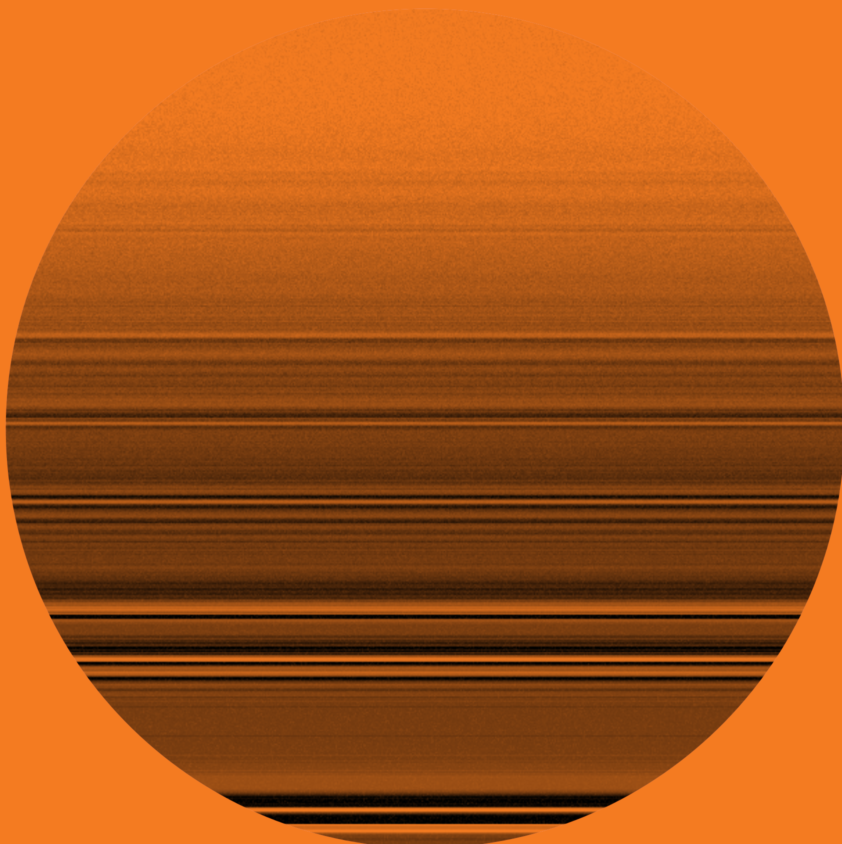
Department of Signal Processing and Acoustics

Dispersive Systems in Musical Audio Signal Processing

Julian D. Parker



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Dispersive Systems in Musical Audio Signal Processing

Julian D. Parker

A doctoral dissertation completed for the degree of Doctor of Science (Technology) (Doctor of Philosophy) to be defended, with the permission of the Aalto University School of Electrical Engineering, at a public examination held at the lecture hall S1 of the school on 25 October 2013 at 12:00

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Abstract

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Abstract

Dispersion is a property seen throughout both nature and the man-made world. By its most basic definition, it simply refers to the spreading out or scattering of some form of wave phenomenon in a medium. In practice, this is usually due to variation in the propagation speed of the wave with respect to frequency or amplitude. Examples of dispersion in the natural world are rainbows, the spreading of water surface waves, and the distinctive sound generated when striking a long metal wire.

This thesis explores the presence of dispersion in signal processing systems designed for musical use, and develops a number of new musical digital signal processing systems which are based around the action of dispersion. The primary focus of the thesis is spring reverberation, an early form of artificial reverberation based on exciting vibrations in helical metal springs. Springs are unique in the world of analog musical technology, in that they derive almost all of their recognized sonic character and desirability from the dispersion they induce.

The first portion of this thesis examines the behavior of spring reverberators through the lens of mathematical models of spring vibration, and develops some important new results about their behavior. These mathematical models are turned directly into a digital model, via the application of finite difference methods. A similar system, that of the larger 'Slinky' spring, is examined and modeled via the same framework.

The second part of this thesis examines an alternative method for emulating spring reverberation, based heavily on the use of the dispersive allpass filter. The result is an efficient and high-quality parametric emulation of the spring reverberator. This model is further extended in several novel ways to improve its efficiency, including via application of multi-rate techniques. The reverberator is further improved by the proposal of a new structure that uses sparse-noise convolution to generate diffusion in the repeating echoes.

Finally, a new algorithm is developed that uses the dispersion of golden-ratio allpass filters as a method for transparently reducing the peak amplitude of musical signals so that their loudness can be maximized.

The novel algorithms developed in this thesis are intended for use in real-time music production or processing applications. Hence, computational efficiency and parametric control are central considerations. These algorithms could be implemented for use in a computer environment, for mobile devices, or for embedded DSP systems.

Keywords Acoustics, music, digital signal processing, digital filters

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Preface

Physics and electronic music have long been passions of mine. This thesis represents some kind of attempt to produce tools for the latter via both the approaches and the insight given by the former. My formal path on the road to this thesis started almost seven years ago at my interview for admission to the MSc. Degree in Acoustics and Music Technology at the University of Edinburgh, where I was asked whether I had any particular topics in mind for the thesis component of the Master's degree. In that moment, it suddenly occurred to me that the behavior of spring reverberators was poorly understood, yet the sound of this piece of 'obsolete' technology was still very much desired by music producers. I expressed that I'd like to investigate this path, and to my great fortune the same topic has managed to carry me through a complete Master's thesis as well as the majority of this Doctoral thesis.

I am greatly indebted to my supervisor and instructor Prof. Vesa Välimäki whose passion, enthusiasm, depth of knowledge and humor have made this thesis possible, as well as a pleasurable experience to complete. I'm deeply thankful for his support, guidance and collaboration. I'd also like to thank Dr. Stefan Bilbao for guiding me down the first steps of this path years ago at Edinburgh, as well as being a continuing source of collaboration, insight, and fun nights out.

I'm very thankful to the co-authors of the papers included in this thesis, Dr. Jonathan S. Abel, Dr. Henri Penttinen, Prof. Lauri Savioja and Prof. Julius O. Smith - your contributions to these works have been invaluable. I'm also greatly appreciative of the efforts of the pre-examiners of this work, Prof. Antoine Chaigne & Dr Maarten van Walstijn - your comments and suggestions were insightful and extremely useful.

I'd like to thank the Academy of Finland (projects no. 126310, 128689 and 122815), GETA (especially Marja Leppäharju & Ari Sihvola) and the

Aalto Department of Signal Processing and Acoustics for their financial support during the time taken to complete this work, as well as the support needed to attend the many international conferences I've been lucky enough to participate in.

I'm very grateful to the DAFx community, not only for providing a great yearly forum to share research, but also for helping make working in this field both fun and inspiring.

