Department of Mathematics and Systems Analysis

Rank-based information in multi-attribute decision and efficiency analysis

Antti Punkka

Aalto University

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Antti Punkka

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Abstract

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Abstract

Additive multi-attribute value models are widely employed in decision and efficiency analysis. Difficulties in specifying preferences for these models have motivated the development of methods that admit incomplete preference information, identify non-dominated alternatives and provide recommendations with heuristic decision rules. These methods accommodate many types of preference statements. Yet, several studies suggest that decision makers prefer to provide rank-based information rather than numerical statements.

First, this thesis defines the notion of incomplete ordinal information, which can capture statements about the relative importance of the attributes and about the achievement levels of alternatives. The thesis then develops an optimization model for identifying non-dominated alternatives when alternatives and preferences are characterized by incomplete ordinal information and possibly by other types of incomplete information. These forms of information can, for example, help stakeholders to arrive at a joint preference characterization.

Second, the thesis shows that the recommendations of many decision rules depend on the selected normalization of value functions. Motivated partly by this, the thesis develops optimization models to determine all the rankings the alternatives attain with the model parameters that are consistent with the stated incomplete information. The resulting ranking intervals help, for example, analyze how sensitive the alternatives' rankings are to the model parameters.

Third, the thesis introduces dominance relations and ranking intervals for the efficiency analysis of decision making units when efficiency is measured through ratios of multi-attribute output and input values, as in the original data envelopment analysis method. These relations and intervals, which can be computed with the optimization models developed in the thesis, make it possible to compare any two decision making units independent of what other units are included in the analysis and to analyze how sensitive the efficiency of a unit is to the output and input attribute weights.

Keywords decision analysis, additive value function, incomplete information, ordinal information, decision recommendations, efficiency analysis, data envelopment analysis

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Tiivistelmä

Tekijä
Antti Punkka

Väitöskirjan nimi
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Tiivistelmä

Additiivisia arvomalleja käytetään laajasti monikriteerisessä päätös- ja tehokkuusanalyysissa. Preferenssien täsmällisen määrittelyn vaikeudesta johtuen on kehitetty menetelmiä, jotka epätäydelliseen preferenssien kuvaukseen perustuen tunnistavat ei-dominoidut vaihtoehdot ja tuottavat suosituksia heuristisilla päätössäännöillä. Nämä menetelmät hyödyntävät erityyppisiä preferenssienilmaisutapoja. Useat tutkimukset ovat kuitenkin osoittaneet päätöksentekijöiden luonnehtivan preferenssejään mieluummin järjestysperustaisesti kuin numeroin.

Väitöskirjassa esitetään epätäydellinen järjestysperäinen informaatio, jolla voidaan mallintaa kriteerien keskinäistä tärkeyttä ja vaihtoehtojen kriteerikohtaisia ominaisuuksia. Tätä varten kehitetään optimointimalli, joka laskee päätössuositukset myös, kun preferenssejä ja vaihtoehtojen ominaisuuksia luonnehditaan samanaikaisesti muillakin tavoin. Epätäydellinen järjestysperäinen informaatio voi esimerkiksi auttaa päätösongelman sidosryhmiä yhteisen preferessien kuvauksen muodostamisessa.

Väitöskirjassa näytetään, että monien päätössääntöjen suositukset voivat riippua arvomallille valitusta normeerauksesta. Tämän ongelman ratkaisemiseksi kehitetään optimointimallit, jotka ratkaisevat kaikki vaihtoehdoille epätäydellisen informaation rajoissa mahdolliset järjestysluvut. Nämä järjestyslukuvaihteluvälit auttavat muun muassa tutkimaan sitä, kuinka herkkiä järjestysluvut ovat mallin parametreille.

Väitöskirjassa sovelletaan dominanssirelaatioita ja järjestyslukuvaihteluvälejä päätöksentekoyksiköiden tehokkuusanalyyseihin, joissa tehokkuutta mitataan monikriteeristen tuotos- ja panosarvojen suhteella, kuten alkuperäisessä DEA-menetelmässä. Nämä relaatiot ja vaihteluvälit, jotka voidaan ratkaista väitöskirjassa kehitetyillä optimointimalleilla, mahdollistavat kahden yksikön muista analyysin yksiköistä riippumattoman vertailun sekä analyysit siitä, kuinka herkkiä päätöksentekoyksiköiden tehokkuudet ovat tuotos- ja panoskriteerien painokertoimille.

Avainsanat päätösanalyysi, additiivinen arvofunktio, epätäydellinen informaatio, järjestysperustainen informaatio, päätössuositukset, tehokkuusanalyysi, data envelopment analysis (DEA)

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The Dissertation consists of the present summary article and the following papers:

[I] Salo, A., Punkka, A. (2005). Rank inclusion in criteria hierarchies. *European Journal of Operational Research* **163** 338–356.

[II] Punkka, A., Salo, A. (2012). Preference programming with ordinal information. *manuscript*, 30 pages.

[III] Punkka, A., Salo, A. (2012). Ranking intervals in additive value models with incomplete preference information. *manuscript*, 36 pages.

