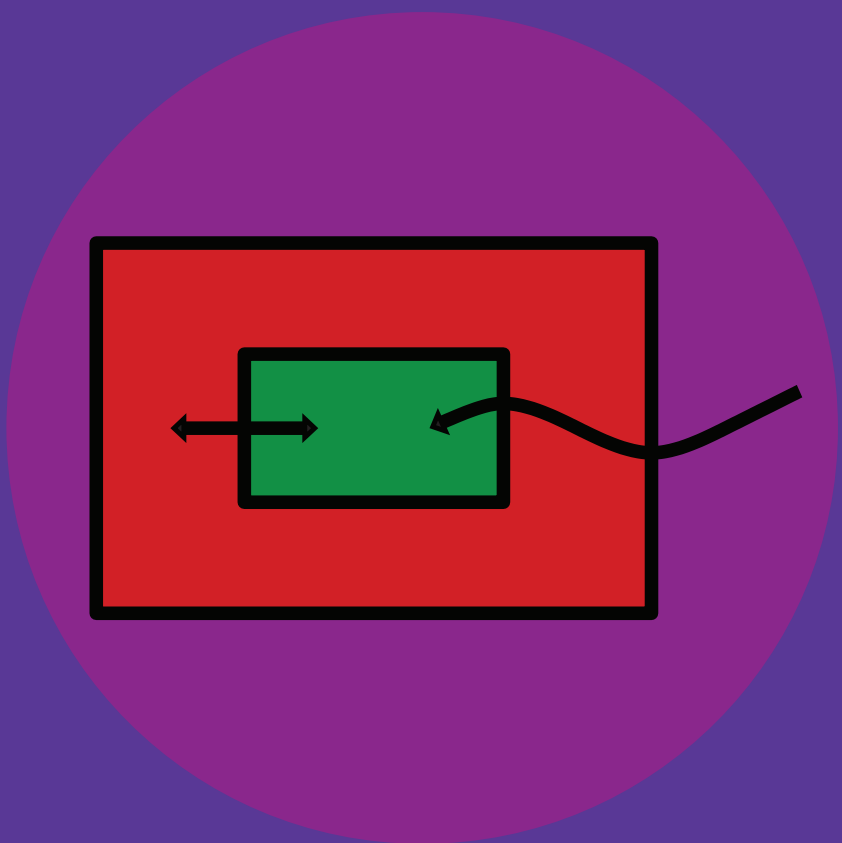
Department of Applied Physics

Control of Open Quantum Systems

Juha Salmilehto



A!

DOCTORAL DISSERTATIONS

Aalto University publication series
DOCTORAL DISSERTATIONS 102/2014

Control of Open Quantum Systems

Juha Salmilehto

A doctoral dissertation completed for the degree of Doctor of Science (Technology) to be defended, with the permission of the Aalto University School of Science, at a public examination held at the lecture hall D (Otakaari 1, Espoo) of the school on 22 August 2014 at 12 o'clock noon.

Aalto University
School of Science
Department of Applied Physics
Quantum Computing and Devices

Supervising professor
Prof. Risto Nieminen

Thesis advisor
Doc. Mikko Möttönen

Preliminary examiners
Prof. Alexander Shnirman, University of Karlsruhe, Germany
Prof. Erkki Thuneberg, University of Oulu, Finland

Opponent
Prof. Steven Girvin, Yale University, USA

Aalto University publication series
DOCTORAL DISSERTATIONS 102/2014

ISBN 978-952-60-5767-5
ISBN 978-952-60-5768-2 (pdf)
ISSN-L 1799-4934
ISSN 1799-4934 (printed)
ISSN 1799-4942 (pdf)
http://urn.fi/URN:ISBN:978-952-60-5768-2

Unigrafia Oy
Helsinki 2014

Finland

Publication orders (printed book):
jsalmilehto@gmail.com

Aalto University

Abstract

Aalto University, P.O. Box 11000, FI-00076 Aalto www.aalto.fi

Author
Juha Salmilehto

Name of the doctoral dissertation
Control of Open Quantum Systems

Publisher School of Science

Unit Department of Applied Physics

Series Aalto University publication series DOCTORAL DISSERTATIONS 102/2014

Field of research Engineering Physics, Theoretical and Computational Physics

Manuscript submitted 13 May 2014 **Date of the defence** 22 August 2014

Permission to publish granted (date) 18 June 2014 **Language** English

☐ **Monograph** ☒ **Article dissertation (summary + original articles)**

Abstract

Accurate control of a quantum system is complicated to achieve partly due to the system being coupled to its surrounding environment. The coupling induces dissipation and decoherence not only destroying the coherent quantum state but making the application of control unpredictable. When the control results from manipulation of external fields, it is usually referred to as driving and its joint effect with decoherence constitutes an active field of study in reduced-density-operator theory. Recently, this field has been pushed forward by its necessity in simulating Cooper pair pumping where the geometric nature of quantum evolution allows for controlled transport of charge carriers in superconducting circuits. Such circuits themselves are under constant investigation as they grant access to fundamental quantum phenomena and facilitate promising applications such as those in quantum information processing.

In this dissertation, driven quantum systems under decoherence are investigated. Extensions and improvements to a recent master equation for nearly adiabatic driving are presented and analyzed. The emergent properties in quantum evolution are studied both analytically and numerically. Related to the findings, a general conservation law of operator current is derived and used to explain observed nonconservation in previous theoretical studies. The derived theory is applied to Cooper-pair pumping and shown to lead to important properties such as superadiabatic ground-state pumping. A recent pumping experiment is simulated and the breakdown point for ideal pumping is found with feasible physical parameters. A study of charge transport in the presence of flux noise is presented leading to detectable dissipative currents and the typical description of the device used for pumping is extended to include a nonvanishing loop inductance.

A novel approach to implement control of quantum systems is proposed based on constructing a tunable coupling to an artificial environment using either a coplanar waveguide cavity or coupled quantum LC resonators. Tunability allowing for both efficient initialization and protected evolution is theoretically demonstrated. Finally, a general framework for quantum driving is constructed without the typical assumption of a classical driving force leading to peculiar results.

This dissertation presents original research on both modeling the control of open quantum systems as well as the realization of such control. The work simulates physical phenomena in superconducting circuits and makes predictions for future experiments. In addition, it introduces novel theoretical tools and approaches that advance the state of the art.

Keywords Quantum system, decoherence, Cooper pair

ISBN (printed) 978-952-60-5767-5 **ISBN (pdf)** 978-952-60-5768-2

ISSN-L 1799-4934 **ISSN (printed)** 1799-4934 **ISSN (pdf)** 1799-4942

Location of publisher Helsinki **Location of printing** Helsinki **Year** 2014

Pages 177 **urn** http://urn.fi/URN:ISBN:978-952-60-5768-2

Aalto-yliopisto

Tiivistelmä

Aalto-yliopisto, PL 11000, 00076 Aalto www.aalto.fi

Tekijä
Juha Salmilehto

Väitöskirjan nimi
Avointen kvanttisysteemien hallinta

Julkaisija Perustieteiden korkeakoulu

Yksikkö Teknillisen fysiikan laitos

Sarja Aalto University publication series DOCTORAL DISSERTATIONS 102/2014

Tutkimusala Teknillinen fysiikka, Teoreettinen ja laskennallinen fysiikka

Käsikirjoituksen pvm 13.05.2014 **Väitöspäivä** 22.08.2014

Julkaisuluvan myöntämispäivä 18.06.2014 **Kieli** Englanti

☐ **Monografia** ☒ **Yhdistelmäväitöskirja (yhteenveto-osa + erillisartikkelit)**

Tiivistelmä

Kvanttisysteemin tarkan hallinnan tekee osaltaan monimutkaiseksi sen kytkeytyminen ympäristöönsä. Tämä kytkentä aiheuttaa dissipaatiota ja dekoherenssia, jotka eivät vain tuhoa koherenttia kvanttitilaa vaan tuovat hallintaan arvaamattomuutta. Kun hallinta on seurausta ulkoisten kenttien käsittelystä, sitä yleensä kutsutaan ajoksi ja sen yhteisvaikutusta dekoherenssin kanssa tutkitaan aktiivisesti redusoidun tiheysoperaattorin teoriassa. Hiljattain tätä tutkimusaihetta on edistänyt sen tarpeellisuus Cooperin parien pumppauksen simuloinnissa. Tässä fysikaalisessa ilmiössä kvanttiaikakehityksen geometrinen luonne mahdollistaa varauksenkuljettajien hallitun siirron suprajohtavissa piireissä. Tällaiset piirit itsessään ovat jatkuvan tutkimuksen alaisina, sillä ne luovat kosketuspinnan perustavanlaatuisiin kvantti-ilmiöihin ja toimivat perustana lupaaville sovelluksille vaikkapa kvantti-informaation käsittelyssä.

Väitöskirjassa tutkitaan dekoherenssille altistuneita ajettuja kvanttisysteemejä. Työssä esitetään laajennuksia ja parannuksia melkein adiabaattisen ajon mestariyhtälöön, ja tutkitaan vastaavia kvanttimekaanisessa aikakehityksessä ilmeneviä ominaisuuksia sekä analyyttisesti että numeerisesti. Löydöksiin liittyvä yleinen operaattorivirran säilymislaki johdetaan, ja sitä käytetään selittämään havaittu säilymättömyys aiemmissa teoreettisissa tutkimuksissa. Tätä teoriaa sovelletaan Cooperin parien pumppaukseen, ja sen näytetään johtavan tärkeisiin ominaisuuksiin. Työssä simuloidaan hiljattain julkaistua pumppauskoetta ja mallinnetaan ihanteellisen pumppauksen hajoamispiste. Varauksensiirtoa tutkitaan myös vuokohinan vaikutuksen alaisena, ja sen huomataan johtavan havaittavissa oleviin dissipatiivisiin virtoihin. Lisäksi väitöskirjassa laajennetaan tyypillistä pumppaukseen käytettävän systeemin kuvausta ottamalla huomioon nollasta poikkeava silmukkainduktanssi.

Työssä esitellään uudenlainen lähestymistapa kvanttisysteemien hallinnan toteuttamiseen. Se pohjautuu säädettävään kytkentään keinotekoisen ympäristön kanssa käyttäen joko koplanaarisen aaltojohtimen kaviteettia tai kvanttimekaanisia LC-resonaattoreita. Teoreettisella tarkastelulla osoitetaan, että säädettävyys mahdollistaa sekä tehokkaan alustamisen että suojatun kehityksen. Lopulta työssä kehitellään kvanttiajon yleinen teoria, joka ei käytä tyypillistä klassisen ajavan voiman oletusta, ja sen huomataan johtavan erityislaatuisiin tuloksiin.

Väitöskirja esittelee uusia tuloksia liittyen hallinnan mallintamiseen ja toteuttamiseen avoimissa kvanttisysteemeissä. Työssä simuloidaan fyysisiä ilmiöitä suprajohtavissa piireissä ja ennustetaan kokeellista käytöstä. Lisäksi siinä esitellään uusia teoreettisia työkaluja ja lähestymistapoja, jotka edistävät nykyistä huipputietämystä.

Avainsanat Kvanttisysteemi, dekoherenssi, Cooperin pari

ISBN (painettu) 978-952-60-5767-5 **ISBN (pdf)** 978-952-60-5768-2

ISSN-L 1799-4934 **ISSN (painettu)** 1799-4934 **ISSN (pdf)** 1799-4942

Julkaisupaikka Helsinki **Painopaikka** Helsinki **Vuosi** 2014

Sivumäärä 177 **urn** http://urn.fi/URN:ISBN:978-952-60-5768-2

Preface

The work presented in this dissertation was performed during the years 2010–2014 in the Quantum Computing and Devices group at the Department of Applied Physics in Aalto University School of Science. During this time, the group has moved a few times, I have moved a few times, and people have popped in and out of my daily life.

I would like to thank Prof. Risto Nieminen for acting as the supervisor for this dissertation and for his important work in generating the daily research infrastructure. My instructor Doc. Mikko Möttönen has been paramount to the completion of this dissertation and I owe him a huge debt of gratitude for everything during these years. I truly appreciate the daily injections of ideas and guidance administered face-to-face. I want to thank Dr. Paolo Solinas and Prof. Jukka Pekola for fruitful collaboration that has carried on to this date. The weekly meetings at the beginning of my doctoral studies were an interesting motivational booster.

The intense collaboration with Dr. Philip Jones that engulfed a few weeks completely is definitely worth thanking him for along with the other co-authors Dr. Jukka Huhtamäki and Dr. Kuan Yen Tan. I also appreciate the conversations with all of them that stemmed from working in the same research group. To that end, I wish to thank some of the current and former members of the QCD group not yet mentioned—M.Sc. Emmi Ruokokoski, Dr. Pekko Kuopanportti, Dr. Harri Mäkelä, Dr. Russel Lake, M.Sc. Joonas Govenius, Dr. Ville Pietilä, M.Sc. Tuomo Tanttu, Dr. Ivan Savenko, Dr. Elsi-Mari Laine, M.Sc. Janne Kokkala, and M.Sc. Konstantin Tiurev—for making a relaxed working environment. I also wish to thank Dr. Joachim Ankerhold for collaboration on one of the publications. Finally, I am thankful for the interesting discussions with Dr. Youngsoo Yoon, M.Sc. Simone Gasparinetti, Prof. Tapio Ala-Nissilä, M.Sc. Samu Suomela, and M.Sc. Aki Kutvonen.

I wish to thank the Vilho, Yrjö and Kalle Väisälä Foundation for the three annual grants for doctoral studies they awarded me with, and greatly appreciate the smaller grants from the Emil Aaltonen Foundation, the KAUTE Foundation, and the National Graduate School in Materials Physics. I acknowledge the Aalto School of Science for computing resources—namely, access to the Triton cluster.

Espoo, June 26, 2014,

Juha Salmilehto

Contents

List of Publications

This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.

I P. Solinas, M. Möttönen, J. Salmilehto, and J. P. Pekola. Decoherence of adiabatically steered quantum systems. *Physical Review B*, **82**, 134517 (12 pages), October 2010.

II J. Salmilehto, P. Solinas, J. Ankerhold, and M. Möttönen. Adiabatically steered open quantum systems: Master equation and optimal phase. *Physical Review A*, **82**, 062112 (7 pages), December 2010.

III J. Salmilehto and M. Möttönen. Superadiabatic theory for Cooper pair pumping under decoherence. *Physical Review B*, **84**, 174507 (11 pages), November 2011.

IV P. Solinas, M. Möttönen, J. Salmilehto, and J. P. Pekola. Cooper-pair current in the presence of flux noise. *Physical Review B*, **85**, 024527 (9 pages), January 2012.

V J. Salmilehto, P. Solinas, and M. Möttönen. Conservation law of operator current in open quantum systems. *Physical Review A*, **85**, 032110 (5 pages), March 2012.

VI J. Salmilehto and M. Möttönen. Quantum effect of inductance on geometric Cooper-pair transport. *Physical Review B*, **86**, 184512 (11 pages), November 2012.

VII P. J. Jones, J. A. M. Huhtamäki, J. Salmilehto, K. Y. Tan, and M. Möttönen. Tunable electromagnetic environment for superconducting quantum bits. *Scientific Reports*, **3**, 1987 (7 pages), June 2013.

VIII P. J. Jones, J. Salmilehto, and M. Möttönen. Highly Controllable Qubit-Bath Coupling Based on a Sequence of Resonators. *Journal of Low Temperature Physics*, **173**, 152 (18 pages), July 2013.

IX J. Salmilehto, P. Solinas, and M. Möttönen. Quantum driving and work. *Physical Review E*, **89**, 052128 (7 pages), May 2014.

Author's Contribution

Publication I: "Decoherence of adiabatically steered quantum systems"

The author produced analytical and numerical results to justify the findings of the article. He actively participated in analyzing the findings and in the writing of the article.

Publication II: "Adiabatically steered open quantum systems: Master equation and optimal phase"

The author produced the theoretical results, analyzed them with his co-authors, and wrote the article.

Publication III: "Superadiabatic theory for Cooper pair pumping under decoherence"

The author produced the theoretical and computational results, analyzed them with his co-author, and wrote the article.

Publication IV: "Cooper-pair current in the presence of flux noise"

The author produced analytical and computational results to justify the findings of the article. He actively participated in analyzing the findings and in the writing of the article.

Publication V: "Conservation law of operator current in open quantum systems"

The author produced the theoretical results and wrote the article.

Publication VI: "Quantum effect of inductance on geometric Cooper-pair transport"

The author deviced some of the theoretical means used, produced the analytical and computational results, analyzed them with his co-author, and wrote the article.

Publication VII: "Tunable electromagnetic environment for superconducting quantum bits"

The author actively participated in deriving and analyzing the results with his co-authors with whom he also prepared the article.

Publication VIII: "Highly Controllable Qubit-Bath Coupling Based on a Sequence of Resonators"

The author deviced some of the theoretical tools needed, derived and analyzed the results with his co-authors, and wrote the article.

Publication IX: "Quantum driving and work"

The author produced most of the theoretical and computational results, and wrote the article.

