin, same energy, roughly the same momentum but, due to the change in particle type, opposite velocity direction. To conserve the number of particles, a Cooper pair appears on the superconducting side so that the net effect is a single electron incident on the interface transforming into a Cooper pair on the S side and a hole on the N side as visualized in Fig. 2.6. Importantly, the excitation energy on the normal side is conserved as well so that while the electronic charge transported across the interface is $2e$, transported energy is zero. The superconductors therefore block heat transport out of the SNS junction at subgap energies and by reducing the efficiency of one of the relaxation channels listed in Sec. 2.1, this phenomenon markedly affects the energy relaxation in the nanosystems presented in this thesis.

Andreev reflection can be conveniently used to explain the nonequilibrium behavior of ballistic SNS junctions. There, in the absence of energy relaxation in the nanoconductor, electrons fed into the system from one end travel across the nanoconductor picking up energy equal to the applied bias voltage eV on the way, experience Andreev reflection at the N-S interfaces and then travel back as holes, again picking up energy $(-e) \times (-V) = eV$. When the voltage is small, it can take several of these trips for the electrons to accumulate energy larger than 2Δ, required to climb over the energy gap and escape into the superconductor on the other side of the junction. This is called the process of *multiple Andreev reflections* (MAR) [18] and it is a convenient way of describing ballistic nanoconductors by simply tracing their paths in energy space.[4]

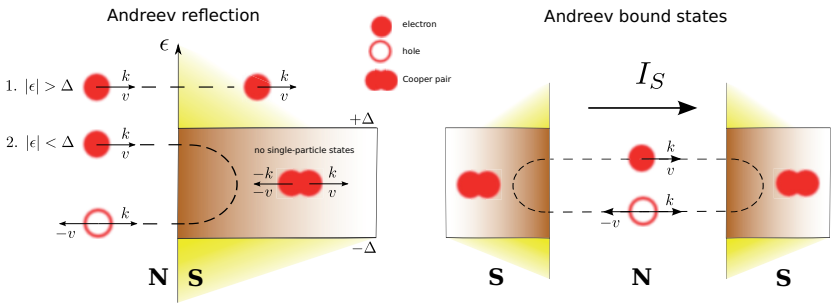

Figure 2.6. Electron arriving at the interface of a normal nanoconductor and a superconductor with velocity v and momentum k either passes through to the superconducting side or, if it does not have enough energy, Andreev reflects as a hole with momentum and energy of the excitation conserved. No energy is transmitted to the superconducting side. When the nanoconductor is enclosed between two superconductors, succession of reflections where energy remains conserved creates Andreev bound states. This enables supercurrent transport across the junction.

The MAR method is not straightforwardly applicable to diffusive SNS systems where random scattering distorts the particle trajectories and the question of phase coherence becomes important when moving away from the interface.[5] In such a case, it is generally simpler to separate the processes taking place at the interface and inside the nanoconductor with techniques I show in the next chapter. By doing this, we neglect the interplay between one- and two-particle coherent phenomena studied quite recently in Ref. [19].

[4]The idea of electrons and holes scattering back and forth in the junction also works for phase-coherent transport in the absence of driving voltage so that the energy states in Fig. 2.5 can be interpreted as *Andreev bound states*: see Fig. 2.6.
[5]It is precisely the decay of the coherence that defines the extent of the proximity effect after it has originated from the Andreev reflections at the interface.

2.2.4 Scattering at the interface

Above, we have considered ideal SNS junctions where the contact between the superconductor and the nanoconductor is transparent. In practice, such ideal contacts are impossible to fabricate and there is always some scattering at the interface. When transparency decreases from its ideal value $\mathcal{T} = 1$, there exists both Andreev reflection which conserves momentum and *normal scattering* where only the momentum component normal to the interface is reversed. This affects the Andreev bound states of Fig. 2.5 in such a way that small gaps open between the levels. When device concepts are thought up, the increased normal scattering can generally be seen as an unwanted consequence since it weakens the proximity effect and superconducting properties in the nanoconductor.[6] We have studied how much these imperfections affect supercurrent transport in ballistic SNS nanowires in Publication II.

Nanodevices are also intentionally prepared so that the interface between the superconductor and the nanoconductor becomes opaque, i.e., $\mathcal{T} \ll 1$. This is done by building up an electrically insulating layer at the S-N interface, typically by oxidization of the contact metal on one side. The resulting S-I(nsulator)-N *tunnel junction*, studied in Publication I, has found applications for example as an electron cooler and as a thermometer at temperatures of the order a few hundred mK and below.[7] These and some other interesting applications of the nanostructures that we discuss in this thesis are reviewed in Ref. [20].

[6] Very recently, however, systems have been found which can be shielded from analogous interaction-induced effects, making them attractive proposals to be used in quantum computing [2].

[7] In this *tunnel limit*, the interface resistance R_I is much larger than the resistance of the nanoconductor, $R_I \gg R_N$. Because of this, any changes in the state of the system are spatially located at the interface and $f(\epsilon)$, μ and T become independent of position inside the nanoconductor(s).

3. Theoretical approaches

In the publications included in this thesis, we have used various theoretical techniques which I now review in this more technical chapter. It is possible to describe the nanosystems starting from a microscopic theory for excitations in a quantum many-body system and solve any particular problem so that full spatial and time dependence is included. However, this is usually not practical since it either conceals the physical phenomena we are particularly interested in or simply requires too much computational power and time. We therefore use different simplifications: consider only weak-driving behavior, include relaxation or superconductivity in the problem using phenomenological models, and more. I describe the methods used by starting from ballistic systems in Sec. 3.1 and then move on to the more complex case of diffusive nanoconductors in Sec. 3.2.

3.1 Ballistic nanoconductors: Bogoliubov-de Gennes equations with BTK method

Perhaps the simplest way to treat superconductivity in a quantum system, in our case a nanoconductor, is to use the *Bogoliubov-de Gennes equations* which are like a two-part Schrödinger wave equation for superconductors, including both electron and hole excitations [10]. It is a mean-field theory for the electron-electron interactions with a complex field $\tilde{\Delta}$ (of magnitude $|\tilde{\Delta}| = \Delta$) describing only those interaction processes which produce superconductivity. While other types of interactions can be later introduced into the formalism by hand, the equation is best suited for describing equilibrium systems and nonequilibrium in ballistic conductors where interactions are absent (or weak).