[IV] Salo, A., Punkka, A. (2011). Ranking intervals and dominance relations for ratio-based efficiency analysis. *Management Science* **57** 200–214.

Contributions of the author

In Paper [I], Punkka developed the computational algorithm, established most mathematical developments and proofs, and performed the computations. Salo proposed the idea for the paper and is the first author of the paper.

Punkka is the first author of Paper [II]. Punkka developed the mathematical formulations based on Salo's initial idea. The proofs and the examples are developed by Punkka.

Punkka proposed the idea for Paper [III] and is the first author of the paper. The mathematical developments, proofs and examples are by Punkka.

In Paper [IV], Punkka proposed the idea of ranking intervals and the ideas behind the computational models. The computational analyses were established by Punkka. Punkka and Salo contributed equally to the mathematical developments and proofs. Salo proposed the idea for the paper and is the first author of the paper.

Preface

This thesis has been made possible by many people who I have the privilege to acknowledge.

First of all, I wish to thank my supervising professor Ahti Salo for all cooperation regarding the thesis. His guidance, ideas, feedback, expertise in the field, and uncompromising attitude towards scientific writing have greatly contributed to the thesis. It's been a pleasure to be part of a research group which is led by an internationally renowned scholar in decision analysis.

To a great extent, the research in this thesis has also benefitted from discussions and brainstorming with my long-time colleague and good friend Dr. Juuso Liesiö. In the same vein, I would like to thank my ex-colleague and good friend Pekka Mild for the atmosphere he, Juuso and I created working together in room U232 with pretty similar research topics. Every doctoral student should have people like you to work with. I also wish to thank the preliminary examiners of the thesis, Professor Luis C. Dias and Professor David L. Olson.

I wish to thank Professor Raimo P. Hämäläinen for the opportunity to work in the Systems Analysis Laboratory. In addition, I'd like to recognize his efforts as the director of the Doctoral programme in systems analysis, decision making and risk analysis and, as the coordinator of this programme, I would like to thank him for all collaboration regarding the programme. I also want to thank all the colleagues I've had at the lab. It's very motivating to work with smart people like you. Special thanks for keeping the Friday floorball tradition alive, for the somewhat legendary camper trip across South Ostrobothnia, and to Ville Brummer, Anssi Käki and Eeva Vilkkumaa for keeping room U232 such a nice place to work in.

For some, science is a way of life. For me, it's not. Therefore, I'd like to thank all my friends for giving me something else to think about during my leisure time. I also wish to thank my parents Heikki and Irmeli for encouragement and support throughout my studies. Finally, I wish to thank my spectacular family. Anne, thank you for everything, especially for not questioning, whether it makes sense to continue studies after a master's degree. Miisa and Lotta, thank you for simply existing, and maybe even understanding that most adults go to work in the weekday mornings and letting me do so in a good mood.

Espoo, October 2012

Antti Punkka

Contents

1 Introduction

Evaluating the value or efficiency of a discrete set of alternatives often involves several criteria. Many methodologies, such as the outranking methods proposed by Roy (1968), the data envelopment analysis (DEA) by Charnes et al. (1978) and the analytic hierarchy process by Saaty (1980), represent advances in such multi-criteria evaluation. Yet, multi-attribute value theory (MAVT) by Keeney and Raiffa (1976) and Dyer and Sarin (1979) is unique in that it is based on an axiomatization of preferences, which establishes a solid theoretical background for multi-criteria evaluation and decision analyses. MAVT methods have received much attention both in literature and in applications, as Corner and Kirkwood (1991), Keefer et al. (2004), and Hämäläinen (2004) note in their reviews.

Based on MAVT applications, additive value functions, in particular, are transparent and easy-to-understand models for analyzing, and deriving decision recommendations in multi-criteria decision problems (e.g., Golabi et al. 1981, Kirkwood and Sarin 1985, Mustajoki et al. 2004, Ewing et al. 2006, Mild and Salo 2009). Such *value tree analysis* makes it possible to represent the objectives and the attributes that measure how alternatives achieve these objectives as a hierarchical 'tree'. Conventionally MAVT captures the decision maker's (DM's) preferences through tradeoff statements in terms of equally preferred (hypothetical) alternatives (Keeney and Raiffa 1976), or through direct evaluation of parameter values (Edwards 1977, Von Winterfeldt and Edwards 1986). Yet, difficulties in providing such preference statements have motivated the development of methods that accommodate incomplete information about the relative importance of the attributes and, moreover, about the alternatives' achievement levels with regard to the attributes (e.g., White et al. 1982, Weber 1987).

