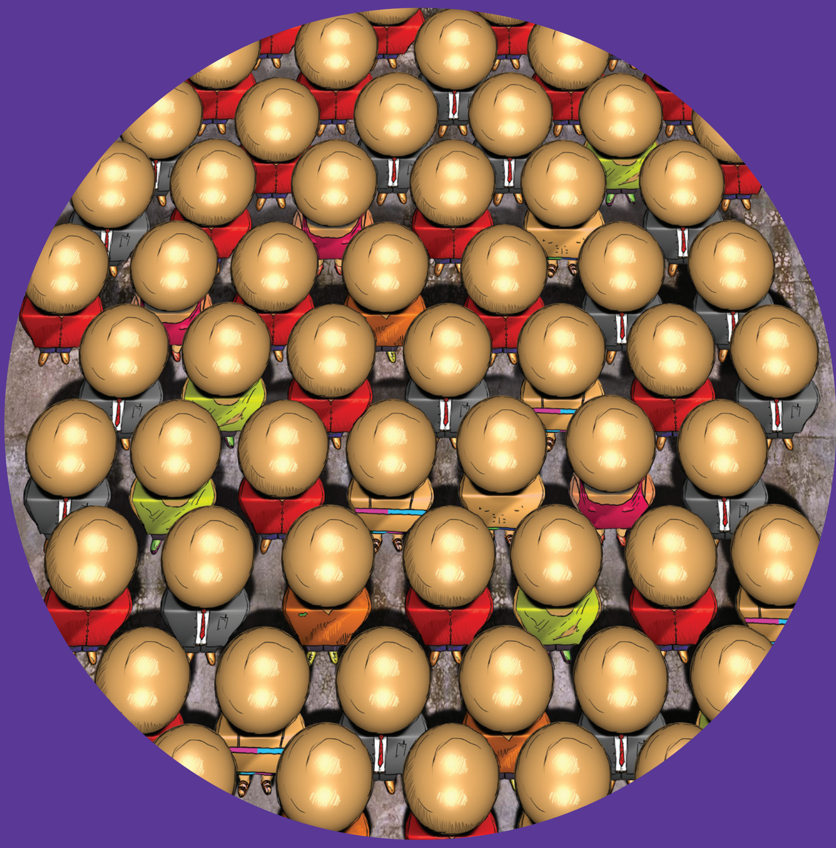
Department of Biomedical Engineering and Computational Science

# Statistical Physics of Opinion and Social Conflict

Gerardo Iñiguez González



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# STATISTICAL PHYSICS OF OPINION AND SOCIAL CONFLICT

**Gerardo Iñiguez González**

A doctoral dissertation completed for the degree of Doctor of Philosophy to be defended, with the permission of the Aalto University School of Science, at a public examination held at the lecture hall F239a of the school on 23 April 2013 at 12.

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# Abstract

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**Abstract**

The rise and development of opinion groups, just as their clash in social conflict, are notoriously difficult to study due to a complex interplay between structure and dynamics. The intricate feedback between psychological and sociological processes, tied with an ample variability of individual traits, makes these systems challenging both intellectually and methodologically. Yet regular patterns do emerge from the collective behavior of dissimilar people, seen in population and crime rates, in protest movements and the adoption of innovations. Statistical physics comes then as an apt and successful framework for their study, characterizing society as the common product of single wills, interactions among people and external effects.

The work in this Thesis provides mathematical descriptions for the evolution of opinions in society, based on simple mechanisms of individual conduct and group influence. Such models abstract the inherent complexity of human behavior by reducing people to opinion variables spread over a network of social interactions, with variables and interactions changing in time at the pace of a handful of equations. Their macroscopic properties are interpreted as the emergence of social groups and of conflict between them due to opinion disagreement, and compared with small controlled experiments or with large online records of social activity.

The extensive analysis of these models, both numerical and analytical, leads to a couple of generic observations on the link between opinion and social conflict. First, the emergence of consensual groups in society may be regulated by well-separated time scales of opinion dynamics and network evolution, and by a distribution of personality traits in the population. Our social environment can then be fragmented as more people turn against the collective mood, ultimately forming minorities as a response to external influence. Second, the exchange of views in collaborative tasks may lead not only to the rise and resolution of opinion issues, but to an intermediate state where conflicts appear periodically. In this way strife and cooperation, so much a part of human nature, can be emulated by surprisingly simple interactions among individuals.

**Keywords** Social dynamics, statistical physics, adaptive networks, mathematical modeling

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# Preface

I can still recall some of that first undergraduate course in Classical Mechanics, back in 2003 at UNAM in Mexico City. During one of our lectures, and before diving deeply into the wonders of frames of reference, forces and energy, Prof. Rafael Barrio asked us a seemingly innocent question, 'how are physicists different from everyone else?' After the expected silence coming from a group of terrified first-year students, he chose to continue. 'A physicist knows just how much is a little. Take that tree for example,' Rafael said, pointing to one of the grandiose trees in between the east and west buildings in our Faculty, 'can you tell me how many leaves it has?'

Well, we could try and guess, or climb and count leaves, yet what a physicist does is make a model. Forget the details for a second, and pretend our tree is nothing but a trunk followed by $z$ branches, each of which is subsequently divided in exactly $z - 1$ branches. Going on and on for $k$ divisions, branches finally give way to $z(z - 1)^{k-1}$ leaves. Now we just have to count branch divisions to set $z$ and $k$ (rather easier than counting foliage) and we end up with a pretty good estimate for the total number of leaves in the surface. Furthermore, we have a *functional* relationship between the properties of our tree, telling us that its surface will increase exponentially as the tree and the number of layers inside grow. Even those with an eye for detail might be satisfied, since we can progressively consider more complicated features (like a varying number of branch divisions) to increase the descriptive power of the model.

Nine years later and with a Ph.D. project on the go, I have learned how to count to try and understand why people behave the way they do. For that is precisely what models give, they are simplified pictures of reality that let us grasp relations between quantities of interest, ultimately allowing us to predict what will happen in the future. So let it be trees, water molecules or the dynamics of social conflict, we could learn so much more than we did while talking about them, if we just start counting and realize how much is a little.

The work summarized in the next pages would not have been possible without the support of a great deal of people in many different places. Most of the research was carried out in the Complex Networks group at the Department of Biomedical Engineering and Computational Science of Aalto University School of Science (or just BECS, formerly the Laboratory of Computational Engineering in Helsinki University of Technology), which functioned as a Centre of Excellence in Computational Complex Systems Research during 2006–2011. There my overall gratitude goes to my supervisor Prof. Kimmo Kaski, who not only gave me the opportunity to join BECS, but worked day by day in making the group an exceptional place to grow academically. This Thesis follows directly from my M.Sc. project developed mainly at Instituto de Física in Universidad Nacional Autónoma de México, with a short visit to the Centre for Mathematical Biology at the University of Oxford. Since then I have counted with the guidance and wisdom of Prof. Rafael Barrio, who has been there from the first equation to the last simulation.

I have been lucky to learn from many leading academics in the field. I thank Prof. János Kertész for his experienced advice and constant help throughout the years, which included a visit to the Budapest University of Technology and Economics. My time at the Institute for Cross-Disciplinary Physics and Complex Systems in Spain would not have been as fruitful without the focused erudition of Prof. Maxi San Miguel. My sincere thanks go to Prof. Julia Tagüeña and everyone at Centro de Investigación en Energía in Mexico for making our small social experiment a reality. I was fortunate to be in the company of Prof. Jari Saramäki and Prof. Santo Fortunato, whose lectures were extremely enlightening and an inspiration to my work. I am also grateful to Prof. Angel Sánchez and Prof. Ingve Simonsen for their useful remarks in the pre-examination of this Thesis. My Ph.D. project, along with its numerous research trips, was supported financially by BECS and by grants from the European Cooperation in Science and Technology and the Finnish Foundation for Technology Promotion.

The (somewhat) younger generation of scientists has also had a strong impact on me over the past few years. It has been a pleasure to work with

Dr. Taha Yasseri and Dr. János Török in an utterly non-conflicting collaboration. My academic roommates deserve compliments as well: Lauri Kovanen for his eternal defense of the scientific method and the grammatical dissection of my Thesis, and Dr. Mikko Kivelä for his passionate arguments never based on emotions. I congratulate both for their ability to publish continually while sharing a room with a noisy Mexican. I have learned a lot from my current collaborators Dr. Márton Karsai and Dr. Raj Kumar Pan, who showed me that the near future is not only bereft of sleep, but full of scientific challenges. Many thanks go also to Ville Lehtola for showing me what it meant to be in Finland, and to Ville-Pekka Backlund for being the first student I actually had to teach something to. My time at BECS has been as inspiring as it was entertaining, from Coffee Time to the Journal Club, and I want to thank everyone at the Complex Networks group for sharing it with me.

Outside academia, friends and family have been there to shape me into the person I am, during my time in Finland and throughout my whole life. I cherish all experiences shared with the groups of friends I have made in the north of the world, which go by the cryptic acronyms HHK, TKK and DSH. A lifelong gratitude will always be devoted to my mother Alma González, who showed me the value of love strengthened by determination, and to my father Ernesto Iñiguez, whose wisdom I carry in my heart and mind. I treasure every moment with my only and favorite sister Daniela Iñiguez, a star in the sky, as well as all ridiculously philosophical discussions with my brother-in-law Alejandro Lastra. To Daniel Arévalo, brother in everything but blood, I am grateful for his tireless awesomeness and a masterful Thesis cover. Finally, in an ending that feels more like a beginning, I thank my dearest Tiina Näsi for being exactly the girl she is. To all the places I have swum, I could not have done it without you.

Espoo, March 21, 2013,

Gerardo Iñiguez González

# Contents

# List of Publications

This Thesis consists of an overview and of the following publications, which are referred to in the text by their Roman numerals.

**I** Gerardo Iñiguez, János Kertész, Kimmo K. Kaski, and Rafael A. Barrio. Opinion and community formation in coevolving networks. *Physical Review E*, Volume 80, Issue 6, 066119, December 2009.

**II** Gerardo Iñiguez, Rafael A. Barrio, János Kertész, Kimmo K. Kaski. Modelling opinion formation driven communities in social networks. *Computer Physics Communications*, Volume 182, Issue 9, 1866–1869, September 2011.

**III** Gerardo Iñiguez, János Kertész, Kimmo K. Kaski, and Rafael A. Barrio. Phase change in an opinion-dynamics model with separation of time scales. *Physical Review E*, Volume 83, Issue 1, 016111, January 2011.

**IV** Gerardo Iñiguez, Julia Tagüeña-Martínez, Kimmo K. Kaski, and Rafael A. Barrio. Are Opinions Based on Science: Modelling Social Response to Scientific Facts. *PLoS ONE*, Volume 7, Issue 8, e42122, August 2012.

**V** János Török, Gerardo Iñiguez, Taha Yasseri, Maxi San Miguel, Kimmo K. Kaski, and János Kertész. Opinions, Conflicts and Consensus: Modeling Social Dynamics in a Collaborative Environment. *Physical Review Letters*, Volume 110, Issue 8, 088701, February 2013.

# Author's Contribution

### Publication I: "Opinion and community formation in coevolving networks"

The author contributed greatly to the conceptual development of the model, its analysis and mean field approximations. He performed all numerical simulations leading to the results presented and actively participated in the writing of the article.

### Publication II: "Modelling opinion formation driven communities in social networks"

The author assisted in devising the scope of the manuscript, performed all numerical simulations and analyzed their results. He also contributed significantly to the writing of the article.

### Publication III: "Phase change in an opinion-dynamics model with separation of time scales"

The author had a significant role in the conceptual development of the model, its analysis and mean field approximations. He performed all numerical simulations leading to the results presented and participated in the writing of the manuscript.