1. Introduction

When the quantum theory was conjured into existence to account for the wisping of electrons around atoms back in the 1920's [1–3], the idea of accurately controlling systems governed by the obscure and unintuitive laws probably seemed like an insurmountable task. In the course of the following decades, the theory claimed its position as the definite means of describing the dynamics of nanoscale systems [4, 5] and spawned some of the basic building blocks of the modern society such as the complex information processing devices based on semiconductor physics [6] that we now depend on. However, there is a difference between exploiting quantum phenomena and truly controlling a quantum system. In a broad sense, control at the quantum level is something one applies to the corehent quantum state rather than to the ensemble properties stemming from quantum behavior. As a result, the biggest enemy of control in quantum systems is decoherence, the destruction of coherence due to the ubiquitous coupling to the surrounding environment [7, 8]. Such quantum systems are usually referred to as *open*.

The concept of driving a quantum system refers to the direct control of its evolution due to the introduction of external time-dependent fields. Such control requires extreme accuracy of the externally applied action and a profound understanding of how the intrigate quantum degrees of freedom are subsequently affected. In recent years, superconducting circuits [9, 10] have emerged as the prime testbed for driven dynamics facilitating complex designs and rapid advance in quantum information processing [11, 12]. They are fundamentally based on the development of the theory of superconductivity [13] and the discovery of the Josephson effect [14, 15] which enable the construction of macroscopic solid-state devices acting as artificial atoms. The devices can be manipulated by externally tuning the characteristic energy scales of the circuit elements

yielding direct access to the quantum state. In addition, superconducting circuits are at the heart of circuit quantum electrodynamics [16, 17] where the coupling between the quantum mechanical cavity modes and other circuit elements introduces additional means of control.

Cooper-pair pumping—the controlled transport of the charge carriers in superconducting devices—has been under extensive review after its connection to the Berry phase accumulated during adiabatic evolution was discovered [18]. A similar connection can be drawn between superconducting circuits and geometric quantum computing [19] in general implying that such circuits can potentially be used for this inherently robust computing scheme. The resulting interest in simulating Cooper-pair pumping has advanced the theory of driven dissipative dynamics starting from the breakthrough findings in Ref. [20] where a novel approach was shown to correctly yield important properties in quantum evolution visible in the pumped charge. Moreover, fluctuation relations for injected work in driven quantum systems [21] have been proposed to be analyzed using superconducting devices, for example, in Refs. [22, 23]. However, such fluctuation relations as well as the current description of driven quantum systems in general implicitly assume an external classical driving force. This potentially limits their range of validity.

This overview has the following structure. Chapter 2 introduces driven quantum systems and discusses the resulting dissipative dynamics. It especially elaborates on the nearly adiabatic driving and discusses the importance of a proper basis selection when formulating a master equation for the reduced-system dynamics. This leads to a study of the combined effect of driving and dissipation as well as motivates a general conservation law of operator current in open quantum systems. Chapter 3 is devoted to applying the derived master equations to superconducting nanocircuits and, more spesifically, to Cooper-pair pumping. The general characteristics of dissipative pumping are explored along with a few important extensions of the theory. In Chapter 4, the methods of circuit quantum electrodynamics are used to engineer a tunable coupling to an artificial environment introducing an additional level of control. Chapter 5 generalizes the previously used concept of driving a quantum system based on external classical fields to a fully quantum description, and investigates the resulting definition of injected work. Chapters 2–5 each begin with a brief introduction to motivate the respective topic and include discussion of the obtained results. Finally, Chapter 6 summarizes the main results

of the dissertation and elaborates on how they contribute to scientific understanding. It introduces further research motivated by the results and proposes future areas of study.

2. Dissipative Dynamics of Driven Quantum Systems

The standard approximations used in the master equation approach to incorporating dissipation into quantum systems fail when the system is externally driven. This chapter goes beyond such approximations and elaborates on the proper inclusion of driving into open system dynamics. Section 2.1 introduces the main concepts and we discuss the details in the nearly adiabatic limit using the results of Publications I–III in Sec. 2.2. The possibly emerging nonphysicality is discussed in Sec. 2.3 on a more general level with the help of Publication V.

2.1 External control of open quantum systems

In this dissertation, one of our focus points is open quantum systems. Such systems are best understood through an averaging process taking us from a complete dynamics to a reduced one. Assume a total quantum system whose dynamics are completely determined by a Hamiltonian $\hat{H}$ yielding the unitary time evolution of the density operator $\hat{\rho}$ through the Schrödinger equation [4]. In principle, such total system accounts for all possible degrees of freedom including those of the measuring device and the thermal bath in an experimental setup. Typically we are only interested in the evolution within certain degrees of freedom making up a confined subsystem and the total evolution is beyond our grasp both theoretically and experimentally. This interest brings about the concept of reduced dynamics where we average over the undesired degrees of freedom to obtain the reduced density operator $\hat{\rho}_S = \mathrm{Tr}_E\{\hat{\rho}\}$ in which the complete information of the system of interest is stored. The notation Tr_E denotes a trace[1] over the remaining degrees of freedom usually referred to as the

[1]The trace of an arbitrary operator $\hat{x}$ is defined as $\mathrm{Tr}_E\{\hat{x}\} = \sum_n \langle n|\hat{x}|n\rangle$, where $\{|n\rangle\}$ is a complete orthonormal set in the Hilbert space of E.

environment E. Note that the open quantum system described by $\hat{\rho}_S$ no longer evolves unitarily as it accounts for both the internal dynamics of the system as well as the interaction with the environment. There exists a myriad of methods for accessing the dynamics of open quantum systems [24, 25] and after applying a set of approximations, usually related to the nature of the system–environment interaction and the properties of the environment, such dynamics can typically be described with some accuracy.

This dissertation studies open quantum systems using the so-called master equation (ME) approach referring to the formulation of an equation of motion for the reduced density. Even though the exact dynamics are formally obtained from the Liouville-von Neumann equation for the total system as $d\hat{\rho}_S/dt = -i\mathrm{Tr}_E\{[\hat{H},\hat{\rho}]\}/\hbar$, the intricacies of actually approximating this in a useful manner are quite challenging. We concentrate on the challenge that arises from accounting for the external control of the system. This control is due to external time-dependent fields generating a perturbative action on the system and manifests itself as driving through the time-dependence of the system Hamiltonian $\hat{H}_S$[2]. Here, $\hat{H}_S$ is defined as the self-Hamiltonian of the open system S corresponding to the environment–decoupled evolution. The total Hamiltonian may generally be taken of the form

$$\hat{H}(t) = \hat{H}_S(t)\otimes\hat{I}_E + \hat{I}_S\otimes\hat{H}_E + \hat{V}(t), \tag{2.1}$$

where we explicitly denote the parametric time-dependence due to the drive. Here $\hat{I}_S$ ($\hat{I}_E$) is the identity operator in the Hilbert space of the system (environment) and $\hat{V}(t)$ is the system–environment coupling operator. Equation (2.1) enables the use of the interaction picture for the von-Neumann equation such that the reduced density obeys

$$\frac{d}{dt}\hat{\rho}_{S,I}(t) = -\frac{i}{\hbar}\mathrm{Tr}_E\{[\hat{V}_I(t),\hat{\rho}_I(0)]\} - \frac{1}{\hbar^2}\int_0^t ds\mathrm{Tr}_E\{[\hat{V}_I(t),[\hat{V}_I(s),\hat{\rho}_I(s)]]\}, \tag{2.2}$$

where $\hat{\rho}_{S,I} = \mathrm{Tr}_E\{\hat{\rho}_I\}$ and the subscript I denotes that the corresponding operator is in the interaction picture [4]. Even though the presentation in Eq. (2.2) is exact, the appearance of the total density operator on its right-hand side does not render it immediately useful. However, it serves

[2]This definition strictly speaking refers to *classical driving* of a quantum system. Chapter 5 and Publication IX are dedicated to formulating driving from a fully quantum point-of-view.

as a convenient starting point for many perturbative approaches where, typically, the system–bath coupling is assumed weak like in the case of the Redfield equation [26].

More to the point, it turns out that the subsequent approximative dynamics based on a further perturbative expansion in the coupling strength neglects important physics when the system is driven. In Sec. 2.2, we explain how this issue is related to an appropriate basis selection and can be alleviated by an additional coordinate transformation step prior to implementing the expansion.

2.2 Nearly adiabatic driving under decoherence

For our purposes, we define adiabaticity in the traditional closed-system sense of Refs. [27,28] related to the temporal change of the instantaneous eigenstates and the local energy gap even though viable open-system extensions have recently appeared [29–31] and consistently applied to quantum information processing [32] and geometric phase evolution [33]. The main motivation for inserting the adiabatic assumption to the control scheme is generally three-fold: concepts such as geometric phase accumulation [34–36] and holonomic [37,38] as well as adiabatic [39–42] quantum computing are well-formulated, the description of the system dynamics obtains simplifications that allow for both more accuracy and accessibility [43,44], and adiabaticity typically provides a level of protection against environmental and parametric noise [45–49].

Even though the protection against control errors [45, 46, 50, 51] seems like the most prominent feature to strive towards with regards to quantum information processing, the requirement of adiabaticity forces a restriction on the use of control protocols. Adiabaticity generally requires that the evolution time is long with respect to the time scales of the open system which potentially leads to decoherence and dissipation destroying the quantum state. An accurate theory of driving and decoherence in the nearly adiabatic limit is required to assess and adapt the adiabatic control schemes.

2.2.1 Motivation

The standard method for treating time-independent systems weakly coupled to the environment relies on applying the Born-Markov approxima-

tion[3] on Eq. (2.2) to obtain the so-called Markovian quantum master equation [25]. A subsequent application of the secular approximation [52–54] to neglect the fast rotating terms[4] in the master equation results in the celebrated Lindblad form [55, 56] guaranteeing certain critical properties of the quantum evolution. Most notably, it guarantees complete positivity [55] and allows for efficient numerical methods such as the quantum trajectory theory [25] to be immediately applied. However, a straightforward application to time-dependent control by external fields potentially leads to issues with conservation of observables during the time-evolution [57, 58] and the dissipation having a non-physical effect on the reduced density [20]. A multitude of approaches was applied to combine driving with dissipation [45, 47, 59–67] until a seminal paper in 2010 by J. P. Pekola *et al.* [20] presented a master equation for an adiabatically steered two-level quantum system coupled to a Markovian bath. In particular, it showed that the non-secular terms are necessary to consistently account for the combined effect of driving and dissipation after a transformation to the adiabatic basis is performed to treat the time-dependence of the system Hamiltonian.

2.2.2 Adiabatic basis selection

In Publication I, we generalize the time-local master equation in Ref. [20] to allow for a generic system–environment coupling operator[5]. To account for the time-dependence of the system Hamiltonian, we apply the first transformation in the adiabatic renormalization procedure [68, 69] and study the dynamics of the transformed density in the fixed basis. To understand the basic procedure, we briefly introduce the idea. Let the Hamiltonian $\hat{H}_S(t)$ be diagonalized in a time-independent basis $\{|m\rangle\}$ using the eigendecomposition as $\hat{\tilde{H}}_S^{(1)}(t) = \hat{D}_1^\dagger(t)\hat{H}_S(t)\hat{D}_1(t)$ corresponding to the eigenproblem $\hat{H}_S(t)\,|m^{(1)}(t)\rangle = E_m^{(1)}(t)\,|m^{(1)}(t)\rangle$ where $\hat{D}_1\,|m\rangle = |m^{(1)}(t)\rangle$ is normalized and nondegenerate for each m. The fixed states

[3] The Born-Markov approximation implements a weak system–environment coupling and the assumption of a memoryless environment. The latter effectively corresponds to temporal coarse graining over times of the order of magnitude of the environment correlation time. See Refs. [24, 25] for details.

[4] The secular approximation essentially corresponds to neglecting terms in the master equation that oscillate fast with respect to the interaction dynamics as they vanish when averaged over [25].

[5] The master equation in Ref. [20] assumes that $\langle g|\hat{Y}|e\rangle \in \mathbb{R}$ where $\hat{Y}$ is the system part of the system–environment coupling operator and $|g\rangle$ and $|e\rangle$ are the instantaneous eigenstates of $\hat{H}_S$ in the two-state representation. This assumption only applies for certain types of coupling operators and is omitted in the derivation of the master equation in Publication I.

$\{|m\rangle\}$ are typically referred to as diabatic. If we similarly transform the total density operator $\hat{\rho}(t)$ in the Schrödinger picture as $\hat{\tilde{\rho}}^{(1)}(t) = \hat{D}_1^\dagger(t)\hat{\rho}(t)\hat{D}_1(t)$, the evolution of $\hat{\tilde{\rho}}^{(1)}(t)$ is governed by the effective Hamiltonian presented in Publications I-III as

$$\hat{\tilde{H}}^{(1)}(t) = \hat{\tilde{H}}_S^{(1)}(t) + \hbar\hat{w}_1(t) + \hat{\tilde{V}}^{(1)}(t) + \hat{H}_E, \tag{2.3}$$

where the transformed coupling operator is $\hat{\tilde{V}}^{(1)}(t) = \hat{D}_1^\dagger(t)\hat{V}(t)\hat{D}_1(t)$ and driving manifests explicitly in the operator $\hat{w}_1(t) = -i\hat{D}_1^\dagger(t)\dot{\hat{D}}_1(t)$. Starting from Eq. (2.2) in the transformed basis, we obtain the integral form of the master equation derived in Ref. [20] and restated in Publication II as

$$\begin{aligned}\frac{d\hat{\tilde{\rho}}_{S,I}^{(1)}(t)}{dt} = {} & i[\hat{\tilde{\rho}}_{S,I}^{(1)}, \hat{w}_{1,I}(t)] - \frac{1}{\hbar^2}\mathrm{Tr}_E\left\{\int_0^t dt'[[\hat{\tilde{\rho}}_I^{(1)}(t), \hat{\tilde{V}}_I^{(1)}(t')], \hat{\tilde{V}}_I^{(1)}(t)]\right\} \\ & + \frac{i}{\hbar^2}\mathrm{Tr}_E\left\{\int_0^t dt'\int_0^{t'} dt''[[\hat{\tilde{\rho}}_I^{(1)}(t), [\hat{w}_{1,I}(t'), \hat{\tilde{V}}_I^{(1)}(t'')]], \hat{\tilde{V}}_I^{(1)}(t)]\right\},\end{aligned} \tag{2.4}$$

where an arbitrary operator in the interaction picture is defined as $\hat{O}_I(t) = e^{i\hat{H}_E t/\hbar}\hat{U}_S^\dagger(t,0)\hat{O}(t)\hat{U}_S(t,0)e^{-i\hat{H}_E t/\hbar}$ and $\hat{U}_S(t,0) = \mathcal{T}e^{-i/\hbar\int_0^t d\tau\hat{\tilde{H}}_S^{(1)}(\tau)}$, where $\mathcal{T}$ denotes time-ordering. It should be noted that the perturbative expansion has been cut so that Eq. (2.4) is accurate up to the second order in the system-evironment coupling strength and to the first order in the local adiabatic parameter $\alpha_1(t) = \hbar||\hat{w}_1(t)||/\Delta(t)$, where $||\hat{w}_1(t)|| = \sqrt{\mathrm{Tr}_S\{\hat{w}_1(t)^\dagger\hat{w}_1(t)\}}$ denotes the Hilbert-Schmidt norm and $\Delta(t)$ is the instantaneous minimum energy gap in the unperturbed system spectrum. The first term after the equality is the decoupled driving term, the second term yields the standard Bloch-Redfield dissipative contribution [25], and the third term combines the effects of the drive and the dissipation, usually neglected in the typical approach [53].

In Publication I, we further take a two-level system and derive a time-local master equation for a generic system–environment coupling operator by assuming that the system is in the Markov regime, the approximation of adiabatic rates applies and the corrections due to the Lamb shift can be neglected. These assumptions are detailed in Publication I and especially in Publication II where we explore the time-scale separation they generate. Rather than presenting the somewhat complicated time-local master equation for the two-level system in this overview, we refer the interested reader to Publications I and II for details. Publication II also

presents an alternative derivation of the time-local master equation by studying a master equation for a nonsteered system and applying higher-order transformations in the renormalization chain. This is potentially more efficient when analytically deriving higher-order master equations than the original method which relies on the treatment of complicated integral expressions arising from Eq. (2.4). One should keep in mind that the master equation derived in Publications I and II is strictly valid for nearly adiabatic evolution and weak system–environment coupling but maintains the cross term between driving and dissipation as well as includes all the non-secular terms traditionally neglected [52]. As a result, the master equation is not in the standard Lindblad form. We study the implications of this inclusion in Sec. 2.2.4.