To my colleagues at the acoustics lab - Antti, Archontis, Bo, Cumhur, Dimitri, Hannu, Heidi-Maria, Henna, Jari, Javier, Jouni, Jussi P, Jussi R, Jyri, Lauri, Leonardo, Magge, Marko, Mikko-Ville, Okko, Olli S, Olli R, Rafael, Rémi, Sami, Sofoklis, Stefano, Symeon, Tapani, Teemu, Tuomo, Ville and others (I'm sure to have forgotten someone) - thanks for being part of such a great working environment. I'm sorry for torturing you with noise music at Pikkujoulu that time. To my colleagues during my recent work at the Media Lab - Callum, Lisa, Till - thanks for making the last year and a half much more enjoyable than it would have been without you. I'd also like to thank all my other friends in Helsinki who haven't yet been mentioned. I can't name you all here for space reasons, but without you my time in Helsinki would have been much poorer.

I am deeply grateful to my parents David and Denise Parker for all their support, both financial and emotional, over the years. Dad - I probably wouldn't have ended up here without the reel-to-reel tape player, the ZX Spectrum, and the lectures about Fourier transforms you gave me when I was 10 (or was it younger?). Lastly, I'd like to thank my wonderful girlfriend Sini - without you looking after me when I broke my leg halfway through this thesis, it would have been immeasurably harder to finish.

Helsinki, October 3, 2013,

Julian Parker

Contents

List of Publications

This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.

I J. Parker and S. Bilbao. Spring Reverberation: A Physical Perspective. In *Proc. 12th Int. Conf. on Digital Audio Effects (DAFx-09)*, pp. 416–421, Como, Italy, Sept. 2009.

II S. Bilbao and J. Parker. A Virtual Model of Spring Reverberation. *IEEE Trans. on Audio, Speech and Language Processing*, Vol. 18, No. 4, pp. 799–808, May 2010.

III J. Parker, H. Penttinen, S. Bilbao and J. S. Abel. Modeling Methods for the Highly Dispersive Slinky Spring: A Novel Musical Toy. In *Proc. of the 13th Int. Conf. on Digital Audio Effects (DAFx-10)*, pp. 123–126, Graz, Austria, Sept. 2010.

IV V. Välimäki, J. Parker and J. S. Abel. Parametric Spring Reverberation Effect. *J. Audio Eng. Soc.*, Vol. 58, No. 7/8, pp. 547–562, July/Aug. 2010.

V J. Parker. Efficient Dispersion Generation Structures for Spring Reverb Emulation. *EURASIP Journal on Advances in Signal Processing*, 8 pages, Vol. 2011, Article ID 646134, Feb. 2011.

VI V. Välimäki, J. Parker, L. Savioja, J. O. Smith and J. S. Abel. Fifty Years of Artificial Reverberation. *IEEE Trans. on Audio, Speech and Language Processing*, Vol. 20, No. 5, pp. 1421–1448, July 2012.

VII J. Parker and V. Välimäki. Linear Dynamic Range Reduction of Musical Audio using an Allpass Filter Chain. *IEEE Signal Processing Letters*, Vol. 20, No. 7, pp. 669–672, July 2013.

Author's Contribution

Publication I: "Spring Reverberation: A Physical Perspective"

The author produced and presented the entire work, with the exception of the finite difference simulation results, which were provided by the second author.

Publication II: "A Virtual Model of Spring Reverberation"

The author conducted the initial investigation of the problem and literature search, and also developed the original explicit finite difference model from which the implicit model presented in the paper was extended. The measurements of real spring reverberation units were conducted by the author. The author also co-wrote Sec. I, Sec. IV and Sec. V and edited the work.

Publication III: "Modeling Methods for the Highly Dispersive Slinky Spring: A Novel Musical Toy"

The author wrote the entirety of the paper, with the exception of Sec. 2 and Sec. 3.1. The author produced the finite difference model, and implemented the software version of the waveguide model. The author also conceptualized the applications of the model described in Sec. 4 and implemented them in software. The author presented the paper at the conference.

Publication IV: "Parametric Spring Reverberation Effect"

The initial conception of the paper was due to the first and third authors. The author collaborated on developing this initial idea in Sec. 1, Sec. 2.1 and Sec. 2.2, and co-authored these sections. The content in Sec. 2.3 – 2.6 was conceived and authored by the author. The author implemented the complete model, calibrated it to measured responses and produced the results presented in Sec. 3. The author also wrote Sec. 3 and provided the measurements of the Leem spring reverberation unit. Sec. 4 was collaboratively written by all the authors.

Publication V: "Efficient Dispersion Generation Structures for Spring Reverb Emulation"

The author conceptualised, implemented and wrote the entirety of the paper.

Publication VI: "Fifty Years of Artificial Reverberation"

The author was responsible for coordinating and editing the work, along with the first author. The author authored Sec. II.A and Sec. VI, and co-authored Sec. I, Sec. II.B and Sec. VII. The author conceived and implemented the model described in Sec. VI.A.2.

Publication VII: "Linear Dynamic Range Reduction of Musical Audio using an Allpass Filter Chain"

The initial idea behind the paper was conceived by the author. The author then conceptualized the details of the technique with the co-author. The author implemented and tested the technique, and wrote the entirety of the paper.

List of Symbols

a_i	Allpass filter coefficients
$A_N(z)$	Allpass filter transfer function of order N
c	Speed of light, in metres per second
D	Delay-line length in samples
f	Frequency, in Hz
F_C	Spring transition frequency, in Hz
$F(\omega)$	Filter frequency response (complex valued function)
g	First-order allpass coefficients
$G(\omega)$	Filter magnitude response
$H(z)$	Filter transfer function
k	Spatial frequency (wavenumber), in radians per metre
M	Filter chain length
N	Filter order
T_D	Spring time delay at DC, in seconds
κ	Stiffness, in newtons per metre
λ	Wavelength, in metres
ν	Wave speed, in meters per second
ν_g	Group velocity, in meters per second
ν_ϕ	Phase velocity, in meters per second
τ_g	Group delay, in meters per second
τ_ϕ	Phase delay, in meters per second
$\Phi(\omega)$	Filter phase response
ω	Temporal frequency, in radians per second

List of Abbreviations

AP	Allpass
DC	Direct Current (0 Hz)
FD	Finite Difference
LHS	Left Hand Side
RHS	Right Hand Side
VA	Virtual Analog

1. Introduction

Dispersion is a property present in wave-motion throughout the physical world, and refers to any situation where the speed of propagation of a wave depends on its frequency [1]. It can be seen in the spreading of waves on water, or in the diffraction of light by water-drops that forms a rainbow (see Fig. 1.1). Similarly, dispersion exists in electrical systems and in discrete digital systems, albeit usually in a less obvious form. In signal processing and in engineering in general, dispersion has often been considered an undesirable quality of a system – one that must be mitigated or eliminated. In this work, we take an opposing viewpoint. We present systems where dispersion is beneficial, whether that be for subjective artistic reasons or for objective technical reasons.