For particle and hole excitations of energy ϵ described by the two-part wave function $\hat{\psi}(x) = \begin{bmatrix} u(x) & v(x) \end{bmatrix}^T$, the Bogoliubov-de Gennes equa-

tions read

$$\begin{bmatrix} H_e + U(x) & \tilde{\Delta} \\ \tilde{\Delta}^* & -(H_e + U(x))^* \end{bmatrix} \hat{\psi}(x) = \epsilon \hat{\psi}(x) \tag{3.1}$$

with single particle kinetic and potential energy expressed by the Hamiltonian $H_e + U(x)$. In constant potential, $U(x) = U$ and solving Eq. (3.1) is equivalent to solving a free-particle equation. The whole SNS system can be described using the method of Blonder, Tinkham and Klapwijk (BTK) [21] where they insightfully model scattering at the S-N interface by a delta function potential $U(x) = I\delta(x)$. For a barrier of arbitrary transparency I, this handles both normal and Andreev scattering and yields all transmission and reflection coefficients.

The above is enough to solve the energy spectrum in ballistic SINIS conductors where, due to the absence of scattering, electrons move coherently and conserve their single-particle phase information. This is done by following an excitation travel across the nanoconductor as its path completes a full cycle with consecutive reflections (normal or Andreev type) at both N-S interfaces as in the bound state of Fig. 2.6. The method, together with a compact solution, is sketched in Ref. [22] and its complex general solution can be simplified for a short nanoconductor ($L \ll \xi$),

$$\epsilon = \pm\Delta\sqrt{1 - \mathcal{T}\sin^2(\varphi/2)}, \tag{3.2}$$

and for the low-lying levels ($|\epsilon| \ll \Delta$) of a long nanoconductor ($L \gg \xi$) with symmetric barriers at N-S interfaces,

$$\epsilon = n\pi\Delta\frac{\xi}{L} \pm \arcsin\sqrt{\mathcal{T}^2\cos^2\varphi/2 + (1-\mathcal{T}^2)\sin^2\tilde{\mu}}. \tag{3.3}$$

Here n is an integer, $|n| \lesssim L/\pi\xi$, and the number of energy levels increases with the length of the nanoconductor, as seen in Chap. 2, Fig. 2.6. The spectrum can be modified by changing either the transparency $\mathcal{T}$ of the N-S interfaces or the parameter $\tilde{\mu}$ which is roughly proportional to the product $\mu \times L$ and can therefore be manipulated by gating the system as explained is Sec. 2.1. An important feature of the spectrum is the appearance of spectral minigaps at $\phi = \pi,\ 3\pi,\ 5\pi\ldots$,

$$\delta\epsilon_\pi = 2\Delta\frac{\xi}{L}\arcsin\left(|\sin\tilde{\mu}|\sqrt{1-\mathcal{T}^2}\right). \tag{3.4}$$

These disappear for perfect transparency $\mathcal{T} = 1$ but they can also be made to vanish by controlling $\tilde{\mu}$ to alter the speed of the electrons moving back and forth in the nanoconductor.

The subgap energy spectrum that we obtain from solving Bogoliubov-de Gennes equations can be directly used to calculate the supercurrent

from Eq. (2.6) both in and out-of equilibrium. In the latter case, we also need to obtain the distribution function $f(\epsilon)$ but this is straightforward for ballistic systems where $f(\epsilon)$ does not change inside the nanoconductor. The continuum states with $|\epsilon| > \Delta$ also contribute to the current but only negligibly for the weak driving $eV \ll \Delta$ that we have studied in Publication II. As such, Bogoliubov-de Gennes theory based on wave functions provides a convenient method of describing subgap transport in ballistic nanoconductors with superconductivity when the problem can be formulated as a single-particle problem. The method can be extended by including some (inelastic) scattering into the system phenomenologically.

3.2 Diffusive nanoconductors: from semiclassical models to quantum formalism

For diffusive systems where elastic interactions dominate, the scattering has to be included in the theory right from the beginning and this increases the complexity of the situation markedly — even if the description is probabilistic. In standard transport theory of metals, the elastic scattering from lattice imperfections is treated using the *Boltzmann transport equation*, assuming that the state of the system is isotropic, and averaging over the momentum directions of the transported electron [23]. The resulting *diffusion equation* does not, as such, deal with superconductivity but it can be separately included by allowing Andreev reflection at the N-S interfaces and accounting for modifications due to the proximity effect inside the nanoconductor. The Boltzmann theory can then be used as an adaptable tool for studying nonequilibrium nanosystems without single-particle coherence, and two-particle coherence included by hand.

The Boltzmann equation is a semiclassical equation which neglects any effects arising from the phase coherence of electrons. An alternative to using it (and adding effects such as superconductivity one by one) is to start from a full quantum field theory for many-body systems (including superconductivity) and to simplify it to account only for the most relevant physical phenomena. This results in a largely analogous quantum diffusion equation, the *Usadel equation*, where nonequilibrium effects are included through the *Keldysh formalism* (for a detailed account, see Ref. [24]). The derivation embodies a quasiclassical approximation which integrates out the dependence on the single-electron phase-coherence, making it look

like we have just arrived at the Boltzmann theory from a different direction. However, two-particle phase coherence *is* still preserved so that superconductivity is intrinsic in the Usadel equation and it can therefore be used to actually determine how the proximity effect modifies the equilibrium properties of a nanoconductor. The results can, in turn, be used with the Boltzmann theory so that we:

1. solve the modified material parameters from the two-particle coherent equilibrium Usadel equation ($N(\epsilon), j_S(\epsilon)$ etc.),
2. determine the nonequilibrium distribution function $f(\epsilon)$ from the incoherent Boltzmann equation,
3. obtain all the physical observables as a combination of these, in the spirit of Eqs. (2.6) and (2.7),

and avoid having to deal with the more complicated problem in the fully formulated Keldysh–Usadel theory which includes both two-particle coherence and nonequilibrium processes. In the remainder of this chapter, I present this method in more depth.