2.2.3 Further transformations and derivations

In Publication III, we go further in the adiabatic renormalization procedure [68, 69] and derive a master equation for an arbitrary number of basis transformations. The explicit form of the corresponding time-local two-level master equation is given in Publication III and we note that it corresponds to that in Publications I and II in the case of a single transformation. After n successive coordinate transformations $\hat{D}_n$ are performed[6], the corresponding density operator takes the form $\hat{\tilde{\rho}}^{(n)} = (\hat{D}_S^{(n)})^\dagger \hat{D}_1^\dagger \hat{\rho} \hat{D}_1 \hat{D}_S^{(n)}$, where $\hat{D}_S^{(n)} = \prod_{i=2}^n \hat{D}_i$ for $n \geq 2$ and $\hat{D}_S^{(n)} = \hat{I}_S$ for $n = 1$, where the operator product is defined as $\prod_{i=2}^n \hat{D}_i = \hat{D}_2 \hat{D}_3 \cdots \hat{D}_{n-1} \hat{D}_n$ and $\hat{I}_S$ is the identity operator acting in the system space. The evolution of $\hat{\tilde{\rho}}^{(n)}$ is subsequently governed by an effective Hamiltonian of the form

$$\hat{\tilde{H}}^{(n)} = \hat{\tilde{H}}_S^{(n)} + \hbar \hat{w}_n + \hat{\tilde{V}}^{(n)} + \hat{H}_E, \tag{2.5}$$

where $\hat{\tilde{H}}_S^{(n)} = \hat{D}_n^\dagger [\hat{\tilde{H}}_S^{(n-1)} + \hbar \hat{w}_{n-1}] \hat{D}_n$, $\hat{\tilde{V}}^{(n)} = (\prod_{i=2}^n \hat{D}_i)^\dagger \hat{D}_1^\dagger \hat{V} \hat{D}_1 (\prod_{i=2}^n \hat{D}_i)$ and $\hat{w}_n = -i \hat{D}_n^\dagger \dot{\hat{D}}_n$. Note that each coordinate transformation defines a set of time-dependent basis states $\{|m^{(n)}\rangle = \hat{D}_1 \hat{D}_S^{(n)} |m\rangle\}$ which better approximate the closed-system evolution in the near-adiabatic limit in the sense that driving-induced transitions between the states are suppressed. These states correspond to the eigenstates of $\hat{H}_S$ for $n = 1$ typically referred to as the adiabatic states, and to the eigenstates of $\hat{D}_1 \hat{D}_S^{(n-1)} [\hat{\tilde{H}}_S^{(n-1)} + \hbar \hat{w}_{n-1}] (\hat{D}_1 \hat{D}_S^{(n-1)})^\dagger$ for $n \geq 2$ referred to as the $(n-1)$th-

[6] The transformations follow the standard iterative procedure [68, 69] and are detailed in Publication III.

order superadiabatic states.

After applying the same set of approximations as in Sec. 2.2.2, we obtain a master equation given in Publication III up to quadratic order in the system–environment coupling and to the first order in the nth superadiabatic parameter $\alpha_n(t) = ||\dot{\hat{w}}_n(t)||/\omega_{01}^{(n)}(t)$, where the superadiabatic energy gap is $E_e^{(n)} - E_g^{(n)} = \hbar\omega_{01}^{(n)}$ such that $E_e^{(n)} = \langle 1|\hat{\tilde{H}}_S^{(n)}|1\rangle$ and $E_g^{(n)} = \langle 0|\hat{\tilde{H}}_S^{(n)}|0\rangle$. Here n denotes the number of basis transformations in the renormalization scheme and $\{|0\rangle, |1\rangle\}$ denotes the selected diabatic basis for the two-level system. As the time-dependence of the transformed system Hamiltonian is suppressed with increasing n, α_n respectively decreases[7] implying that the corresponding nth-order master equation describes the dynamics more accurately. We will study the implications of this more precisely in Sec. 2.2.4.

Nearly adiabatically driven systems weakly coupled to an environment have since been studied using the same perturbative methodology by replacing the basis transformations defined above by those related to modified Floquet modes. Essentially this line of thinking constitutes to being able to describe the closed-system evolution exactly with the price of additionally asserting that the driving is cyclic, i.e., the system Hamiltonian has a periodic time-dependence [70–72].[8] This has spawned a series of papers ultimately aiming to improve the description [77–81] by a proper selection of the modes. Whether the secular approximation is performed during the derivation and whether it is performed fully or partially generally determines if the Lindblad form is obtained and which properties of the reduced system one can expect to retrieve in a physically feasible manner. Modified superadiabatic states have also been used in Ref. [82] in association with the full secular approximation to capture the main features of dissipative Landau–Zener transitions [83] and the addition of the Lamb shift has been studied in detail in Ref. [84]. Finally, we note that the master equations derived in Publications I–III allow for an arbitrary phase selection of the basis states. We tackle this issue in Publication II by introducing an optimal phase selection procedure for the adiabatic basis states which minimizes the local adiabatic parameter at all times and leads to a simple renormalization of some matrix elements in the

[7] This possibly only applies to a certain n after which the following basis rotations no longer offer further improvement [34,68]. However, this point was not reached in any of the numerical analysis in this dissertation.

[8] The Floquet theory provides convenient physical insight into the effect of the driving field and has been extensively used to study the driven dynamics of closed systems in the past. We direct the reader to Refs. [73–76] and references therein for recent advances in the population dynamics of driven quantum systems.

corresponding master equation. The procedure is closely related to the selection of basis states in Ref. [82] and can be straightforwadly adapted to higher-order bases in the renormalization scheme.

2.2.4 Interplay between driving and dissipation

The basis selection procedure and the corresponding master equations derived in Publications I–III and introduced in Secs. 2.2.2 and 2.2.3 allow for an insight into the combined effect of driving and dissipation. More accurately, they allow us to consider the best possible time-dependent basis for describing the dissipation. Considering a zero-temperature environment offers a physical viewpoint: The norm of $\hat{w}_1$ is roughly inversely proportional to the evolution time and, consequently, usually neglected in Eq. (2.3) for adiabatic evolution [32]. This means that the basis transformation is neglected in the perturbative calculation of the dissipative rates [20, 45] and, consequently, the relaxation is to the instantaneous ground state while the transformation induces excitations. The resulting state does not correspond to either the adiabatic or superadiabatic states and, thus, is not physically justified. On the contrary, Eq. (2.3) suggests that the relaxation is to the first superadiabatic ground state which, up to the second order in α_1, approximates the closed-system evolution. Up to this order, the master equation is given in Eq. (2.4) and necessarily includes the cross-term. The inclusion of the cross-term has also been previously studied in Ref. [53].

Let us adopt the two-state notation such that $\rho_{gg}^{(n)} = \langle 0|\hat{\tilde{\rho}}_S^{(n)}|0\rangle$, $\rho_{ge}^{(n)} = \langle 0|\hat{\tilde{\rho}}_S^{(n)}|1\rangle$, and $w_{ge}^{(n)} = -i\,\langle 0|\hat{D}_n^\dagger \dot{\hat{D}}_n|1\rangle$. In Publication I, we derive the solution of the time-local master equation in the quasistationary limit and, especially, show that assuming a zero-temperature environment results in $\rho_{gg}^{(1)} = 1 + O(\alpha_1^2)$ and $\rho_{ge}^{(1)} = -w_{ge}^{(1)}/\omega_{01}^{(1)} + O(\alpha_1^2)$ indicating that the ground-state evolution is not affected by the zero-temperature environment. The result in the adiabatic basis translates to $\rho_{gg}^{(2)} = 1 + O(\alpha_1^2)$ and $\rho_{ge}^{(2)} = 0 + O(\alpha_1^2)$ showing that in the first order in α_1, the density matrix $\hat{\tilde{\rho}}^{(2)}$ describes the evolution of a pure state. This is a remarkable result validating that the master equation in Publications I and II ensures relaxation to $|g^{(2)}\rangle$ up to the first order in α_1. Similarly, the master equation in Publication III for $\hat{\tilde{\rho}}^{(n)}$ ensures that the relaxation takes the system to $|g^{(n+1)}\rangle$ up to the first order in α_n. Especially, in the limit $n \to \infty$, the rotational terms $w_{kl}^{(n)}$, $k, l \in \{g, e\}$, in the master equation vanish and the basis $\{|g^{(n)}\rangle, |e^{(n)}\rangle\}|_{n\to\infty}$ fully describes the steering as-

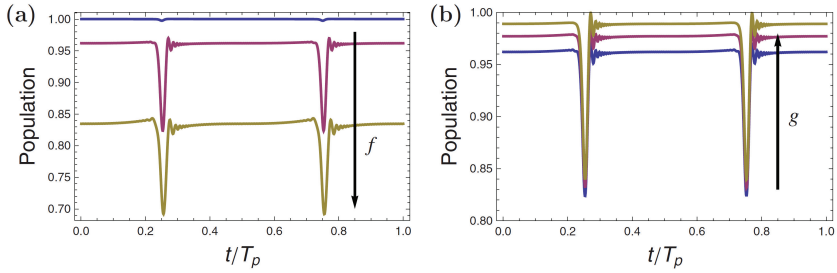


Figure 2.1. Population of the first superadiabatic ground state of the Cooper-pair sluice coupled to a zero-temperature environment in the quasistationary limit using the master equation derived in Publications I and II. (a) Time-evolution with coupling strength $g = 0.01$ and driving frequency $f = 10$, 75 and 100 MHz from top to bottom. (b) Time-evolution for $f = 75$ MHz and $g = 0.01$, 0.0125, and 0.015 from bottom to top. Reprinted from Publication I with permission.

suming that the process of basis rotations converges[9]. This corresponds to the physical intuition that the dissipation takes place between states best approximating the closed-system state. Beyond the strict adiabatic limit, the zero-temperature environment has a stabilizing effect on the ground state evolution. This is illustrated in Fig. 2.1 using the time-local master equation in Publication I for the so-called Cooper-pair sluice[10] in a zero-temperature environment. Note how increasing the pumping frequency f, which corresponds to increasing the driving speed, decreases the population of the first superadiabatic ground state in Fig. 2.1(a) due to more prominent driving-induced transitions whereas increasing the environmental coupling strength g in Fig. 2.1(b) stabilizes the ground-state evolution due to the described effect.

Crucially, we include all the nonsecular contributions to the time-local master equations in Publications I-III. For comparison, we also implement the secular approximation in Publication I and show that the corresponding master equation has a quasistationary limit solution $(\rho_{gg}^{(1)})^{\mathrm{sec}} = 1+O(\alpha_1^2)$ and $(\rho_{ge}^{(1)})^{\mathrm{sec}} = -2iw_{ge}^{(1)}/(2i\omega_{01}^{(1)}-\Gamma)+O(\alpha_1^2)$ where Γ is a joint dissipation rate presented in Publication I. The off-diagonal density matrix element is directly affected by the coupling to the environment which contradicts the physical picture discussed above and results in an artificial loss of robustness of the ground-state evolution. Especially, neglecting the nonsecular terms has an effect on the physical observables dependent on $\rho_{ge}^{(1)}$ which potentially leads to nonconservation of physical quantities. We will cover this more generally in Sec. 2.3.

[9] As decribed earlier, the renormalization scheme does not necessarily offer improvements after a certain n. The convergence is a complex phenomenon studied in Refs. [34, 68].

[10] We will describe the system in detail in Sec. 3.

As mentioned briefly in Sec. 2.2.2, the master equation derived in Publications I and II and generalized in Publication III is not of the Lindblad form [55, 56]. This is due to the inclusion of the non-secular terms and, indeed, the secular counterpart derived in Publication I can be shown to be of the Lindblad form. As a result, our non-secular master equation does not guarantee complete positivity which may lead to nonphysical reduced density. In Publication III, we show that this manifests as negative eigenvalues of the reduced density operator for the Cooper-pair sluice. However, utilizing superadiabatic bases is found to alleviate the nonphysicality. This is because the superadiabatic bases track the exact evolving closed-system state more accurately causing the nonadiabatic transitions to disturb the mixed state less. In general, including the nonsecular terms involves a trade-off: the secular approximation yields a positive map and, in this sense, approximates the density accurately but may neglect important contributions related to the dynamics of physical observables [78,82].

2.3 Conservation laws in open systems

As we briefly mentioned in Sec. 2.2.4, implementing the secular approximation in the derivation of the reduced dynamics potentially leads to non-conservation of important physical observables. This has manifested, for example, as the non-conservation of electric charge in transport through quantum dots [57, 58] and in Cooper-pair pumping [20]. We also directly observe the non-conservation for the Cooper-pair sluice in Publication I where we study the charge pumped through the two adjustable junctions composing the sluice using both our non-secular and secular master equations in the adiabatic basis. Figure 2.2 presents the main results of such study. The full master equation including the non-secular terms yields a pumped charge close to the ideal result that is expected for ground-state pumping near the adiabatic limit. However, the secular solution yields clearly decreasing charge as a function of the environmental coupling strength which contradicts the expected stabilizing behaviour. More importantly, the pumping is periodic implying that the system returns to the same charge state after each cycle and, as a result, the charge through each individual junction is necessarily equal. The secular master equation fails to capture this feature and hence yields an apparent non-physical charge accumulation. We also present an analytical study of the non-conservation in relation to the system parameters in Publication I.

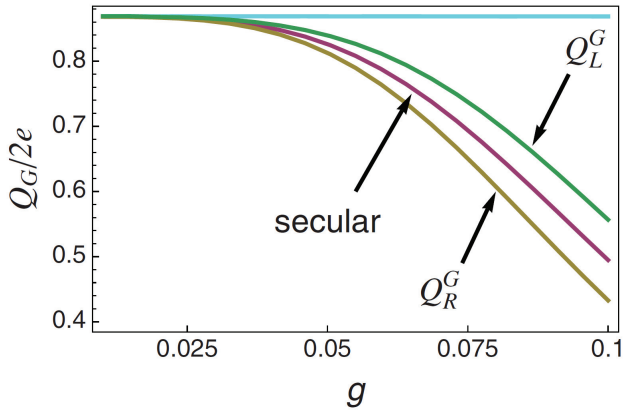


Figure 2.2. Charge pumped through the Cooper-pair sluice, Q_G, obtained using both the non-secular and secular master equations as a function of the environmental coupling strength g. Straight blue line is the solution of the full master equation including the non-secular terms, the green line is the charge pumped through the left junction Q_L^G using the secular master equation, the brown line is the charge pumped through the right junction Q_R^G using the secular master equation, and the red line is $(Q_L^G+Q_R^G)/2$. Reprinted from Publication I with permission.

Motivated by the emergent lack of charge conservation and the studies on the continuity equation for current in coarse-graining [85–87], we set out to investigate the corresponding general phenomenon for open quantum systems in Publication V. Denoting the time derivative of the expectation value of an arbitrary system observable $\hat{G}$ as *operator current*, we can derive it using both the complete dynamics referring to the unitary time-evolution stemming from Eq. (2.1) and the reduced dynamics given by a general master equation. The master equation can be written as $d\hat{\rho}_S/dt = -i[\hat{H}_S, \hat{\rho}_S]/\hbar + \hat{D}$, where the right-hand side has been separated into the part relating to unitary closed-system evolution and $\hat{D} = \hat{D}(\hat{\rho}_S, t)$ representing a generalized dissipator, that is, it also accounts for any unitary contribution stemming from the system–environment interaction. The operator currents derived by these two means have the same terms relating to the evolution of the closed system affected by the environment only through $\hat{\rho}_S$, but differ in their representation of *the dissipative current*, the current induced directly by the environment. Asserting that the definitions of dissipative currents are the same for exact dynamics results in

$$-\frac{i}{\hbar}\mathrm{Tr}\{\hat{\rho}[\hat{G}, \hat{V}]\} = \mathrm{Tr}_S\{\hat{D}\hat{G}\}, \tag{2.6}$$

where Tr_S denotes a trace over the system degrees of freedom and Tr is the

total trace. For approximative reduced dynamics, Eq. (2.6) still provides a measure of the reliability and accuracy of the applied approximations even when the complete dynamics cannot be solved. As we show in Publication V, if Eq. (2.6) is not obeyed naturally by the approximate master equation, an artificial effective Hamiltonian emerges in the complete dynamics.

The significance of Eq. (2.6) for the conservation of operator current can be explained using the following simple example. In Publication V, we investigate the decoupling of the eigenstate populations and the coherences between them, such as that occurring when implementing the secular approximation. We then show that such decoupling generally leads to dynamics yielding a temporal change of the expectation value of an almost arbitrary $\hat{G}$ that is different from that given by the usual definition of the corresponding subsystem current operator [4]. Hence, the local conservation of the operator current breaks down in the sense that the current operator cannot accurately describe the temporal change of the observable. Equation (2.6) now provides a necessary and sufficient condition for the conservation of the operator current, and in Publication V we show that the non-conservation of current in the example immediately manifests through the condition.

In Publication V, we further explore the general properties of the dissipative current and study its relation to some of the typical approximations used for open quantum systems. Most importantly, we show that the master equations in the Lindblad form are not intrinsically protected from nonconservation. We study nonsteered systems in the singular and weak-coupling limits [25, 88] and find that the application of the secular approximation generally destroyes current conservation in the case of the vanishing dissipative current. We find the same to be true for adiabatically driven systems in the weak-coupling limit using the master equation assuming cyclicity derived in Ref. [78]. As a result, the nonconservation of geometric charge was correspondingly observed in Ref. [78] and contributed to the secular approximation in accordance with Publication V. In Publication IV, we study charge transport through the Cooper-pair sluice subjected to both charge and flux noise consistently accounting for the dissipative current defined earlier. For charge noise, the dissipative current vanishes and the conservation law is only shown to hold for the nonsecular master equation derived in Publications I and II. For flux noise, the dissipative current is nonvanishing and can be accounted for by

using Eq. (2.6). We provide a more detailed comparison of the two noise environments as well as study the charge transport for the flux noise in Sec. 3.3

3. Cooper-Pair Pumping

In this chapter, we explore the theory of driven, dissipative quantum systems by introducing an example also interesting by its own merit: Cooper-pair pumping. We motivate our work and introduce our model system in Sec. 3.1 and, aided by Publications I, III, IV, and VI, explore some interesting features and further advances in Secs. 3.2–3.4.