With the increasing prevalence of cheap and plentiful computational power, the world of music production and performance is becoming increasingly computer-based and hence digital. The traditional strength of digital audio processing has been in its predictability and its lack of the supposedly undesirable qualities of physical or analog systems - noise, non-linearities, and indeed dispersion. Whilst this behavior is ideal in engineering applications where a specific and predictable result is desired, in systems designed to be used creatively by artists it is less desirable.

Figure 1.1. Photos of surface water waves, and a rainbow – both examples of dispersive wave propagation. (images in public domain)

Systems which exhibit some complexity, unpredictability and emergence of behaviors are arguably more suited for producing interesting artistic outcomes.

Recent years have seen an explosion of interest by musicians, producers and artists in digital models of analog musical equipment and instruments. This interest is fueled partially by nostalgia, but also by realization of the conclusion given above. In response to this demand, there has been significant amounts of research dedicated to producing these models, termed 'virtual analog' (VA). This work has encompassed many areas, from guitar pickups [2], distortion pedals [3, 4, 5, 6] and amplifiers [7, 8, 9] to synthesizer filters [10, 11, 12, 13, 14], oscillators [15, 16, 17, 18, 19], and ring modulators [20, 21] to audio effects such as analog echo/delay [22, 23], modulation effects [24, 25], compressors and limiters [26, 27, 28] and plate-based electromechanical reverberation [29, 30, 31, 32, 33]. VA work considers both complete structures, and individual components [34, 35]. Similar work has even emulated non-musical analog audio systems, such as carbon-microphones [36] and early recording media and sound reproduction equipment [37]. There are several excellent reviews of VA research available [38, 39, 40]. This research can be seen as having developed from the longer tradition of research into digital modeling of musical acoustic systems (called *physical modeling*) [41, 42, 43], from circuit theory via *wave-digital filters* [44], and also from the broader computer music tradition.

VA research can be thought of as addressing three main goals:

1. *Emulation* – to produce exact digital copies of particular analog systems. These copies can be produced just by replicating input-output relationships (called a *black-box* model), or by attempting to model the mechanisms which produce these relationships.

2. *Artifact reduction* – to produce digital sound processing or generation blocks which behave like their ideal continuous-time equivalents by reducing or eliminating the undesirable side-effects of digital signal processing, such as aliasing or frequency response warping.

3. *Analog feel* – to produce techniques or structures that bring some of the above-mentioned artistically desirable qualities of the analog realm into digital systems without necessarily exactly reproducing a particu-

lar system. These qualities include unpredictability, drift and emergent behavior, dynamic non-linear behavior etc.

The majority of work presented in this dissertation addresses the first and third of these goals, and hence can be thought of as largely (but not exclusively) being within the remit of VA research.

The main work of this thesis is concerned with examining an important class of analog audio processing device, the spring reverberator, which derives much of its distinctive sound from the effects of dispersion. By examining the behavior of this device and understanding the mechanisms which produce it, we can both derive digital models that replicate it and learn of properties that might be interesting to include in a more general non-emulative digital system. We also examine a particular technical problem in audio engineering, that of reducing the peak amplitude of a musical signal in order to maximize its loudness, and show how a dispersive system can be used to address it.

The contents of the thesis consist of this introductory document, along with seven peer-reviewed publications. Five have been published in international journals, and two in international conferences. Publications I and II present background work on spring reverberation, including measurements, mathematical models and results derived from both. They also present a digital model of spring reverberation constructed using direct discretization of a continuous mathematical model using finite differences. Publication III presents analysis and modeling of a related system, the large *Slinky* springs used as children's toys. Publications IV - VI present a different approach to the modeling of spring reverberation, using chains of allpass filters, and develop this model further to improve its computational efficiency and realism. Finally, publication VII presents a novel system that uses the dispersive property of allpass filters to reduce the peak amplitude of transients in musical audio.

The introductory document is structured as follows. In Chapter 2, we explain the concept of dispersion and discuss its presence in many kinds of wave-motion phenomena. In Chapter 3 we examine a digital signal processing structure for which dispersion is the main property, the *allpass filter*, and summarize its applications. This gives background information necessary for Publications IV, V and VII. In Chapter 4 we examine the major example of a highly dispersive system used in audio processing, the spring reverberator. We summarize the mathematical models of spring vibration, examine measurements taken from a spring reverberator, and review the work on digital models of spring reverberation. This places Publications I – VI in context. In Chapter 5 we give a summary of the novel scientific contributions of each publication. In Chapter 6, we conclude.

2. Dispersion

All of the publications associated with this work describe systems whos defining property is dispersion. In this section we give a brief review of the concept of dispersion, both in physical systems and in signal-processing systems.

2.1 Dispersion as a generalized wave-motion phenomena

In a general system, dispersion is described by the *dispersion relation*, a function that relates the wavelength of a particular wave to its temporal frequency. The general dispersion relation is of the form:

$$\omega(k) = \nu(k)k \tag{2.1}$$

where the LHS is an arbitrary function of the *wavenumber* k (the reciprocal of wavelength) which describes the temporal frequency ω, and the RHS is an arbitrary function of k which describes the wave speed ν. To understand this relationship intuitively, it is useful to consider what these terms mean individually. Wavelength is the distance between crests of the wave in space, and its reciprocal k (best understood as spatial frequency) describes how many cycles of a wave are contained within a unit spatial interval. Temporal frequency describes how many cycles per unit time the wave-motion completes at a fixed spatial point. It should be clear that the ratio between the spatial frequency and the temporal frequency gives the time taken for a crest of the wave-motion to cross a unit spatial interval. Hence the ratio of temporal frequency to spatial frequency gives the speed of propagation of the wave-crest. This quantity is known as *phase velocity*:

$$\nu_\phi(\omega) = \frac{\omega(k)}{k} \tag{2.2}$$

If the dispersion relation is linear, this implies that the phase-velocity is constant for all combinations of temporal and spatial frequency, and an arbitrary time-limited wave packet will propagate at a constant speed without changing its shape. This is the un-dispersive case, and is seen for example in the propagation of electromagnetic waves in a vacuum:

$$c = f\lambda \tag{2.3}$$

where c is the speed of light, f is the frequency of wave and λ is its wavelength.

If the dispersion relation is non-linear, then the phase velocity associated with a particular combination of temporal and spatial frequencies is no longer constant. This has important ramifications. Consider a time-constrained wave-packet consisting of a single frequency component with a Gaussian amplitude envelope. In the frequency domain, the amplitude envelope has the effect of spreading the single frequency component over a range of frequencies. If the phase velocity is not constant over this range, we see a phenomenon where the crests of the sinusoidal component of the wave-packet appear to move at a different velocity to that of the amplitude envelope.