### Publication IV: "Are Opinions Based on Science: Modelling Social Response to Scientific Facts"

The author performed all numerical simulations, analyzed their outcomes and mainly devised the mean field approximation included here. He aided in the poll data analysis and actively participated in the writing of the article.

### Publication V: "Opinions, Conflicts and Consensus: Modeling Social Dynamics in a Collaborative Environment"

The author contributed significantly to the conceptual development of the model and its scope. He performed numerical simulations in the case of a fixed agent pool and analyzed their results. He participated in writing and revising the manuscript.

# List of Figures

# List of symbols

In order of appearance.

| | |
|---|---|
| $A_{ij}$ | Adjacency matrix |
| $i$ | Agent |
| $j$ | First neighbor |
| $N$ | Network size |
| $p$ | Edge probability in a random network |
| $\rho_k$ | Degree distribution |
| $k$ | Degree |
| $\langle k \rangle$ | Average degree |
| $\langle C \rangle$ | Average clustering coefficient |
| $\langle \ell \rangle$ | Average path length |
| $\langle k_{nn} \rangle$ | Average nearest-neighbors' degree |
| $\sigma$ | Microscopic state of system |
| $t$ | Time |
| $r$ | Transition rate between states |
| $\beta$ | Inverse temperature |
| $Z$ | Partition function |
| $p_1$ | Probability to equal opinions |
| $q$ | Relative frequency of agreement |
| $\phi$ | Rewiring probability |
| $x_i$ | Opinion of agent $i$ |
| $dt$ | Time scale of transactions |
| $T$ | Time scale of generations |
| $\hat{O}$ | Network change operator |
| $g$ | Ratio between time scales |
| $\{x_j\}_s$ | Subset of close agents |
| $f_s$ | Short-range interaction |
| $\{x_j\}_l$ | Subset of far agents |

| | |
|---|---|
| $f_l$ | Long-range interaction |
| $\ell_{\max}$ | Longest shortest-path length for agent $i$ |
| $l$ | Second neighbor |
| $p_{ij}$ | Opinion difference weight |
| $q_{il}$ | Opinion similarity for triadic closure |
| $\langle n_{\mathrm{und}} \rangle$ | Average number of undecided agents |
| $\langle s \rangle$ | Average susceptibility |
| $\alpha_i$ | Attitude parameter for agent $i$ |
| $N_c$ | Attitude distribution in group of size $c$ |
| $\langle \alpha \rangle$ | Average attitude |
| $n_\alpha$ | Number of opinion groups |
| $g_c$ | Critical value of ratio $g$ |
| $c$ | Size of opinion group |
| $h$ | Strength of external field |
| $h_0$ | Critical field strength |
| $\mu_A$ | Article convergence |
| $A$ | Opinion expressed by article |
| $\tau$ | Relaxation time to consensus |
| $M$ | Controversiality measure |
| $p_{\mathrm{new}}$ | Agent renewal rate |
| $S$ | Cumulative amount of conflict |

# 1. Introduction

## 1.1 Background

We are, after all, only human. We digress and transgress, so to speak, naturally forming opinions and discussing with those of a different mind. As individuals we perceive bits and pieces of reality, interpret the facts through our ideas and emotions, and if willful enough, support the resulting subjective beliefs by reasoned arguments. Yet the same set of facts may lead to opposing opinions, luring us to compare arguments and decide which one is better. Our only choice left at the end is whether to join a group of like-minded fellows, or to enjoy the continual strife of disagreement. How do communities of similar opinion form? What are the main traits determining their development? What is the social response to information proclaimed as fact? How do conflicts of opinion emerge and get resolved?

The work in this Thesis aims at answering such questions through mathematical modeling and physical insight. Opinion and community formation, like most social phenomena, are notoriously difficult to study due to their complex structure and adaptive dynamics. The intricate feedback between psychological and sociological processes, coupled with the inherent ambiguity of language, makes these systems challenging both intellectually and methodologically. Even worse, most statements about them seem as obvious and common-sense as their opposites, making knowledge difficult to validate. As D. Watts points out [206, 207], 'everyone has experience being human, and so the vast majority of findings in social science coincide with something that we have either experienced or can imagine experiencing.' It is in this context that a modeling-based approach is relevant, since the rigorous analysis of rule-bound dynamics (and the validation of the underlying hypotheses through comparison with empirical data) can relieve us from our own intuition.

The statistical physics approach in modeling a system formed by a large number of constituents, say a kettle full with boiling water, aims at characterizing the system's macroscopic properties in terms of the dynamical evolution of its basic elements, like the water molecules in the kettle. The fact that two systems with differing components may have the same macroscopic behavior only due to their large size has prompted physicists to go out of physics and into the realms of biology, economics and sociology. Who is not, after all, tempted to use the same general framework and modeling principles to study spins in ferromagnets as well as humans in society? We should be careful though and remember, as D. Stauffer does [188], that 'people are not atoms.' While electrons are identical, individuals are highly heterogeneous in personality, with interactions variable in time through will and experience, which may indicate some limitations for the 'simple' transfer of methods from physics [178].

Yet striking regularities at the societal level do exist [31]. Birth and death rates, the development of protest movements, crime statistics and the adoption of innovations, all show definite patterns emerging from the collective behavior of dissimilar people. Thus, suitable modeling of processes like community formation and opinion conflict should follow two steps. First we need to establish simple yet realistic rules for the microscopic dynamics of individuals, mathematically inferred from sociological studies and small controlled experiments. Then we can derive the macroscopic behavior of the system through analytical and numerical calculations, aimed at a comparison with real data on large-scale social phenomena. The availability of recent opinion surveys at the country level, as well as detailed temporal records of conflict in collaborative websites such as Wikipedia, are invaluable in this respect.

A common set of underlying mechanisms tied to broad interdisciplinary applications have made the statistical physics of social dynamics a trendy field with emergent success, as can be seen from the growing number of reviews in the literature [36, 186, 209]. Despite the interest shown by the scientific community, a majority of results favor theoretical description over empirical verification, choosing the analytical tractability of simple dynamical rules over the sociological relevance of more complicated mechanisms, and with conclusions accepted by plausibility rather than by comparison with observations. It seems pertinent then to focus our research on this unfortunate gap, in an attempt to answer the 'call for a closer link with reality' of P. Sobkowicz [182] by a sensible combination of theoretical modeling and real data analysis.

## 1.2 Objectives and scope

With such background in mind, the goal of this Thesis is to model the evolution of opinions in society and compare outcomes with empirical data on opinion conflict. Under the general framework of statistical physics, the models included here characterize individuals with a reduced set of variables and parameters, describe their social structure as a dynamic network of interactions, and consist in equations for the coupled time development of opinions and society. The macroscopic properties of the system are then interpreted as the emergence of social groups and of conflict between them due to opinion disagreement.

The following chapters comprise an overview of the field and a summary of the results in Publications I–V, ordered as to answer these research questions:

- *How do communities of similar opinion form?*
  Opinion formation is mediated by social interactions and at the same time influences the structure of society itself. In Publication I we model this coevolution of opinion and network structure by considering discussions between individuals, personal attitudes towards the mood of the majority, and rewiring of social links among people. The dynamical rules are motivated by known sociological mechanisms (such as homophily and network closures) and by our own small experiment regarding agreement on a polemic issue. We argue that the separation of time scales between fast opinion dynamics and slow network rewiring may control the emergence of communities of similar opinion.

- *What are the main traits determining their development?*
  In Publication II we show that individuals with opposing attitude towards the majority's opinion tend to form small groups, while those with agreeing attitude constitute larger communities. Thus, our modeled society becomes fragmented as more people go against the collective mood. We further confirm this claim in Publication III, where a simplified version of the model is used to extend the ana-

lytical treatment of the dynamics. Overall, both the ratio between time scales and the diversity of personal attitudes may determine the development of heterogeneous community structure in society.

- *What is the social response to information proclaimed as fact?*
  Opinions are based not only on personality traits and discussions with our peers, but also on information flow channeled by the media. This input can be divulged as fact, often creating groups with contrary views on the subject. In Publication IV we extend our model to consider the social response to scientific facts, finding that concepts promoted by the media may be more difficult to acquire than those opposed by it, since disagreeing individuals form tight communities that prevent opinion consensus. Additionally, we use scientific perception surveys to adjust parameters in the model and pinpoint cultural differences between two real populations.

- *How do conflicts of opinion emerge and get resolved?*
  Finally, we focus our attention on the rise, persistence and resolution of opinion conflicts in tasks achieved by cooperation. In Publication V we develop a simple model where individuals interact directly through discussions, indirectly by making changes to the common product, and might decide to abandon the project altogether. The dynamics allows for a state of mainly consensus and one of perpetual conflict, as well as an intermediate regime where small conflicts continually emerge and get resolved. These scenarios of strife agree qualitatively with data on the collaborative website Wikipedia, where people edit articles on numerous topics and discuss about their contents.

# 2. The physics of society

Models are not reality. Among their myriad of definitions, types and applications [80], models are fictional objects aimed at representing a piece of the world around us, where the goal is to achieve a level of isomorphism with measurable quantities of interest [51, 198]. More often than not, however, they are simplified pictures of nature with an incomplete account of relevant variables and interactions, which in view of their objective would strike us as nothing but wrong. Why should we care about false models then? Well, simply because they are useful tools to get at the truth [217]. Simple models might be used as a starting point for more complex and accurate descriptions of reality, just as incomplete models could let us focus on particular properties of intricate phenomena or assess the importance of missing variables.

Above all, false models allow us to understand. In the study of social dynamics, for example, they may help in determining causal relations and driving mechanisms behind empirical observations, lifting the burden of explanation from a common sense that seems to fail as often as it succeeds. Models in the field of complex networks reveal the structural and dynamical similarities between systems with very different functions, just as the models of statistical physics show that size often comes along with simpler descriptions in terms of macroscopic variables. So let us jump right in and review some of the roles modeling has taken in these fields.

## 2.1 Social dynamics

The first philosophical discussion of a science of social phenomena is usually attributed to the 19th century positivism of A. Comte [45], who argued the inevitable coming of sociology as a consequence of mankind's quest to describe systems of increasing complexity with mathematical tools. Indeed, the success of newtonian mechanics in predicting the motion of macroscopic bodies, both earthly and celestial, led many to believe in the existence of quantitative laws governing the behavior of individuals. Some were more skeptical, L. Tolstoy among them [196], thinking that our freedom of choice as conscious beings would allow us to invalidate any proposed social mechanism. This paradox of free will [71] was partly circumvented by the empirical work of A. Quetelet [161] and other scholars of the time, who found that laws of statistical nature could be applied to societies as a whole instead of particular individuals.

The statistical perspective of such ‘social physics’ was useful in other fields of science as well, most prominently when the likes of J. Maxwell, L. Boltzmann and J. Gibbs accounted for the macroscopic behavior of gases in terms of the properties of large ensembles of particles, laying the foundations of statistical physics on the way [83]. Efforts to describe social structures with mathematical models followed suit in the mid 1900s, as the field of sociometry started to develop around the main concept used in this Thesis: a *social network* [27, 179, 205]. Introduced as a sociogram by J. Moreno in psychology [141] and as a social network by J. Barnes in anthropology [12], it is a mathematical representation of the pattern of relationships, or *ties*, among a group of social entities known as *actors*. These social units (individuals, groups of people or even entire societies) are characterized by attribute data capturing their behavior, such as opinions and attitudes. Relational ties, on the other hand, imply the existence of any kind of interaction between actors, like discussions and friendship. An example of a social network describing the pattern of discussions between individuals with different opinions is portrayed in Figure 2.1.