3.1 Josephson junction arrays and geometric phases

3.1.1 Motivation

The study of superconducting nanocircuits emerged from the discovery of the Josephson effect [13–15] and they have since become a staple in the toolbox for detecting and harnessing quantum phenomena [9, 89, 90]. The basic building block adding nonlinearity in such circuits is the Josephson junction based on connecting two superconducting electrodes by a weak link [13] and enabling the construction of large circuits capable of performing tasks in the quantum regime [9]. The circuit picture typically reduces to a study of Josephson junction arrays consisting of a collection of superconducting islands for which the phase–number duality applies: $[\hat{\phi}, \hat{n}] = i$, where $\hat{\phi}$ is the superconducting phase operator of the island[1] and $\hat{n}$ is the conjugate Cooper-pair number operator [13]. In quantum information processing, superconducting qubits based on Josephson-junction arrays form their own field of study[2] usually subdivided by the mode of operation to charge, flux, and phase qubits [9, 90]. More efficient

[1] Related to the superconducting phase parameter in the Ginzburg-Landau theory [13, 91].

[2] We direct the interested reader to two excellent review articles on superconducting quantum bits and quantum information processing [11, 12] and references therein.

designs such as the transmon [92] or the fluxonium [93] qubits are developed at an increasing rate with experimental systems approaching the treshold for fault-tolerant quantum computing [94–96].

In the 1980's, the interest in adiabatically steered systems brought about the concept of geometric phase accumulation during quantum evolution in the form of the celebrated Berry phase [34, 35, 97, 98] generated during the traversal of a cyclic path in the projective Hilbert space. The Berry phase was soon generalized to non-Abelian gauge structures [36], to include analytical corrections from nearly adiabatic steering [68, 99], to non-adiabatic cylic steering [100], and to adiabatic noncylic evolution [101]. Further extensions and approaches to defining geometric phases have since emerged[3], and the Berry phase and its corrections have been analyzed in a variety of systems[4]. One of the most prominent applications of geometric phase evolution is based on the emergent non-Abelian structures in Ref. [36] which can be used to generate quantum gates in the so-called geometric quantum computing formulated in Refs. [37, 38, 113] and proposed for an abundance of physical systems [114–125] in recent years[5]. Since the gate construction is geometric in nature, the computing scheme is expected to exhibit inherent robustness against control errors [46, 51, 126] and even environmental noise [47, 67, 127]. Different methods to mitigate logical errors in such a scheme have also been considered [48, 59, 128, 129].

A few methods to directly generate and detect geometric phases in superconducting circuits have been proposed and implemented [110, 130–133], but interestingly the study of charge transport through the superconducting structures provides an alternative approach[6]. The different contributions to the charge transported through Josephson junction arrays has been studied for adiabatic evolution [134, 135] allowing it to be immediately connected to the bare system properties through the instantaneous eigenstates of the corresponding tunneling-charging Hamiltonian [13]. The different contributions are typically referred to as supercurrent and pumped current where the latter refers to the typical naming convention for current control at the level of single charge carriers [136].

[3] See for example Refs. [102–107] for studies on phase evolution with mixed states and non-unitary maps, and Refs. [33, 53, 60] for systematic open-system extensions.

[4] We direct the reader to some important examples in Refs. [50, 61, 108–112].

[5] A useful review of phase accumulation and quantum computing is given in Ref. [19].

[6] An extensive review on Berry phase with relation to electronic properties is provided by Ref. [98].

The connection between the accumulated Berry phase and the pumped current is first drawn in Ref. [18] resulting in a multitude of experimental proposals [137–140] additionally extending the theory. In general, the discovered connection states that the pumped charge is $Q_G = 2e\partial_\varphi \theta_B$, where φ is the superconducting phase difference over the device in question and θ_B is the accumulated Berry phase. Similarly, the supercurrent through the device corresponds to a charge $Q_S = -2e\partial_\varphi \theta_d$ where θ_d is the accumulated dynamical phase. Hence, the total charge transported through the superconducting device within the Josephson junction array consists of both dynamical and geometric contributions providing an immediate measurable observable for detecting the corresponding phases[7].

Even though the pumped charge in Josephson junction arrays corresponds to geometric phase evolution, it does not generally have the ideal form $Q_P = 2en^*$, where n^* is the desired number of charge carriers transported during a cycle. Such ideal transport is consistent with incoherent Cooper-pair tunneling through the superconducting device and generally obtains a quantum correction due to coherent tunneling events [142]. We analyze dissipative pumping in Sec. 3.2 and further advances can be found in Refs. [20, 77, 78, 80–82, 84, 143, 144]. We additionally analyze different noise environments in Sec. 3.3 and go beyond the assumption of exact phase biasing for pumping in Sec. 3.4. In addition, Cooper-pair pumping and its connection to the geometric nature of the time-evolution has been further studied in numerous publications. For example, Cooper-pair pumping has been used to realize Landau–Zener interferometry in Ref. [80], optimized for accuracy in Ref. [145], studied for nonadiabatic evolution in Refs. [146,147], analyzed for topological properties in Ref. [148], and proposed as a quantized current source for metrology [149] in Ref. [150].

3.1.2 Model system

We introduce a flux assisted Cooper-pair pump, the so-called Cooper-pair sluice first proposed in Ref. [138], theoretically analyzed in Ref. [151] and initially experimentally measured in Refs. [141, 152, 153]. We use this device to study the driven dissipative dynamics in Publications I, III, and IV, as well as the effect of nonvanishing loop inductance in Publi-

[7] Reversing the pumping protocol generally reverses the pumped current while the leakage supercurrent remains the same due to a constant phase bias. This can be used to detect the different charge contributions [141].

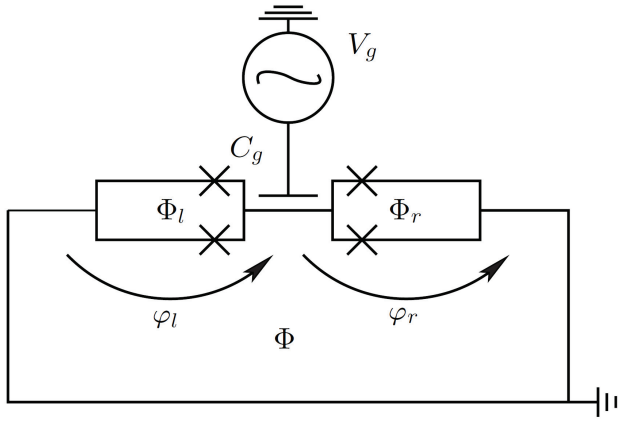


Figure 3.1. Circuit diagram of the Cooper pair sluice. The fluxes threading the left and right SQUIDs are denoted by Φ_l and Φ_r, and Φ denotes the total flux threading the large superconducting loop. The phase differences over the SQUIDs are marked by φ_l and φ_r and they are defined in the direction specified by the arrows in the figure. The gate capacitance C_g is used to manipulate the island charge with the gate voltage V_g. Reprinted from Publication III with permission.

cation VI. It is also the system of choice for numerous other publications [20, 77, 78, 80–82, 84, 143, 144, 147] due to its well-known pumping characteristics and relative simplicity. The circuit diagram of the Cooper-pair sluice is given in Fig. 3.1. By manipulation of the gate voltage V_g and the magnetic fluxes penetrating the superconducting quantum interference devices (SQUIDs) [13] Φ_l, Φ_r, charge can be carried through the device in a controlled manner [151]. The standard parameter cycle used for pumping is presented in Publications I, III, IV, and VI. Note that the two SQUIDs operate as junctions with tunable Josephson energies and the gauge-invariant phase difference over the device $\varphi = \varphi_r + \varphi_l$ is fixed by the total flux threading the large loop[8] $\varphi = 2\pi\Phi/\Phi_0$ where the flux quantum is $\Phi_0 = h/(2e)$.

The current operator of the kth SQUID takes the form[9] $\hat{I}_k = (2e/\hbar)\partial_{\varphi_k}\hat{H}_S$ and in the charging regime near the charge degeneracy point, the dynamics are accurately described using the two lowest charge states allowing the two-state master equations discussed in Sec. 2.2 to be used. The pumped charge, including the quantum correction, becomes [151]

$$Q_G = 2e(1 - 2\epsilon\cos\varphi) + O(\delta^2) + O(\epsilon^2) \tag{3.1}$$

[8]This assertion of the exact phase bias requires a vanishing total inductance of the superconducting loop. We will lift this requirement in Sec. 3.4 when discussing Publication VI.

[9]This is the correct form for a closed system or when the corresponding dissipative current vanishes as discussed in Sec. 2.3, i.e., $[\hat{n}_k, \hat{V}] = 0$ where $\hat{n}_k$ is the Cooper-pair number operator of the kth SQUID.

where ϵ is proportional to the leakage of the pump, that is, the residual Josephson energy from incomplete operation of the SQUIDs and δ is small in the charging regime. Correspondingly, the Berry phase is $\theta_B = \varphi - 2\epsilon \sin\varphi + O(\delta^2) + O(\epsilon^2)$. The phase dependence of the pumped current has been experimentally demonstrated in Ref. [141]. Finally, Cooper-pair pumps can be used to implement non-Abelian phases [154, 155] and, correspondingly, structures similar to the Cooper-pair sluice have been proposed to implement geometric quantum computing [119–121].

3.2 Dissipative pumping

3.2.1 Pumped charge

In Publication I, we study dissipative Cooper-pair pumping using the master equation in the adiabatic basis introduced in Sec. 2.2. We use the Cooper-pair sluice introduced in Sec. 3.1 and study how the general properties emergent in our treatment of driven dissipative dynamics manifest in the pumped charge in the quasi-stationary state. We assume exact phase-biasing for the main superconducting loop and charging regime operation near the degeneracy point, apply the standard parameter cycle for transport [20], and assume that the noise is due to gate voltage fluctuations capacitively coupled to the sluice island. By calculating the pumped charge with respect to the total phase difference of the sluice for zero-temperature environment, we reproduce the pumped charge in Eq. (3.1) in the adiabatic limit. For increased driving speed, the phase dependence is modified as the pumped charge decreases but is restored when the environmental coupling strength is increased. This is a direct consequence of ground-state stabilization described in Sec. 2.2.4: any driving-induced transitions would decrease the pumped charge as the excited state corresponds to pumping in the opposite direction and, hence, the ground-state stabilization yielded by our master equation restores the ideal adiabatic pumping.

In Publications I and III, we set the total superconducting phase difference to $\varphi = \pi/2$ and show that our dissipative master equation reproduces $Q_G = 2e$ in the adiabatic limit for any environmental coupling strength. As evident in Fig. 3.2(a) for $n = 1$ and the zero-temperature limit, increasing the driving frequency f causes the pumped charge in the

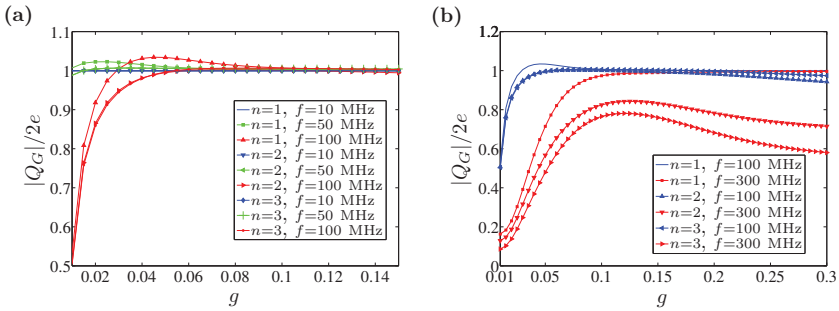


Figure 3.2. Charge pumped through the Cooper-pair sluice during a cycle assuming a zero-temperature environment and a fixed phase bias $\varphi = \pi/2$ as a function of the environmental coupling strength. Results are given for multiple pumping frequencies f and numbers of basis rotations n in the derivation of the corresponding master equation. Reprinted from Publication III with permission.

quasi-stationary limit[10] to behave differently depending on the environmental coupling strength g.[11] For small g, the nonadiabatic transitions dominate decreasing the pumped charge and, for large g, relaxation dominates yielding the stabilization to ground-state pumping. In the intermediate region, our master equation in the adiabatic basis unphysically overestimates the pumped charge $Q_G > 1$ which results from the lack of positivity of the reduced density. As explained in Sec. 2.2.4, the use of the superadiabatic bases alleviates this issue and, similarly, the overestimation is signifigantly reduced for $n > 1$ in Fig. 3.2(a). In Fig. 3.2(b), we highlight that beyond the strict adiabatic limit the ideal adiabatic pumping is only obtained as a result of the relaxation to the ground state for the master equation in the adiabatic basis, i.e., $n = 1$. The higher-order master equations ensure relaxation to higher-order superadiabatic bases that generally carry less pumped charge. We additionally derive finite-temperature pumping results in Publication I and show that the main features are similar to the zero-temperature limit. Furthermore, we propose a way of modifying the gate voltage noise spectrum by introducing a tunable environment in Publication I.

[10]The pumped charge is recorded in the quasi-stationary state, that is, after sufficiently many cycles such that the evolution of consecutive cycles is identical.
[11]We remind the reader that the parameter n corresponds to the number of basis rotations in the renormalization scheme leading to the master equation as detailed in Sec. 2.2 and, consequently, $n = 1$ corresponds to the two-state master equation in Publications I and II.

3.2.2 Breakdown of adiabaticity

In Publication III, we use our master equations to study the breakdown of adiabaticity indicated by the pumped current I_G. Our two-state master equation does not allow us to simulate changes in the pumping amplitude similar to the experimental results in Ref. [141], but we can still simulate the breakdown by changing the pumping frequency. We find that the effect of the basis selection for the master equation increases with increasing pumping frequency, as one would expect, and that the breakdown point becomes more robust against changes in the environment with an increasing number of basis rotations. In addition, there appears an optimal coupling strength with which the ideal ground-state pumping is conserved up to the highest frequency[12].

Finally, we model the breakdown characteristics of the experimentally pumped current of Ref. [141] by establishing an equality between the experimental pumping speed and the pumping speed in our simulation[13]. In Publication III, we are able to simulate the breakdown frequency with feasible system parameters, especially with strong dephasing, but find weak predictability of the exact breakdown characteristics due to strong oscillatory behavior. To explain the behavior, we perform a detailed analysis of the phase accumulation during the driving protocol using the general nth effective Hamiltonian in Eq. (2.5). We calculate the difference in the total phase accumulated by the nth eigenstates between two successive driving-induced excitations $\Delta\Theta_T^{(n)}(t_i, t_{i+1})$ at times t_i and t_{i+1} and derive a condition for the maximum constructive interference between such excitations as $\Delta\Theta_T^{(n)}(t_i, t_{i+1}) \approx 2\pi(N + 1/2)$, where N is an integer number and we assume large n.[14] Figure 3.3 shows that the downwards resonance peaks in the simulated pumped current match the condition with excellent accuracy. The same idea of quantum interference between driving-induced excitations generated at different times was proposed to be used for geometric Landau-Zener-Stückelberg interferometry [44] in Ref. [143] and experimentally realized in Ref. [158].

[12] This could potentially be exploited with an environment tuning setup such as that presented in Publication I.

[13] This is a crude approximation as the experimental pumping speed is modified by adjusting the pumping amplitude whereas the pumping speed in the simulation is changed by altering the pumping frequency. Nevertheless, this allows us to study the breakdown point beyond which our approach is no longer valid.

[14] The total accumulated phase difference includes both dynamic and noncyclic geometric [103] contributions, and the condition accounts for the impulsive phase shift at the Landau–Zener transitions [156] using the adiabatic impulse model [44, 157].

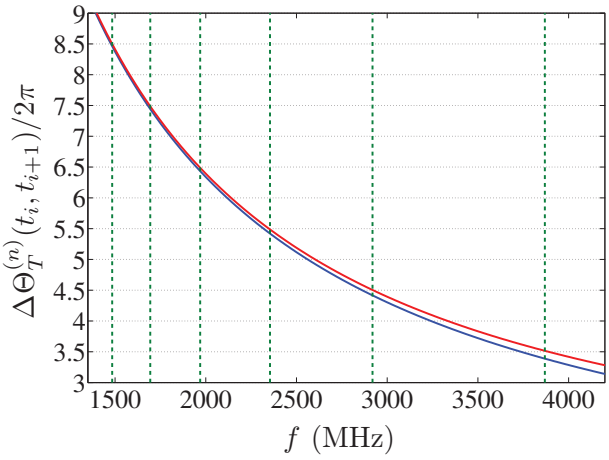


Figure 3.3. Comparison between the accumulated phase difference and the resonance peaks observed in the pumped current. The solid lines depict $\Delta\Theta_T^{(n)}(t_i, t_{i+1})$ for $n = 1$ and $n = 3$ from bottom to top. The dashed vertical lines are the resonance peak positions determined from the simulation for $n = 3$. Reprinted from Publication III with permission.