In order to describe better this phenomenon, we need a further concept – one that describes how quickly the envelope of the wave-packet itself moves, rather than how the individual components move. This quantity is known as group velocity:

$$\nu_g(\omega) = \frac{\partial \omega}{\partial k} \tag{2.4}$$

Group velocity describes the speed at which the amplitude envelope of a wave-packet of a certain frequency moves. In the case of a non-linear dispersion relation, group-velocity will vary with frequency. Therefore, in an arbitrary wave-packet consisting of many sinusoidal components and their associated amplitude envelopes, the amplitude envelopes of the individual components will move at different speeds – resulting in distortion of the overall envelope of the wave packet. In the special case where the dispersion relation is affine, the components will move at a different rate than their envelopes, but the overall envelope will not be distorted.

In summary, for an arbitrary wave-packet consisting of multiple frequencies:

- If the dispersion relation is *linear*, the wave packet will maintain its exact shape as it propagates.

- If the dispersion relation is *affine*, the envelope of the wave-packet will stay the same, but the instantaneous amplitudes will change as the phase of the components shifts relative to each other.

- If the dispersion relation is *non-linear*, the envelope of the wave-packet will change as it propagates.

Note that the dispersion relation is not required to be a one-to-one relation – a particular temporal frequency could be associated with several spatial frequencies, meaning that it would propagate at several velocities simultaneously.

2.2 Dispersion in signal-processing systems

In systems which are not distributed in space, the concepts of phase-velocity and group-velocity have no meaning. Instead, we consider how an input to the system is changed by the system.

The frequency response $F(\omega)$ of a general LTI filter $H(z)$ can be expressed as the product of two functions, magnitude response $G(\omega)$ and phase response $\Phi(\omega)$:

$$H(e^{i\omega}) = |H(e^{i\omega})| \arg(H(e^{i\omega})) = F(\omega) = G(\omega)\Phi(\omega) \tag{2.5}$$

The magnitude response $G(\omega)$ describes how the amplitude of any frequency component changes between the input and output of the system. The phase response $\Phi(\omega)$ describes how the phase of any frequency component changes between the input and output of the system.

The phase response is the analog to the dispersion relation seen in spatially distributed systems. At first glance, it would seem that it is also the analog of the phase velocity, but this is not the case. In a digital signal processing system, the phase of a sinusoidal component will change during a sampling interval by an amount equal to its angular frequency, ω, divided by the sampling rate. This means that the raw change in phase between the input and output of the system, given by $\Phi(\omega)$, also includes this quantity. To see the true contribution of the system to the output phase, we must remove this quantity. This is done simply by dividing $\Phi(\omega)$ by ω, giving:

$$\tau_\phi(\omega) = \frac{\Phi(\omega)}{\omega}. \tag{2.6}$$

This quantity is known as the *phase-delay*, and describes how the system itself effects the phase of the input. Hence, it is the analog to phase-velocity. The phase-delay can be thought of as describing the delay which the system imparts on a sinusoidal component of a certain frequency.

Similarly to the spatially-distributed case, we also need a measure of how the system changes the amplitude envelope of a time-constrained sinusoid of a certain frequency. This is calculated similarly to the group-velocity:

$$\tau_g = \frac{\partial \Phi(\omega)}{\partial \omega}. \tag{2.7}$$

This is known as the *group-delay* of the system, and describes how the amplitude envelope of the input is changed by the system. The group-delay can be thought of as the delay imparted by the system on the amplitude envelope associated with a specific frequency component. If the group-delay is not constant, then different frequencies in the input will differ in the time they take to appear at the output of the system. This means the system would, for example, bend an impulse at the input into a chirp. The group-delay exactly describes the shape of this chirp.

The majority of digital filters have non-constant group-delay, and hence are dispersive to a certain extent. The exception to this are *linear phase* filters, which have a purely linear phase-response, and hence constant phase-delay and group-delay.

3. Allpass Filters

Whilst many digital filters exhibit dispersion of some kind, the filter for which this property is most fundamental is the allpass (AP) filter. An AP filter is by definition a filter that does not have any effect on the magnitude spectrum of a signal. The most trivial digital AP filters are the identity operation 1 and the unit delay z^{-1}, which also have no effect the relative phase of the components of the signal. Beyond these trivial cases, there is a large class of more complex filters which preserve the magnitude spectrum of a signal, but alter its phase in some way. It is this class of filter which are generally referred to as AP filters. The digital AP filter originates with Manfred Schroeder [45]. Later important works include Mitra & Hirano's derivation of a multitude of networks that implement the basic first and second order AP filters [46], and Gerzon's description of both vector-valued and nested AP systems [47].

The generalized transfer function of a digital AP filter of order N is given by:

$$A_N(z) = \frac{a_N + a_{N-1}z^{-1} \cdots + a_1 z^{-(N-1)} + z^{-N}}{1 + a_1 z^{-1} + \cdots + a_{N-1}z^{-(N-1)} + a_N z^{-N}} \tag{3.1}$$

where the a_i are the set of *allpass coefficients*. This reduces in the first-order case to:

$$A_1(z) = \frac{g + z^{-1}}{1 + gz^{-1}} \tag{3.2}$$

where we have taken $a_1 = g$ as the single allpass coefficient.

It is simple to prove that this describes an allpass filter by assuming unit sampling interval and calculating the magnitude of the transfer function on the unit circle:

$$|A_1(e^{i\omega})| = \left|\frac{g + e^{-i\omega}}{1 + ge^{-i\omega}}\right| = \frac{|g + e^{-i\omega}|}{|1 + ge^{-i\omega}|} = \frac{\sqrt{g^2 + 2ge^{-i\omega} + 1}}{\sqrt{1 + 2ge^{-i\omega} + g^2}} = 1 \tag{3.3}$$

Similarly, the phase response is given by:

$$\begin{aligned}\arg[A_1(e^{i\omega})] &= \arg\left[\frac{g+e^{i\omega}}{1+ge^{i\omega}}\right] = \frac{\arg[g+e^{i\omega}]}{\arg[1+ge^{i\omega}]} = \arg[g+e^{i\omega}] - \arg[1+ge^{i\omega}] \\ &= \arctan\left(\frac{\sin(\omega)}{g+\cos(\omega)}\right) - \arctan\left(\frac{g\sin(\omega)}{1+g\cos(\omega)}\right) \\ &= 2\arctan\left(\frac{g\sin(\omega)}{1+g\cos(\omega)}\right) - \omega \end{aligned} \quad (3.4)$$

The group delay can be calculated by taking the derivative of the phase response, giving:

$$\tau_g(\omega) = \frac{1-g^2}{1+2g\cos\omega+g^2}. \quad (3.5)$$

From these properties, we can make two important observations. Firstly, we can see that the unity magnitude response of the filter implies that it is lossless – the energy entering the filter is equal to the energy exiting it. Secondly, we can see that the group-delay of the filter is a function of frequency, ω, implying that whilst energy is preserved by the system, that energy may not all arrive at the output at the same time. In other words, an allpass filter can bend an impulse into a *chirp*. This chirp may be too short to be perceivable as a chirp in the case of a single allpass filter, but if many are cascaded then the chirp becomes very prominent. This property is central to Publications IV – VI.