Social networks are useful models due to their incompleteness and simple definition. By disregarding most information about the social environment apart from some chosen attribute and relational data, we can simplify the study of its structure to answer a particular question we might have. A direct consequence is that even the same set of actors may lead to very different network structures, depending on the definition of ties and the topic one is interested in. Also, similar tools and measures are usually appropriate to probe networks made up of any kind of actors and ties, leading to a generic methodology for the study of social and behavioral phenomena based on structural analysis. The flexibility of this framework has given rise to an extensive number of research studies, regarding topics as varied as friendship networks in terms of typical size, chains of

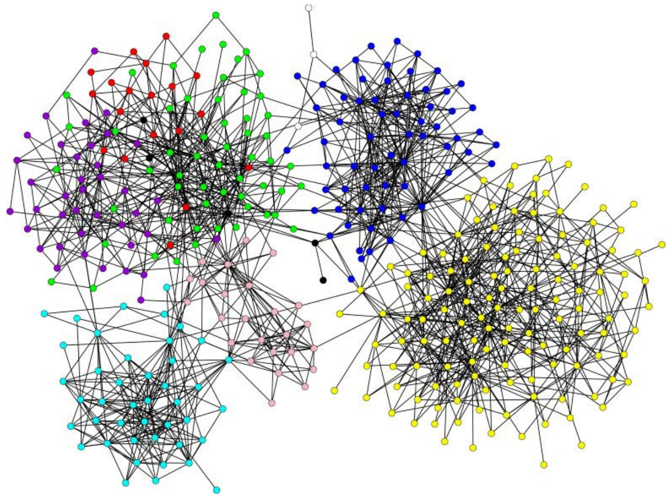


**Figure 2.1. A social network.** Representation of the pattern of relationships, or ties, among a group of social entities known as actors. This case corresponds to the model of Publication I where actors are individuals with a single attribute, their opinion on a given topic, and ties imply the presence of related discussions. By disregarding other social features, the network reveals the presence of well-connected groups of actors with similar attributes, pictured here as separate colors.

acquaintances and group segregation [64, 113, 134, 140], sexual and romantic networks [19, 124], networks of collaboration among scientists or movie actors [5, 144], and even temporal phenomena like the spreading of innovation in professional networks [44] and the stability of corporate networks over time [53].

The modeling of dynamical features in social systems is arguably as fascinating as it is challenging. Time-resolved networks (usually called longitudinal studies in the literature [204, 212]) allow us to discern the temporal variation of social processes, with the goal of identifying causality relations and ultimately predicting future behavior. Given that the description of physical phenomena has similar objectives, the last few decades have witnessed an increasing use of the 'methods of physics' [158] in social network analysis. Indeed, the field of social dynamics [6, 36] deals with the generic transition between order and disorder due to the presence of networked interactions among social entities, where order is identified as a state of consensus or homogeneity in the attributes of actors, and disorder implies an opposite state of conflict or heterogeneity.

There are many models in social dynamics, each tailored to describe a particular topic ranging from the emergence of languages [213] to the formation of hierarchies [40]. In the Axelrod model for culture dissemination [7, 38], for example, neighboring actors in a social network are more likely to interact if they share many cultural features (a principle known as *homophily* [130]), and by doing so they exert social influence and become more similar. Despite this drive for homogeneity, the dynamics may end up in a stable state where many different cultural regions coexist. Another equally surprising scenario of social polarization is found in the Schelling model of residential segregation [172, 173], where even a mild preference of actors to relocate in urban areas with alike neighbors leads to a society of fully segregated ghettos [52, 192].

Social dynamics are often deeply influenced by the network underneath, although notable exceptions do occur [88, 93, 94]. The temporal evolution of individual attributes like opinions, attitudes and beliefs may be affected by particularly short chains of acquaintances speeding up the flow of information, or by actors with a large number of ties capable of influencing entire social groups [144]. The study of the properties of different network structures lies at the heart of the field of complex networks.

## 2.2 Complex networks

Yes, networks are everywhere. From the technological and informational backbone of the world that is the Internet and the WWW [157] to the abundance of social networking services like Facebook [123] and Twitter [58], networks have gone beyond their academic status as abstractions of human interactions to become an iconic concept in our everyday lives. Recent years have witnessed an equally drastic shift in their use as modeling tools: networks describe not only social groups but food chains, neural and metabolic processes, product distribution structures and any other system made up of a very large number of linked parts, fit to be analyzed with generic statistical methods and often sharing properties despite their distinct origins as social, biological or technological systems. The emerging field of complex networks is now well established in the literature, with several introductory and reference books on the matter [15, 33, 61, 63, 66, 147, 148], as well as scientific reviews dealing with its main concepts and applications [2, 24, 48, 62, 146].

In its simplest definition, a network is a collection of *nodes* (or vertices) connected by *links* (or edges), quite equivalent to the pattern of actors and ties of a social structure. It can be mathematically represented by the elements $A_{ij}$ of an *adjacency matrix*, equal to 1 when nodes $i$ and $j$

are connected and 0 otherwise. The network idea is usually attributed to the celebrated mathematician L. Euler [3], who used a small one to prove that it was impossible to come up with a closed route along the bridges of 18th century Königsberg by crossing each bridge only once. The study of networks as purely abstract entities has gone a long way since then, forming an entire branch of discrete mathematics known as graph theory [25, 57, 85]. A typical task in proving some graph-theoretical statements, from coloring to the analysis of flows, is to estimate the proportion of networks having a certain property by means of deterministic, combinatorial techniques. Yet another route is to approximate exact results with probabilistic methods, an often useful approach that led to the development of random networks [67, 183] in the mid 1900s.

A minimal model for any pattern of connections, a *random network* can be constructed from a set of $N$ nodes where a link between any pair of them is placed with independent probability $p$. The ensemble of all networks devised in this way has many analytical properties, such as the *degree distribution* $\rho_k$ giving the probability that a randomly chosen node has $k$ links to other vertices (called first neighbors or just neighbors), and the mean value of $\rho_k$ or average *degree* $\langle k \rangle = (N-1)p$. When $N$ becomes really large and $\langle k \rangle$ stays constant, $\rho_k$ takes the Poissonian shape $\rho_k = e^{-\langle k \rangle} \langle k \rangle^k / k!$, meaning that the probability of having large degrees in the network decays exponentially fast with $k$. We can consider properties that depend on a couple of nodes as well, like the average *clustering coefficient* $\langle C \rangle$ (the probability that two neighbors of a vertex are also connected, forming a triangle) and the average *path length* $\langle \ell \rangle$ (the size of the shortest chain of edges linking two vertices). The limit $N \to \infty$ with constant $\langle k \rangle$ gives zero clustering and a path length scaling as $\ln N / \ln \langle k \rangle$, meaning that large random networks have almost no triangles and relatively short distances between nodes.

If anything, random networks might strike us as a bit 'too simple' to describe the pattern of interactions in a real system, more fit as a neutral model [217] with the explicit purpose of assessing the importance of variables or mechanisms not considered by it. On one hand, patterns of social interactions such as friendship [134] and corporate relations [53] usually have small path lengths (just like random networks), but significantly higher clustering than their random counterparts [143]. On the other, degree distributions of many networks (scientific citation webs [160], for example) tend to be broad, decaying slower than exponentially.

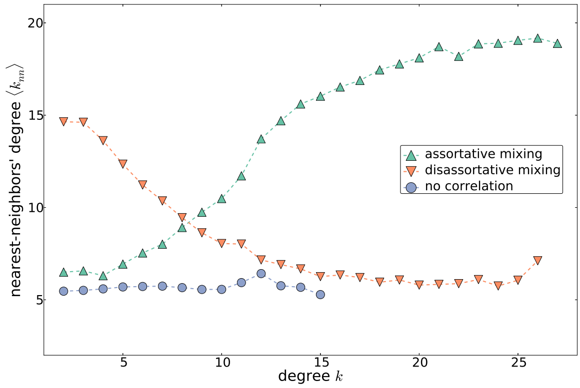



**Figure 2.2. Mixing patterns in networks.** Average nearest-neighbors' degree $\langle k_{nn} \rangle$ as a function of the degree $k$ of a node for three mixing patterns commonly found in networks, calculated here for different conditions in the model of Publication I. A tendency of actors to connect with others of the same opinion leads to assortative mixing, while the opposite creates disassortative behavior. The neutral pattern of edges in random networks gives no correlation between degrees.

These shortcomings prompted the development of two more realistic models, generally considered as the starting point of the field of complex networks [193]. The first is the small-world model of D. Watts and S. Strogatz [16, 208], where short distances and many triangles coexist in a network formed by randomly rewiring some edges in a regular lattice. The second is the preferential-attachment model of A. Barabási and R. Albert [11], a model of growing networks where new nodes connect to old ones with probability linearly proportional to their degree, giving rise to an asymptotically scale-free distribution $\rho_k \sim k^{-3}$. Broad degree distributions are indeed ubiquitous in nature [2], although the scale-free hypothesis in particular is often not tested in a rigorous way [42].

Networks can also be characterized globally by considering properties of several vertices at the same time. Motifs, for example, are recurrent small patterns of connections usually associated with particular functions in the system [136], while mixing patterns measure the amount of selective linking between nodes with the same attributes [145]. Structural mixing in the form of degree correlations is normally quantified by the average *nearest-neighbors' degree* $\langle k_{nn} \rangle$ and its overall dependence on the degree of a node, like it is pictured in Figure 2.2. When $\langle k_{nn} \rangle$ grows with $k$ (meaning that nodes of similar degrees tend to be connected), we have a so-called *assortative* mixing readily identified with the homophilic structure of social networks [82, 140]. Other systems like biological and technological networks [128, 156] are *disassortative* instead, containing many edges between high- and low-degree vertices.

Yet another concept related to sets of many nodes is that of *community*. In social networks, it is intuitively defined as a group of actors belonging together due to some criteria we are interested in, such as sharing attribute data or similar patterns of ties [26, 205]. In one of the most commonly used definitions, the group has more links between members than with the rest of the network [163]. The related field of community detection is an active research topic [74, 171] that shares several features and methods with the problem of data clustering [106], where points in arbitrary spaces are organized in clusters according to their similarity. When looking for communities most approaches are operational, defining groups in the network as the result of a given algorithm.

There is a lot more. The use of networks as models for empirical systems has greatly increased the number of tools used to analyze them. Simple patterns of nodes and links can be substituted by directed [1] or weighted [13, 14] networks, where edges have directions or values representing the strength of interactions [154]. The random network has been generalized to contain any degree distribution in the configuration model [139, 150], without losing its analytical tractability thanks to the (cunning) use of generating functions [215]. Networks may be embedded in a geographical space [17], considered as dynamical structures [100] to account for temporal activity like bursty human behavior [10], and even be interconnected to other networks [32]. Still, let us turn the page now and jump into the field of statistical physics, a useful framework for studying the macroscopic properties of large systems of interacting components.

## 2.3 Statistical physics

Our world seems to be quite hierarchical in terms of length and time scales, allowing us to classify systems in levels of different size and duration and even offer independent mathematical descriptions for their behavior. Yet higher levels do not come with additional laws, just with new phenomena understandable by a proper reformulation of the rules of lower levels [121]. Statistical physics, as a prime example, explains how the macroscopic behavior of large physical systems arises from the multitude of microscopic interactions of their components [60, 83, 89, 109]. In a very generic summary of the statistical physics method, the dynamical properties of a system's elements are enclosed in *state variables*, all of which determine the microscopic state $\sigma$ of the system. As time goes by, the system will jump between different configurations due to the interactions among components and any external effect that might be present, such as a field or a thermal reservoir. But instead of solving a very large number of equations of motion for the individual elements, we may take a statistical approach and consider the probability $\rho(\sigma, t)$ of finding the system in state $\sigma$ at time $t$. Its temporal evolution is then governed by the (conveniently called) master equation [15, 149],

$$\frac{\partial \rho(\sigma, t)}{\partial t} = \sum_{\sigma'} \left[ \rho(\sigma', t) r(\sigma' \to \sigma) - \rho(\sigma, t) r(\sigma \to \sigma') \right], \tag{2.1}$$

where $r$ represents the transition rates between states. By formally solving this equation, any macroscopic property that takes the value $m(\sigma)$ in a given state can be averaged to obtain the expectation $\langle m \rangle = \sum_\sigma m(\sigma)\rho(\sigma, t)$, a good approximation for measurements made on the system as a whole.