3.3 Resistively and inductively coupled environments

In Publications I and III, we assume that the environment is resistively coupled to the Cooper-pair sluice inducing gate voltage noise. We extend the dissipative analysis to account for an inductive coupling yielding noise in the magnetic flux Φ penetrating the total superconducting loop in Publication IV. This immediately translates to noise in the total phase across the sluice φ, i.e., noise in the biasing phase. The flux noise has an interesting connection to the analysis in Sec. 2.3: unlike for the gate voltage noise, for the flux noise it applies that $[\hat{n}_k, \hat{V}] \neq 0$ implying that the dissipative current through the kth SQUID is nonvanishing and must be accounted for. As mentioned in Sec. 2.3, we show that the vanishing dissipative current for gate voltage noise is properly accounted for by our nonsecular master equation and yields current nonconservation if the secular approximation is additionally applied.

In Publication IV, we calculate the dissipative current through the kth SQUID for the flux noise using Eq. (2.6) as $\langle \hat{I}_k^{\mathrm{diss}} \rangle = -2e\mathrm{Tr}_S\{\hat{D}\hat{n}_k\}$, where $\hat{D}$ corresponds to the dissipator in our two-state master equation. From this expression, the dynamical and geometric current contributions can be separated similarly to the closed-system current. Since the master equation is defined in the weak-coupling limit, we expect the resulting dissipative current to be small but possibly detectable if the standard closed-system contributions are reduced by an appropriate selection of the

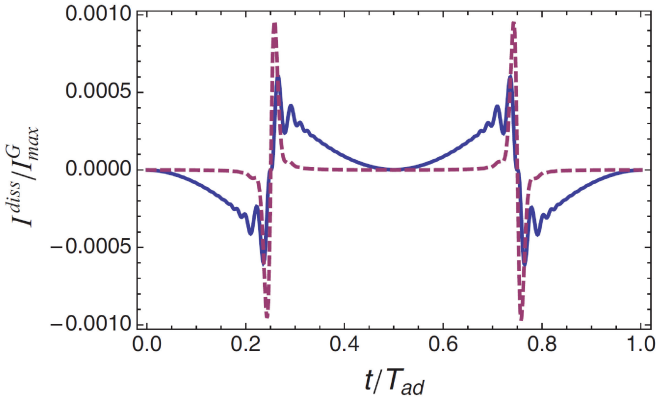


Figure 3.4. Dynamic dissipative current $I^{D,\mathrm{diss}}$ (solid line) and geometric dissipative current $I^{G,\mathrm{diss}}$ (dashed line) normalized to the maximum geometric current I^{G}_{max} during the pumping cycle of time T_{ad} assuming a zero-temperature flux noise environment. Reprinted from Publication IV with permission.

system parameters. We present the average dissipative currents $I^{\mathrm{diss}} = (I_l^{\mathrm{diss}} + I_r^{\mathrm{diss}})/2$ in Fig. 3.4 for the Cooper-pair sluice using the first-order master equation at the quasi-stationary state. The dissipative currents are approximately 3 orders of magnitude smaller than the total geometric current and obtain a complicated time-dependence corresponding to the interplay between driving and relaxation.

In Publication IV, we also introduce a scheme for tuning the phase bias noise environment similar to that presented for the gate voltage noise in Publication I. It is based on coupling the sluice to an artificial environment circuit through mutual inductance and we show that the resulting phase bias noise spectrum can be carefully modified by adjusting the flux through a control SQUID in the circuit. The modifications are seen to directly affect the pumped charge and we analyze the details related to changing the control flux.

3.4 Beyond exact phase bias

Above, all the results presented for the Cooper-pair sluice have assumed an exact phase bias. This is equivalent to assuming that the inductance of the superconducting loop is small enough for the total phase difference over the sluice φ to be exactly fixed by the penetrating magnetic flux Φ. In fact, this simplifying assumption is made for the sluice in all the references presented in Sec. 3.1 and typically exact phase biasing is assumed in some form for any system implementing Cooper-pair pumping. Even

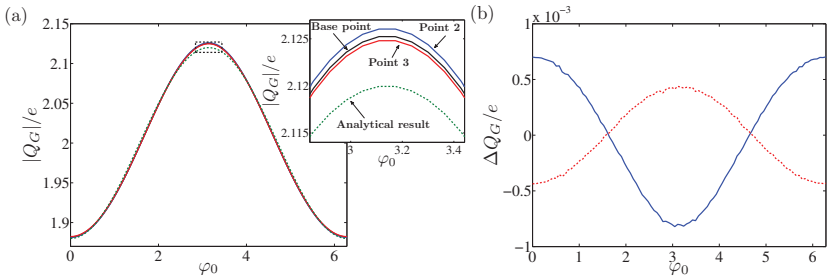


Figure 3.5. (a) Pumped charges and (b) charge differences as a function of the scaled external magnetic flux bias $\varphi_0 = \frac{2\pi\Phi_{\rm ext}}{\Phi_0}$. The parameter points in (a) are $E_L = 10^2 E_C$ and $E_\varphi = E_C$ (base point), $E_L = E_C$ and $E_\varphi = 10^{-2}E_C$ (point 2), and $E_L = E_C$ and $E_\varphi = 4E_C$ (point 3), where E_C is the island charging energy scale. The dashed line depicts the analytical solution Eq. (3.1) and the dashed rectangle indicates the area of the inset. In (b), we give the geometric charge differences between the base point and point 2, and between the base point and point 3 (from bottom to top at $\varphi_0 = \pi$). Reprinted from Publication VI with permission.

though the measurements of the sluice [141, 147] are in agreement with the standard model using exact phase biasing, they are lacking in high-precision and the inclusion of a finite realistic loop inductance should be analyzed. We tackle this problem in Publication VI where we introduce a theoretical model for the sluice which allows for a nonvanishing loop inductance[15].

The Hamiltonian corresponding to the model is constructed from first principles and accounts for the emerging quantum degree of freedom evident in the total phase difference now being described by an operator $\hat{\varphi}$. The total phase difference operator is canonically conjugate to the so-called feed Cooper-pair number operator formulated to correspond to the average charge imbalance between the superconducting islands that constitute the left and right leads. The Hamiltonian reduces to the one in the exact phase bias model when we go to the limit of vanishing loop inductance.

We derive the current operators for different parts of the system without resorting to operator derivatives and analyze adiabatic closed-system ground-state pumping using the same premises as the previous works. We perform a detailed perturbative analysis and show that there is no first-order correction in $g_0 = [E_\varphi/(32E_L)]^{1/4}$, where E_φ is the charging energy energy scale for the feed and E_L is the inductive energy scale due

[15]The self-inductance of the large loop is assumed dominant over the self-inductances of the SQUIDs in the transported charge. In addition, the derivation accounts for both geometric and kinetic [159] inductances. See Publication VI for details.

to finite loop inductance[16], to the geometric and dynamic currents across the island. We further select feasible physical parameters and numerically calculate the pumped charge with respect to the external magnetic flux bias Φ_{ext} in Fig. 3.5(a). We establish a base point roughly corresponding to the energy scales used in the experiments [141] and find that the result from the exact phase bias model is reproduced within the accuracy of the simulation.

Moving in the (E_φ, E_L) space while retaining an experimentally feasible parameter range yields weak changes in the pumped charge of $\Delta Q_G/e \sim 10^{-3}$. Note that these changes are still orders of magnitude larger than the metrological accuracy. Furthermore, the behaviour of the charge difference between the base point and the two other points in Fig. 3.5(b) cannot be accurately captured by treating the inductance classically within the exact phase bias model. We also analyze the instantaneous eigenstructure of our model and show that it depends signifigantly on the parameters emerging from the inclusion of the nonvanishing loop inductance.

[16]The inductive energy scale is defined as $E_L = (1/L)(\frac{\Phi_0}{2\pi})^2$ where L is the total loop inductance and, hence, g_0 vanishes for vanishing L.

4. Tunable Environment for Quantum Bits

This chapter introduces the idea of benefitting from a strong but tunable coupling between a qubit and an artificial environment. Such coupling allows for control over the effective qubit temperature as well as enables swithing between efficient ground-state initialization and protected evolution. The idea is realized using a coplanar-waveguide cavity in Sec. 4.1 and LC resonators in Sec. 4.2 corresponding to Publications VII and VIII, respectively.

4.1 Cavity-embedded setup

In this section, we move away from developing the theoretical means of studying open quantum systems and formulate a way of actually benefitting from the openness of the system. To showcase our idea, we use the architecture and methods of circuit quantum electrodynamics (cQED) [10, 16, 160–163] where quantum mechanical cavity modes are realized using superconducting circuit elements, typically in the so-called superconducting coplanar waveguide (CPW) cavity structures. The modes couple to other circuit elements, such as superconducting qubits [92, 93, 164–166], providing a promising platform for quantum computing [17, 167–171]. In Publication VII, we use this coupling to construct a setup which provides an alternative and complementary method to accurate qubit initialization[1] and protection, as well as allows us to signifigantly lower the effective qubit temperature.

Our setup in Publication VII is based on inserting a superconducting qubit[2] and a resistor into a high-quality-factor coplanar waveguide cav-

[1] Accurate state preparation is one of the prerequisities of quantum computing [90, 172], and recently several advanced methods in superconducting qubits have been proposed [173–179] for its implementation.

[2] The general setup works, in principle, for a variety of qubit designs but we study a Cooper-pair box operated in the transmon regime [92].

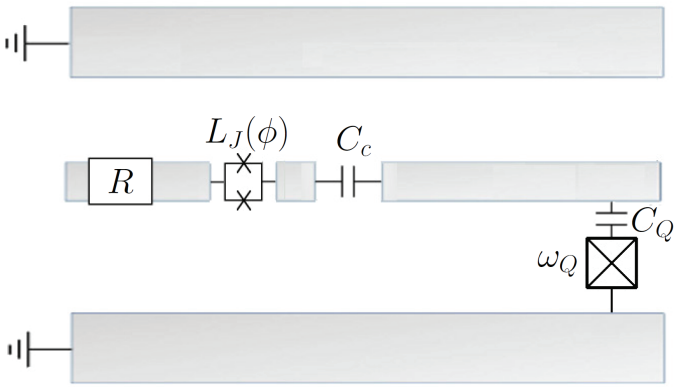


Figure 4.1. Schematic representation of the cavity setup consisting of a center conductor located between two gound planes. The cavity is divided by a capacitor of capacitance C_c, resistor of resistance R is placed left of the capacitor, and C_Q (ω_Q) gives the coupling capacitance (transition frequency) of the qubit inserted right of the dividing capacitor. The inductance $L_J(\phi)$ describes one or more SQUIDs placed evenly into the left region and controlled by a penetrating magnetic flux ϕ. Reprinted from Publication VII with permission.

ity which is then divided into two coupled cavities by the insertion of a capacitor into the central conducting strip[3]. A schematic representation of our system is shown in Fig. 4.1. We carefully derive the Hamiltonian for the complete cavity–qubit–resistor system by applying the distributed element model [168, 184, 185] in a manner that accounts for the modification of the cavity modes by the dividing capacitor. We additionally account for the insertion of evenly distributed SQUIDs into the left cavity region to realize a tuning of the effective inductance, ℓ_L, of the center conductor in that region. We show that by tuning the flux through the SQUIDs we can a priori modify the spatial profile of the modified cavity modes which allows us to change the effective coupling between the resistive artificial environment and the qubit.

In Publication VII, we derive the modified voltage and current operators for the different modes which allows us to account for the cavity-qubit and cavity-resistor couplings. We then identify qubit-like and photon-like eigenstates and calculate their decay rates to the ground state. The qubit-like decay rate is presented in Fig. 4.2. We immediately see that the cavity setup intermediates the interaction between the resistive environment and the qubit in a manner that allows the decay rate to be changed by several orders of magnitude by changing the qubit, ω_Q, and/or bare left cavity, ω_L^B, frequencies.[4] Note that the bare left cavity frequency scales

[3]Coupled-cavity systems have been previously used with good results, see for example Refs. [180–183].

[4]The bare left cavity frequency corresponds to considering the cavity region left

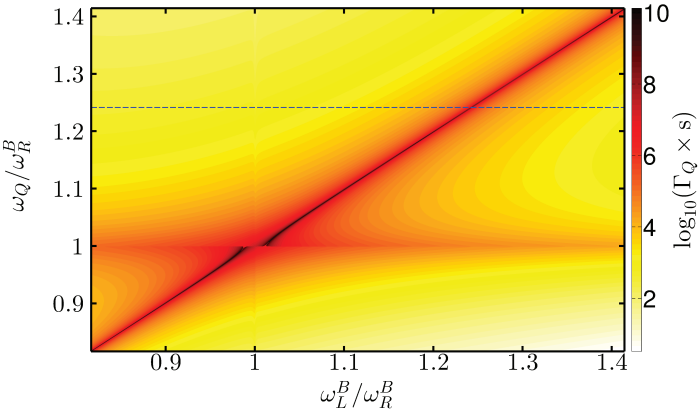


Figure 4.2. Contour plot of the qubit-like decay rate Γ_Q to the ground state from the excited state. Given as a function of the qubit, ω_Q, and bare left cavity, ω_L^B, frequencies. Reprinted from Publication VII with permission.

as $\omega_L^B \sim 1/\sqrt{\ell_L}$ implying that the qubit-like decay rate can be modified by simply changing the magnetic flux through the SQUIDs. This tunability means that we can either move the system to the left-cavity–qubit resonance point $\omega_L^B = \omega_Q$ and quickly initialize the qubit, or detune the qubit from the left cavity protecting its state[5]. We show similar tuning characteristics for the photonic state of the right cavity and, most notably, predict qubit lifetimes of only a few nanoseconds around full resonance $\omega_Q = \omega_L^B = \omega_R^B$. The system parameters are selected so that the tunability should be experimentally achievable with current technology [186].

Finally, our setup can be straightforwardly adapted to use SIN tunnel junction thermometry [187, 188] to accurately control the resistor temperature. We show that if the qubit is far-detuned from the left cavity, changing the resistor temperature has little effect on the effective qubit temperature [8] due to the effective decoupling from the artificial resistive environment. On the other hand, near the left-cavity–qubit resonance the effective coupling is strong and the effective qubit temperature can be decreased signifigantly by lowering the resistor temperature. Note that in this case the artificial environment is strongly coupled to the qubit and acts as the dominant noise source governing the efficient cooling of the qubit[6].

---

of the dividing capacitor as an isolated cavity. Similarly, we denote the bare right cavity frequency as ω_R^B.

[5]This allows the qubit state to evolve unhindered during possible gate operations.

[6]This facilitates more efficient initialization and adds to nanoscale temperature control [173–176, 187–189].

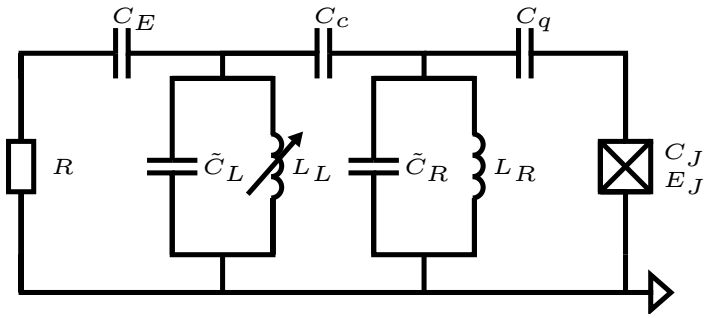


Figure 4.3. Effective circuit diagram of the quantum-LC-resonator system. The left and right LC resonators are coupled by a capacitor C_c. A resistor of resistance R is capacitively coupled to the left resonator via a capacitance C_E and acts as a noise source. The right resonator is coupled via capacitance C_q to a Cooper pair box whose Josephson capacitance is C_J and Josephson energy E_J. Reprinted from Publication VIII with permission.

4.2 Coupled quantum LC resonators

In publication Publication VIII, we propose an alternative setup for controlling the environmental coupling of a qubit. Instead of using a cQED architecture to intermediate the environment–qubit interaction like in Publication VII, we replace the CPW cavities with a pair of coupled quantum LC resonator. The corresponding circuit diagram is presented in Fig. 4.3. Here, the inductive tuning of the center conductor is replaced by a tunable inductor of inductance L_L. The system is composed of well-known circuit elements in superconducting nanoelectronics [190] and, specifically, implements the resistor coupling capacitively rather then galvanically as in Publication VII. As a result, it lacks some of the adjustability of the setup in Publication VII but is potentially easier to experimentally implement. In addition, we show in Publication VIII that the system still retains the main characteristics of tuning the environmental coupling.

Publication VIII presents a derivation of the total Hamiltonian of the system[7] and studies the decay rates due to the coupling to the resistive environment. The transition rates are more conveniently accessible than those of the CPW system and we derive analytical results for several experimentally important cases that can be exploited in the design of the device. Most notably, we find that if all the system couplings are weak and all the device components are far-detuned, the excited qubit state is doubly protected from the environmental noise: firstly by the weak resonator–

[7] We neglect terms of the second-order in the resistor voltage fluctuations and assume that the internal coupling capacitances are much smaller than other system capacitances. The assumptions and the derivation are explained in detail in Publication VIII.

qubit coupling and secondly by the weak resonator–resonator coupling. In addition, we numerically study the decay rates of the qubit and the right resonator and show that externally tuning L_L allows the decay rates to be varied over many orders of magnitude. The tuning principle is identical to that presented in Sec. 4.1 for the CPW system and makes it possible to both rapidly initilize the qubit to the ground state and to protect it from dissipation depending on the matching of the frequencies between the different circuit elements. We additionally expect similar control over the effective qubit temperature.