A corollary of this observation is that if the input energy is preserved, but does not arrive simultaneously, the allpass filter must reduce the peak amplitude of an impulse. The amount of reduction is dependent on the coefficient of the filter. This property is exploited in Publication VII, as a method of reducing peak the peak amplitude of a more general signal.

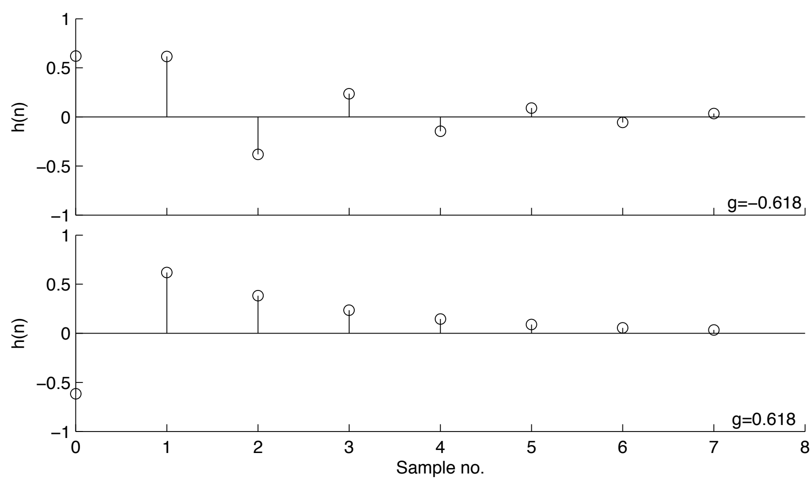

Figure 3.1. First 8 samples of the impulse response of a first-order allpass filter, with golden-ratio allpass coefficient $g = \pm 0.618$.

We can illustrate this property by examining the impulse response produced by a first-order AP filter. Fig. 3.1 shows the first 8 samples of such a response. As is described in Publication VII, the peak value of the impulse response depends on the value of the allpass coefficient g. In Fig. 3.2 we show how the absolute value of the first 8 samples of the impulse response of a first-order allpass filter vary as g is varied. As can be clearly seen, the absolute value of the first sample of the impulse response starts low and increases with $|g|$ whereas the value of the second sample is high with low $|g|$, and increases as $|g|$ increases. Therefore, values of $|g|$ that are close to 0 or 1 produce a larger peak output value, with the maximum being at the second sample for low $|g|$ and at the first sample for high $|g|$. The value of $|g|$ which produces the lowest peak output value is one where the values of both the first and second sample are mediated (and the most energy is transferred into the third sample). This point can be clearly seen in Fig. 3.2. Publication VII give a full derivation of this behavior, and shows that this point corresponds with g being equal to ($\pm$) the *golden ratio*.

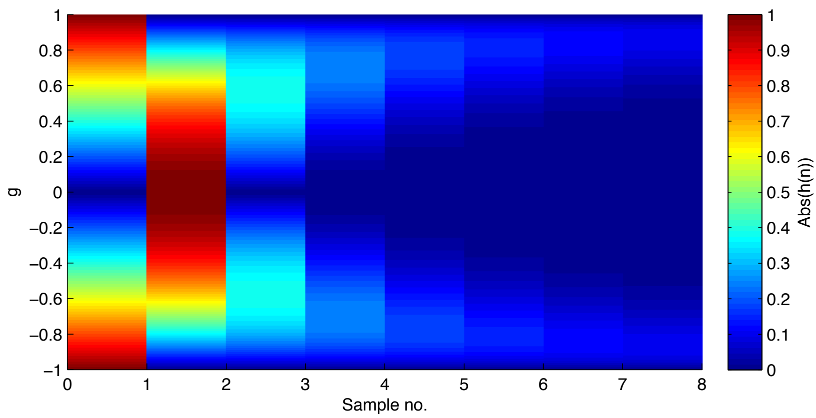

Figure 3.2. Diagram showing absolute value of the first 8 samples of the impulse response of a first-order allpass filter, with varying values of allpass coefficient g.

Allpass filters with golden-ratio coefficients can be cascaded to further reduce the peak amplitude of the impulse response. In order to provide maximum reduction, non-zero elements of the allpass filters impulse response must not be coincident. This can be achieved by replacing the unit delay inside the allpass filter with a longer delay-line. This results in a transfer function for a single section of:

$$A_1(z^D) = \frac{g + z^{-D}}{1 + gz^{-D}} \tag{3.6}$$

where D denotes the length of the embedded delay-line in samples. This

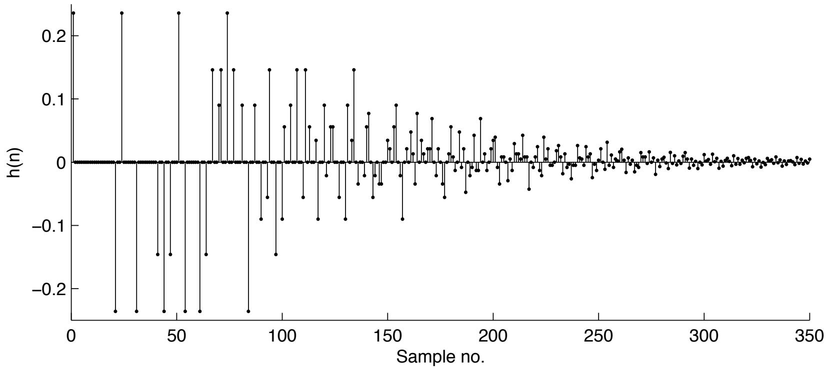

Figure 3.3. Impulse response of a cascade of three first-order allpass filters, with allpass coefficient $g = \pm 0.618$ and delay-lengths of 20, 23 and 30 samples.

replacement has the effect of stretching the impulse response of the system by inserting zeros between the previously consecutive samples of the impulse response. If the the delay-lines embedded in the allpass filters are given mutually prime lengths, the period at which overlapping samples appear is maximized.

An example of the impulse response of such a filter cascade is given in Fig. 3.3. If the delay-line lengths are chosen correctly, such a chain will have an peak impulse response amplitude of g^M, where M is the length of the cascade.

3.1 Applications

Digital AP filters are used throughout the field of signal processing, for many different applications [48]. They were first suggested by Manfred Schroeder [45], as a system which could extend an input in time without altering its frequency content. Schroeder applied the allpass filter to construct the first digital artificial reverberator. Since this time, allpass filters have been used for constructing equalizers [49, 50, 51], for interpolation of delay lines [52, 53], for warping the frequency scale of general digital filters to provide better resolution [54, 55, 56], for simulating inharmonicity in physical models of stiff strings [57, 58, 59, 60] or bells [61, 62], and for construction of audio effects such as phasing [63] and forms of distortion [64, 65], and for abstract sound-synthesis [66, 67, 68, 69]. They have also been used for the implementation of Kautz and Laguerre filters [70], and for decorrelation of audio for spatial purposes [71, 72].