If we are fortunate and patient enough, however, there may be a moment when all terms in Eq. (2.1) cancel each other and $\rho(\sigma, t)$ remains constant at an *equilibrium* value $\rho_\sigma$. A system in contact with a thermal reservoir follows in this case the Boltzmann-Gibbs distribution $\rho_\sigma = Z^{-1} e^{-\beta H_\sigma}$ [83], where $\beta$ is an inverse temperature and the Hamiltonian $H_\sigma$ gives the energy associated with state $\sigma$. The so-called partition function $Z$, being much more than just a normalization constant, can be used directly to calculate all the macroscopic properties studied in thermodynamics [34, 116].

Just like in the fields of social dynamics and complex networks, models come to our rescue as simplifications of reality to focus on a given feature of interest. In statistical physics, models are explicit expressions for the Hamiltonian that consider some microscopic interactions in a minimal way. One of the simplest and most studied examples is the Ising model [29] that aims to describe the ferromagnetic properties of materials in terms of the magnetic dipoles of their atoms. Here, the state variables are $N$ spins $\sigma_i = \pm 1$ that tend to align with their neighbors $j$ on a network via interactions of strength $J > 0$, implying $H_\sigma = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j$. A useful macroscopic property is the magnetization per spin $m(\sigma) = \sum_i \sigma_i / N$, measuring the level of alignment (or order) in the system. For certain network structures and as the temperature goes down, the system undergoes a transition from a disordered and symmetric phase with $\langle m \rangle = 0$ to an ordered one with spontaneous magnetization $\langle m \rangle \neq 0$. The study of phase transitions and symmetry breaking through dynamics like the

Ising model is one of the main goals of statistical physics. Near the transition point many models (and the systems they describe) show the same critical behavior despite their different definitions, a concept known as universality [86, 184, 185].

Yet things are not always that simple. The partition function might not have an explicit expression from where to calculate macroscopic quantities directly, or we may be interested in dynamical behavior before reaching equilibrium, leaving us only with a master equation often impossible to solve. One alternative is *agent-based* modeling [47, 177], a broad set of computational and analytical techniques used in and out of physics to describe seemingly disparate topics like Brownian motion, structure formation in biological systems, pedestrian movement and urban growth. Under this approach a system's elements (let them be actors, nodes or particles) are substituted with *agents* moving between a predefined set of states available to them. The dynamics of each agent depends on others via a network of interactions, and follows equations mimicking the expected microscopic behavior of the system. The effect of any unknown mechanism is usually considered with stochastic rules akin to those of Monte Carlo methods [119, 133, 149]. Finally, global regularities are studied by averaging over all agents in computer simulations and, when possible, by comparing with analytical approximations. These techniques are quite multidisciplinary and also used in population biology [129], artificial intelligence [211] and computational sociology [126].

Overall, agent-based models take concepts and tools from statistical physics and network theory to describe collective phenomena like social dynamics, allowing us to understand the development of society as the common product of individual wills, interactions among people and external effects. It is now time to narrow down and move on to the main topic of this Thesis: opinion formation and its relationship with the structure of society.

# 3. The dynamics of opinion

Everyone has an opinion. Many, in fact. Whether it is politics, art or the unfathomable void of religion, we all feel compelled to have a say and defend our positions, either through arguments or by pure dismissal of alternative points of view. Yet opinions do sway. There are many studies on the psychological and sociological factors determining opinion change, ranging from personality and resistance traits to persuasive communication [69]. In the work of H. Kelman, for example, opinion shifts are classified into groups according to the level of private acceptance that goes with public conformity [112]. We may be influenced and change our minds either by compliance (to get social approval), identification (to establish relationships with others), or by internalization (when the new position is congruent with our values). Another example is B. Latané's social impact theory [120, 152], where opinions change due to social forces exerted by people around us and with strength proportional to their number, importance and immediacy.

As the size of the studied social group increases, however, it becomes increasingly difficult to setup face-to-face experiments, follow individual interactions and deduce their combined effects on the group as a whole. A complementary approach is the use of computer simulations via agent-based modeling, referred to in the literature as opinion dynamics [36, 188, 190]. Here, individuals are described as agents shifting between opinion states due to elementary social mechanisms, and the ultimate goal is to characterize the rise of complete agreement or disagreement in the system. What follows is a brief review of the most famous attempts at modeling opinion formation, both over static social structures and when the network of interactions varies over time.

## 3.1 Modeling opinion formation

Models for the rise, exchange and shift of opinions simplify human interactions by assuming a limited or bounded set of states available to agents, and by proposing dynamical rules that enforce a particular social behavior. The simplest of all is arguably the voter model [98], an extension of the Ising model where opinions are binary variables $x_i = \pm 1$. For each interaction an agent $i$ is randomly selected along with one of its neighbors $j$ and $x_i = x_j$, that is, the agent copies the state of the neighbor. Although usually just an approximation, binary opinions may be the outcome of a lengthy and complex discussion, or even the only alternative in processes like elections and product selection. The mechanism of state copying, on the other hand, mimics the homophily principle [130] by promoting neighborhoods of similar opinion.

The voter model can be solved exactly with a master equation formalism [78], showing that consensus is reached in regular lattices of dimension 2 and lower, but not in more than 2 dimensions. Extensions of the voter dynamics are numerous [54], considering other ingredients like noise [175] and zealous individuals [138], the effect of more than two opinion states [39, 181, 201] or of strategic imitation [203]. The voter model behaves somewhat differently over complex structures such as small-world and scale-free networks [37, 194], showing variations in the time required to attain consensus and temporary stages with no opinion change.

Opinion formation models are as varied as the social mechanisms or individual personalities they intend to mimic. In the majority rule model [81], for example, groups of a given size are chosen and the majority opinion takes over all its agents, implementing a principle of social inertia or validation [41, 79] by which pressure groups compete for political influence [20]. A nice classification of models in terms of individual personalities is that of D. Stauffer [187], where agents take the roles of 'missionaries', 'negotiators' or 'opportunists'. Social validation is a key factor for the missionaries of the Sznajd model [195], in which agents manage to convince all neighbors only if their own opinions are similar.

When our choices are not black and white but gray, like with political tendencies or economic utilities, opinions may be better approximated by continuous variables $x_i \in [0, 1]$ rather than by a couple of options. This extension brings about two new features to the modeling of opinion dynamics. First, equilibrium states can show total agreement or be frag-

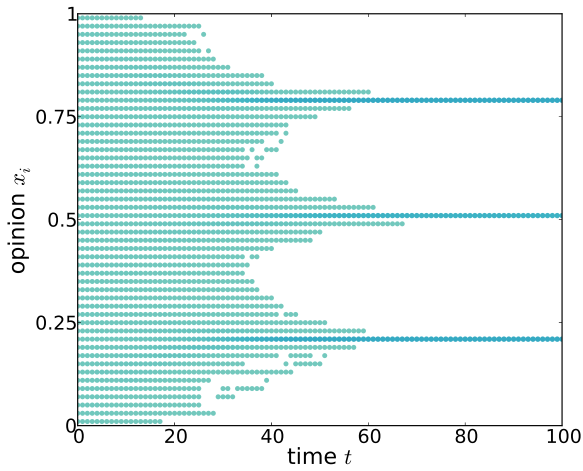



**Figure 3.1. Bounded confidence in opinion dynamics.** Opinions $x_i$ as a function of time $t$ for the Deffuant-Weisbuch model in a system of size $N = 10^4$ with $(\epsilon_T, \mu_T) = (0.17, 0.2)$. Darker dots imply a larger density of opinions. As time goes on, agents with opinions closer than $\epsilon_T$ negotiate and converge their views by the relative amount $\mu_T$, leading to the formation of distinct opinion groups. This dynamics constitutes the basic social interaction in the extended model of Publication V.

mented into several groups with more than two values of opinion. Second, the distance between agents' states is a measure of their similarity and can be used to define the occurrence of social interactions in the model. One example is the concept of *bounded confidence*, which assumes that individuals discuss only if they are close in opinion to each other [125]. Bounded confidence is the main ingredient of the Deffuant-Weisbuch [56] and Hegselmann-Krause [95] models. In the former, agents take the role of negotiators that meet in pairs to find a compromise, while in the latter agents are opportunists that adopt the average opinion of all of their similar neighbors.

As is common in opinion dynamics, the Deffuant-Weisbuch model starts by setting up $N$ agents in a given network of social interactions. The dynamics depends on only two parameters: a *tolerance* $\epsilon_T \in [0, 1]$ that determines the reach of agent similarity, and a *convergence* $\mu_T \in [0, 1/2]$ that defines the amount of compromise after discussions. Every interaction involves a random pair $(i, j)$ of neighbors, and if $|x_i - x_j| < \epsilon_T$ their opinions get updated in the following manner:

$$(x_i, x_j) \mapsto (x_i + \mu_T[x_j - x_i], x_j + \mu_T[x_i - x_j]). \tag{3.1}$$

These rules give rise to a dynamic process where similarly-minded individuals negotiate their positions and move symmetrically in opinion space, except for those near the boundaries that can only drift towards the center. Such instability leads to the formation of a number of disjoint opinion groups, approximately $1/(2\epsilon_T)$ for the completely connected network used in Figure 3.1. The properties of this model have been studied extensively, first in the physical context of inelastic collisions [9, 21] by means of master equations [22], and then by considering societies with stubborn individuals [117, 159] or open to external effects [87]. Some of the original results have recently been questioned as well, regarding the conditions for particular final states of the dynamics [131]. The mechanism of bounded confidence for both negotiators and opportunists has been extended to vector opinions [77, 105], where discussions cover more than one single topic, and to various underlying network structures [72, 73, 210].

Perhaps one of the most distinguishing features of the field of opinion dynamics is the clear disproportion between theoretical models and research validating their sociological assumptions through empirical data. One notable exception is the study of elections. The Brazilian elections of 1998 [49], in particular, have been described as the transient state in a modified Sznajd model over scale-free networks [23]. Interestingly enough, the distribution of votes received by candidates seems not to depend on countries and years, once factors like the total number of candidates and votes in party lists are taken into account [75, 76]. Another feature of real opinion formation processes that has attracted interest recently is the fact that social networks are not static, but change in time with their own dynamics. The explicit coupling between agents' states and network evolution, along with its new effects like fragmentation and group formation, is our next topic to discuss.

## 3.2 Coevolution of network and opinions

Opinions may change, but sometimes people just don't. A persistent conflict of views can push us to break social contacts and look for more amiable relationships, modifying the surrounding social structure and thus affecting the way our opinions are shaped. From the point of view of the statistical physics of social phenomena, the network of interactions may change because of the agent-based model defined on top of it, creating a feedback loop between structure and dynamics that has come to be known as *coevolution* [90, 92, 96, 225]. Coevolving or adaptive networks (as they are often called nowadays) have been studied across a broad range of disciplines, with applications to chemical and neural networks [28, 104, 107], cooperation in game theory [222, 223], disease spreading [91, 127, 180] and even the dynamics of swarming insects [101, 202]. In addition to the typical ingredients of agent-based models, a theoretical description of coevolving systems includes microscopic rules for the removal, creation or rewiring of edges that depend on the state of agents. Both node and link dynamics come with characteristic *time scales*, telling us the speed of their time evolution. When these time scales are comparable, adaptive networks can organize themselves in highly nontrivial patterns and show transitions due to the interplay between structure and dynamics.