3.2.1 Boltzmann equation with superconductors

The Boltzmann equation essentially tells how a distribution of classical particles obeying Newton's laws changes under external forces and random scattering. For a distribution $f(x, p, t)$ which depends on position x, momentum p and time t, the state at a later time $t + dt$ can be straightforwardly predicted,

$$f(x + v \cdot dt, p + F \cdot dt, t + dt) = f(r, p, t) + I_{\text{coll}}[f]dt. \tag{3.5}$$

This is because the state continuously depends on the velocity v with which the particles move, (electromagnetic) forces F affecting the particles and potential scattering $I_{\text{coll}}[f] = I_{\text{el}}[f] + I_{\text{inel}}[f]$ which contains both elastic and inelastic processes and mixes the occupation probabilities. In the absence of collisions, i.e., when $I_{\text{coll}} = 0$, this equation can be used to study ballistic nanoconductors, albeit without phase-coherence or superconductivity. The collisions are modeled by terms of the form

$$I\left[f(p)\right] = \sum_{\text{states}} K\left[I^{\text{in}}[f(p)] - I^{\text{out}}[f(p)]\right], \tag{3.6}$$

where we include all the in-scattering states from which the particle can scatter *into* state p and the out-scattering states where the particle can

scatter *from* state p, increasing and decreasing the expected occupation of the state, respectively. Terms $I^{\text{in/out}}$ contain all the corresponding distribution functions while kernel K depends on the states involved and the specifics of the interaction. The actual Boltzmann equation is obtained from Eq. (3.5) at the limit $dt \to 0$ but we go a step further and take the diffusive limit, where f is expanded over the lowest order in dependence on the momentum directions and then integrated over them. This largely simplifies the elastic scattering term and gives the diffusion equation [25]

$$(\partial_t - D\partial_x^2) f(x, \epsilon, t) = I_{\text{inel}}[f], \tag{3.7}$$

where f is now isotropic in momentum and consequently dependent only on the magnitude $|p|$, directly related to energy. Elastic scattering is now included in the diffusion constant $D = v^2\tau/\dim$ which contains the elastic scattering time τ and the dimensionality $\dim$ of the system. Since we are interested in phenomena close to Fermi level, $\epsilon_F/k_B \sim 10000$ K, we always have $v \approx v_F$, the Fermi velocity.

Inelastic scattering: electrons, phonons and photons

The inelastic part of Eq. (3.7) needs to be determined on a case-by-case basis depending on the physical processes that produce the scattering. For this thesis, we consider non-magnetic systems and the most important processes are the electron-electron scattering mediated by direct Coulomb interaction, the material-dependent electron-phonon scattering and the electron-photon scattering which can be either due to specific radiation sources or electromagnetic environment of the nanoconductor.

Electron-electron interaction in disordered (dirty) normal conductors can be described with a *collision integral* [20], i.e., an isotropic version of Eq. (3.6) in the continuum limit:

$$I_{\text{e-e}}(\epsilon) = \kappa_{\text{e-e}} \int_{-\infty}^{\infty} d\omega \; d\epsilon' \; \omega^{\alpha} \left[I_{\text{e-e}}^{\text{in}}(\epsilon, \epsilon', \omega) - I_{\text{e-e}}^{\text{out}}(\epsilon, \epsilon', \omega) \right] \tag{3.8}$$

with in- and out-scattering terms

$$I_{\text{e-e}}^{\text{in}} = \left[1 - f(\epsilon)\right] \left[1 - f(\epsilon')\right] f(\epsilon - \hbar\omega) f(\epsilon' + \hbar\omega), \tag{3.9a}$$

$$I_{\text{e-e}}^{\text{out}} = f(\epsilon) f(\epsilon') \left[1 - f(\epsilon - \hbar\omega)\right] \left[1 - f(\epsilon' + \hbar\omega)\right]. \tag{3.9b}$$

Both the exponent $\alpha = \frac{\dim}{2} - 2$ of the integration kernel and the prefactor $\kappa_{\text{e-e}}$ describing the interaction strength depend on the dimensionality $\dim$ of the system. For nanoconductors, this is an ambiguous quantity, and we obtain it effectively by comparing the dimensions of the sys-

tem to the energy-dependent coherence length $\xi = \sqrt{\hbar D/\epsilon}$ of Sec. 2.2.1.[1] The electron-electron scattering acts as a force to thermalize the electron system towards the quasiequilibrium distribution of Eq. (2.1) so that $I_{\mathrm{e-e}}(\epsilon) = 0$ whenever $f = f_0$.

For electron-phonon scattering, the collision integral is closely reminiscent of Eq. (3.8) and it can be found in Ref. [20]. Whereas electron-electron collisions conserve the total energy within the electron system and only mix the relative population of different energy levels, the electron-phonon collisions actually act as a heat sink (or source) for the electron system. It is therefore often enough to know how much energy flows from phonons to electrons altogether without separating this flow to specific electron energy states. This total heat flow, in units of power, is obtained from the collision integral by integration,

$$\dot{Q} = N_F \Omega \int_{-\infty}^{\infty} d\epsilon \, \epsilon \, I(\epsilon), \tag{3.10}$$

for a conductor of volume Ω. For all-electron interactions, it always holds that $\dot{Q}_{\mathrm{e-e}} = 0$ whereas the power escaping to phonons, $\dot{Q}_{\mathrm{e-ph}}$, can in principle vary depending mainly on the type of phonons and the ratio of the phonon wavelength λ_{ph} to the (elastic) mean free path l. In practice, it has however been observed that at sub-Kelvin temperatures where acoustic phonons dominate, the experimental results follow well the theoretical prediction for the clean-system limit $\lambda_{\mathrm{ph}} \ll l$, derived for example in Ref. [26]. While this has so far been confirmed only for three-dimensional conductors, we can expect that in metallic nanoconductors, in quasiequilibrium with electron temperature T_e, the electron-phonon channel transfers power

$$\dot{Q}_{\mathrm{e-ph}} = \Sigma \Omega (T_e^{\alpha} - T_{\mathrm{bath}}^{\alpha}). \tag{3.11}$$

While the exponent α may depend on the dimensionality of both electrons and phonons [27], we typically use the value for clean three-dimensional conductors: $\alpha = 5$. The material-dependent prefactor Σ has been measured on many occasions for three-dimensional conductors with some of the results tabulated in Ref. [20].

Finally, the electron system in the nanoconductor is also sensitive to any external electromagnetic fields. The fields can either be caused by direct irradiation or by fluctuations in the electromagnetic environment which

[1] Because of the energy dependence, it is not always a priori evident which value for $\dim$ best describes the system as a whole and it may even occasionally become necessary to leave $\dim$ as a free parameter to be determined by comparing theory to experiments.

couples to the nanoconductor through the photons transmitted through the leads (see Fig. 2.1). Either way, the coupling is rarely perfect due to differing source and load impedances, Z_S and Z_L, whose matching can be described by the parameter

$$r_0 = \frac{\mathrm{Re(Z_S)Re(Z_L)}}{|Z_S + Z_L|^2} \tag{3.12}$$

which ranges from 0 (large difference in impedances) to 1 (perfect matching). Typically, the photonic channel into the leads does not contribute to the thermal relaxation of the system since its thermal conductance $G^Q_{e-\gamma} = r_0 \pi^2 k_B^2 T_e / 3h$ [31, 32][2] is very weak compared to its phononic counterpart

$$G^Q_{\mathrm{e-ph}} = \frac{\partial \dot{Q}_{\mathrm{e-ph}}}{\partial T_e} = 5\Sigma\Omega T_e^4. \tag{3.13}$$