The particular application of allpass filters that is relevant to this dissertation is their use to specifically produce dispersion. This is seen in work on *spectral delay filters* [73], and in emulation of spring reverberation. Abel et al. [74] proposed the application of allpass filters in this context, based on a method described further in another of their works [75]. Later work, seen in Publications IV–VI, developed this idea further to provide a full parametric model of spring reverberation based on cascades of allpass filters.

As discussed above for the case of a single impulse, allpass filters have also been used to reduce the peak amplitude of transient signals. This work is presented in Publication VII, and is achieved by using a large parallel collection of allpass filter cascades. Each cascade has its delay-line lengths chosen randomly. The transient signal is passed through this system, and the output of the parallel system which produces the lowest peak amplitude is chosen to be the main output. If the impulse response is kept short enough, the effect of the phase-change on the transient should be inaudible. Publication VII does not test this inaudibility via listening tests, but instead uses previously established thresholds [76] from the literature.

4. Spring Reverberation

Since the advent of recorded and electrically generated sound in the late 19th and early 20th century, there has been a desire for the ability to add the impression of an acoustic space to a sound that otherwise lacks it. This process is known as *artificial reverberation*, and was first developed by RCA in the 1920s [77]. The earliest methods of solving this problem relied on transmitting the sound into an empty acoustic space, and recording the result [78]. However, this method was inherantly costly, inflexible and tied to a particular location. To address these deficiencies other methods of approximating the sound of an acoustic space were developed, initially using electro-mechanical systems and later using digital signal processing. A summary of the history of artificial reverberation is given in Sec. II of Publication VI.

An early form of electro-mechanical artificial reverberation was the spring reverberator, conceived by Laurens Hammond in the early 1940s [79, 80]. Hammond invented the method as a compact and inexpensive way of adding a reverberation-like effect to his electric organs. A typical spring reverb has a fairly simple topology consisting of a number of small helical metal springs connected in series or parallel [81, 82]. Extra damping of the system was introduced when necessary by various methods, including immersing the springs in oil [79, 83]. A photo of a typical small spring reverberation unit, in this case intended for automotive use, is shown in

Figure 4.1. Typical spring reverberation unit.

Fig. 4.1.

The modern style of spring reverberator was introduced in the late 1950s, and early 1960s, and it based on exciting small diameter springs torsionally using a magnetic driving system [84]. Each spring has small magnets attached to their ends. A signal passed through a nearby electromagnetic coil applies a force to the magnet at one end of the springs, and hence excites vibrations. At the other end, the movement of the similar magnetic bead induces a current in another electromagnetic coil, producing the output signal. This output signal is related to the input signal, but possesses a diffuse and acoustic-like quality introduced by the vibration of the springs. A more detailed description of the physical construction of a spring reverberation unit is given in Sec. 2 of Publication I.

Spring reverberation gained wide popularity starting in the 1950s and 1960s, due to its low cost and small size. It was particularly prominently used within guitar amplifiers. As mentioned above, the sound of spring reverberation is somewhat like that of an acoustic space. However, it has some important differences. Firstly, a single spring will produce much more prominent and repetitive echoes than most acoustic spaces. Secondly, the quality of these echoes is very different to those produced in acoustic spaces, they possess a quality often described as 'wet'. This property is of particular relevance to this collection of work, as it is caused by the dispersive nature of wave propagation on a helical spring. Publications I, II, IV are primarily focused on analyzing and replicating this behavior.

The springs used in spring reverberation units are generally constructed from thin wire, and have a small helix radius. There is also a precedent for larger springs with thick, stiff wire being used both for processing audio and as an alternative musical instrument or sound effect generator. These larger springs are generally those sold as child's toys under the trade name *Slinky*. These springs vibrate in a different manner to the small springs used in spring reverberation units, primarily because of the stiffness of the wire used. Their behavior is more related to that of a metal bar. Publication III examines the behavior of these larger springs, and explores some methods by which their behavior can be modeled digitally for artistic or musical use.

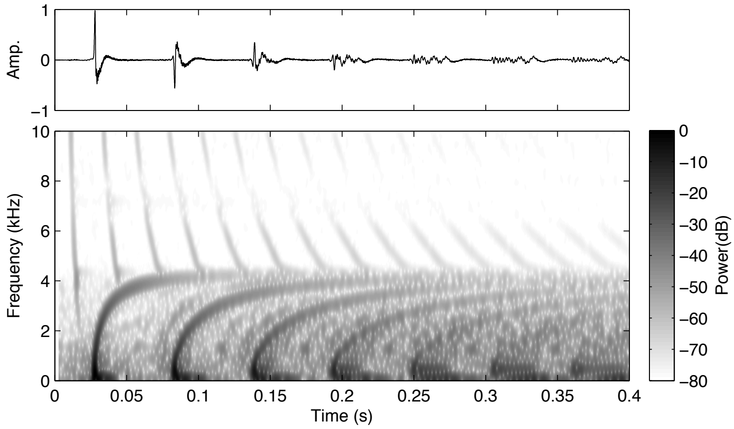

Figure 4.2. Impulse response of typical spring used for spring reverberation, presented in time-domain and as spectrogram.

4.1 Measurements

A typical impulse response of a spring reverberation unit is shown in Fig 4.2. The measurement has been made using a sine-sweep technique [85, 86]. The impulse response can be seen to contain a number of interesting features. Firstly, it is clearly made up of a number of repeating echoes. These echoes are not straight reflections of the input, but are instead curved into chirps – with different frequencies arriving at the output at different times. Furthermore, there are two clear separate sequences of these chirps. One is present below around 5kHz, with the fastest propagation happening at low frequency and increasing to an asymptote. The second series of chirped echoes propagates fastest at high frequency, and seems to extend over the entire audio band.

From this response, we can identify two important parameters of a spring reverberator. The first is the time-delay between the low-frequency chirped echoes, measured at DC. We call this quantity T_D. The second is the frequency after which the low-frequency chirped echoes are no longer present. We call this quantity the transition frequency F_c. These parameters are first identified in Publication I, where expressions for their calculation from physical parameters are also derived.

More subtle features of some spring-reverb responses have also been identified, including frequency-dependent blending between multiple sets of modes [87] and small pre-echoes before the main echoes (see Publication IV).

4.2 Mathematical Models

The ideal spring has a long history in classical mechanics as a lumped element used to represent a generalized elastic connection between two objects, and governed by Hooke's Law:

$$F = \kappa X \tag{4.1}$$

where F is the force exerted by the spring, measured in N, κ is the stiffness of the spring, measured in Nm^{-1} and X is the distance in m by which the spring has been compressed or stretched. In this general sense, a spring is merely a store of potential energy.