As for the coevolution of network and opinions, one of the first contribution is the minimal model by S. Gil and D. Zanette [84, 221]. In this model the state variables and edges change to enhance the contact between agents with the same opinion, leading to a variety of network structures. The model starts off by distributing binary opinions randomly over a completely connected network. Then, disagreeing neighbors equal opinions with probability $p_1$ (in a voter-like fashion), or keep their states and get disconnected with probability $(1-p_1)p_2$. The final state of the dynamics is determined by the parameter $q = p_1/[p_1 + (1-p_1)p_2]$, measuring the relative frequency of these two processes. When $q$ is small, the system gets divided into a couple of components with many internal links, similar sizes and opposite opinions. For large $q$ the network either remains connected and adopts a single opinion, or fragments into a large agreeing component and several poorly-connected, smaller ones. Such regimes are loosely separated by a minimum in the fraction of remaining links in the network.

In the light of real social behavior, link deletion may be substituted by a rewiring process describing how individuals look around for more fitting relationships. The model of P. Holme and M. Newman [65, 99], for example, uses link rewiring to represent the formation of new acquaintances between people of similar views, just as they influence each other due to their friendship. Starting from a random network of agents with a finite set of different opinion values, with probability $\phi$ a randomly selected node $i$ rewires one of its edges to another node with the same opinion. Otherwise $i$ leaves the network structure unchanged and adopts the opinion of the neighbor. The final state of the dynamics is then tunable by the parameter $\phi$. The limit $\phi \to 1$ leads to separate components formed by the initial holders of each opinion value, while for $\phi \to 0$ agreement is promoted inside the original components of the random network. Nicely enough, these regimes are separated by a phase transition where the distribution of component sizes is broad.

Coevolving models in the study of opinion formation and other social phenomena are numerous, usually aimed at investigating the effect of an adaptive network in generic dynamics like the voter [142, 199], Deffuant-Weisbuch [115, 191] or Axelrod [200] models. As with the previous two examples, their typical result is the existence of transitions as the relative rate of node and link updates is varied. This finally takes us to the starting point of Publication I, a generic equation describing the coupling between state variables and network structure in a coevolving model,

$$\frac{dx_i}{dt} = \frac{\partial x_i}{\partial t} + \sum_j \hat{O}(x_i, x_j, g) A_{ij}. \tag{3.2}$$

In other words, the time evolution of an opinion $x_i$ is determined by two terms: a dynamics of *transactions* $\partial x_i/\partial t$ specifying opinion change due to the existing interactions at time $t$, and a dynamics of *generations* $\sum_j \hat{O} A_{ij}$ that tells how network variations affect $x_i$. While transactions happen at a fast time scale $dt$, generations develop over a slow time scale $T$ and correspond to an operator $\hat{O}$ modifying the entries of the adjacency matrix. The interplay between structure and dynamics is then regulated by the ratio of these two time scales, $g = T/dt$.

The next chapter deals with the main results of Publications I–III, a series of specific implementations for the dynamics of transactions and generations in Eq. (3.2). Intermediate values of $g$ lead to a system that may not only be fragmented, but present a heterogeneous community structure of agents with the same opinion. In this way, the coupling between node and link dynamics turns out to be an appropriate mechanism to describe the emergence of a multitude of opinion groups in society.

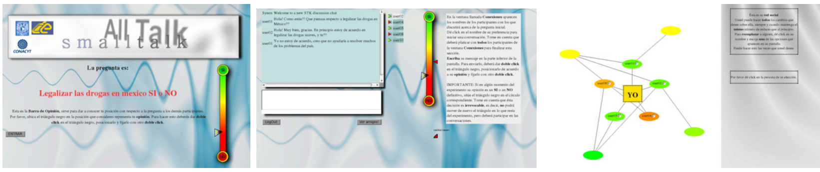
# 4. Opinions and communities

## 4.1 Opinion and community formation

As natural as it is to have opinions and share arguments with peers, it is our tendency to stop discussing with someone who clearly disagrees, choosing instead the less stubborn or more akin to our thoughts. At the same time, the flow of information from new acquaintances may help reshape our arguments and modify our views. In the terms of Section 3.2, there is a coevolution of network and opinions leading to heterogeneous community structure in the system. How do these groups of individuals sharing the same opinion develop in time? Well, it often helps to look at the tree before attempting to draw it. Equipped with the sociological insight of Section 2.1, we can prepare a controlled experiment where a small group of people may discuss freely about a given topic, as well as end conversations and start new ones with other participants. Such experiment can help us establish rules for the dynamics of individuals and network links, leading to a model for the time evolution of opinion and community formation.

### 4.1.1 A small social experiment

The warm and amiable city of Temixco in central Mexico is as good a place as any to perform our small social experiment of opinion spreading, inventively called 'SmAll Talk' [102]. In it, 20 university students with scientific background were asked to share arguments and periodically state their agreement or disagreement with a polemic statement, namely that drugs should be legalized in Mexico. The issue of illegal drug trafficking raises deep and mixed feelings in Mexican society, that with the so-called 'Drug War' [167, 168] has seen a 65% raise in homicide rates from 2005 to



**(a)** Statement **(b)** Discussion **(c)** Rewiring

**Figure 4.1. Experimental setup of SmAll Talk.** (a) Participants are presented with a statement, record their initial position with the opinion bar, and are arranged in a random network. (b) Then discussions with all first neighbors take place for a certain number of rounds, as well as possible changes in the opinion bar. (c) Finally the subjects may rewire links from first to second neighbors if dissatisfied with their neighborhood. The last two steps are repeated until most of the group acknowledge their final position on the subject.

2010 [197]. Overall, the topic of drug legalization is not only controversial enough as to keep the interest of participants in lengthy discussions, but comes from such a multifaceted problem that a simple yes/no answer is unlikely to prevail, thus preserving a symmetry between extreme opinions.

Figure 4.1 summarizes the experimental setup of SmAll Talk. Participants are first presented with the statement and asked about their initial position on the matter by means of a colored 'opinion bar' (Fig. 4.1a). In this way, the opinion of individual $i$ is measured by a continuous variable $x_i \in [-1, 1]$ where $\pm 1$ implies total agreement or disagreement. Then everyone is arranged in a random network of social contacts and allowed to discuss with all of their first neighbors for a given number of rounds (Fig. 4.1b). The subjects are also encouraged to record any change in opinion by using the opinion bar. Finally, participants are given the opportunity to simultaneously rewire their immediate neighborhood due to any dissatisfaction by exchanging links from first to second neighbors (Fig. 4.1c). This parallel rewiring scheme allows for pairs of subjects to cut the same edge and create a couple of new ones, thus increasing the average degree in the network. The discussion and rewiring steps are repeated until most individuals reach definite opinions and decide to stop the process.

For those with an eye for detail, here is a brief description on how SmAll Talk was carried out. All participants signed an informed consent form to enroll, while their anonymity was preserved by providing them with random usernames and passwords to use in the system. The experiment was undertaken on a single evening at the Centro de Investigación en Energía of UNAM in Temixco, Mexico, with volunteers from the postgraduate pro-

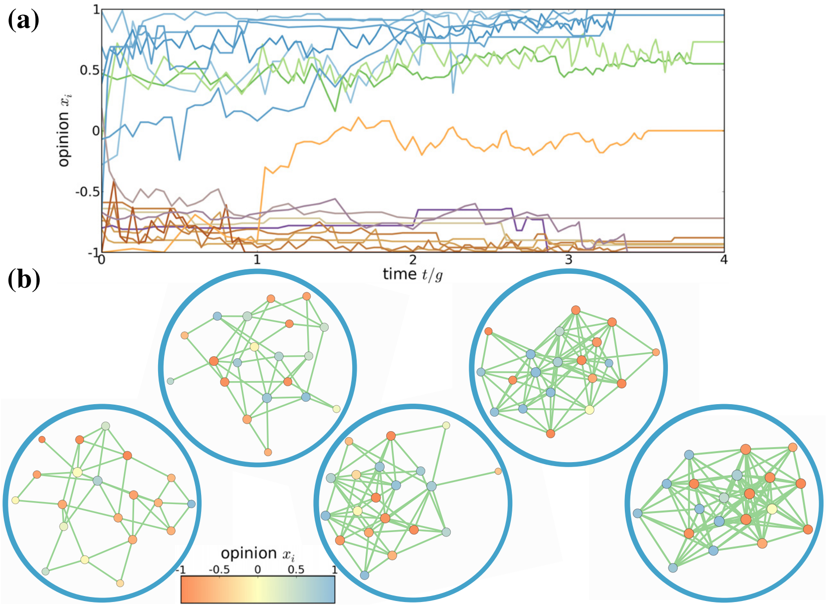



**Figure 4.2. Small experiment of opinion spreading.** (a) Time series of opinions $x_i$ as a function of the number of generations $t/g$, where most individuals eventually attain extreme positions close to $\pm 1$ and a few intermediate opinions remain. (b) Coevolution of opinions and network structure for every generation, leading to the formation of two communities with opposite positions.

gram in Energy Engineering (area of Renewable Energies). After a call for participation we selected 20 students coming from several provinces in the country, with ages 22–35, middle-class income and a 50–50 ratio of male to female. Subjects enrolled voluntarily with no incentive other than a certificate of attendance, snacks for the event and an explanation of the results at the end. The experiment was arranged in a large hall with a single computer per individual, where several coordinators prevented face-to-face chatting and helped with the on-screen instructions. Each round of discussion between neighbors consisted of a message of arbitrary length and its answer, while the rewiring step happened simultaneously for all participants. The event lasted roughly 4 hours, with brief pauses every hour to eat and rest.

The outcome of our social experiment is shown in Figure 4.2. Following Section 3.2 we see that many discussions take place before a change in the network structure occurs, so that the time scale for a typical transfer of information (transaction) is $dt$ while the time scale for network rewiring (generation) is $T = gdt$. The ratio $g$ sets then the number of discussions per rewiring and describes the separation between the fast transaction and slow generation processes in the system. The time series depicted in

Fig. 4.2a reveals how an initial distribution of opinions spread over the interval $[-1, 1]$ evolves into a state where most individuals have extreme values close to $\pm 1$ and a few intermediate positions linger. Some participants present small but erratic fluctuations, others seldom modify their opinions, and a minority even undertakes a radical change of mind by flipping the sign of $x_i$. Moreover, in Fig. 4.2b we see how the network structure coevolves with the opinion formation process. The average degree increases, undecided individuals with $x_i \sim 0$ have neighbors of both extreme positions, and the initial random topology evolves into a network with two distinct opinion groups. This segregation of communities based on individual traits is also seen in larger empirical friendship networks [140].

### 4.1.2 A coevolving opinion formation model

Armed with these generic observations, we now turn to model the coevolution of opinions and network structure by summarizing the results of Publication I. In the agent-based framework of Section 2.3, we model opinions as state variables of agents and ongoing discussions as links between them. The coupled time evolution of state variables and network links should follow Eq. (3.2), where the exchange of information through discussions in a fast time scale $dt$ is described by a differential equation for $x_i$. Then, the outcome of our social experiment prompt us to write,

$$\frac{\partial x_i}{\partial t} = f_s(\{x_j\}_s)x_i + f_l(\{x_j\}_l)\alpha_i, \tag{4.1}$$

defining the dynamics of transactions. Here, the flow of opinions towards the extreme values $\pm 1$ seen in Fig. 4.2a can be considered by a short-range interaction term $f_s(\{x_j\}_s)x_i$, so that $x_i$ grows exponentially towards $\pm 1$ with a rate determined by the opinions $\{x_j\}_s$ of a subset of 'close' agents, namely the first neighbors of $i$. Moreover, negative values of $f_s$ allow for an asymptotic approach to an opinion $|x_i| < 1$ identified with a state of indecision. The flip of opinion sign of some participants in Fig. 4.2a is also allowed by Eq. (4.1) due to the long-range interaction term $f_l(\{x_j\}_l)\alpha_i$, comprising the indirect effect of the opinions $\{x_j\}_l$ of all remaining 'far' agents in the network. This overall or public influence contained in $f_l$ is weighted by the so-called attitude $\alpha_i \in [-1, 1]$, a fixed parameter (for each agent $i$) that tells us how personality traits make an individual oppose or agree with the perceived mood of society.