These become comparable when the electron temperature is lowered below the transition temperature $T_{\mathrm{tr}} = \left[r_0 \pi k_B^2/(30\hbar\Sigma\Omega)\right])^{1/3}$, which for typical metallic nanoconductors of size $\Omega \approx 10^{-20}$ m^3 corresponds to 100-200 mK. Because of the cubic root dependence, this value is relatively rigid with respect to any changes in the parameters and photonic relaxation is therefore significant only at the lowest end of temperatures for the present-day nanoelectronics research. While typically negligible in terms of relaxation, electron-photon coupling becomes important when photons act to inject energy into the system. In this case, the excess energy modifies the electron distribution function and can therefore either decrease or, perhaps more surprisingly, increase the supercurrent of Eqs. (2.6) and (2.7). The latter effect was first predicted by Eliashberg in 1970 for superconductors [28] and the corresponding theory for SNS systems has been proposed only recently in Ref. [29]. To account for electron-photon processes, we can use a collision integral similar to Eq. (3.8) with a kernel that depends on the matching circuit and the frequency band of the radiation. We give examples of this in Publications III and V.

How to use the Boltzmann method in practice

Random scattering makes the diffusion equation irreversible and thus applicable for transport problems with energy dissipation. In addition, the theory has to have a way to accommodate a bias, a favored direction of transport for the flow of both charge and energy. This can be done with the standard method of introducing reservoirs that are large enough not to be

[2] ... which remarkably equals the quantum limiting value for a single channel (when $r_0 = 1$) [33]...

affected by the exchange of energy with the nanoconductor so that they are always in equilibrium with a well-defined temperature. In Fig. 2.1, these reservoirs correspond to the leads where the electrons assume distribution $f_0(\epsilon)$ of Eq. (2.1) and the (phononic) heat bath at temperature T_{bath}. The phononic reservoir is defined through the electron-phonon collision integral whereas the inclusion of leads for a clean ($\mathcal{T} = 1$) interface is carried out mathematically by imposing boundary conditions [30]

$$f(\epsilon) = f_0(\epsilon) \quad , \quad |\epsilon| > \Delta, \tag{3.14a}$$

$$\left.\begin{aligned} f(\epsilon) &= 1 - f(-\epsilon) \\ \partial_x f(\epsilon) &= \partial_x f(-\epsilon) \end{aligned}\right\} \quad |\epsilon| < \Delta, \tag{3.14b}$$

at each nanoconductor-lead interface in the system ($\mu = \mu_S$). For normal-conducting reservoirs $\Delta = 0$ and the set of equations reduces to Eq. (3.14a). For superconducting reservoirs, Eqs. (3.14b) state that there is a balance between positive and negative energy excitations inside the superconductors (this always holds for equilibrium superconductors) and that there is no subgap energy flux into/out of the superconductors. These conditions describe Andreev reflection as discussed in Sec. 2.2.3 so that by solving the diffusion equation Eq. (3.7) with boundary conditions of Eqs. (3.14), we obtain:

spatial and temporal dependence of the nonequilibrium distribution function $f(\epsilon)$ with specified inelastic scattering inside an incoherent, diffusive nanoconductor with Andreev reflection at the interfaces.

The two-particle quantum coherence that manifests as the proximity effect can be included in the theory by solving the collision integrals and material parameters such as the density of states within the equilibrium quantum theory.

In this thesis work, we focus predominantly on steady-state phenomena for which $\partial_t f(x, \epsilon, t) = 0$ in Eq. (3.7). For the spatial dependence, we follow two routes: In Publication IV, we determine the full spatial profile of $f(x, \epsilon)$ in a nanosystem, whereas in Publication III we simplify and consider the behavior of the nanoconductor on average so that $\partial_x f(x, \epsilon) = 0$. We then use a simple model to account for the spatially-independent diffusion of electrons and energy out of the nanoconductor: a collision integral $I_{\mathrm{diff}} = 8/\tau_D[f(\epsilon) - f_0(\epsilon, T_{\mathrm{bath}})]$ for energies $|\epsilon| > \Delta$. Below the superconducting gap, where outdiffusion is blocked, $I_{\mathrm{diff}} = 0$. With concessions

such as these to attain computational speedup, the Boltzmann method can be versatilely applied to model nanosystems with superconductors.

3.2.2 Nonequilibrium quantum dynamics in dirty nanoconductors: Keldysh–Usadel technique

From the Boltzmann equation we can only deduce the nonequilibrium electron distribution function. The full quantum dynamics of a nanosystem out of equilibrium *and* the spectral properties independent of the state of the system can be obtained using the *Green function techniques* for many-body quantum systems.[3] The starting point is the *Gor'kov equation of motion* which includes superconductivity in a 2×2 matrix structure and thus resembles the Bogoliubov-de Gennes equation, Eq. (3.1). As the field theoretical version of the latter, the advantage of the Gor'kov equation over the wave function methods is that it is a genuine many-particle equation. It is thus readily applicable to problems with various types of scattering, and to account for nonequilibrium effects, it can be supplemented with an additional 2×2 matrix structure that creates a *Keldysh space*. The complicated Gor'kov equation can be simplified by the so-called quasiclassical approximation to give the *transport-like Eilenberger equation* which can in turn be likened to the Boltzmann equation. Also here, we can average over momentum directions to obtain the *Usadel equation*

$$D\partial_x \cdot (\check{G}\partial_x\check{G}) = \left[-i\epsilon\check{\tau}_3 + \check{\Delta} + \check{\Sigma}, \check{G}\right], \tag{3.15}$$

a field-theoretical counterpart of the diffusion equation Eq. (3.7) (written here for the static case). The Green functions $\check{G} = \check{G}(x,\epsilon)$ are 4×4 matrices which contain both the spectral structure[4] and the population of energy levels, i.e., the electron distribution function $f(x,\epsilon)$. It is enough to know that matrix $\check{\Delta} = \check{\Delta}(x)$ describes the pair potential and that the remaining inelastic scattering can be included in the self-energy $\check{\Sigma} = \check{\Sigma}(x)$, apart from which the details of the compact notation of Eq. (3.15) are irrelevant here. The equation can in principle be solved in the full nonequilibrium setting but in this thesis work, we mostly adhere to solving only the spectral properties of the proximity-affected nanoconductors following the method outlined in Sec. 3.2.1.

[3] The tools of this section and their relation to the typical problems of nanoscale superconductivity are elaborated in Refs. [24] and [20]. For a complete book on theory of nonequilibrium superconductivity, see Ref. [34].