Real-world springs that approximate this behavior can be constructed in many different ways, for use with respect to different kinds of motion. One example of a such a device is a gas spring, consisting of a sealed piston and cylinder filled with compressed gas, and designed to act as a spring with respect to axial movement of the piston. Another example is the mainspring used in analog watch mechanisms, constructed from a flat spiral of wire and used to store the potential energy that powers the watch.

Perhaps the most common type of spring, so common that it has become synonymous with the term in general speech, is the helical metal spring. This is a device constructed from metal wire, bent into a cylindrical helix. Helical metal springs behave like an ideal spring with respect to small amounts of compression or tension in the direction of the axis of the spring. This kind of spring is widely used, for purposes ranging from vehicle suspension to mattresses.

The behavior of a helical metal spring is, as mentioned above, only like that of an ideal spring with respect to small deformations along its axis. To examine its behavior in response to more general forces (and crucially, vibrations), a more complete model is needed. Early attempts to model the helical spring as a distributed object attempted to approximate its behavior by representing it as an elastic rod or beam [88]. The behaviour could then be characterized by the well-known Euler-Bernoulli or Timoshenko theories of beam mechanics.

Wittrick [89] proposed a full mathematical model of spring vibration that includes the helical geometry of the spring, consisting of a coupled set of partial differential equations in 12 variables. This model has been further analysed in the context of axial excitations [90] and extended in various ways, notably to allow for loading of the spring [91]. Fletcher &

Tarnopolskaya [92, 93] proposed a simplified model of model of wave propagation on a helix, consisting of a coupled set of only two partial differential equations. This simplification is reached by neglecting the helix angle of the spring, essentially collapsing the spring into two dimensions along its axis, leaving a circular bar wound many times around a single point. This simplified model retains a surprisingly large amount of the interesting behaviors of the spring, particularly the shape of the dispersive echoes produced by the system. Recent work [87] [94], has developed a new simplified model based on a reduction of the work of Della Pietra [90], which is itself a simplification of the work of Wittrick. This new model achieves the simplification by neglecting Timoshenko effects in the wire, whose main influence seems to be significantly above the audio range. The resulting model is a system in two equations, like that of Fletcher & Tarnopolskaya [92, 93]. However, it appears to preserve more of the important dynamics of the spring vibration, notably the process which leads to the diffuseness of the echoes, and hence the reverberance of the sound.

4.3 Digital Models

Spring reverberation, as a popular analog audio effect, is a desirable target for modeling in the *virtual analog* tradition. The first paper on the topic developed a waveguide-like model of the low-frequency chirp sequences described above, with the dispersion generated by a set of second-order allpass filters fitted to match a measured response [74]. This initial analysis had a few drawbacks. Firstly, it was based on exact fitting to a measured response, with no provision for parametric variation of the sound. Secondly, the method used to fit the set of second-order allpass filters to the measured dispersion characteristics is not publically described in detail and hence is not reproducible. Thirdly, no serious attempt was made to relate the model to the physical mechanism which produce the unusual characteristics of spring vibration, or to the geometry of individual springs. The author's initial work on spring reverberation was initiated in an attempt to address these issues.

The first step towards this goal was the identification of suitable mathematical models, and their discretization as an explicit finite difference (FD) scheme [95]. The simplified model of Fletcher & Tarnopolskaya [92, 93] was chosen for this initial implementation. This explicit FD scheme exhibits poor numerical properties – notably stability problems and significant extra numerical dispersion. These problems were addressed by deriving an implicit scheme for the solution of the equations. This implicit scheme is described and analyzed in Publications I and II.

This approach has two main drawbacks. Firstly, it is currently computationally too expensive to implement for real-time musical use. Secondly, the output of the model manages to reproduce accurately the chirped echoes produced by a real spring, but fails to reproduce the progressive blurring and diffusion of these chirps which gives the spring its reverberant sound. This limitation is caused by the neglection of helix angle which is inherent in the continuous model which the digital model is based on. Recent work by Bilbao [94] has presented a new FD model based on a slightly different continuous model, which appears to do better at producing this reverberant quality. In applications where computational power is plentiful, and access to arbitrary points within the modeled system (for input, output or manipulation) is required, finite difference techniques would seem to be optimal. However, for use in computer music environments, computational load should ideally be very low and access to arbitrary points on the spring may not be necessary. It is therefore interesting to also explore other techniques for modeling spring reverberation.

Another such approach which better fits these criteria is to construct a system which consists of a network of delay-lines and allpass filters, as seen in standard digital reverberators [45, 96, 97, 98] and in digital waveguides [52, 99, 100]. To this system we add some kind of dispersion generating element. This is the approach taken by Publication IV. In this case, the dispersion generating element chosen is long cascades of allpass filters. These cascades are placed within a feedback structure with a delay line, to generate a sequence of repeating chirped echoes. Modulation of the delay-line is used to progressively diffuse the chirps, and hence produce the reverberant quality. An advantage of this approach is its parametric nature – the behavior of the system can be tuned with respect to either perceptual or physical parameters, rather than being fitted to a particular measured response. As has been pointed out in the case of physical modeling of musical instruments [101], this is an important feature.

This approach was refined in Publication V, where a number of modifications to the structure were suggested in order to greatly reduce its computational load. The main modification is the selective straightening of parts of the chirps, by utilizing a crossover network to isolate the dispersion only to certain important ranges of the spectrum. The design requirements of the dispersion generating filter cascade can then be relaxed somewhat, as the behavior in the straightened region is no longer important. This allows a more efficient formulation of the dispersion generating filter. The second modification applies multi-rate techniques to run the band-limited low-frequency chirp structure at a lower sampling rate, hence reducing computational cost.

The allpass-based approach to spring reverberation was further developed in Publication VI, where a new mechanism for generating the diffusion of the chirps was proposed, based on convolution with time-varying sparse noise [102, 103]. This refinement improves the sound quality of the model, with little additional computation.

Recent theoretical work has started to explore the modal structure of the helical metal spring [87]. Following this line of inquiry, it is also logical to explore the possibility of digitally emulating a spring reverberator via the application of modal synthesis [104] or the functional transformation method [43]. Spring reverb is perhaps well suited to this kind of emulation, as the physical system has only a single fixed input and output point - thus bypassing the principal drawback of these methods. Additionally, dispersion is dealt with easily and with no extra cost using modal or functional transformation methods [105], whereas waveguide-based physical models are more natural for non-dispersive systems. However, the relative viability of this approach depends on the number of modes needed to emulate the system satisfactorily and the ease with they can be derived from the mathematical model of the system. This topic has the potential for interesting future work.

Other work has considered how a model based on this approach can be automatically calibrated to measurements of a real device, via application of optimization techniques [106].

4.3.1 A note on perceptual evaluation of models

In the works presented in this thesis, no formal efforts have been made to evaluate the perceptual similarity between the sounds produced by the model and those from the original device via systematic listening tests or similar. Instead, the view has been taken that the provision of audio examples should sufficiently allow the reader to evaluate the quality of the algorithm. It is the opinion of the author that traditional listening tests, usually focussed the perception of differences between the real and modeled signals, are not appropriate for evaluating digital models that are intended for musical use. In fact, they encourage over-fitting of the models. In musical contexts, it is more important that the model captures the broad behavior of the original device and also provides for parametric variation of its main perceptual characteristics. This viewpoint is in agreement with the historical literature on creative audio effects and computer music.