In many methods for incomplete specification of preferences, the DM expresses preferences with numbers, such as score intervals (White et al. 1982) or intervals for attribute weight ratios (Salo and Hämäläinen 1992). Several studies, however, suggest that ordinal comparison of actual or hypothetical alternatives is more suitable for eliciting the DM's preferences, because (i) alternatives' achievement levels are often described verbally due to lack of natural measurement scale (Larichev 1992), (ii) value judgements can be easier to express in words than through numbers (Sarabando and Dias 2009), and (iii) a group of DMs attempting to obtain a joint preference representation may disagree about the numerical statements (Kirkwood and Sarin 1985) or even the appropriate measurement scales (Grushka-Cockayne et al. 2008), but they may still agree on a rank-ordering of the attributes' relative importance or the alternatives' achievements with regard to the attributes. Indeed, Larichev (1992) and Edwards and Barron (1994) argue that numerical evaluation affects negatively the reliability of the analysis. Moshkovich et al. (2002) observed in their review that discrete scales with verbal explanations are often applied even, when the attributes have a natural numerical measurement scale. They argue that this is because procedures for numerical parameter estimation are time-consuming and not necessarily well understood by the DMs. Indeed, Moshkovich et al. (2002) conclude that ordinal information is less complex and expect it to more accurately reflect the DM's preferences. This view is shared by Larichev et al. (1995) who note that attempts to solve decision tasks through more 'exact' (quantitative) judgments may lead to erroneous results, thus suggesting use of ordinal judgments. Yet, ordinal information may need further quantification so that the precision of the preference specification better matches the intensions and 'true' preferences of the DM (Sage and White 1984) and, moreover, provides decision recommendations that discriminate between the alternatives.

With an incomplete specification of preferences and alternatives, there are typically several value functions and characterizations of the alternatives' achievement levels that are consistent with the stated information. Based on combinations of parameters that correspond to these value functions and achievement levels, the non-dominated (White et al. 1982; see also Hannan 1981) and potentially optimal alternatives (e.g. Hazen 1986) can be identified and proposed as 'good' decision candidates. Further decision support can be provided by applying heuristic decision rules that recommend a single alternative. Suggested rules are based on comparing (i) the 'sizes' of the parameter sets that favor an alternative (Eiselt and Laporte 1992), (ii) the magnitudes of value differences (Park and Kim 1997, Dias and Clímaco 2000, Salo and Hämäläinen 2001, Sarabando and Dias 2009) and (iii) sums of these value differences (Ahn et al. 2000). In addition to such rules, approaches to describe the alternatives' sensitivity to the DM's preference statements have been developed (e.g., Rios Insua and French 1991, Kämpke 1996, Butler et al. 1997).

The DEA method by Charnes et al. (1978) (referred to as CCR-DEA) resembles MAVT in that it models the efficiency of *decision making units* (DMUs) by examining the ratio of additive output value and additive input value. As its primary results, CCR-DEA distinguishes between *efficient* and *inefficient* DMUs. Further results are provided by *efficiency scores* that convey information about how efficient a DMU can *at best* be, when it is compared with all output and input weights to the DMU that is the most efficient with those weights. However, this measure does not discriminate between efficient DMUs. In addition, the efficiency score is based on one combination of weights, which is typically different for each DMU and also depends on what other DMUs are included in the analysis. These features have motivated the development of, for example, cross-efficiency analysis (Sexton et al. 1986) in which the DMUs' efficiencies are evaluated by using an aggregate measure that is based on several combinations of weights.

This dissertation extends possibilities of using ordinal information in value tree analysis and efficiency analysis. Specifically, Paper [I] introduces the notion of *incomplete ordinal information* which is specified through statements that associate a set of attributes or alternatives with a set of rankings. For example, the DM can state that attributes 1 and 2 are among the three most important ones in the preference model. Paper [II] develops a model to characterize the corresponding feasible region of value function parameters so that other kinds of statements can be used to complement ordinal statements, and to provide decision recommendations in this setting.

Paper [III] shows that many proposed decision rules and concepts for multi-parameter sensitivity analysis can exhibit *rank reversals* (Belton and Gear 1983) so that changing the normalization of the additive value functions can change the recommendations of these rules and the results of the sensitivity analyses. Furthermore, Paper [III] develops a model to compute all the rankings that the alternatives can attain under incomplete preference specification and characterization of alternatives. The resulting *ranking intervals* do not depend on the selected normalization. They can be used as complementary ordinal information alongside dominance relations and they help, for example, in conducting sensitivity analyses. Paper [IV] develops the ratio-based efficiency analysis methodology, which makes it possible to use the ordinal comparison concepts of dominance and ranking intervals to compare DMUs, when their efficiency is measured through ratios of additive output and input values, as in CCR-DEA.

The rest of this summary article is structured as follows. Section 2 discusses the relevant theory and methods for value tree analysis with incomplete information and data envelopment analysis. Section 3 summarizes the contribution of this dissertation. Section 4 discusses the implications of the methodological developments of the dissertation and outlines some ideas for future research.

2 Theoretical and methodological foundations

2.1 Additive value in multi-attribute value theory

Decision problems with several objectives are generally referred to as multi-criteria decision making problems. Miettinen (1999) divides these further into two categories (see also Korhonen et al. 1992): In multi-objective optimization, the problem's feasible solutions are in general implicitly defined, whereas multi-criteria decision analysis (MCDA) deals with problems with a finite number of predefined solution candidates. Methodologies for solving MCDA problems include for example (i) the analytic hierarchy process (AHP) by Saaty (1980) (see Ishizaka and Labib 2011 for a review), (ii) the outranking methods, such as the family of ELECTRE methods (see Roy 1968 for the seminal paper in French; see Roy 1991 and Roy and Vanderpooten 1996 for reviews), and the PROMETHEE methods (Vincke and Brans 1985; see Behzadian et al. 2010 for a review) and (iii) multi-attribute utility theory (MAUT) and multi-attribute value theory (MAVT) by Keeney and Raiffa (1976).