We can express the interaction terms of Eq. (4.1) in a simple way by summing over opinions and weighting by distance,

$$f_s = \mathrm{sgn}(x_i) \sum_{j \in m_1(i)} x_j \qquad \text{and} \qquad f_l = \sum_{\ell=2}^{\ell_{\max}} \frac{1}{\ell} \sum_{j \in m_\ell(i)} x_j, \tag{4.2}$$

where $m_\ell(i)$ is the set of $\ell^{\text{th}}$ neighbors of agent $i$, and $\ell_{\max}$ is the shortest path length to its most distant neighbor. The sign in $f_s$ keeps a symmetry between positive and negative opinions, reasonable in the description of controversial topics such as drug legalization where no opinion is clearly the correct one. While $f_s x_i$ is an homophilic term that tends to homogenize opinion in the neighborhood of $i$, a negative attitude in $f_l \alpha_i$ promotes disparity of opinions, thus allowing for competing interactions.

In order to complete the model set by Eq. (3.2), we need rules defining how agents rewire links between them in a slow time scale $T$, namely the dynamics of generations. We choose a deterministic rewiring scheme that implements the triadic closure mechanism [114] of acquainting ourselves with the 'friends of a friend' to form triangles, as used by the participants of our social experiment. Although sociologically plausible, this local scheme may be relaxed to include a focal closure mechanism where links between any two nodes are created, as discussed in Section 5.1.

When cutting bonds, an agent $i$ preferentially breaks its link with a first neighbor $j$ if there is large disagreement. We accomplish this by selecting agents $j$ in decreasing order of the opinion difference $p_{ij} = |x_i - x_j|/2$, as long as $p_{ij} > 0$. At the same time $i$ chooses to connect with a second neighbor $l$ if the new link may help the agent in reaching an extreme opinion $\pm 1$. This means that we create the same number of new links (as cut in the first phase) with agents $l$ in decreasing order of the opinion similarity $q_{il} = |x_i + x_l|/2$, as long as $q_{il} > 0$. For each $i$ the number of deleted edges and of created links are always equal, but a parallel rewiring implies that the simultaneous actions of a couple of agents may lead to the net creation or deletion of an edge. Quite nicely, the homophilic coupling between opinions and network structure in the weights $p_{ij}$ and $q_{il}$ promotes assortative degree correlations like the ones seen in Figure 2.2, as is expected for real social networks [145].

Figure 4.3 summarizes the main result found in Publication I. Our model is let to run over a network of size $N$ with initially random links and opinions, as well as random attitudes, while the dynamics modifies state variables and topology with a fixed ratio $g = T/dt$. We freeze the opinion of decided agents in the extreme positions $\pm 1$, allowing for a stationary final state where no more changes occur in the system. In Fig. 4.3a we can see

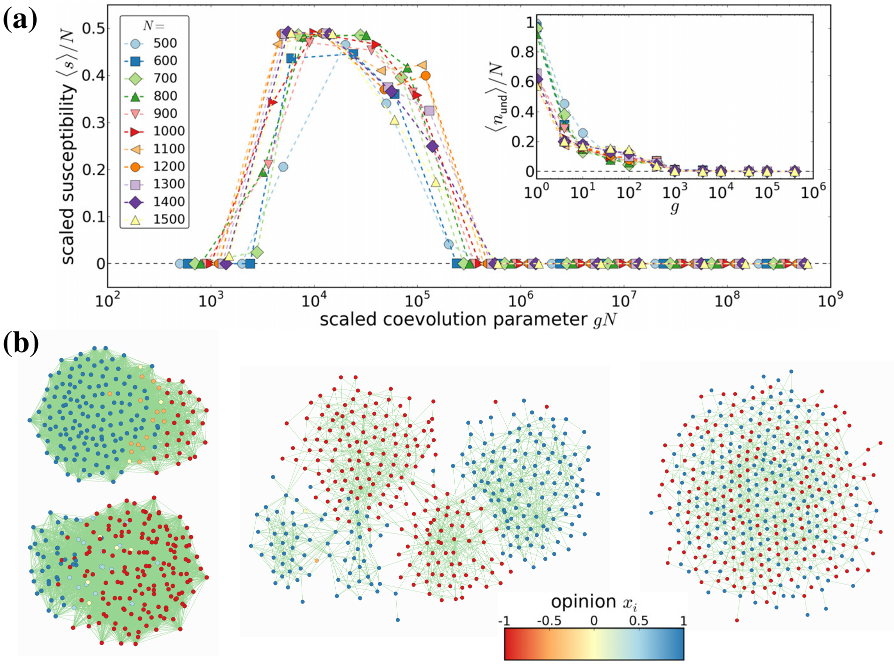



**Figure 4.3. Model of opinion and community formation.** (a) Average susceptibility $\langle s \rangle$ (main) and number of undecided agents $\langle n_{\mathrm{und}} \rangle$ (inset) as a function of the coevolution parameter $g$ for varying system size $N$, signaling fragmentation and merging transitions in the final state of the dynamics. (b) Example networks for $N = 400$ and $g = 5$, $10^3$ and $10^5$, corresponding to the two transitions and a dynamics with no rewiring.

how the coevolution parameter determines such final state in terms of the susceptibility $\langle s \rangle = \sum s^2 / \sum s$, i.e. the average size of a small component (other than the largest one) to which a randomly selected agent belongs [189], and the average number of undecided agents $\langle n_{\mathrm{und}} \rangle$ (i.e. those with $|x_i| < 1$). When $g$ increases the system undergoes a fragmentation transition, as evidenced by the increase in $\langle s \rangle$, followed by a merging transition where $\langle n_{\mathrm{und}} \rangle$ goes to zero. As shown in Fig. 4.3b, the intermediate regime corresponding to a moderate interplay between opinion dynamics and network evolution also leads to an inner structure of well-connected groups with the same opinion, discoverable either by network drawing techniques [97] or by community detection methods [118].

Publication I delves deeper into the analysis of our model by characterizing this regime with other topological properties, as well as with analytical approximations for the time evolution of $\langle n_{\mathrm{und}} \rangle$ and the functional relation between average degree and clustering coefficient in the final state of the dynamics. It is time to move on, though, so let us concentrate on the way individual attitudes shape opinion groups in our modeled society.

## 4.2 Attitudes and opinion groups

'No man is an island', J. Donne would say [59], and as such, everything we do takes and reflects on what those around us think. Social psychology follows the lead with its social impact theory [120, 152], describing the influence on individual thoughts exerted by the presence of others. Our model encloses this effect in a minimal way via the long-range interaction term $f_l \alpha_i$, that together with one-to-one discussions determines the time evolution of opinions between generations. As opposed to the homophilic effect assumed for direct discussions, a negative attitude $\alpha_i$ can enhance opinion differences among agents. Perhaps another way of explaining this is through the anecdotal comment of a participant in SmAll Talk, whom after being inquired about his continual attempts at interacting with people of the opposite opinion, chose to answer: "it's not about me being right, but about showing them that they are wrong". Some may prefer strife over agreement, and their presence will undoubtedly play a role in the formation of social groups. How does attitude determine the development of communities in our model?

In Publication II we tackle this question by analyzing the link between attitude and group size. A straightforward way of doing so is first to identify communities in a stationary final state corresponding to the intermediate $g$ regime (like the center network in Fig. 4.3b), and then to calculate the distribution of attitude values in each opinion group. In Fig. 4.4a we plot such agent number distribution $N_c$ as a function of $\alpha_i$ for several relative group sizes $c/N$. Quite clearly the small community is composed of agents with $\alpha_i < 0$, while the medium-sized group has attitudes of both signs and the large one has mostly agents with $\alpha_i > 0$. In other words, a negative attitude parameter drives the formation of small groups of individuals with the same attitude and opinion sign, segregated from those who feel comfortable in the majority.

Let us go further and consider the effect of the average attitude $\langle \alpha \rangle$ on the number of communities in the network. The results in Figure 4.3 and Fig. 4.4a correspond to attitudes uniformly distributed in $[-1, 1]$, so we can move this interval around to test the effect of varying $\langle \alpha \rangle$ on the final state of the dynamics. Indeed, Fig. 4.4b shows that when all attitudes are positive ($\langle \alpha \rangle \gg 0$) there is only one community with full consensus, but as $\langle \alpha \rangle$ decreases the number of groups $n_\alpha$ gets maximized. For $\langle \alpha \rangle \ll 0$ such heterogeneous structure is lost due to the merging of most groups. The

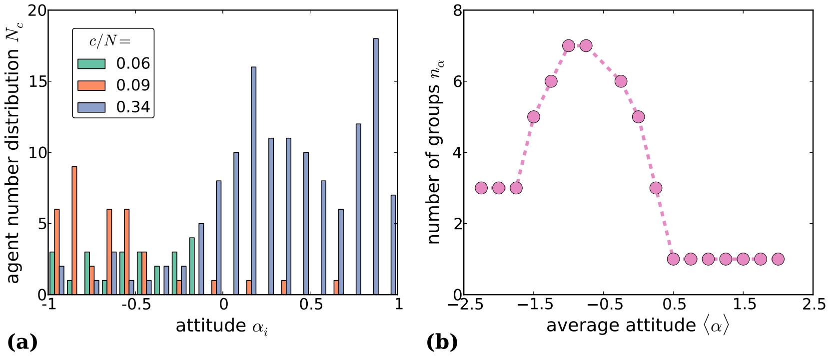



**Figure 4.4. Attitudes and opinion groups.** (a) Histogram of the agent number distribution $N_c$ as a function of $\alpha_i$ for varying relative group size $c/N$, with $N = 400$ and $g = 10^3$. Small groups have only negative attitudes, while big ones are mostly formed by agents with $\alpha_i > 0$. (b) Number of opinion groups $n_\alpha$ as a function of the average attitude $\langle\alpha\rangle$, showing a maximal segregation of communities for $\langle\alpha\rangle < 0$.

existence of an optimal attitude value can be intuitively understood as frustration in the system: for decreasing $\langle\alpha\rangle$ agents tend to form smaller and smaller communities, until at some point numerous groups of the same opinion are close by and detected as a single one. It is worth noting that for $\langle\alpha\rangle \ll 0$ the visualized network tool [97] lets us distinguish only two groups of opposite opinion, while the community detection algorithm [118] finds three instead. Overall, it is clear that a measured portion of agents with opposing attitude towards the mood of society is necessary to break communities apart and enhance structural heterogeneity in the network.

This link between attitude and group size can be further corroborated by averaging over an ensemble of initial conditions, and by verifying the robustness of the identified communities against several detection algorithms. Yet another way is to devise a simplified version of the coevolving model of Section 4.1.2, one where the realism of complex topologies is traded for an unambiguous definition of opinion group, while still preserving some of the same macroscopic properties of its interplay between opinion and network structure. The latter is indeed the focus of Publication III, as well as the next section.

### 4.2.1 A simplified model of coevolution

Picture for a moment that fancy dinner where everyone was seated around a big wide table, and you just couldn't find a topic of conversation with your neighbors. Would it have been better to choose the other side of the

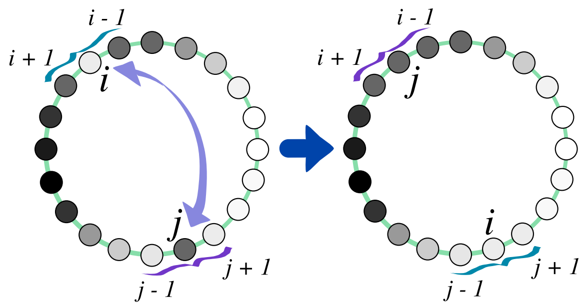



**Figure 4.5. A simplified model of coevolution.** Diagram of generation dynamics through location exchange in the chain model of Publication III. A pair of agents $i$ and $j$ may swap places if opinion differences in their original neighborhoods (left) are larger than in the alternative configuration (right), thus increasing opinion homogeneity in the system.

table? Publication III deals with an opinion formation model of agents in a closed chain, where the time evolution of opinions is coupled to location exchanges between agents. The dynamics of transactions follows Eq. (4.1) as before, but the interaction terms of Eq. (4.2) deal with opinion averages over the sets of close and far agents, that is, $f_s = \mathrm{sgn}(x_i)\langle x \rangle_s$ and $f_l = \langle x \rangle_l$.