[4] For example, the density of states can be effectively obtained as a real part of one of the matrix elements.

To solve the equilibrium equation where the Keldysh structure is omitted, I have exploited ready-made solvers such as the one available at Ref. [35]. This is done in Publications III, IV, and V. In Publication I, we have solved the finite-voltage Keldysh–Usadel equation in tunnel approximation which simplifies the problem considerably since the high-resistance N-S interfaces make the problem spatially independent inside the nanoconductor(s) as discussed in Sec. 2.2.4

To close the theoretical discussion of this chapter, I briefly summarize the usefulness of the Keldysh approach in separating the spectral part and the distribution part of a problem. The Keldysh matrix structure is

$$\check{G} = \begin{bmatrix} \hat{G}^R & \hat{G}^K \\ 0 & \hat{G}^A \end{bmatrix} \tag{3.16}$$

with normalization $\check{G}^2 = 1$. The *Keldysh part* can be parametrized as $\hat{G}^K = \hat{G}^R\hat{h} - \hat{h}\hat{G}^A$, where $\hat{h}$ is a 2×2 matrix (when superconductivity is included). Moreover, $\hat{h}$ corresponds essentially to the (nonequilibrium) energy distribution function $f(\epsilon)$ which can then be obtained by solving $\hat{G}^K$. On the other hand, the *retarded and advanced parts* $\hat{G}^{R,A}$ define the energy spectrum of the system, including all the spectral quantities like j_S of Eq. (2.7). When the matrices are in the upper-triangular form of Eq. (3.16), it follows that any product of two matrices in Keldysh space satisfies

$$\check{G}_1\check{G}_2 = \begin{bmatrix} \hat{G}_1^R & \hat{G}_1^K \\ 0 & \hat{G}_1^A \end{bmatrix} \begin{bmatrix} \hat{G}_2^R & \hat{G}_2^K \\ 0 & \hat{G}_2^A \end{bmatrix} = \begin{bmatrix} \hat{G}_1^R\hat{G}_2^R & \hat{G}_1^R\hat{G}_2^K + \hat{G}_1^K\hat{G}_2^A \\ 0 & \hat{G}_1^A\hat{G}_2^A \end{bmatrix}.$$

As a result, the retarded and advanced parts can be solved independently from the Keldysh part (with some exceptions). The former then provide the spectral solution and the latter yields the distribution function *or* it can be replaced with the Boltzmann equation for the same purpose. This justifies the use of Boltzmann equation ($f(\epsilon)$) together with the equilibrium Usadel equation ($G^{R,A}$) as an approximation to the full nonequilibrium transport problem ($G^{R,A,K}$).

4. Application to nanoelectronic circuits

Previous chapters present the fundamental phenomena affecting electric and thermal transport in SNS systems and show the methods we can use to model these systems. In this chapter, I go through specific examples we have studied and we can see the variety of nanosystems where the nonequilibrium state may build up with important consequences. The focus here is on the characterizing property of each piece of research. In most of the papers, we have studied the system at hand from more than one perspective and to get insight on the specifics of each system one should read up on the Publications.

4.1 Relaxation in up-and-coming nanodevices

We start with two systems with potential for future real-life applications of nanotechnology. The first is devised around graphene, a two-dimensional form of carbon which rose to scientific spotlight in 2004 when it was first successfully produced in a research laboratory [36]. A graphene flake corresponds to a single sheet of graphite and as a genuine one-atom thick material, it has garnered significant multi-disciplinary interest due to its (potentially) groundbreaking mechanical, optical and electronic properties with well over 10000 scientific articles published between 2005 and 2011 [37].[1] From the electronic transport perspective that we take in this thesis, the most important aspect of graphene is that it is a semi-metal: while it has no gap in its energy spectrum, it does have a *charge neutrality point* where the density of states vanishes. Mathematically, the spectrum is a linear function of momentum k, $\epsilon = \hbar v_F k - \mu$, as opposed to the parabolic spectrum of ideal metals and semiconductors. The com-

[1] ... with over 5000 of these published in 2011 alone, amounting to an average of 15 graphene (or graphene-related) articles published each day.

plete theory of the electronic properties of graphene, as known so far, has been already outlined in several review articles such as Ref. [38] which focuses on graphene fundamentals and Ref. [39] with an eye on the latest discoveries.

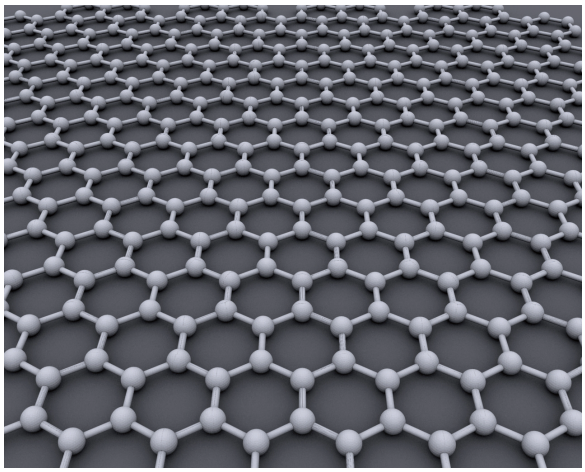

Figure 4.1. Graphene lattice is shaped like a honeycomb with bonds of length 0.14 nm connecting individual carbon atoms
(By AlexanderAlUS, released under CC BY-SA 3.0 license)

An attractive feature of graphene in terms of applications is its high carrier mobility and large part of early graphene research has therefore focused on the electrical conductivity of the material. This quantity is mostly unaffected by inelastic scattering, and since the many-body problems of inelastic interaction are theoretically very challenging, many questions regarding the topic are still unclear. This is particularly true for the electron-electron problem [40] whereas for electron-phonon interaction, the heat transfer between electrons and phonons in graphene has been solved when the system is in quasiequilibrium [41]. It has been recently shown that while graphene is not intrinsically superconductive, it can be contacted to superconducting leads to form a proximity SNS nanoconductor (in most cases a diffusive one) and drive supercurrent through it [42]. This allows to probe the inelastic interaction in graphene, since while the direct conductivity does not depend on the strength of inelastic interactions, the induced superconducting features do - through the direct dependence of the supercurrent on the distribution function as in Eqs. (2.6) and (2.7). Of course, to observe supercurrent, temperatures and driving voltages need to be sufficiently low and for this reason the method is applicable specifically to studying the low-energy end of the energy-dependent interactions. In Publication IV we have done exactly this and estimated

the strength of inelastic interactions in graphene at sub-Kelvin temperatures. In particular, we have measured the strength of electron-electron interaction since at such low temperatures, electron-phonon relaxation is expected to be negligible, which we also noted. To do this, we have devised a *graphene thermometer*, pictured in Fig. 4.2, where energy from a