No doubt a robust method for testing the audio quality of creative effects, modeled instruments or synthesis techniques is possible - some recent publications have described work in that direction [107]. However, at the time of writing of the works included in this thesis, such a method was not available.

5. Summary of Main Results

Publication I: "Spring Reverberation: A Physical Perspective"

Publication I gives the necessary background information on spring reverberation, which had not been previously published. Sec. 2 describes the construction of a generic spring reverberation. Appendix A presents measurements of two typical spring reverberation units, and Tables 1 and 2 give the physical parameters of the springs used in these units.

Sec. 3 analyses a typical spring impulse response, and proposes the two important perceptual parameters F_C and T_D which are used throughout the following writing on spring reverberation. Sec. 4 reviews the available mathematical models of spring vibration, and draws some conclusions about the nature of spring vibration from them. Sec. 4.1 shows how the above proposed perceptual parameters F_C and T_D can be calculated from easily-measured physical dimensions of a spring. The derivation of these relations from one of the mathematical models is given in Appendix B.

Sec. 5 briefly describes how finite differences can be used to discretize one of the mathematical models given in Sec. 4, and gives some preliminary results of this technique.

Publication II: "A Virtual Model of Spring Reverberation"

Publication II expands on Sec. 4 and Sec. 5 of Publication I, and gives a full description of an implicit finite difference based digital model of spring reverberation. Sec. II describes the continuous-domain model which the digital model is based on, the simplest form of which is given in Eqs. 2a and 2b. Sec. IV shows the results of the digital model, and compares them to measurements of real spring reverberation units

Publication III: "Modeling Methods For The Highly Dispersive Slinky Spring: A Novel Musical Toy"

Sec. 3 makes the observation that the measured impulse response of the Slinky spring is reminiscent of that of an ideal bar. It is then suggested that its vibration could be modeled by the Euler-Bernoulli bar equation (given in Eq. 1), with added terms to deal with loss. The dispersion characteristics of this continous model are fitted to those observed in the measured impulse response by adjusting the dimensionless stiffness parameter, κ.

Sec. 3.1 describes how this continuous model can be discretized via application of finite differences. Sec. 4 describes two novel musical signal processing structures, one effect and one instrument, that are based on the Slinky model.

Publication IV: "Parametric Spring Reverberation Effect"

Publication IV describes a new spring reverberation model based on chains of allpass filters placed into a feedback system. Sec. 1 describes how the characteristic low-frequency chirped echo of the spring reverberator can be emulated with a chain of stretched allpass filters with further embedded interpolation allpass filters. This process is illustrated most clearly in Fig. 4 and Fig. 6. Sec. 1.5 explains how a chain of normal first-order allpass filters can be equivalently applied to emulate the dispersion characteristics of the high-frequency chirped echoes.

Sec. 2 describes how these filter chains can be placed within a feedback structure to produce a series of dispersive echoes. Sec. 2.2 and Sec. 2.3 present some techniques for producing increasing diffusiveness in this series of echoes, by tapping the delay-line within the feedback loop at multiple points, and modulating the output position with a random modulation source. Sec. 2.6 relates the parameters of the allpass-based spring reverberator to the physical parameters of spring.

Sec. 3 presents the results of using this model to emulate measured impulse responses taken from a number or real spring reverberation units.

Publication V: "Efficient Dispersion Generation Structures for Spring Reverb Emulation"

Publication V describes a number of novel extensions of the spring reverberation model originally described in Publication IV which greatly reduces the model's computational complexity. This is achieved via two approaches. The first approach halves the length of the allpass filter cascade needed to produce the required dispersion, by doubling the stretch factor K of the allpass filters and inverting their coefficient. This modification produces an extra peak in the group delay at DC. This peak is hidden by using a crossover network to split the chirp into two sections, the upper of which is processed by the allpass filter chain whilst the lower is not. This unprocessed band is time-aligned with the processed band using a novel method which utilizes a first-order allpass filter to exactly match the phase of the unprocessed band with the processed band at the crossover frequency. This process is applied to both the low-frequency and high-frequency chirp structures. The second approach involves running the low-frequency chirp structure at an integer division of the sampling rate of the system, chosen to allow F_C to still be varied over the necessary range.

The combination of these modifications reduces the computational complexity to about 1/3 of its original value, as shown in Table 2.

Publication VI: "Fifty Years of Artificial Reverberation"

Publication VI is a large review paper covering all research on digital signal processing based artificial reverberation since its begininng with the works of Schroeder in the 1960s. As such, only a small subset of the material is novel and contributed by the author. Only this material is discussed here.

The novel material is contained in Sec. VI.A.2, where a new extension of the spring reverberation model developed in Publication IV is described. This extension consists of a new mechanism for introducing diffuseness into the echoes produced by the system by embedding a switched FIR filter generated from velvet noise sequences into the feedback loop.

Publication VII: "Linear Dynamic Range Reduction of Musical Audio using an Allpass Filter Chain"

Publication VII presents a novel method of reducing the peak amplitude of musical signals by applying the dispersive properties of golden-ratio allpass filters. Sec. II presents a derivation of the optimal allpass coefficient which minimises the peak amplitude of the impulse response of the filter. Sec. II.A presents a novel structure consisting of many parallel allpass filter chains, with randomly generated embedded delay line lengths. The output from this structure that corresponds with the lowest peak amplitude is selected. The results of this system applied to isolated musical sounds are shown. Sec. II.B shows how this approach can be extended to mixed musical audio via segmenting the signal based on transient positions and processing the segments seperately. This approach allows many transient signals to be reduced in peak amplitude by up to 5dB.

6. Conclusions

This work explores the use of dispersive systems in audio signal processing, primarily for musical use. Dispersive systems are those in which the speed of propagation of energy is dependent on frequency. This quality is present in many real-world systems, from mechanical to electrical to fluid. Examples from the natural world include gravity-waves on the surface of water, electro-magnetic waves propagating through the ionosphere, and pressure-waves in most solids.

In this introductory document, we have discussed dispersive systems in general, and gone into more detail examining two particular examples -– the spring reverberator and the allpass filter. These discussions set the context for the included publications. Publications I and II present important background work on spring reverberation, and a novel digital model constructed using direct discretization of a continuous mathematical model using finite differences. Publication III presents analysis and modeling of the larger *Slinky* springs used as children's toys. Publications IV - VI present a different and new approach to modeling spring reverberation that employs chains of allpass filters, and develop this model further to improve its computational efficiency and realism. Publication VII presents a novel system that uses the dispersive property of allpass filter cascades to reduce the peak amplitude of transients in audio.

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