In MAVT, alternatives are described as vectors of attribute-specific achievement levels, and the DM's preferences are captured by a relation so that '$x = (x_1, \ldots, x_n) \succeq (y_1, \ldots, y_n) = y$' is interpreted as "$x$ is preferred or indifferent to y" (Keeney and Raiffa 1976). The aim is to form a value function V which captures this relation so that $V(x) \geq V(y)$ if and only if $x \succeq y$. If both $x \succeq y$ and $y \succeq x$ hold, then the DM is indifferent between x and y, that is, they are equally preferred. Dyer and Sarin (1979) extend MAVT by presenting requisite conditions for comparing differences in the strength of preference between *pairs* of alternatives through relation '$\succeq_d$'. This establishes *measurable* value functions so that $V(x')-V(x) \geq V(y')-V(y)$ if and only if $x \to x' \succeq_d y \to y'$, that is, "the preference difference for x' over x is greater than or equal to the preference difference for y' over y". Measurable value functions are unique up to positive affine transformations. Hence, the rank-orderings of values and value differences do not depend on how the value function is normalized.

The form of a value function depends on the DM's preferences. If the requisite conditions – most notably mutual preference independence (Keeney and Raiffa 1976) and difference independence (Dyer and Sarin 1979) – hold, then the DM's preferences can be captured by a measurable additive value function $V(x) = \sum_{i=1}^{n} v_i(x_i)$, in which v_i is the attribute-specific value function for the i-th attribute. The additive value function is often represented in the normalized form $V^N(x) = \sum_{i=1}^{n} w_i v_i^N(x_i)$, in which positive *attribute weights* w_i reflect the value differences between predefined achievement levels x_i° and x_i^*, and v_i^N are normalized so that $v_i^N(x_i^\circ) = 0$ and $v_i^N(x_i^*) = 1$.

2.2 Preference elicitation and incomplete information

The assumptions of the additive value representation make it possible to elicit attribute-specific value functions independently of each other; for elicitation methods, see Keeney and Raiffa (1976) and Von Winterfeldt and Edwards (1986). The elicitation of attribute weights can be carried out by constructing pairs of equally preferred alternatives. These statements imply trade-offs between the attributes (Keeney and Raiffa 1976). Technically, these statements lead to a system of linear equalities from which weight ratios w_i/w_j can be solved. Methods that elicit weight ratios directly have also been proposed, for example the SMART method by Edwards (1977) and the subsequent SMARTS method by Edwards and Barron (1994), and the SWING method by Von Winterfeldt and Edwards (1986). The weights can be normalized to sum up to one, for example, to come up with numerical values for the weights.

Yet, complete specification of the value function parameters can be time-consuming (White et al. 1982) or require knowledge that is not available (Weber 1987). The DM may also be unable or unwilling to provide precise trade-off statements that are required for such a complete specification (Hazen 1986) or he may feel uncomfortable with giving them (Sage and White 1984). Complete specification can even be unneccessary, if less information would lead to an unequivocal decision recommendation. These reasons, among others, have motivated the development of methods that derive decision recommendations based on incomplete characterization of preferences and alternatives (e.g., White et al. 1982, 1983, 1984, Kirkwood and Sarin 1985, Weber 1985, Hazen 1986, Salo and Hämäläinen 1992, 2001; for reviews, see Weber 1987, Salo and Hämäläinen 2010).

Most of the methods for dealing with incomplete information build on two assumptions. First, the DM is able provide complete trade-off statements between alternatives that differ along a single attribute (Hazen 1986), that is, specify the attribute-specific value functions. Second, the DM is able to provide preference information about the relative importance of the attributes through statements $x \succeq y$, in which x and y are different with regard to two attributes (e.g., White et al. 1984, Kirkwood and Sarin 1985, Weber 1985, Hazen 1986, Pearman 1993, Malakooti 2000, Salo and Hämäläinen 2001). Also methods that employ incomplete weight or other value difference ratios (such as $v_2(x_2^*) - v_2(x_2^\circ) \leq [v_1(x_1^*) - v_1(x_1^\circ)] \leq 2[v_2(x_2^*) - v_2(x_2^\circ)]$; e.g., Salo and Hämäläinen 1992, 2001, Mustajoki et al. 2005), and methods that admit any kind of linear constraints on the weights (e.g., $0.4 \leq w_1 \leq 0.6$) or allow the DM to adjust the alternatives' achievement levels so that one becomes preferred to the other (e.g., Sage and White 1984, Park et al. 1996, Malakooti 2000) have been developed.