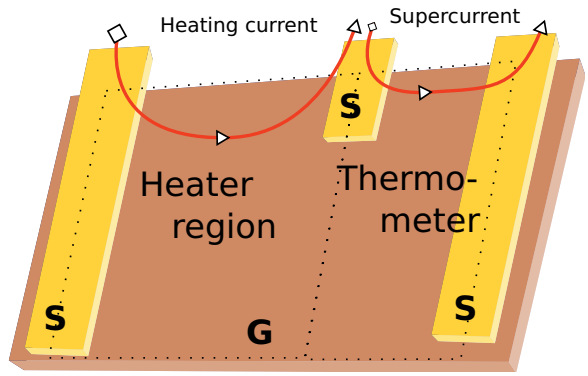

Figure 4.2. Graphene thermometer is composed of a heater and a thermometer junction. Superconducting leads (S) deposited on top of graphene (G) allow both heating normal current and probing supercurrent to be run through the device.

heated SGS junction (the nanoconductor here is graphene, in short: G) is allowed to leak partly into a SGS thermometer junction and the increase in energy content of the electron system (or temperature) in this junction is seen as a drop in the measured supercurrent. How much energy leaks into the thermometer junction depends strongly on the electron-electron relaxation inside the heater junction and we have modeled this relaxation with the dirty-limit collision integral of Eq. (3.8). This integral, while not graphene-specific, is expected to describe the properties of graphene in practical situations where the system does not rest at its charge neutrality point. By doing this, we obtained prefactors $\kappa_{\mathrm{e-e}}$ over 100 times larger than our initial estimate based on the material parameters, especially the normal-state resistance of graphene. This suggests that electron-electron interaction in graphene is much stronger than in comparable two-dimensional metallic conductors and despite the extreme thinness of the material, it can be described using thermal quasiequilibrium models with satisfactory accuracy.

The second nanosystem with direct potential for future applications is an actual device concept for infrared radiation detection. This *Proximity Josephson sensor* (PJS) portrayed in Fig. 4.3 was proposed in 2008 [43] and it demonstrates how proximity effect in nanoconductors can be exploited in practice. It is based on the idea of *bolometric* detection where

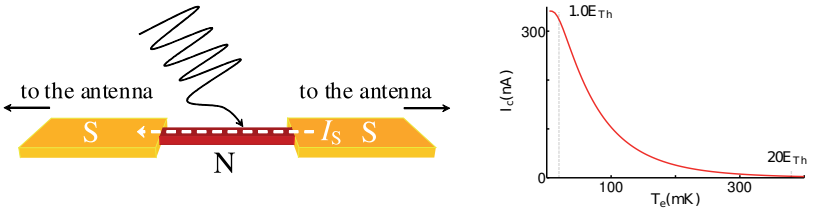

Figure 4.3. In the Proximity Josephson sensor external antennas couple incident radiation to the detector, composed of a proximity nanoconductor. The critical supercurrent I_c is a sensitive function of the nanoconductor electron temperature T_e.

the temperature increase caused by incident radiation can be measured with high accuracy. Bolometers have found use especially in the infrared and millimeter-wave regime of the electromagnetic spectrum[2] in astronomy [45] and recently, in the quest for high sensitivity, nanofabricated superconducting detectors in particular have become the most popular [44]. Since the field of such "nanoastronomy" is still very young, several different approaches to detector fabrication exist and currently the most promising directions are *kinetic-inductance detectors* (KIDs), which are based on radiation breaking Cooper pairs in a superconducting detector so that the inductance of the device changes [46], and *resistive hot-electron bolometers* (HEBs), which are diffusive SNS devices where radiation heats up the nanoconductor and outflow of energy is blocked with superconductors [47].[3] The proposed Proximity Josephson sensor resembles the latter HEB concept but instead of a resistive temperature readout, it is projected to operate in a dissipationless state of proximity-induced superconductivity where increase in temperature results in a change of proximity-induced inductance making them akin to KIDs in terms of readout. The advantage that results is that the PJS can be made much smaller than KIDs, increasing sensitivity while at the same time removing the resistive Johnson noise present in HEBs.

In the original work of Ref. [43], the authors provided a bare concept of the PJS where the device physics have been greatly simplified. In Publication III, we have analyzed the PJS in detail and used recent results on the properties of proximity SNS systems such as the heat capacity [49] and

[2]...at wavelengths $\sim$ 100 μm-1 mm corresponding to frequencies of the order 100 GHz - 1 THz.

[3]In addition, there are *superconducting transition edge sensors* (TESs) where detection is based on the sharp rise in superconductor resistance when they absorb energy at the superconducting transition temperature. They may also exploit the hot-electron mechanism of HEBs and are therefore not distinguished above [48].

the thermal conductance of the heat link [50] to see how the device performance is affected by the superconductivity induced in the nanoconductor. While we observed that the performance-defining noise characteristics remain essentially the same, we also noticed that paying special attention to the energy relaxation process is essential to understanding the operation of the device. For sensitivity, the nanoconductor in PJS should be made as small as possible, but we also note that there is a lower limit to this, after which shrinking the device only results in increased noise.[4] By composing a spatially independent steady-state Boltzmann equation to balance the energy flow in and out of the nanoconductor, we study both *monochromatic driving* with electromagnetic field of fixed frequency (Publication III) and *thermal driving* with a continuous spectrum of frequencies determined by the radiator temperature (Publication V). We estimate that electron-electron interaction in the small sensor is typically too weak to equilibrate the system into a thermal distribution and a large part of the radiation energy escapes to phonons or the leads. We expect this phenomenon to be ubiquitous in nanoscopic *thermal detectors* (including HEBs) which are by definition assumed to operate in a quasiequilibrium state and consequently suggest that when device size is reduced, one should be increasingly aware of the nonequilibrium processes taking place in the system.