The generation dynamics is pictured in Figure 4.5. Instead of rewiring, a pair of undecided agents may exchange places if opinion differences in the original neighborhoods $\{i, i \pm 1\}$ and $\{j, j \pm 1\}$ are larger than in the proposed configurations $\{j, i \pm 1\}$ and $\{i, j \pm 1\}$. Therefore agents move around to find a better discussing environment, in a manner reminiscent of T. Schelling's model for residential segregation [172, 173]. Moreover, the simplified ring topology permits us to define boundaries between communities where a flip of opinion sign takes place.

Despite the differences, this model exhibits a transition in the number of undecided agents $\langle n_{\mathrm{und}} \rangle$ akin to the inset in Fig. 4.3a. For $g$ larger than a critical value $g_c$, all agents reach the extreme values $\pm 1$ and form large communities with the same opinion. Yet below $g_c$ homophily gives way to frustration, as an increasing number of undecided agents with $\alpha_i < 0$ get exchanged perpetually at the borders of decided opinion groups. Publication III includes an extensive analytical treatment of the model, with approximations for $\langle n_{\mathrm{und}} \rangle$, $g_c$ and the time evolution of the average absolute opinion in the network.

Finally, we can confirm the scenario of Fig. 4.4a by calculating the attitude distribution $N_c$ over a large ensemble of realizations of the dynamics. When $g > g_c$ all distributions have roughly a Gaussian shape centered at $\langle \alpha \rangle = 0$ regardless of $c$, implying no correlation between attitude and group size. However, with $g < g_c$ the distributions for small and large $c$ peak below and above $\langle \alpha \rangle$, respectively, while for intermediate sizes the shape of $N_c$ is bimodal.

All in all, the research in Publications I–III allows us to conclude that a heterogeneous community structure in society may arise due to the interplay between opinion dynamics and network evolution in well-separated time scales, and to the frustration arising from a diversity of personal attitudes. The situation we describe is somehow ideal though, as the setup of SmAll Talk implies, since individuals interact in an isolated setting and suffer no consequences from breaking connections. So let us move on and consider more realistic scenarios, such as the presence of external information and the effect of disagreement in cooperation.

# 5. Opinions and conflict

## 5.1 Social response to scientific facts

Modern times are all about information. Let it be television, radio, newspapers, or the web and its myriad of social networks, our daily lives are immersed in an endless flow of data product of a society deeply intertwined with modern science and various emerging technologies. Opinions are subject not only to discussions and personality traits, but to the far-reaching effect of media. What is the social response to information proclaimed as fact, such as scientific knowledge? The public perception of science and its relationship with people's opinions is in itself a topic of debate, with descriptions in terms of scientific literacy [103, 135] or cultural cognition [110]. Empirical evidence tends to favor the latter [4, 151], where the perception of a fact is mainly influenced by moral values, beliefs and cultural traits shared with others rather than by technical understanding. Indeed, in polemic issues such as climate change the science literate are often the most culturally polarized [111], actively dismissing facts due to their values.

The culprits are our friends. Accepting a piece of scientific evidence has no social value on its own, but the risk of disagreeing and be shunned by peers may be huge [43]. Since this group influence depends on the structure of the underlying social network, in Publication IV we describe the effect of external information on opinion formation within the coevolutionary framework of Section 3.2. There we take $x_i$ as the opinion of agent $i$ regarding the validity of a given scientific fact, such that total agreement ($x_i = 1$) would correspond to the correct position of accepting it as true. The constant flow of data coming from the media is described by a new parameter $h$, where its magnitude is proportional to the amount of information and its sign tells us if media is promoting the fact or not. The symmetry between extreme positions $\pm 1$ in the model of Section 4.1.2 is then broken by the effect of the associated external field for each agent, $h_i = h_i(x_i; h)$, constituting a drive towards the scientific truth ($h_i > 0$) or away from it ($h_i < 0$).

We can take into account these factors in a minimal way by writing the dynamics of transactions as,

$$\frac{\partial x_i}{\partial t} = f_s x_i + f_l \alpha_i + h(1 - x_i), \tag{5.1}$$

where the long- and short-range interaction terms are given by Eq. (4.2) as before. Those fully accepting the scientific fact will disregard the media, but the rest may shape their views on its validity through an interplay of opinions, attitudes and external information. Finally, group influence is enhanced by the same generation dynamics of Section 4.1.2, describing a process of opinion homogenization in social communities by means of link rewiring.

The main features of the final state of the dynamics are shown in Figure 5.1, where we explore the effects of a varying field strength $h$ in a typical network configuration for the intermediate $g$ regime. As the media increases its opposition to the scientific fact through a negative field with growing magnitude, more agents comply with $x_i = -1$ and lead to an asymptotic consensus of disagreement with the fact in the limit $h \to -\infty$. There are no undecided individuals to be found, and the few agreeing agents get scattered over a random topology. If the media turns supportive with $h > 0$, however, the situation changes drastically. Small disagreeing groups linger for considerably large values of $h$, while undecided agents slow down the approach to positive consensus. In terms of swaying individuals into its position, opposing media ends up being more effective than a supportive one [50].

Publication IV contains an extensive analysis of this effect by using ensemble averages and analytical approximations. There we explain the asymmetry of Figure 5.1 with an approximate solution to Eq. (5.1), $x_i(t) = [x_i(0) - x^*]e^{\lambda t} + x^*$, where the fixed point $x^* = -(h + \alpha_i f_l)/\lambda$ and its associated eigenvalue $\lambda = f_s - h$ have explicit expressions for their average values. This solution implies the existence of a critical field strength $h_0 > 0$ where the eigenvalue changes sign, separating regimes where the fixed point is either repulsive or attractive. Additionally, we extend the generation dynamics of Section 4.1.2 by considering a focal closure mechanism

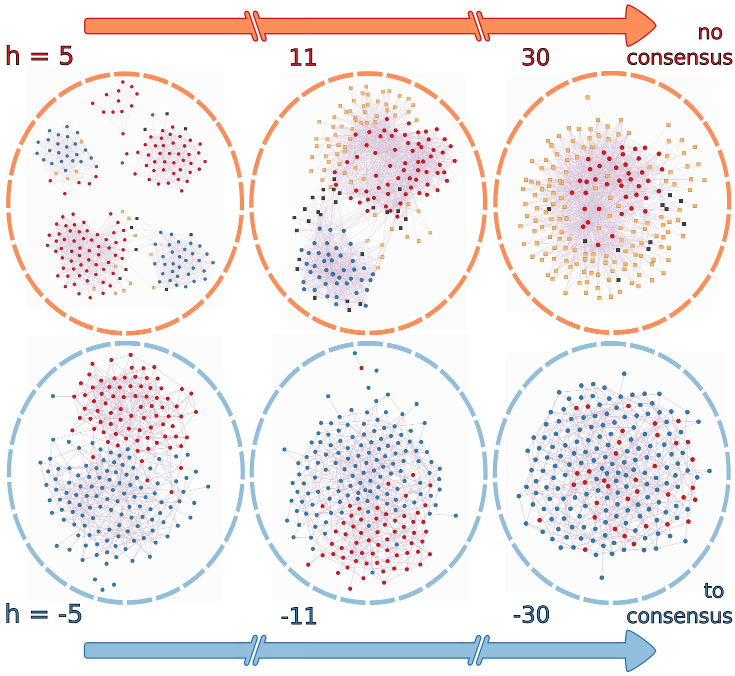



**Figure 5.1. Social response to external information.** Effect of the external field strength $h$ on the final state of the dynamics with $N = 200$ and $g = 10^3$. Decided agents are drawn as red ($x_i = 1$) or blue ($x_i = -1$) circles, and undecided individuals as yellow ($0 < x_i < 1$) or black ($-1 < x_i < 0$) squares. The social response to a fact represented by the position $+1$ is characterized by a critical field value $h_0 > 0$, above which external information fails to sway small disagreeing groups and undecided agents into accepting the fact.

[114], where agents are allowed to rewire links with anyone in the network (not only second neighbors) due to opinion similarity. The presence of this mechanism results in a loss of heterogeneous network structure, less undecided agents and an increase in the critical value $h_0$.

At this stage, an ideal step would be to gather large-scale data concerning temporal features of the public perception of science and compare it with the previous model. Unfortunately the methodologies used in most scientific perception surveys have changed continually, shifting their focus from literacy to the link between science and society [18], thus making the integration of data in different time snapshots quite difficult but for isolated cases [170]. Moreover, polls at the country level do not usually record details about social interactions, leaving a direct measurement of the coupling between opinions and network structure to small controlled experiments like the one in Section 4.1.1.

An alternative and less ambitious approach is considered in Publication IV, where we use a couple of surveys from the European Union [68] and

Mexico [46] to adjust the field strength in our model and quantify differences between the two populations. We first select 15 equivalent statements from both surveys, classify them as facts or fallacies (according to the subjective judgment of the authors), and give them $h$ values that are compatible with the level of agreement found in each population. Then we focus, for example, on the set of fallacies with $h > 0$ that correspond to an unfavorable perception of science at the matters at hand. It turns out that the European poll has a larger set, yet with smaller assigned $h$ values, implying a Mexican population with particular yet pronounced scientific misconceptions. A detailed account on the process of adjusting field strength with survey data (as well as the classification of statements as facts or fallacies) can be found in Publication IV.

The public understanding of science is a multifaceted phenomenon dealing with the susceptibility of social groups to scientific and technological notions in the presence of the same human cognitive abilities, different cultural traits, and the politics of a globalized world, where opinions develop under the effect of ever-changing external information. We now turn to yet another situation in which individual opinions are forced to clash against the common product of a larger group, that of the tasks achieved by cooperation.

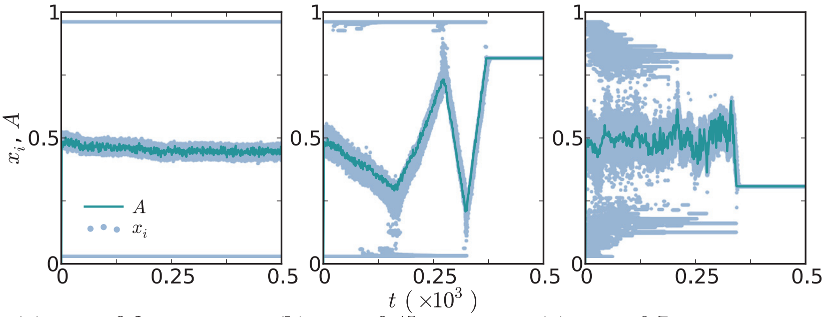
## 5.2 Conflict in collaborative dynamics

'Two heads are better than one', or so the old proverb goes. Indeed, the efforts of many usually outweigh those of a single individual when it comes to efficiency and the ability to solve complex tasks, leading to higher levels of organization and social structure. Cooperation is no less fascinating in its origin and consequences, arising despite the competition of natural selection [8, 153] and often ending up in strife due to disagreements of any kind [174]. The potential for conflict among cooperating individuals is commonplace in insect species [166] and in groups of primates [55, 70], the latter usually managed through policing and negotiation. Humans are the masters of the trade, so to speak, with a gregarious nature that has taken us from hunter-gatherer groups to societies entwined at the global level [30, 132, 164], and a taste for conflict all the way from personal struggles to all-out war. Let them be partnerships [137], teamwork in operating rooms [169], open source software development [122] or public policy making [162], collaborative endeavors are prone to differences in attitudes, approaches and emphases, that is, opinions. How do such conflicts of opinion emerge and get resolved?