Finally, Publication VIII studies the mapping between the LC resonator system and the CPW system used in Publication VII. The former is described using the so-called lumped element model [185] where each resonator is comprised of a lumped inductor and a lumped capacitor. We applied the distributed element model in Publication VII because the lumped element model cannot immediately account for the positioning of the resistor and the qubit in the divided cavity and, consequently, there is generally no exact mapping to the LC resonator system. To establish a meaningful mapping that would allow the simplistic lumped element model along with results of Publication VIII to be used for the CPW system, we propose a modified cavity system with a capacitive bath coupling. We subsequently execute a bare mode mapping scheme for frequencies and voltage operators, and show that the spectra of the two models exactly match when the qubit is placed at the end of the cavity. In addition, the CPW system is well-approximated in the vicinity of the exact matching point by defining an effective resonator–qubit coupling capacitance.

5. Quantum Description of Driving and Work

This chapter goes beyond the assumption of external classical fields generating the driven quantum dynamics and treats driving as a true quantum effect arising from interaction with a separate quantum system. To this end, this chapter studies the related work injection and fluctuation relations. Section 5.1 establishes the general framework for quantum driving and studies the resulting work injection whereas Sec. 5.2 elaborates on the implications to non-equilibrium work relations.

5.1 Quantum driving and work injection

Chapter 2 explored the issues related to incorporating external control in the form of driving to dissipative dynamics and provided the ground work for the Cooper-pair pumping studies in Chap. 3. It was built upon the basis of a time-dependent system Hamiltonian $\hat{H}_S$ appearing as a result of an action being imposed by an external driving force. The action exactly defines the closed-system driving protocol and, as a result, quantifies any ideal temporal change related to the corresponding path traversed in the projective Hilbert space [21, 191]. However, this typical description of driving is approximative in nature as it entails the underlying assumption that the time-dependent control is brought about by a classical entity whereas in reality, the driving protocol is due to an actual interaction with a separate quantum system. As a result, this typical approach cannot account for quantum-mechanical backaction resulting from the interaction or include all dissipative channels. Considering the emerging experimental accuracy in, for example, quantum information processing [94, 96, 192, 193] or fluctuation relations in steered evolution [21, 194, 195], a more fundamental theoretical description of driving is potentially required.

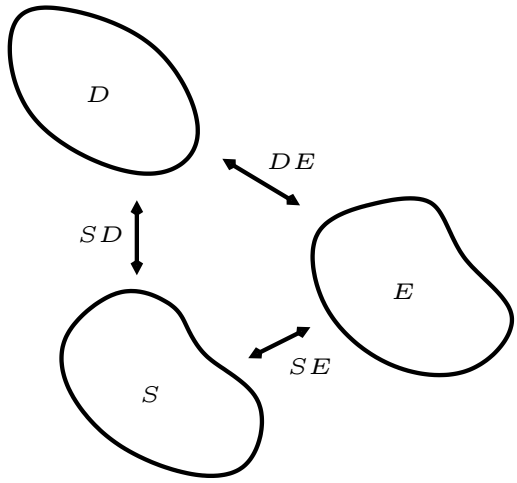


Figure 5.1. Schematic representation of the total composite system implementing quantum driving. S denotes the subsystem of interest, D denotes the drive subsystem, and E denotes the environment. Furthermore, SD, SE, and DE denote the interactions between the subsystems. Reprinted from Publication IX with permission.

In Publication IX, we introduce a general framework for driving a quantum system by dividing the total system into its constituent parts in accordance with Fig. 5.1. In this composite framework, the driving protocol is assigned to a specific subsystem D and quantum backaction is naturally allowed. The total Hamiltonian is

$$\begin{aligned}\hat{H} =&\hat{I}_D \otimes \hat{H}_S \otimes \hat{I}_E + \hat{H}_D \otimes \hat{I}_S \otimes \hat{I}_E + \hat{I}_D \otimes \hat{I}_S \\ &\otimes \hat{H}_E + \hat{H}_{SD} \otimes \hat{I}_E + \hat{I}_D \otimes \hat{H}_{SE} + \hat{H}_{DE},\end{aligned} \tag{5.1}$$

where $\hat{I}_i$ is the identity operator in the Hilbert space of the ith subsystem, $\hat{H}_i$ is the corresponding Hamiltonian and $\hat{H}_{ij}$ is the interaction Hamiltonian between the ith and the jth subsystems. There is no explicit time-dependence for any of the subsystem Hamiltonians but the full information of the driving protocol on S is encoded in the internal interactions of the composite and the initial state of D. This bypasses the basis selection problem detailed in Secs. 2.1 and 2.2, and turns it into a search for drive interactions that allow for the desired complexity in driving protocols. Publication IX shows how this framework reduces to *classical driving* of a quantum system when the drive acts as a classical entity, that is, its state is unaffected by the internal interactions of the composite. As a result, the explicit time-dependence of the system Hamiltonian is introduced by the time-evolution of the uncorrelated drive subsystem ap-

pearing in the effective system-drive interaction Hamiltonian[1].

Using our general framework, we tackle the issue of defining the work injected into a quantum system during a driving protocol and, correspondingly, calculating possible energy flow to a coupled environment. Even though this issue has been resolved for classical thermodynamics [199, 200], it is under constant debate for quantum systems as work relates to a process rather than a quantum observable even in the typical approach to driving [201–212]. In Publication IX, we propose a natural definition for the average work W_Q injected into the system during the driving protocol as the change in the internal energy of the system and its interaction energy with the drive combined with the energy dissipated directly from the system to the environment[2]. We obtain the corresponding instantaneous injected power as

$$\frac{d}{dt}W_Q = -\frac{i}{\hbar}\mathrm{Tr}_{S+D}\{\hat{\rho}_{SD}[\hat{H}_{SD}, \hat{H}_D \otimes \hat{I}_S]\}, \tag{5.2}$$

where $\hat{\rho}_{SD} = \mathrm{Tr}_E\{\hat{\rho}\}$, $\hat{\rho}$ is the total density operator, Tr_E is the trace over the environment, and Tr_{S+D} is the combined trace over the system and the drive. Furthermore, we derive the average powers dissipated directly from the system and the drive using both the complete and reduced dynamics. Importantly, the drive does not have to be rapidly self-equilibrating and quantum backaction is allowed in our analysis. Taking the system to the classical driving limit reduces the injected power in Eq. (5.2) to the recently derived result for classical injection into a quantum system [212] corresponding to the power operator in analogy to the fully classical case [215].

Finally, Publication IX studies the resonant single-mode Jaynes–Cummings model [216] as an example and takes the driving of the two-level system to be induced by the photonic bath. We additionally include a coupling to a reservoir inducing Markovian decay [25] and examine the behaviour of different energy flows as any initial excitation in the system–drive composite is absorbed into the environment. In particular, the injected work assumes a nonzero asymptotic value due to direct heat transfer from the drive to the environment. Such heat transfer is always ne-

[1]Our definition of a classical drive is related to the bipartite considerations in Refs. [196, 197], and a good example of the quantum to classical cross-over is studied for a spin-$\frac{1}{2}$ system in Ref. [198]. See Publication IX for further details.

[2]This corresponds to the *inclusive work* in the typical notation [21, 191], and extends the notion of the dynamical work agent [213], corresponding to the idea of the work source in the inclusive classical Hamiltonian approach [214], to a fully consistent quantum description.

glected in the classical driving limit.

5.2 Non-equilibrium quantum work relations

Defining the work performed on a quantum system during a driving protocol is closely related to the so-called quantum fluctuation relations [21, 191] which extend the ideas of classical stochastic thermodynamics [217] to quantum systems. They essentially allow one to connect individual stochastic trajectories determined by the dynamics to equilibrium properties of the system. When work is defined as the relevant stochastic variable, the corresponding relations are generally referred to as non-equilibrium quantum work relations[3]. Recent measurements of fluctuation relations in classical systems [194, 195, 224–227] as well as proposals for experiments in classically driven quantum systems [22, 23, 228–232][4] indicate that the experimental setups are approaching the limit of extreme accuracy and, hence, the full description of quantum driving might be necessary.

To study how fluctuation relations emerge from our general framework, we study the Bochkov–Kuzovlev identity [21, 234] concerning the *exclusive work* W_{excl} performed on the system during the driving. Such work specifically excludes the temporal change in the system–drive interaction energy [21]. In Publication IX, we apply the well-established two-measurement approach (TMA) [218, 219] for a non-degenerate discrete system and assume that the system–drive composite is decoupled from the environment during the driving protocol[5]. Applying the usual as-

[3]The quantum extensions of non-equilibrium work relations are extensively studied in literature. Extensive reviews are provided by Refs. [21, 191] and we direct the interested reader to, for example, Refs. [203–206, 208, 209, 211, 218, 219] for quantum extensions of the closed-system relations and Refs. [201, 202, 207, 210, 213, 220–223] for recent extensions to dissipative dynamics.

[4]In addition, there exists a very recent experimental study on fluctuation relations in classically driven quantum systems [233].

[5]The TMA is the established method for defining trajectory-dependent work required to generalize fluctuation relations to classically driven quantum systems [21, 191]. However, there are other recent methods for obtaining work distributions such as non-demolition measurements [235], wave function ensembles [236], and quantum jumps [237, 238]. The connection between fluctuation relations and generalized measurements has also been considered in literature, see Ref. [239] and references therein.

sumption of initialization to a Gibbs state, we obtain

$$\begin{aligned}\langle e^{-\beta W_{\mathrm{excl}}}\rangle &= \sum_k \frac{e^{-\beta\epsilon_k}}{Z_S}\sum_n P_{k,n} \\ &= \sum_k \frac{e^{-\beta\epsilon_k}}{Z_S}\mathrm{Tr}_D\{\langle k^S|\hat{U}(T,0)\hat{\rho}_D(0)\otimes\hat{I}_S\hat{U}^\dagger(T,0)|k^S\rangle\},\end{aligned} \tag{5.3}$$

where $\langle\ldots\rangle$ denotes the ensemble average over the distribution of exclusive work[6], $Z_S = \mathrm{Tr}_S\{e^{-\beta\hat{H}_S}\}$ is the partition function of the bare system [21, 191, 217], β is the inverse temperature of the initial stabilizing environment, T is the duration of the driving protocol corresponding to the time elapsed between the two measurements in the TMA, and $P_{k,n}$ is the probability of obtaining ϵ_k from the second projective measurement of $\hat{H}_S$ assuming that the first measurement has yielded ϵ_n. The eigenenergies ϵ_k and ϵ_n correspond to the system eigenstates $|k^S\rangle$ and $|n^S\rangle$, respectively. In addition, $\hat{\rho}_D(0)$ is the initial drive state and $\hat{U}(t,0)$ is the time-evolution operator for the composite. We show that in the classical driving limit, this expression reduces to the traditional Bochkoz-Kuzovlev identity $\langle e^{-\beta W_{\mathrm{excl}}}\rangle = 1$, but is generally not unity because of the correlations accumulated during the protocol due to quantum backaction.

Finally, Eq. (5.3) can be interpreted in terms of a partition function corresponding to the temporal evolution of the drive-averaged canonical state of the system. The resulting effective partition function defines the drive-averaged thermodynamic quantities in a self-consistent manner. For the Jaynes–Cummings model, we define $\bar{n}$ as the average photon number of the assumed initial coherent state for the drive, and find that the correction to the traditional Bochkov–Kuzovlev identity scales as $1/\bar{n}$ when the system approaches the classical driving limit.

[6]The two-measurement approach defines the exclusive work as a stochastic variable with each of its possible realizations corresponding to a quantum trajectory [21, 191] and, consequently, the exclusive work has a distinct probability distribution. See Publication IX for further details.

6. Summary and Conclusions

In the research presented in this dissertation, the primary objective was to explore the control of quantum systems. Such control is always hindered by the presence of an environment introducing dissipation and decoherence which disturb the quantum state making its exploitation a taunting task. To facilitate control, one requires a comprehensive description of the combined effect of dissipation and the desired driving of the quantum state. In some sense, such description is always required as an isolated driven system is a logical paradox. The research in this dissertation delved into this issue using reduced-density-operator theory, discovered improvements to previous works, and used them to study Cooper-pair pumping. It additionally improved on the underlying theory of the devices used for pumping by accounting for phenomena previously omitted. Furthermore, a method to realize built-in control by means of tunable environments using a coplanar-waveguide cavity as well as coupled quantum LC resonators was proposed. Finally, the idea of control was generalized beyond the typical classical driving assumption so that quantum backaction can be included, and the resulting implications for injected work were studied.

In Publications I–III, a consistent scheme for deriving and improving time-local master equations for driven systems was constructed. The presented research was based on the careful basis selection procedure first introduced in Ref. [20] but signifigantly generalized and extended the ideas and results. The master equations accurately display the interplay between driving and dissipation making them a valuable tool especially in cases where small changes in coherence determine the dynamics of an observable. Due to the intuitive design and physical predictivity of the master equations, Publications I–III have enticed research into further advancements in driven dissipative dynamics [64, 77, 78, 80, 82, 84, 240] as

well as added to the understanding of the general construction of master equations in the nearly adiabatic limit [241–244]. Even though the theoretical understanding of driving an open quantum system constantly develops to alleviate requirements such as slow driving, Markovianity, and non-degeneracy, the current picture is by no means perfect and invites future improvements.

The role played by different approximations with respect to the conservation of operator current was studied in Publication V drawing a direct link to the application of the secular approximation in Publications I–III. The derived conservation law acts both as a means of explaining the previously obtained nonconservation as well as a tool to test future approximative dynamics. On top of having direct use for specific systems, such as those in superconducting nanoelectronics [245], it has attracted some interest for its applicability in general open-system theory [82, 243, 244, 246, 247]. Studying whether the conservation law itself can be used as a starting point to build up the dynamics could potentially be beneficial and bring about an interesting field of study.

Cooper-pair pumping provides a test bed for the derived reduced dynamics and was used to study related phenomena in Publications I–III. The results in these publications illustrate the predicted robustness of ground-state pumping in the nearly adiabatic limit along with the changes in pumping provided by the superadiabatic bases. Later theoretical work has partially built upon the obtained pumping results [77, 78, 81, 82, 84], and further explored the applications in interferometry [143], geometric phase evolution [248, 249], and in the general theory of pumping [250]. Even though the experimental breakdown of adiabaticity [150] was simulated in Publication III, the results are only suggestive as the adiabatic two-state theory is not suited for changes in the pumping amplitude or fast pumping. With complementary experimental results now available beyond the adiabatic limit [147], an extended theory could be straightforwardly tested.

The general robustness of pumping has prompted new proposals for superconducting devices [144, 146] and is likely to do so in the future. Considering that extreme accuracy is desired in their operation, the accuracy of the exact phase biasing assumption should potentially be investigated in their design and the resulting error either mitigated or exploited. Even if the device at hand is not the Cooper-pair sluice, the methodology in Publication VI can be straightforwardly adapted to study and design more

sophisticated systems. This is especially important for the closely related proposals for geometric quantum computing [120, 121] which assume exact phase biasing. Similarly, the effect of the non-vanishing loop inductance is likely more significant when flux noise is dominant or reveals prominent features such as the dissipative currents as in Publication IV. Using the flux noise engineering scheme formulated in the publication, the dissipative currents could immediately be experimentally probed.

In Publications VII and VIII, the idea of obtaining control by externally tuning the coupling to the environment was introduced. In Publication VII, the tuning was implemented by a coplanar waveguide cavity intermediating the qubit–environment interaction. Moreover, it was shown that such tuning realizes both efficient ground-state initialization and protected operation when experimentally feasible parameters [186] are used. Implementing SIN tunnel junction thermometry adds the device to the toolbox of qubit temperature control [188]. Note that the proposal in Publication VII draws on Refs. [251, 252] in the sense that it provides a useful and efficient way to incorporate normal-metal components into the framework of circuit quantum electrodynamics and, as such, is likely to produce further theoretical and experimental activity. The device in Publication VIII uses the same principle of tunability but replaces the cavity by a pair of coupled quantum LC resonators. It is potentially easier to experimentally implement while still retaining the tunability of the qubit decay rate over several orders of magnitude.

The concept of control by driving was re-examined in Publication IX. Instead of having ideal classical driving of a quantum system, a general framework that takes into account both dissipation and system–drive backaction was constructed. This not only provides a more fundamental understanding of the driven quantum dynamics and hence opens a new way of treating and designing driven systems, but is also likely necessary for the operation of future high-precision quantum devices. In Publication IX, a definition for the work injected during the driving protocol was additionally proposed, and it was shown that one of most well-known fluctuation relations, the Bochkov-Kuzovlev identity, is modified by the quantum backaction allowed by the framework. As the explicit time dependence of the system Hamiltonian is removed in the framework formulated in Publication IX, driven dissipative dynamics avoids the earlier basis selection issue but must now obtain the desired driving protocol through engineering the system–drive interactions. Finally, future theoretical work should

attempt to uncover corresponding modifications to other fluctuation relations, investigate the details of the basis selection issue using the general framework, and explore the quantum to classical cross-over possibly exploiting the recent experimental ideas [22, 23, 230–232].

Bibliography

[1] W. Heisenberg, Z. Phys. **33**, 879 (1925).

[2] E. Schrödinger, Ann. Phys. (Leipzig) **384**, 361 (1926).

[3] W. Heisenberg, Z. Phys. **43**, 172 (1927).