4.2 Superconductors out of equilibrium

In Sec. 2.2.4, I introduce the possibility of cooling the electron system below the bath temperature in a diffusive nanoconductor by contacting it to the leads with high-resistance tunnel barriers. In this SINIS system, the superconductors selectively remove energetic excitations from the nanoconductor and this is presently exploited predominantly by cooling electrons in the temperature range $300 \ldots 100$ mK which is otherwise unattainable without expensive-to-operate dilution refrigerators or adiabatic demagnetization refrigerators [51]. The performance is limited by processes that affect both the spectrum and the energy distribution

[4] The limit results directly from Eq. (3.11) when the incident radiation power P is balanced with the power flowing to the phonons: $P = \dot{Q}_{\mathrm{e-ph}}$ and threshold volume $\Omega_{\mathrm{tr}} \approx \frac{P}{\Sigma T_{\mathrm{bath}}^5}$. In large absorbers, the noise is dominated by the phonon base temperature T_{bath} whereas for absorbers smaller than Ω_{tr}, it is the heated electron temperature $T_e \sim \Omega^{-1/5}$ that determines the noise level.

of excitations in the superconductors [52]: Spectrum-wise, the cooling is hampered by the appearance of states inside the superconducting gap of Fig. 2.4(b)[5] while the electron energy distribution is modified because of the inability of the superconductors to effectively conduct the extracted heat away from the NIS interface. This makes them imperfect reservoirs (in contrast to the ideal ones defined in Sec. 3.2.1), which do not remain in the bath temperature but heat up, resulting in back flow of energy into the nanoconductor. We focus on the latter limitation as it is not only that the superconductors tend to heat up but they do it in a way that produces nonequilibrium distributions.

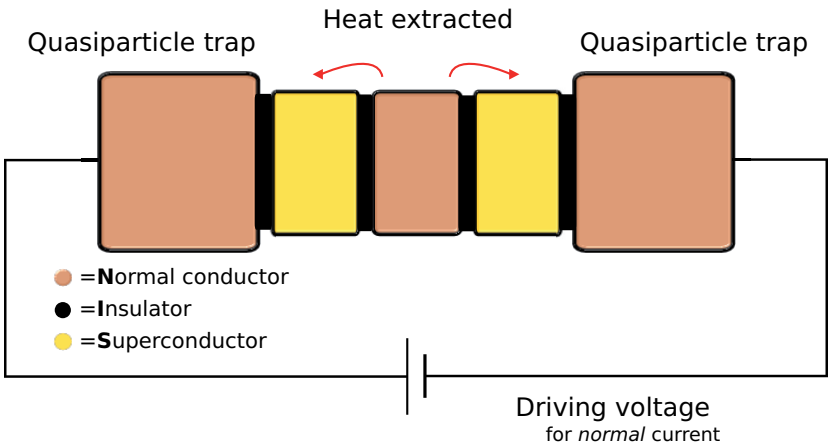

Figure 4.4. SINIS electron cooler can be complemented by an additional layer of N electrodes which act as quasiparticle traps, i.e., as perfect reservoirs which aim to remove energy from the superconductors.

There have been attempts to prevent the superconductors from warming up and in Publication I, we study an approach where additional normal-conducting electrodes are attached to the superconductors to function as *quasiparticle traps* [54]. The purpose of these traps, illustrated in Fig. 4.4, is to provide a channel for the excitation energy in the superconductors to relax in. In our quantitative analysis, valid within the tunnel approximation of dominating barrier-resistance, we observe that if such traps are introduced, maximal cooling is achieved when the contact between the traps and the superconductors is made as transparent as possible. This implies that there is no added benefit from the more complicated system geome-

[5]This *Dynes density of states* is actually a phenomenological model that can describe the effect of various physical processes such as increased tunneling into the superconductor due to a high-temperature environment. With current techniques, this effective DOS can be suppressed below $10^{-7}N_F$ for $|\epsilon| < \Delta$ (in aluminum) [53].

try, and the sole purpose of the traps is to provide the superconductors an environment where effective energy relaxation is possible. This effect has lately been produced in an alternative way using magnetic fields to partly suppress superconductivity in order to improve relaxation of extracted energy [55]. We also determine the quantitative nonequilibrium distributions that form in the superconductors in the absence of relaxation and observe that they differ markedly from the Fermi–Dirac distribution of Eq. (2.1) having the same sharp features as the superconducting density of states in Fig. 2.4. This raises the question if such peculiarities can be exploited in any way. While we cannot answer this yet, electron cooling remains as a topic of active study and provides a convenient testbed for research in nonequilibrium superconductivity [56].

4.3 Exploiting nonequilibrium: current pump

The final nanosystem I present is an SNS structure where a ballistic wire constitutes the nanoconductor. In total absence of scattering both inside the nanoconductor and at the N-S interfaces, the dissipative current response of the junction differs dramatically from SNS systems where such scattering does occur, as noted above in Sec. 2.2. For example, the (zero-temperature) current-voltage relation of an NIS junction in a tunnel limit reads $(eR_N)^{-1}\sqrt{(eV)^2-\Delta^2}$ when $|eV|>\Delta$, and at subgap voltages the current is absent altogether. This demonstrates the impossibility of single-particle tunneling into superconductors with no low-energy states. In fully ballistic SNS junctions, however, the subgap current emerges solely from multiparticle tunneling processes made possible by Andreev

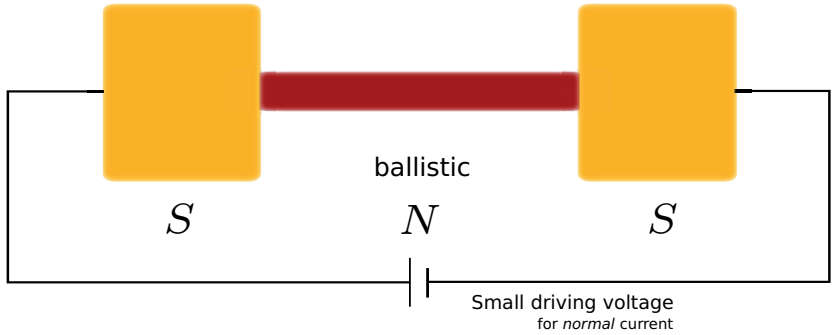

Figure 4.5. In an ideal ballistic SNS nanoconductor scattering is absent both inside the nanoconductor and at the N-S interfaces. We study, how departure from this ideal affects transport at the low-voltage limit.

reflection and, as a result, there is a roughly linear dissipative current at all voltages $V > 0$ [57]. Between these two extreme limits, the subgap current can vary considerably depending on how effectively the many-particle current is suppressed by interaction phenomena. The region is also rich in physics, governed by junction parameters including temperature, nanoconductor length L, interface transparency $\mathcal{T}$, and degree of inelastic scattering [58]. In Publication II, we study what happens in the low-voltage end of the subgap regime.