Incomplete information about the alternatives leads to constraints for feasible characterizations of the alternatives. Theoretically, such information corresponds to incompletely characterized achievement levels, for example, through intervals ($10 \leq x_1 \leq 15$; e.g., Sage and White 1984, Weber 1985, Salo and Hämäläinen 1992), direct evaluation of the alternatives' normalized attribute-specific values through intervals ($0.15 \leq v_1^N(x_1) \leq 0.2$; e.g., White et al. 1982), or ordinal pairwise comparisons of alternatives' attribute-specific values ($v_1(x_1) \geq v_1(y_1)$; e.g., Salo and Hämäläinen 2001).

Some methods elicit mostly ordinal information about the DM's preferences and alternatives. For example, the ZAPROS-LM method by Larichev and Moshkovich (1995) captures attribute-specific preferences by eliciting a ranking of a finite number of possible achievement levels, and admits preference information about the relative importance of the attributes through ordinal comparisons of hypothetical alternatives. The method of Kirkwood and Sarin (1985) admits a rank-ordering of hypothetical alternatives, whose overall values correspond to attribute weights (e.g., $w_1 \geq w_2 \geq \ldots \geq w_n$). In the ordered metric method of Pearman (1993), the DM ranks differences between these weights, too (e.g., $w_1 - w_2 \geq w_3 - w_4 \geq \ldots$). Park et al. (1996) extend this model to evaluation of alternatives. The models by Cook and Kress (1996, 2002) complement ordinal information by discrimination factors between attribute weights and alternatives' normalized attribute-specific values (e.g., $w_1 \geq w_2 + 0.02$).

2.3 Decision recommendations under incomplete information

White et al. (1982) propose that alternatives should be compared based on (pairwise) *dominance* so that an alternative dominates another if all its feasible characterizations are preferred to all of those of the latter one, with all value functions that are consistent with the preference information (see also Hannan 1981, Kirkwood and Sarin 1985, Hazen 1986, Salo and Hämäläinen 1992). Because the dominance relation is irreflexive, asymmetrical, and transitive (e.g., Weber 1987), the dominance relations among the alternatives under analysis can shown as a domination digraph (White et al. 1982).

Mathematically, incomplete information leads to linear inequalities on the attribute weights w_i, and the alternatives' normalized attribute-specific values $v_i^N(x_i)$ and defines a convex *feasible region* of these model parameters. Based on this idea of *set inclusion* (White et al. 1982), the feasible region includes the parameters that correspond to the DM's 'true' value function and alternatives' true achievement levels. The dominance relations can be solved by examining the alternatives' minimum and maximum value difference over the feasible region. Several algorithms for computing dominance relations have been developed. Especially the early ones are based on enumerating the extreme points of the feasible region (e.g., Kirkwood and Sarin 1985, Hazen 1986, Carrizosa et al. 1995, Cook and Kress 2002, Mustajoki and Hämäläinen 2005), but due to the recent growth in computational power the emphasis has shifted towards formulations of linear programs (LPs; see e.g., Ahn et al. 2000, Salo and Hämäläinen 2001, Kim and Han 2000, Park 2004).

With incomplete information, there can be several *non-dominated* alternatives. White et al. (1982, 1984) show that with the specification additional statements, there are fewer value functions or characterizations of alternatives which are compatible with the statements, and that this, in turn, can lead to fewer non-dominated alternatives. Liesiö et al. (2007) present conditions under which the set of non-dominated alternatives cannot be enlarged as a result of additional information. Some interactive preference elicitation methods – such as the PAIRS method by Salo and Hämäläinen (1992) – provide guidance to the DM in keeping new preference statements consistent with earlier ones. Some methods even suggest preference statements that could efficiently reduce the set of nondominated alternatives (Mustajoki and Hämäläinen 2005).

Potentially optimal alternatives, too, have been proposed as good candidates (see e.g., Hazen 1986, Weber 1987, Rios Insua and French 1991). For these alternatives there exists a feasible characterization of alternatives so that they have the highest value for some value function that is consistent with the DM's preference statements. LPs can be used to solve the potentially optimal alternatives (e.g., Hazen 1986, Rios Insua and French 1991, Lee et al. 2001, 2002, Park 2004).

In addition to identification of non-dominated and potentially optimal alternatives, determination of alternatives' rankings over the feasible region have been proposed. The model of Kämpke (1996) solves rank variability for a set of alternatives, when preferences are captured through holistic comparisons among these alternatives. Butler et al. (1997) simulate random value functions to explore the robustness of the alternatives' rankings. The flexible ranking approach by Köksalan et al. (2010) first estimates precise achievement levels for the alternatives and then determines the most favorable rankings for them, when attribute weights are constrained by linear inequalities.

To support the selection of a single (non-dominated or potentially optimal) alternative, heuristic decision rules and 'tighter' dominance concepts have been proposed. These rules include the *domain criterion* by Eiselt and Laporte (1992) (cf. *acceptability index* of Lahdelma et al. 1998), *weak dominance* by Park and Kim (1997) (equal to *minimax regret* rule by Salo and Hämäläinen 2001), *quasi-dominance* by Dias and Clímaco (2000) and related *quasi-optimality* and *quasi-dominance* rules by Sarabando and Dias (2009), and *maximax*, *maximin*, and *central values* rules by Salo and Hämäläinen (2001). Moreover, following the ideas of outranking methods, Ahn et al. (2000) propose the *net dominance value* to be used as a measure for a decision rule. Sarabando and Dias (2009) provide a comparison of such decision rules. In a related stream of proposed decision rules, heuristics have been developed to obtain 'representative attribute weights' from the feasible region, based on which the alternatives are then compared; see Stillwell et al. (1981) and Barron and Barrett (1996) for comparisons of such methods.