[4] F. Schwabl, *Quantum Mechanics,* 4th ed. (Springer, New York, 2007).

[5] F. Schwabl, *Advanced Quantum Mechanics,* 3rd ed. (Springer, New York, 2005).

[6] K. Seeger, *Semiconductor Physics: An Introduction,* 9th ed. (Springer, New York, 2004).

[7] M. Schlosshauer, Rev. Mod. Phys. **76**, 1267 (2005).

[8] A. A. Clerk, M. H. Devoret, S. M. Girvin, F. Marquardt, and R. J. Schoelkopf, Rev. Mod. Phys. **82**, 1155 (2010).

[9] Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).

[10] J. Q. You and F. Nori, Nature (London) **474**, 589 (2011).

[11] J. Clarke and F. K. Wilhelm, Nature (London) **453**, 1031 (2008).

[12] M. H. Devoret and R. J. Schoelkopf, Science **339**, 1169 (2013).

[13] M. Tinkham, *Introduction to Superconductivity,* 2nd ed. (Dover, New York, 2004).

[14] B. D. Josephson, Phys. Lett. **1**, 251 (1962).

[15] B. D. Josephson, Rev. Mod. Phys. **46**, 251 (1974).

[16] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R. S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **431**, 162 (2004).

[17] A. Blais, J. Gambetta, A. Wallraff, D. I. Schuster, S. M. Girvin, M. H. Devoret, and R. J. Schoelkopf, Phys. Rev. A. **75**, 032329 (2007).

[18] M. Aunola and J. J. Toppari, Phys. Rev. B **68**, 020502(R) (2003).

[19] E. Sjöqvist, Physics **1**, 35 (2008).

[20] J. P. Pekola, V. Brosco, M. Möttönen, P. Solinas, and A. Shnirman, Phys. Rev. Lett. **105**, 030401 (2010).

[21] M. Campisi, P. Hänggi, and P. Talkner, Rev. Mod. Phys. **83**, 771 (2011).

[22] L. Mazzola, G. De Chiara, and M. Paternostro, Phys. Rev. Lett. **110**, 230602 (2013).

[23] J. P. Pekola, P. Solinas, A. Shnirman, and D. V. Averin, New J. Phys. **15**, 115006 (2013).

[24] U. Weiss, *Quantum Dissipative Systems,* 4th ed. (World Scientific, Singapore, 2012).

[25] H.-P. Breuer and F. Pettrucione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2002).

[26] A. G. Redfield, IBM J. Res. Dev. **1**, 19 (1957).

[27] M. Born and V. A. Fock, Z. Phys. **51**, 165 (1928).

[28] T. Kato, Phys. Soc. Jap. **5**, 435 (1950).

[29] M. S. Sarandy and D. A. Lidar, Phys. Rev. A **71**, 012331 (2005).

[30] P. Thunström, J. Åberg, and E. Sjöqvist, Phys. Rev. A **72**, 022328 (2005).

[31] X. X. Yi, D. M. Tong, L. C. Kwek, and C. H. Oh, J. Phys. B: At. Mol. Opt. Phys. **40**, 281 (2007).

[32] M. S. Sarandy and D. A. Lidar, Phys. Rev. Lett. **95**, 250503 (2005).

[33] M. S. Sarandy and D. A. Lidar, Phys. Rev. A **73**, 062101 (2006).

[34] M. V. Berry, Proc. R. Soc. Lond. A **392**, 45 (1984).

[35] B. Simon, Phys. Rev. Lett. **51**, 2167 (1983).

[36] F. Wilczek and A. Zee, Phys. Rev. Lett. **52**, 2111 (1984).

[37] P. Zanardi and M. Rasetti, Phys. Lett. A **264**, 94 (1999).

[38] J. Pachos, P. Zanardi, and M. Rasetti, Phys. Rev. A **61**, 010305(R) (1999).

[39] E. Farhi, J. Goldstone, S. Gutmann, and M. Sipser, arXiv:quant-ph/0001106 (2000).

[40] E. Farhi, J. Goldstone, S. Gutmann, J. Lapan, A. Lundgren, and D. Preda, Science **292**, 472 (2001).

[41] D. Aharonov, W. Van Dam, J. Kempe, Z. Landau, S. Lloyd, and O. Regev, SIAM J. Comput. **37**, 166 (2007).

[42] D. Bacon and S. T. Flammia, Phys. Rev. Lett. **103**, 120504 (2009).

[43] M. Born and R. Oppenheimer, Ann. Phys. (Leipzig) **389**, 457 (1927).

[44] S. N. Shevchenko, S. Ashhab, and F. Nori, Phys. Rep. **492**, 1 (2010).

[45] A. M. Childs, E. Farhi, and J. Preskill, Phys. Rev. A **65**, 012322 (2001).

[46] P. Solinas, P. Zanardi, and N. Zanghi, Phys. Rev. A **70**, 042316 (2004).

[47] G. Florio, P. Facchi, R. Fazio, V. Giovannetti, and S. Pascazio, Phys. Rev. A **73**, 022327 (2006).

[48] D. Parodi, M. Sassetti, P. Solinas, and N. Zanghì, Phys. Rev. A **76**, 012337 (2007).

[49] A. Fubini, G. Falci, and A. Osterloh, New J. Phys. **9**, 134 (2007).

[50] G. De Chiara and G. M. Palma, Phys. Rev. Lett. **91**, 090404 (2003).

[51] S.-L. Zhu and P. Zanardi, Phys. Rev. A **72**, 020301(R) (2005).

[52] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions* (Wiley, New York, 1992).

[53] R. S. Whitney, Y. Makhlin, A. Shnirman, and Y. Gefen, Phys. Rev. Lett. **94**, 070407 (2005).

[54] K. Blum, *Density Matrix Theory and Applications* (Plenum Press, New York, 1996).

[55] G. Lindblad, Commun. Math. Phys. **48**, 119 (1976).

[56] V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, J. Math. Phys. **17**, 821 (1976).

[57] G. Schaller, G. Kießlich, and T. Brandes, Phys. Rev. B **80**, 245107 (2009).

[58] J. Prachař and T. Novotný, Physica E **42**, 565 (2010).

[59] D. Parodi, M. Sassetti, P. Solinas, P. Zanardi, and N. Zanghì, Phys. Rev. A **73**, 052304 (2006).

[60] A. Carollo, I. Fuentes-Guridi, M. F. Santos, and V. Vedral, Phys. Rev. Lett. **90**, 160402 (2003).

[61] A. Carollo, I. Fuentes-Guridi, M. F. Santos, and V. Vedral, Phys. Rev. Lett. **92**, 020402 (2004).

[62] E. M. Gauger, S. C. Benjamin, A. Nazir, and B. W. Lovett, Phys. Rev. B **77**, 115322 (2008).

[63] P. Pawlus and E. Sjöqvist, Phys. Rev. A **82**, 052107 (2010).

[64] P. Haikka, J. D. Cresser, and S. Maniscalco, Phys. Rev. A **83**, 012112 (2011).

[65] J. Hausinger and M. Grifoni, Phys. Rev. A **81**, 022117 (2010).

[66] J. C. Escher and J. Ankerhold, Phys. Rev. A **83**, 032122 (2011).

[67] I. Fuentes-Guridi, F. Girelli, and E. Livine, Phys. Rev. Lett. **94**, 020503 (2005).

[68] M. V. Berry, Proc. R. Soc. Lond. A **414**, 31 (1987).

[69] M. V. Berry, in *Geometric Phases in Physics* (World Scientific, Singapore, 1988).

[70] R. Blümel, A. Buchleitner, R. Graham, L. Sirko, U. Smilansky, and H. Walther, Phys. Rev. A **44**, 4521 (1991).

[71] M. Grifoni and P. Hängi, Phys. Rep. **304**, 229 (1998).

[72] S. Guérin and H. R. Jauslin, Adv. Chem. Phys. **125**, 147 (2003).

[73] J. Tuorila, M. Silveri, M. Sillanpää, E. Thuneberg, Y. Makhlin, and P. Hakonen, Phys. Rev. Lett. **105**, 257003 (2010).

[74] J. Li, M. P. Silveri, K. S. Kumar, J.-M. Pirkkalainen, A. Vepsäläinen, W. C. Chien, J. Tuorila, M. A. Sillanpää, P. J. Hakonen, E. V. Thuneberg, and G. S. Paraoanu, Nature Commun. **4**, 1420 (2013).

[75] M. Silveri, J. Tuorila, M. Kemppainen, and E. Thuneberg, Phys. Rev. B **87**, 134505 (2013).

[76] J. Tuorila, M. Silveri, M. Sillanpää, E. Thuneberg, Y. Makhlin, and P. Hakonen, Supercond. Sci. Technol. **26**, 124001 (2013).

[77] A. Russomanno, S. Pugnetti, V. Brosco, and R. Fazio, Phys. Rev. B **83**, 214508 (2011).

[78] I. Kamleitner and A. Shnirman, Phys. Rev. B **84**, 235140 (2011).

[79] I. Kamleitner and A. Shnirman, Phys. Rev. B **87**, 039903(E) (2013).

[80] S. Gasparinetti, P. Solinas, S. Pugnetti, R. Fazio, and J. P. Pekola, Phys. Rev. Lett. **110**, 150403 (2013).

[81] P. Wollfarth, I. Kamleitner, and A. Shnirman, Phys. Rev. B **87**, 064511 (2013).

[82] I. Kamleitner, Phys. Rev. A **87**, 042111 (2013).

[83] R. S. Whitney, M. Clusel, and T. Ziman, Phys. Rev. Lett. **107**, 210402 (2011).

[84] V. Gramich, P. Solinas, M. Möttönen, J. P. Pekola, and J. Ankerhold, Phys. Rev. A **84**, 052103 (2011).

[85] R. Gebauer and R. Car, Phys. Rev. Lett. **93**, 160404 (2004).

[86] A. Bodor and L. Diósi, Phys. Rev. A **73**, 064101 (2006).

[87] R. Englman and A. Yahalom, in *Advances in Classical Field Theory* (Bentham, Sharjah, 2011).

[88] M. G. Schultz and F. von Oppen, Phys. Rev. B **80**, 033302 (2009).

[89] R. Fazio and H. van der Zant, Phys. Rep. **355**, 235 (2001).

[90] M. Nakahara and T. Ohmi, *Quantum Computing: From Linear Algebra to Physical Realizations* (CRC Press, Boca Raton, 2008).

[91] V. L. Ginzburg and L. D. Landau, Zh. Eksp. Teor. Fiz. **20**, 1064 (1950).

[92] J. Koch, T. M. Yu, J. Gambetta, A. A. Houck, D. I. Schuster, J. Majer, A. Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A **76**, 042319 (2007).

[93] V. E. Manucharyan, J. Koch, L. I. Glazman, and M. H. Devoret, Science **326**, 113 (2009).

[94] E. Lucero, M. Hofheinz, M. Ansmann, R. C. Bialczak, N. Katz, M. Neeley, A. D. O'Connell, H. Wang, A. N. Cleland, and J. M. Martinis, Phys. Rev. Lett. **100**, 247001 (2008).

[95] E. Lucero, R. Barends, Y. Chen, J. Kelly, M. Mariantoni, A. Megrant, P. O'Malley, D. Sank, A. Vainsencher, J. Wenner, T. White, Y. Yin, A. N. Cleland, and J. M. Martinis, Nature Phys. **8**, 719 (2012).

[96] J. M. Chow, J. M. Gambetta, A. D. Córcoles, S. T. Merkel, J. A. Smolin, C. Rigetti, S. Poletto, G. A. Keefe, M. B. Rothwell, J. R. Rozen, M. B. Ketchen, and M. Steffen, Phys. Rev. Lett. **109**, 060501 (2012).

[97] A. Shapere and F. Wilczek, *Geometric Phases in Physics* (World Scientific, Singapore, 1989).

[98] D. Xiao, M.-C. Chang, and Q. Niu, Rev. Mod. Phys. **82**, 1959 (2010).

[99] M. V. Berry and R. Lim, J. Phys. A: Math. Gen. **26**, 4737 (1993).

[100] Y. Aharonov and J. Anandan, Phys. Rev. Lett. **58**, 1593 (1987).

[101] J. Samuel and R. Bhandari, Phys. Rev. Lett. **60**, 2339 (1988).

[102] N. Mukunda and R. Simon, Ann. Phys. (New York) **228**, 205 (1993).

[103] G. G. de Polavieja and E. Sjöqvist, Am. J. Phys. **66**, 431 (1998).

[104] E. Sjöqvist, A. K. Pati, A. Ekert, J. S. Anandan, M. Ericsson, D. K. L. Oi, and V. Vedral, Phys. Rev. Lett. **85**, 2845 (2000).

[105] M. Ericsson, E. Sjöqvist, J. Brännlund, D. K. L. Oi, and A. K. Pati, Phys. Rev. A **67**, 020101(R) (2003).

[106] J. G. Peixoto de Faria, A. F. R. de Toledo Piza, and M. C. Nemes, Europhys. Lett. **62**, 782 (2003).

[107] D. M. Tong, E. Sjöqvist, L. C. Kwek, and C. H. Oh, Phys. Rev. Lett. **93**, 080405 (2004).

[108] I. Fuentes-Guridi, J. Pachos, S. Bose, V. Vedral, and S. Choi, Phys. Rev. A **66**, 022102 (2002).

[109] I. Fuentes-Guridi, A. Carollo, S. Bose, and V. Vedral, Phys. Rev. Lett. **89**, 220404 (2002).

[110] M. Pechal, S. Berger, A. A. Abdumalikov, Jr., J. M. Fink, J. A. Mlynek, L. Steffen, A. Wallraff, and S. Filipp, Phys. Rev. Lett. **108**, 170401 (2012).

[111] F. C. Lombardo and P. I. Villar, Phys. Rev. A **89**, 012110 (2014).

[112] V. A. Mousolou and E. Sjöqvist, Phys. Rev. A **89**, 022117 (2014).

[113] R. G. Unanyan, B. W. Shore, and K. Bergmann, Phys. Rev. A **59**, 2910 (1999).

[114] L.-M. Duan, J. I. Cirac, and P. Zoller, Science **292**, 1695 (2001).

[115] J. A. Jones, V. Vedral, A. Ekert, and G. Castagnoli, Nature **403**, 869 (2000).

[116] A. Recati, T. Calarco, P. Zanardi, J. I. Cirac, and P. Zoller, Phys. Rev. A **66**, 032309 (2002).

[117] P. Solinas, P. Zanardi, N. Zanghì, and F. Rossi, Phys. Rev. B **67**, 121307(R) (2003).

[118] P. Solinas, P. Zanardi, N. Zanghì, and F. Rossi, Phys. Rev. A **67**, 062315 (2003).

[119] J.-M. Pirkkalainen, P. Solinas, J. P. Pekola, and M. Möttönen, Phys. Rev. B **81**, 174506 (2010).

[120] P. Solinas, J.-M. Pirkkalainen, and M. Möttönen, Phys. Rev. A **82**, 052304 (2010).

[121] I. Kamleitner, P. Solinas, C. Müller, A. Shnirman, and M. Möttönen, Phys. Rev. B **83**, 214518 (2011).

[122] H. Wu, E. M. Gauger, R. E. George, M. Möttönen, H. Riemann, N. V. Abrosimov, P. Becker, H.-J. Pohl, K. M. Itoh, M. L. W. Thewalt, and J. J. L. Morton, Phys. Rev. A **87**, 032326 (2013).

[123] K. Toyoda, K. Uchida, A. Noguchi, S. Haze, and S. Urabe, Phys. Rev. A **87**, 052307 (2013).

[124] A. A. Abdumalikov, Jr., J. M. Fink, K. Juliusson, M. Pechal, M. Berger, A. Wallraff, and S. Filipp, Nature (London) **496**, 482 (2013).

[125] J. Spiegelberg and E. Sjöqvist, Phys. Rev. A **88**, 054301 (2013).

[126] D. Ellinas and J. Pachos, Phys. Rev. A **64**, 022310 (2001).

[127] A. Blais and A.-M. S. Tremblay, Phys. Rev. A **67**, 012308 (2003).

[128] M. Johansson, E. Sjöqvist, L. M. Andersson, M. Ericsson, B. Hessmo, K. Singh, and D. M. Tong, Phys. Rev. A **86**, 062322 (2012).

[129] G. F. Xu, J. Zhang, D. M. Tong, E. Sjöqvist, and L. C. Kwek, Phys. Rev. Lett. **109**, 170501 (2012).

[130] G. Falci, R. Fazio, G. M. Palma, J. Siewert, and V. Vedral, Nature (London) **407**, 355 (2000).

[131] P. J. Leek, J. M. Fink, A. Blais, R. Bianchetti, M. Göppl, J. M. Gambetta, D. I. Schuster, L. Frunzio, R. J. Schoelkopf, and A. Wallraff, Science **318**, 1889 (2007).

[132] M. Neeley, M. Ansmann, R. C. Bialczak, M. Hofheinz, E. Lucero, A. O'Connel, D. Sank, W. Haohua, J. Wenner, A. N. Cleland, M. R. Geller, and J. M. Martinis, Science **325**, 722 (2009).

[133] S. Berger, M. Pechal, S. Pugnetti, A. A. Abdumalikov, Jr., L. Steffen, A. Fedorov, A. Wallraff, and S. Filipp, Phys. Rev. B **85**, 220502(R) (2012).