Typically, transport in ballistic SNS systems is discussed within the framework of multiple Andreev reflections mentioned in Sec. 2.2.3 but since the number of possible reflections $n = 2\Delta/eV$ grows without a limit when voltage V is lowered, this becomes increasingly complicated at $eV \ll \Delta$. In Ref. [59], the authors have shown that transport in such a case can alternatively be viewed using the concept of *spectral flow* in energy space. In this process, depicted in Fig. 4.6, electrons are taken from en-

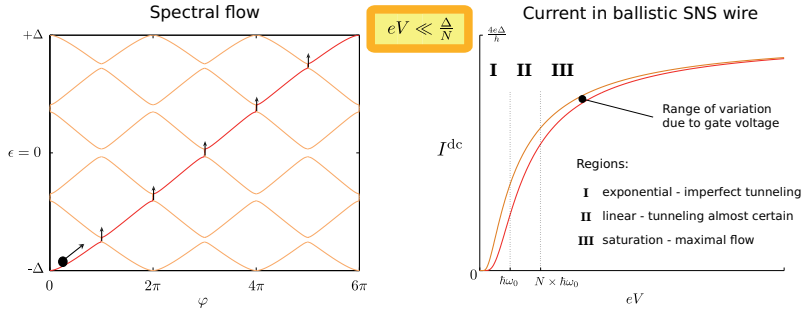

Figure 4.6. (On the left) Spectral flow in energy space is realized when discrete energy levels in a nanoconductor driven by a small voltage V change in time (here $\varphi = 2eVt/h$) and an electron follows these levels from $-\Delta$ to $+\Delta$. The process is adiabatic apart from points (here when φ is a multiple of π) where the electron tunnels onto another level. In the process, electric current flows through the junction and when the electron reaches the continuum of states at $\epsilon = \pm\Delta$, the accumulated energy is dissipated.
(On the right) Current resulting from the spectral flow is the dominating current component in a long ballistic SNS wire. It has three distinct regimes, depending on the relative magnitude of the driving voltage and the energy scale $\omega_0 = \pi v_F(1-\mathcal{T}^2)/L\mathcal{T}$ which determines the inter-level tunneling probability. The current can be controlled with a gate voltage which affects the tunneling probabilities and is particularly effective at small voltages $eV < \hbar\omega_0$.

ergy $\epsilon = -\Delta$ across the discrete energy levels formed in the nanoconductor (numbering roughly $N = L \times 2\Delta/hv_F$)[6] all the way to $\epsilon = \Delta$, after which the electron can escape into the leads. In the process of generat-

[6]The exact spectrum can be obtained using the methods of Sec. 3.1 and it resembles those in Fig. 2.5, depending on the nanoconductor length. The phase difference φ of the superconductors changes linearly in time as in Eq. (2.3).

ing the electric current, energy of 2Δ is dissipated. Larger probability for inter-level tunneling results in a larger current which can be maximized by changing the chemical potential of the nanoconductor periodically using a gate voltage, in effect realizing a current pump as suggested in Ref. [22]. However, the spectral-flow concept only works for essentially one-dimensional wires[7] where the longitudinal velocity component of the electrons moving along the conductor is unambiguously defined for a given energy [59].

In Ref. [22], the authors show that the nonequilibrium state formed inside the superconducting gap of a long ($N \gg 1$) ballistic SNS wire can be exploited by pumping to give a low-voltage ($eV \ll \Delta/N$) *dc* current $I^{\rm dc} =\gg I_c$. In Publication II, we extend this discussion by considering spectral flow in the system without the pumping action. This means that the chemical potential is fixed and, in case of imperfect boundary transparency ($1-\mathcal{T} \ll 1$), gaps open in the spectrum reducing the probability of inter-level tunneling necessary for the spectral flow. Assuming $T_{\rm bath} \ll \Delta$, our results are then comparable to previous studies for low-voltage behavior of ballistic SNS wires, also when we include variable amount of inelastic scattering in the nanoconductor [57, 58]. We find out that, in the low-voltage end, the build-up of current can be separated into three regimes as in Fig. 4.6 with a maximum of $I^{\rm dc} = N \times I_c \gg I_c$. These conclusions support and supplement the earlier results but while theoretical understanding on the properties of ballistic SNS nanoconductors can be considered good, the long junctions studied here are yet to be realized in experiments. One possible candidate for realization is a *carbon nanotube*, where the Andreev bound states have already been observed [60] but the current experiments are still solidly in the short junction regime. If (or when) realized, these "large" ballistic conductors will stand out as an extreme example of nonequilibrium nanosystems with neither elastic or inelastic relaxation in conductors of 1 micron or longer.

[7] The conductor is here considered one-dimensional when it only supports a single quantum-mechanical conduction channel in the transverse direction.

5. Summary: what next?

Previous chapters outline the background of our research on nonequilibrium nanoconductors with superconducting contacts. The nanosystems I have presented in Chap. 4 show the variation of situations where nonequilibrium state may be formed and they demonstrate that when we operate in conditions where temperature is low and the system size is microscopic, we generally *have to* consider the effects which incomplete relaxation and the accompanying nonequilibrium state may entail. In modern sub-Kelvin, sub-micron nanoelectronics, this should be considered above all a blessing. Despite the increased complexity of nanoscale physics, the diversity of new phenomena encourages us to expect that there are ways to exploit these findings in not-so-distant future. In this thesis work, I consider transport of charge and energy in systems with superconductors, variable degree of relaxation and variable dimensionality (1, 2, and 3) but this still leaves systems with zero-dimensionality, magnetism, strong electromagnetic interactions and one-electron quantum coherence untouched. Like in all fundamental research, the question that remains is still not *"why?"*, but rather *"how?"*. The following publications aim to help answering this in a representative sample of driven SNS nanoconductors with limited relaxation.

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See also: D-Wave quantum-computer project, *http://www.dwavesys.com* While the scientific merits of the project are still controversial, it stands out as an intriguing *commercialized* approach to quantum computation and low-temperature nanoelectronics as a whole.

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Imagine something as small as the thickness of a human hair. Now imagine 1 percent of it. One thousandth? Modern nanoelectronics operates with conductors fabricated as small as this and, to boost, there exists a branch of low-temperature nanoelectronics which goes to temperatures less than one degree above the absolute zero to exploit the quantum phenomena which show up in absence of thermal noise. Such nanoscience has become reality in research laboratories around the world and the field is constantly foraging for real-life applications with quantum computer as one of the primary goals. However, in such extreme conditions even a slight input of external energy may change the properties of the conductor altogether - and energy is everywhere. To be able to reliably use the future applications, we need to understand how the conductor exchanges energy with its surroundings and what happens to the energy it absorbs. This work presents several nanoscale devices where this energy may become a nuisance, or an asset, and shows in detail how a non-equilibrium state forms inside the devices.

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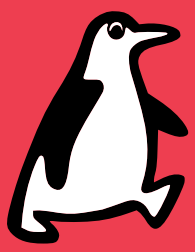

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