Following the approach described throughout this Thesis, in Publication V we take a particular collaborative environment and analyze its generic dynamics with the agent-based modeling techniques of Section 2.3. In the era of Internet and fast remote communications, the access to data regarding cooperation between large groups of people is more feasible than ever. Our chosen example is Wikipedia, a free, web-based encyclopedia where volunteering individuals jointly write articles about any topic imaginable, with all records of edits and discussions open to the public [216]. Although the writing process is usually peaceful and constructive, some controversial topics prompt users to disagree profoundly about the contents of the articles. The ensuing 'edit wars', silly as they might seem [214], result in a complex interplay between disparate opinions and a common product, where editors continually override each other's contributions instead of building a consensual article.

We can model this situation in a minimal way by considering the coevolving dynamics between a set of $N$ continuous opinions $x_i \in [0,1]$ and a single article value $A \in [0,1]$, representing the views of agents on a given topic covered by the article and the particular position written on it. Editors on the real Wikipedia can propose changes in an open forum or 'talk page' [176], yet instead of constructive discussions, most comments are simply appraised by similarly-minded individuals or ignored by the rest. Then, it seems pertinent to describe this clash of minds by the bounded-confidence dynamics of Eq. (3.1), otherwise known as the Deffuant-Weisbuch model, where only pairs of opinions differing less than a given tolerance $\epsilon_T \in [0,1]$ can get even closer by the relative amount $\mu_T \in [0,1/2]$. Additionally editors may modify the article when dissatisfied by it, effectively coupling the opinion and common product dynamics in a second bounded-confidence process with tolerance and convergence $\epsilon_A, \mu_A \in [0,1]$. In other words, if $|x_i - A| > \epsilon_A$ agent $i$ edits the article to its liking ($A \mapsto A + \mu_A[x_i - A]$), else it agrees and adopts the current state of the product ($x_i \mapsto x_i + \mu_A[A - x_i]$).

Figure 5.2 summarizes one of the main results of Publication V, regarding the time evolution of opinions and article state when $\mu_A$ is varied. In the simplest yet non-trivial scenario in parameter space $(\epsilon_T, \mu_T, \epsilon_A)$, the initial stages of the dynamics are characterized by one large mainstream group with opinions $x_i \sim 1/2$ and two small extremist groups near the

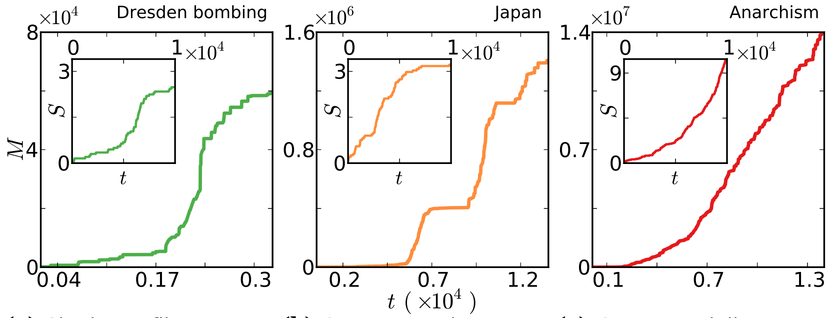



**(a)** $\mu_A$ =0.2 **(b)** $\mu_A$ =0.45 **(c)** $\mu_A$ =0.7

**Figure 5.2. Symmetry breaking in collaborative dynamics.** Time evolution of opinions $x_i$ and article $A$ for different values of the convergence $\mu_A$. (a) For low $\mu_A$ groups are stable and the article follows the majority in an intermediate position. (b) As the convergence increases $A$ begins to oscillate between extremes and reaches consensus in a slanted opinion. (c) For large enough $\mu_A$ the extremists spread out and converge to the mainstream group.

borders of the interval $[0, 1]$. For low convergence $\mu_A$ the article remains close to a stable mainstream opinion, meaning that the small changes made by dissident agents are not enough to overcome the majority's view on the topic. As $\mu_A$ increases, however, $A$ begins to oscillate between extreme opinions and ends up in a consensus state different from $1/2$. In more technical terms, the system undergoes a symmetry-breaking transition due to a local bifurcation in the speed of the mainstream group, causing the entirety of editors to agree on an article expressing a slanted view on the subject. Larger convergence undermines this effect, causing the extremist groups to spread out and converge to the majority.

These regimes can also be discerned with the relaxation time $\tau$ of the dynamics. As shown in Publication V both numerically and with analytical arguments, a system of finite size $N$ always reaches a state of consensus where all opinions and article share the same value. Yet in the regime of Fig. 5.2a the relaxation time is very large, while for Fig. 5.2b it reduces drastically to a quantity independent of $N$. Somewhat curiously, the open conflict between groups (signaled by a persistent interaction through the article) actually accelerates the convergence to consensus. Is it then possible to describe a conflictual scenario where consensus is only temporary or nonexistent at all? We now turn to this question by extending our model and comparing it with real data from Wikipedia.




**(a)** Single conflict **(b)** Consensus plateaus **(c)** Controversiality

**Figure 5.3. Modeling controversy in Wikipedia.** Empirical controversiality measure $M$ as a function of the number of edits $t$ for three different conflict regimes in Wikipedia. As more and more editors arrive to change an article, the resolution of a single conflict (a) can be replaced by cycles of strife and compromise (b) or by pure, uninterrupted controversiality (c). Insets show the theoretical analogue $S$ for $Np_{\text{new}} = 4$ and $\epsilon_A = 0.47, 0.46, 0.44$, respectively, where the transition between regimes is modulated by the time scales of relaxation and renewal.

### 5.2.1 Controversy in Wikipedia

The online project Wikipedia has been subject to intensive research on recent times, mainly due to its large scale, data availability and the diversity of social phenomena it encompasses. The focus of research has changed as well, moving slowly from the topological analysis of growing networks of articles and links between them [35, 224] to the study of temporal features such as online content popularity [165] and circadian patterns of editorial activity [219]. As for the detection of conflicts in the collaborative creation of articles, the selection of appropriate metrics is far from trivial, given the diversity of features that correlate with an article's controversiality. A suitable option relies on counting mutual reverts by pairs of editors [220], that is, situations where each individual chooses to delete an edit made by the other. The associated controversiality measure $M$ disregards non-conflictive scenarios (like vandalism and mistakes due to inexperience) by considering the total number of edits of each reverting individual and the amount of editors involved in the conflict [218].

Figure 5.3 shows the temporal evolution of $M$ for three different conflict regimes found in Wikipedia, with time $t$ measured in number of edits for a given article. Although the definition of the controversiality measure makes it a monotonically increasing quantity, the distribution of mutual reverts in time varies greatly. In the scenario of Fig. 5.3a we find topics like the bombing of Dresden in World War II, where the rise and resolution of a single conflict is signaled by a smooth increase to a constant value. The arrival of new editors dissatisfied with the current state of the article may lead subjects such as Japan to the intermediate regime of Fig. 5.3b, characterized by stages of conflict and plateaus of consensus. Finally, the popularity of the Anarchism article makes it fall into the extreme case of Fig. 5.3c, a scenario of never-ending controversiality where mutual reverts happen all the time.

In the second part of Publication V we extend our bounded-confidence model to account for a nonzero flux of agents and capture these conflict regimes. In order to keep $N$ fixed and simplify the treatment of the dynamics, we introduce a new parameter $p_{\mathrm{new}}$ as the probability for an old editor to be replaced by a new one with random opinion. Then, the system never reaches a state of permanent consensus and we can measure conflict by following the time evolution of the sum of absolute changes in the article, $S(t) = \sum_{t'=1}^{t} \sum_{i=1}^{N} |A(i) - A(i-1)|$. Indeed, $S$ turns out to be a qualitative analogue of the empirical measure $M$, as inferred from the inset curves in Figure 5.3. The interplay between the time scales of relaxation to consensus and agent renewal gives rise to the peaceful and warring scenarios of Fig. 5.3a and c, separated by a narrow regime in parameter space where the time scales are similar ($N p_{\mathrm{new}} \tau \sim 1$) and the density of consensus plateaus gets maximized.

It is often surprising and soothing to find models that, simple as they may be, are still capable of emulating properties of complex phenomena such as conflict resolution, where the opinions of many clash together in an environment full of perspectives with the oft impossible goal of agreeing on a task. Instead of losing ourselves in the myriad of differences between individuals, we can concentrate on a few mechanisms and describe the global behavior of the system in terms of physical concepts, like interactions and time scales. Yet reality is never that easy; a fitting model does not disprove that other mechanisms are at play, and the qualitative information gained might not be enough to allow prediction or control over the system. To conclude with the overview in this Thesis, we now continue with a brief outlook on the statistical physics approach in social dynamics.

# 6. Final remarks

We have all felt the sway of our opinions and the pull of friends, the way our gregarious nature leads to circles of supporting relationships and yet profound strife. The work summarized in this Thesis provides mathematical descriptions for the evolution of opinions in society, based on simple mechanisms of individual conduct and group influence. Such models abstract the inherent complexity of human behavior by reducing people to opinion variables spread over a network of social interactions, with variables and interactions changing in time at the pace of a handful of equations. While the rules of the models are motivated by sociological studies or small controlled experiments, their behavior at the system level can be analyzed with statistical physics tools and compared with available data on large-scale social phenomena.

From the extensive analysis of these models, a couple of generic conclusions are in order. First, the emergence of groups of agreeing individuals in society may be regulated by well-separated time scales of opinion dynamics and network evolution, and by a distribution of personality traits in the population. Our social environment can then be fragmented as more people turn against the collective mood, ultimately forming minorities as a response to external influence. Second, the exchange of views in tasks achieved by cooperation may lead not only to the rise and resolution of opinion issues, but to an intermediate state where conflicts get solved just to appear again and again. This aspect of human behavior, seen in collaborative websites like Wikipedia, can be emulated by surprisingly simple interactions among individuals.

The results presented here are but a piece in the intricate puzzle that is the understanding of the interplay between structure and dynamics in society, a recent collective effort known as computational social science [47]. The field aims at characterizing temporal human behavior in different scales through interdisciplinary approaches, and as grand and difficult as that sounds, a series of steps to follow seems clear enough. We should first ask socially relevant questions, beyond simple curiosity or common sense, allowing for proper descriptions at the individual level. Alternative models can then be statistically treated, compared and validated with empirical studies, like the development of groups in large social networks [155]. The final and most challenging step is a successful data-driven modeling of social phenomena, where real data analysis and theoretical simulations may be run in unison to give quantitative predictions and control over the system.

Research studies in the statistical physics of opinion and social conflict, including this Thesis, fulfill such requirements just to some extent. Apart from foreseeable improvements like quantitative comparisons with current or future data sets, a basic difficulty lies in the collaboration between social and physical scientists, who tend to regard the other field as little more than descriptive [27]. While the physics approach revolves around simple models and the search for universal properties across many systems, the social sciences emphasize variations among individuals and interactions. Better representations of reality probably lie somewhere in between, and these disciplines should try to share more than just a common interest in social behavior.

An ambitious step in this direction would be a stringent use of the scientific method, beyond the conceptual exercises that abound in the study of opinion formation. The systematic gathering of data through controlled online social experiments, tied to a variety of modeling tools, could allow us to measure the effect of different mechanisms and replicate results [93, 108]. If so, we will be in the position to formulate and reject hypotheses like those underlying the agent-based formalism, or even predict and control the response of a social group to carefully designed perturbations. Perhaps then we will cease, as W. Hazlitt once said, to be the slaves of our own opinions.

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