2.4 Ratio-based data envelopment analysis

The seminal work of Charnes et al. (1978) has preceded the development of a variety of *data envelopment analysis* (DEA) methods to compare decision making units (DMUs) that differ in the amounts of outputs they produce, and the amounts of inputs they use to produce the outputs. The original CCR-DEA method proposed by Charnes et al. (1978) models the efficiency of a DMU by its *efficiency ratio*, the ratio of additive virtual output value and additive virtual input value. It thus assumes constant returns to scale; DEA methods that assume variable returns to scale have been developed by Banker et al. (1984) and Charnes et al. (1985).

The CCR-DEA method is non-parametric in the sense that it identifies *efficient* (potentially optimal in MAVT literature) and *inefficient* DMUs based on the output and input data of the DMUs that are included the analysis. Yet, several models accommodate preference information through weight constraints (i) to provide results which are not based on weights that reflect too large a compensation of one output (or input) over another output (input) (Thompson et al. 1986), and (ii) to add discrimination among the DMUs by obtaining fewer efficient DMUs (e.g., Adler et al. 2002). In their review, Allen et al. (1997) distinguish between (i) assurance regions type I (Thompson et al. 1986), which are constraints on the relative values among different outputs or inputs, (ii) assurance regions type II, which apply constraints also between outputs and inputs (Thompson et al. 1990, Khalili et al. 2010), and (iii) absolute weight restrictions (Dyson and Thanassoulis 1988).

Technically, such preference information imposes linear constraints on the output and input weights, and thus resembles incomplete preference specification for additive value functions. Cooper et al. (1999, 2001) develop models that allow use of intervals in describing the DMUs' inputs and outputs. Other similarities between DEA and MCDA or MCDM have been discussed by several authors (e.g., Doyle and Green 1993, Stewart 1996, Athanassopoulos and Podinovski 1997, Joro et al. 1998). These observations have underpinned the development of methods that compare DMUs with the help of value functions, for example (e.g., Halme et al. 1999, Gouveia et al. 2008, de Almeida and Dias 2012).

In conventional CCR-DEA, the DMUs' efficiencies are characterized by evaluating them with the output and input weights that are most favorable to them, in the sense that their efficiency ratio divided by that of the most efficient DMU is maximized over the set output and input weights. As a result, the efficient DMUs are assigned an efficiency score of one, and inefficient DMUs' efficiency scores are between zero and one. The conventional DEA concepts thus do not discriminate among the efficient DMUs. According to Adler et al. 2002, this has partly motivated the development of models and efficiency measures that provide a full ranking for the DMUs. Of these, for example *super-efficiencies* indicate how much more efficient a DMU can be than the most efficient of other DMUs (Andersen and Petersen 1993). *Benchmark ranking* by Torgersen et al. (1996) is based on the extent to which a DMU affects other DMUs' efficiency scores. *Cross-efficiency* analysis by Sexton et al. (1986) differs from other concepts in that it evaluates DMUs' efficiency ratios with *several* combinations of output and input weights, and uses the average of these ratios in comparing the DMUs (see also Doyle and Green 1994). Cross-efficiency analysis indeed differs from the other above concepts in that it employs several different weights to evaluate the efficiency of a DMU. However, these weights are determined based on which specific DMUs are included in the analysis.

3 Results

3.1 Incomplete ordinal information in preference modeling

Paper [I] introduces the notion of incomplete ordinal information for capturing preference information. This information is obtained through paired statements of attributes and rankings; for example, the DM can state that attributes *cost* and *environmental aspects* are among the three most important attributes; or that either *cost* or *environmental aspects* is the most important attribute. The paper shows how the feasible region of attribute weights can be reduced by revising the provided preference statements. It also presents conditions under which this feasible region is non-convex. To compute decision recommendations over a non-convex feasible region, Paper [I] develops an algorithm to enumerate those attribute weights whose convex hull is equal to that of the feasible region, and shows how dominance relations can be determined by computing the alternatives' value differences at these points. This computational algorithm can be applied also in presence of common, absolute lower bounds for the attribute weights, and when alternatives' achievement levels are specified through intervals.

Paper [II] develops a computational model which makes it possible to give incomplete ordinal preference statements *also* about the *alternatives' performance* with regard to any set of attributes. For example, the DM can state that alternative A is among the two most preferred ones with regard to *environmental aspects*; or that either alternative A or B is the most preferred one in view of attributes *cost* and *environmental aspects* together. The corresponding feasible region is modeled with a set of linear constraints on the model parameters and auxiliary binary variables. This makes it possible to complement incomplete ordinal statements by any incomplete cardinal preference statements which correspond linear constraints on the model parameters. As a result, it is possible to admit incompletely specified attribute weight ratios or ordinal comparisons between alternatives' achievement levels in preference specification, for example. The number of binary variables employed in the mixed integer linear programs (MILPs) developed for solving decision recommendations depends on the given preference statements. For example, if the feasible region is convex, the optimization problems simplify from MILPs to LPs.

3.2 Rank-based results for value trees and CCR-DEA based efficiency analysis

Paper [III] focuses on ordinal results of value tree analysis under incomplete information. First, it shows that recommendations of some comparison concepts and decision rules that compare preference differences across value functions that describe different preferences (e.g., Eiselt and Laporte 1992, Park and Kim 1997, Dias and Clímaco 2000, Ahn et al. 2000, Salo and Hämäläinen 2001, Sarabando and Dias 2009) as well as sensitivity analysis results based on the size of the feasible region or distances within it (Rios Insua and French 1991, Lahdelma et al. 1998) can depend on how the additive value functions are normalized. These recommendations and results can thus exhibit rank reversals (Belton and Gear 1983) in the sense that changing the normalization of the value functions can change the relative ranking of two non-dominated alternatives. Furthermore, for maximax, maximin and weak dominance decision rules Paper [III] presents sufficient conditions, under which the normalization can always be selected so that a non-dominated alternative is favored over another.

Second, as a partial solution to this problematic phenomenon, Paper [III] develops MILPs for computing all rankings that the alternatives can attain over a convex feasible region of all those model parameters that correspond to the DM's incompletely specified preferences and incompletely characterized alternatives. Like dominance relations, the resulting *ranking intervals* do not depend on the selected normalization of the value functions.

Paper [IV] develops the *Ratio-based Efficiency Analysis* (REA) methodology, which follows the CCR-DEA method in that it models DMUs' efficiencies with their efficiency ratios. It differs from earlier methods in that it derives results based on, and for *all* feasible output and input weights, which fulfill possible statements about the relative values of different inputs and outputs in terms of assurance regions type I statements. REA extends conventional efficiency scores by developing LPs to compute *efficiency bounds*, which communicate how efficient a DMU can be related to a benchmark group of DMUs, for *all* feasible output and input weights. In addition to this generalization, REA adopts the ordinal comparison concepts of dominance and ranking intervals from the MCDA literature and develops MILPs and LPs for computing:

- What *rankings* can a DMU attain in comparison with other DMUs, based on the comparison of their efficiency ratios for *all* feasible output and input weights?
- Does a DMU *dominate* another DMU in the sense that its efficiency ratio is higher than or equal to that of the other for *all* feasible output and input weights?

The results provided by the REA methodology coincide with some well-known results of CCR-DEA-based methods as special cases. Specifically, (i) the best ranking of an efficient DMU is one, and, conversely, a DMU whose best ranking is one has efficiency score of one, (ii) if all DMUs are in the benchmark group, the upper efficiency bound of a DMU is equal to its efficiency score, (iii) if all other DMUs are in the benchmark group, the upper efficiency bound of a DMU is equal to its super-efficiency. The REA results offer new possibilities to set performance targets for the DMUs. For example, Paper [IV] develops MILPs to compute the smallest radial improvement in outputs required for a DMU to improve its best or worst ranking to some target ranking.

4 Discussion

Incomplete ordinal preference information has been used in modeling the relative importance of attributes in many applications, for example by Ojanen et al. (2005), Salo and Liesiö (2006), Mild and Salo (2009), and Mild (2006) (in Finnish; a very similar case study is found in Liesiö et al. 2007). In all of the above applications, the preference information has represented the preferences of a group of DMs (or, stakeholders). Indeed, incomplete ordinal information makes it possible to construct preference statements even from group members who disagree. For example, each DM can be asked to specify the two most preferred alternatives with regard to an attribute, after which the group's preferences are expressed by a statement that the two most preferred alternatives are among the ones specified by the group members.

If the DMs cannot agree on the attributes' numerical measurement scales, or if natural scales do not exist, one way to describe preferences between the alternatives is to divide them into classes for which numerical values – perhaps together with verbal expressions describing preferences between these classes – are assigned (e.g., Salo and Liesiö 2006, Könnölä et al. 2007). Such preference information ranks the classes, but the fixed numerical values do not necessarily reflect strength of preference between the classes. In addition, the DMs may be prepared to provide additional preference statements between the alternatives in the same class for example through pairwise comparisons. Incomplete ordinal information helps model such classification as ordinal information, yet making it possible to define bounds for the values associated with the classes and to constrain value differences between the classes. This way, incomplete ordinal information can be used to perform *ex ante* sensitivity analysis on the values associated with the classes, and to allow alternatives in the same class to differ in values. Such possibilities for preference elicitation can be particularly beneficial in large problems with dozens alternatives in which data is available for only some attributes. In these settings, the available data together with incomplete ordinal information with regard to the other attributes can be sufficient to establish dominance relations that reduce the set of non-dominated alternatives. This, in turn, can lead to resource savings as fewer alternatives need be thoroughly evaluated.

Ranking intervals are suitable for this kind of *screening* of alternatives, especially if the aim is to choose several alternatives (referred to as 'pick k out of n' by Stillwell et al. 1981). Indeed, Butler et al. (1997) note that multi-criteria analysis is often performed in order to select a subset of alternatives, and they suggest that the ranking intervals should be examined to get insights about the robustness of the alternatives' rankings. Specifically, the ranking intervals identify which alternatives are among the K most preferred ones (i) for all, (ii) for some, and (iii) for no combinations of feasible parameters. These results are obtained simultaneously for all 'budgets' K, thus making it possible to analyze how decision recommendations change as a function of the budget. The above categorization is closely connected to recent advances in multi-criteria *portfolio* decision analysis (Salo et al. 2011). More precisely – following the terminology of the robust portfolio modeling (Liesiö et al. 2007, 2008) – if feasible portfolios are characterized only by the number of alternatives they include, the ranking intervals identify core, borderline and exterior alternatives among all potentially optimal portfolios. Paper [III] illustrates this connection by revisiting an application by Könnölä et al. (2007).

Some fifteen years ago, Butler et al. (1997) noted that exploration of all feasible parameter combinations to compute ranking intervals would be "extremely tedious". In this regard, the MILPs developed in Paper [III] are computationally effective as they can compute the ranking intervals among hundreds of alternatives, as shown in the sensitivity analysis of university rankings in Paper [III]. The use of ranking intervals as a tool for multi-parameter sensitivity analysis is supported by the observation in Paper [III] that ranking intervals do not depend on the selected normalization of the value functions, unlike many other results. From the perspective of decision support, practitioners can be given a holistic view through these intervals, independently of the number of attributes.

The ranking intervals and efficiency bounds are novel concepts in CCR-DEA based efficiency analysis in that in addition to communicating how 'good' a DMU can be at best, they also provide information about how 'bad' it can be at worst. They can be used to compare efficient DMUs, unlike conventional efficiency scores, for example. More specifically, they can help identify (i) those efficient DMUs, which perform 'well' compared to other DMUs across the entire set of feasible weights, and (ii) those inefficient DMUs, which do not perform 'extremely badly' compared to other DMUs for any feasible weights. On the other hand, the results can help identify the ones, whose relative efficiency varies 'much' in the set of feasible weights. This may help identify the outputs and inputs that should be bettered in order to improve the worst possible ranking, for example.

Many efficiency measures, such as efficiency scores, cross efficiencies, and super-efficiencies, are computed relative to the other DMUs included in the analysis. These measures discriminate between the efficiencies of the DMUs only on the condition that the number of DMUs is large enough compared to the number of outputs and inputs (Cooper et al. 2000). Furthermore, they can exhibit rank reversals, if the set of DMUs included in the analysis is manipulated. These concerns do not apply to dominance relations which compare pairs of DMUs independently of any other DMUs

Although the proposed concepts for ratio-based efficiency analysis are new, efficiency scores, super-efficiencies, and division into efficient and inefficient DMUs are obtained as special cases of the new results. They are also intuitive in that additional preference information in terms of new weight constraints (i) keeps previous dominance relations intact, but can establish new ones, (ii) does not widen the ranking intervals or the intervals bound by the efficiency bounds, but can make them narrower. These appealing features together with the relations to earlier efficiency measures can catalyze the adoption of the REA methodology by researchers and practitioners.

The thesis suggests some future research directions. First, preference elicitation procedures that accommodate incomplete ordinal information should be designed and tested. These procedures should give the DMs the possibility to express their preferences with the accuracy they feel confident with, but deploy also more discriminative numerical information to obtain decision recommendations. One possibility could be to extend the classification procedure discussed in Section 4 so that it would admit incomplete assignments; for example, when evaluating research proposals, a proposal's attribute-specific performance could be evaluated to belong to either class 'excellent' or to class 'very good'.

Second, the REA methodology could be extended to admit interval-valued data about the DMUs (Cooper et al. 1999, 2001). Furthermore, some of the proposed results for REA could be applied to DEA models with other returns-to-scale assumptions, such as the BCC model by Banker et al. (1984).

Third, the observation that comparing value differences' magnitudes across value functions that describe *different* preferences can result in rank reversals has implications outside the scope of this thesis. For example, many simulation studies have used the average loss of value (or, utility) – which is effectively a sum of value differences over different value functions – to evaluate the quality of decision recommendations in comparing (i) attribute weight (Barron and Barrett 1996) and multi-attribute utility function approximations (Durbach and Stewart 2012) and (ii) multi-attribute value function elicitation procedures that are based on incomplete preference information (Salo and Hämäläinen 2001, Paper [I], Mustajoki et al. 2005). Furthermore, in the context of resource allocation, Liesiö et al. (2008) suggest that budgeting decisions could be based on the minimum value of the portfolio suggested by the maximin decision rule over different budgets. Thus, one research question raised by this thesis is how – if at all – should strengths of preferences between different value functions be measured?

And, subsequently, if such a measure were to be found, can it be used (i) to evaluate the robustness of the alternatives, (ii) to act as a basis for decision rules, and (iii) to characterize incompleteness of preference specification? Or is rank-based information all there is, when we are comparing alternatives across different value functions?

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