[134] J. P. Pekola, J. J. Toppari, M. Aunola, M. T. Savolainen, and D. V. Averin, Phys. Rev. B **60**, R9931 (1999).

[135] M. Aunola, Phys. Rev. B **63**, 132508 (2001).

[136] L. J. Geerligs, S. M. Verbrugh, P. Hadley, J. E. Mooij, H. Pothier, P. Lafarge, C. Urbina, D. Estéve, and M. H. Devoret, Z. Phys. B **85**, 349 (1991).

[137] J. P. Pekola and J. J. Toppari, Phys. Rev. B **64**, 172509 (2001).

[138] A. O. Niskanen, J. P. Pekola, and H. Seppä, Phys. Rev. Lett. **91**, 177003 (2003).

[139] M. Governale, F. Taddei, R. Fazio, and F. W. J. Hekking, Phys. Rev. Lett. **95**, 256801 (2005).

[140] M. Cholascinski and R. W. Chhajlany, Phys. Rev. Lett. **98**, 127001 (2007).

[141] M. Möttönen, J. J. Vartiainen, and J. P. Pekola, Phys. Rev. Lett. **100**, 177201 (2008).

[142] R. Fazio, F. W. J. Hekking, and J. P. Pekola, Phys. Rev. B **68**, 054510 (2003).

[143] S. Gasparinetti, P. Solinas, and J. P. Pekola, Phys. Rev. Lett. **107**, 207002 (2011).

[144] S. Gasparinetti and I. Kamleitner, Phys. Rev. B **86**, 224510 (2012).

[145] S. Safaei, S. Montangero, F. Taddei, and R. Fazio, Phys. Rev. B **77**, 144522 (2008).

[146] F. Hoehne, Y. A. Pashkin, O. V. Astafiev, M. Möttönen, J. P. Pekola, and J. Tsai, Phys. Rev. B **85**, 140504(R) (2012).

[147] S. Gasparinetti, P. Solinas, Y. Yoon, and J. P. Pekola, Phys. Rev. B **86**, 060502(R) (2012).

[148] R. Leone and L. Lévy, Phys. Rev. B **77**, 064524 (2008).

[149] J. P. Pekola, O.-P. Saira, V. F. Maisi, A. Kemppinen, M. Möttönen, Y. A. Pashkin, and D. V. Averin, Rev. Mod. Phys. **85**, 1421 (2013).

[150] R. Leone, L. P. Lévy, and P. Lafarge, Phys. Rev. Lett. **100**, 117001 (2008).

[151] M. Möttönen, J. P. Pekola, J. J. Vartiainen, V. Brosco, and F. W. J. Hekking, Phys. Rev. B **73**, 214523 (2006).

[152] A. O. Niskanen, J. M. Kivioja, H. Seppä, and J. P. Pekola, Phys. Rev. B **71**, 012513 (2005).

[153] J. J. Vartiainen, M. Möttönen, J. P. Pekola, and A. Kemppinen, Appl. Phys. Lett. **90**, 082102 (2007).

[154] L. Faoro, J. Siewert, and R. Fazio, Phys. Rev. Lett. **90**, 028301 (2003).

[155] V. Brosco, R. Fazio, F. W. J. Hekking, and A. Joye, Phys. Rev. Lett. **100**, 027002 (2008).

[156] C. Zener, Proc. R. Soc. Lond. A **137**, 696 (1932).

[157] B. Damski and W. H. Zurek, Phys. Rev. A **73**, 063405 (2006).

[158] X. Tan, D.-W. Zhang, Z. Zhang, Y. Yu, S. Han, and S.-L. Zhu, Phys. Rev. Lett. **112**, 027001 (2014).

[159] A. J. Annunziata, D. F. Santavicca, L. Frunzio, G. Catelani, M. J. Rooks, A. Frydman, and D. E. Prober, Nanotechnol. **21**, 445202 (2010).

[160] F. Deppe, M. Mariantoni, E. P. Menzel, A. Marx, S. Saito, K. Kakuyanagi, H. Tanaka, T. Meno, K. Semba, H. Takayanagi, E. Solano, and R. Gross, Nature Phys. **4**, 686 (2008).

[161] J. M. Fink, M. Goeppl, M. Baur, R. Bianchetti, P. J. Leek, A. Blais, and A. Wallraff, Nature (London) **454**, 315 (2008).

[162] Y.-x. Liu, J. Q. You, L. F. Wei, C. P. Sun, and F. Nori, Phys. Rev. Lett. **95**, 087001 (2005).

[163] T. Niemczyk, F. Deppe, H. Huebl, E. P. Menzel, F. Hocke, M. J. Schwarz, J. J. Garcia-Ripoll, D. Zueco, T. Huemmer, E. Solano, A. Marx, and R. Gross, Nature Phys. **6**, 772 (2010).

[164] Z.-L. Xiang, S. Ashhab, J. Q. You, and F. Nori, Rev. Mod. Phys **85**, 623 (2013).

[165] H. Paik, D. I. Schuster, L. S. Bishop, G. Kirchmair, G. Catelani, A. P. Sears, B. R. Johnson, M. J. Reagor, L. Frunzio, L. I. Glazman, S. M. Girvin, M. H. Devoret, and R. J. Schoelkopf, Phys. Rev. Lett. **107**, 240501 (2011).

[166] P. Nataf and C. Ciuti, Phys. Rev. Lett. **107**, 190402 (2011).

[167] J. Q. You and F. Nori, Phys. Rev. B **68**, 064509 (2003).

[168] A. Blais, R. S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A **69**, 062320 (2004).

[169] J. Q. You and F. Nori, Phys. Today **58**, 42 (2005).

[170] I. Buluta and F. Nori, Science **326**, 108 (2009).

[171] I. Buluta, S. Ashhab, and F. Nori, Rep. Prog. Phys. **74**, 104401 (2011).

[172] D. Di Vincenzo, Fortschr. Phys. **48**, 771 (2006).

[173] S. O. Valenzuela, W. D. Oliver, D. M. Berns, K. K. Berggren, L. S. Levitov, and T. P. Orlando, Science **314**, 1589 (2006).

[174] J. Q. You, Y.-x. Liu, and F. Nori, Phys. Rev. Lett. **100**, 047001 (2008).

[175] M. Grajcar, S. Ashhab, J. R. Johansson, and F. Nori, Phys. Rev. B **78**, 035406 (2008).

[176] M. Grajcar, S. H. W. van der Ploeg, A. Izmalkov, E. Il'ichev, H.-G. Meyer, A. Fedorov, A. Shnirman, and G. Schön, Nature Phys **4**, 612 (2008).

[177] M. D. Reed, B. R. Johnson, A. A. Houck, L. DiCarlo, J. M. Chow, D. I. Schuster, L. Frunzio, and R. J. Schoelkopf, Appl. Phys. Lett. **96**, 203110 (2010).

[178] J. E. Johnson, C. Macklin, D. H. Slichter, R. Vijay, E. B. Weingarten, J. Clarke, and I. Siddiqi, Phys. Rev. Lett. **109**, 050506 (2012).

[179] D. Ristè, J. G. van Leeuwen, H.-S. Ku, K. W. Lehnert, and L. DiCarlo, Phys. Rev. Lett. **109**, 050507 (2012).

[180] L. Zhou, Z. R. Gong, Y.-x. Liu, C. P. Sun, and F. Nori, Phys. Rev. Lett. **101**, 100501 (2008).

[181] L. Zhou, H. Dong, Y.-x. Liu, C. P. Sun, and F. Nori, Phys. Rev. A **78**, 063827 (2008).

[182] L. Zhou, S. Yang, Y.-x. Liu, C. P. Sun, and F. Nori, Phys. Rev. A **80**, 062109 (2009).

[183] J.-Q. Liao, Z. R. Gong, L. Zhou, Y.-x. Liu, C. P. Sun, and F. Nori, Phys. Rev. A **81**, 042304 (2010).

[184] B. Yurke and J. Denker, Phys. Rev. A **29**, 1419 (1984).

[185] D. Pozar, *Microwave Engineering* (Wiley, Hoboken, 2004).

[186] M. Göppl, A. Fragner, M. Baur, R. Bianchetti, S. Filipp, J. M. Fink, P. J. Leek, G. Puebla, L. Steffen, and A. Wallraff, J. Appl. Phys. **104**, 113904 (2008).

[187] J. T. Muhonen, M. Meschke, and J. P. Pekola, Rep. Prog. Phys. **75**, 046501 (2012).

[188] F. Giazotto, T. T. Heikkilä, A. Luukanen, A. M. Savin, and J. P. Pekola, Rev. Mod. Phys. **78**, 217 (2006).

[189] H. Courtois, F. W. J. Hekking, H. Q. Nguyen, and C. B. Winkelmann, J. Low Temp. Phys. **175**, 799 (2014).

[190] D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. H. Devoret, Science **296**, 886 (2002).

[191] M. Esposito, U. Harbola, and S. Mukamel, Rev. Mod. Phys. **81**, 1665 (2009).

[192] J. J. Pla, K. Y. Tan, J. P. Dehollain, W. H. Lim, J. J. L. Morton, D. N. Jamieson, A. S. Dzurak, and A. Morello, Nature (London) **489**, 541 (2012).

[193] J. J. Pla, K. Y. Tan, J. P. Dehollain, W. H. Lim, J. J. L. Morton, F. A. Zwanenburg, D. N. Jamieson, A. S. Dzurak, and A. Morello, Nature (London) **496**, 334 (2013).

[194] O.-P. Saira, Y. Yoon, T. Tanttu, M. Möttönen, D. V. Averin, and J. P. Pekola, Phys. Rev. Lett. **109**, 180601 (2012).

[195] J. V. Koski, T. Sagawa, O.-P. Saira, Y. Yoon, A. Kutvonen, P. Solinas, M. Möttönen, T. Ala-Nissilä, and J. P. Pekola, Nature Phys. **9**, 644 (2013).

[196] H. Schröder, J. Teifel, and G. Mahler, Eur. Phys. J. Special Topics **151**, 181 (2007).

[197] H. Weimer, M. J. Henrich, F. Rempp, H. Schröder, and G. Mahler, Europhys. Lett. **83**, 30008 (2008).

[198] D. Calvani, A. Cuccoli, N. I. Gidopoulos, and P. Verrucchi, Proc. Natl. Acad. Sci. USA **110**, 6748 (2013).

[199] H. Callen, *Thermodynamics and an Introduction to Thermostatistics* (Wiley, New York, 1985).

[200] H. Goldstein, *Classical Mechanics,* 2nd ed. (Addison-Wesley, Reading, 1980).

[201] T. Albash, D. A. Lidar, M. Marvian, and P. Zanardi, Phys. Rev. E **88**, 032146 (2013).

[202] M. Campisi, P. Talkner, and P. Hänggi, Phys. Rev. Lett. **102**, 210401 (2009).

[203] S. Yukawa, J. Phys. Soc. Jpn. **69**, 2367 (2000).

[204] S. Mukamel, Phys. Rev. Lett. **90**, 170604 (2003).

[205] V. Chernyak and S. Mukamel, Phys. Rev. Lett. **93**, 048302 (2004).

[206] A. E. Allahverdyan and T. M. Nieuwenhuizen, Phys. Rev. E **71**, 066102 (2005).

[207] M. Esposito and S. Mukamel, Phys. Rev. E **73**, 046129 (2006).

[208] A. Engel and R. Nolte, Europhys. Lett. **79**, 10003 (2007).

[209] P. Talkner, E. Lutz, and P. Hänggi, Phys. Rev. E **75**, 050102(R) (2007).

[210] P. Talkner, M. Campisi, and P. Hänggi, J. Stat. Mech. (2009) P02025.

[211] D. Andrieux and P. Gaspard, Phys. Rev. Lett. **100**, 230404 (2008).

[212] P. Solinas, D. V. Averin, and J. P. Pekola, Phys. Rev. B **87**, 060508(R) (2013).

[213] D. Cohen and Y. Imry, Phys. Rev. E **86**, 011111 (2012).

[214] S. Deffner and C. Jarzynski, Phys. Rev. X **3**, 041003 (2013).

[215] C. Jarzynski, Phys. Rev. Lett. **78**, 2690 (1997).

[216] B. W. Shore and P. L. Knight, J. Mod. Opt. **40**, 1195 (1993).

[217] U. Seifert, Rep. Prog. Phys. **75**, 126001 (2012).

[218] H. Tasaki, arXiv:cond-mat/0009244.

[219] J. Kurchan, arXiv:cond-mat/0007360.

[220] G. E. Crooks, J. Stat. Mech. (2008) P10023.

[221] R. Chetrite and K. Mallick, J. Stat. Phys. **148**, 480 (2012).

[222] A. E. Rastegin, J. Stat. Mech (2013) P06016.

[223] A. E. Rastegin and K. Życzkowski, Phys. Rev. E **89**, 012127 (2014).

[224] J. Liphardt, S. Dumont, S. B. Smith, I. Tinoco Jr, and C. Bustamante, Science **296**, 1832 (2002).

[225] D. Collin, F. Ritort, C. Jarzynski, S. B. Smith, and C. Tinoco Jr, I Bustamante, Nature (London) **437**, 231 (2005).

[226] F. Douarche, S. Ciliberto, A. Petrosyan, and I. Rabbiosi, Europhys. Lett. **70**, 593 (2005).

[227] S. Toyabe, T. Sagawa, M. Ueda, E. Muneyuki, and M. Sano, Nature Phys. **6**, 988 (2010).

[228] G. Huber, F. Schmidt-Kaler, S. Deffner, and E. Lutz, Phys. Rev. Lett. **101**, 070403 (2008).

[229] M. Heyl and S. Kehrein, Phys. Rev. Lett. **108**, 190601 (2012).

[230] R. Dorner, S. R. Clark, L. Heaney, R. Fazio, J. Goold, and V. Vedral, Phys. Rev. Lett. **110**, 230601 (2013).

[231] J. P. Pekola, A. Kutvonen, and T. Ala-Nissilä, J. Stat. Mech. (2013) P02033.

[232] M. Campisi, R. Blattmann, S. Kohler, D. Zueco, and P. Hänggi, New J. Phys. **15**, 105028 (2013).

[233] T. Batalhão, A. M. Souza, L. Mazzola, R. Auccaise, R. S. Sarthour, I. S. Oliveira, J. Goold, G. De Chiara, M. Paternostro, and R. M. Serra, arXiv:1308.3241.

[234] G. N. Bochkov and Y. E. Kuzovlev, Sov. Phys. JETP **45**, 125 (1977).

[235] D. Gelbwaser-Klimowsky, N. Erez, R. Alicki, and G. Kurizki, Phys. Rev. A **88**, 022112 (2013).

[236] M. Campisi, New J. Phys. **15**, 115008 (2013).

[237] F. W. J. Hekking and J. P. Pekola, Phys. Rev. Lett. **111**, 093602 (2013).

[238] P. Strasberg, G. Schaller, T. Brandes, and M. Esposito, Phys. Rev. E **88**, 062107 (2013).

[239] G. Watanabe, B. P. Venkatesh, P. Talkner, M. Campisi, and P. Hänggi, Phys. Rev. E **89**, 032114 (2014).

[240] D. Kast and J. Ankerhold, Phys. Rev. Lett. **110**, 010402 (2013).

[241] T. Albash, S. Boixo, D. A. Lidar, and P. Zanardi, New J. Phys. **14**, 123016 (2012).

[242] S. Ibáñez, X. Chen, and J. G. Muga, Phys. Rev. A **87**, 043402 (2013).

[243] J. Larson, Phys. Scr. **T153**, 014040 (2013).

[244] H. Mäkelä and M. Möttönen, Phys. Rev. A **88**, 052111 (2013).

[245] M. Hell, M. R. Wegewijs, and D. P. DiVincenzo, Phys. Rev. B **89**, 195405 (2014).

[246] R. D'Agosta and M. Di Ventra, Phys. Rev. B **87**, 155129 (2013).

[247] M. Tokman and M. Erukhimova, J. Lumin. **137**, 148 (2013).

[248] S. Berger, M. Pechal, A. A. Abdumalikov, Jr., C. Eichler, L. Steffen, A. Fedorov, A. Wallraff, and S. Filipp, Phys. Rev. A **87**, 060303 (2013).

[249] J. Larson, Phys. Rev. Lett. **108**, 033601 (2013).

[250] F. Pellegrini, C. Negri, F. Pistolesi, N. Manini, G. E. Santoro, and E. Tosatti, Phys. Rev. Lett. **107**, 060401 (2011).

[251] P. J. Jones, J. A. M. Huhtamäki, K. Y. Tan, and M. Möttönen, Phys. Rev. B **85**, 075413 (2012).

[252] P. J. Jones, J. A. M. Huhtamäki, M. Partanen, K. Y. Tan, and M. Möttönen, Phys. Rev. B **86**, 035313 (2012).

The ubiquitous coupling to the surrounding environment makes controlling quantum systems a challenging task. One needs to both employ efficient means of control as well as to have a good understanding of its joint effect with dissipation. The field of superconducting nanocircuits offers a convenient premise to implement the control and has established a strong track record in experimentally realizing complex quantum systems. In this dissertation, several aspects of control in open quantum systems are theoretically investigated and applied to superconducting devices paying special attention to the geometric transfer of Cooper pairs. This work simulates important physical phenomena, makes predictions for future experiments, and introduces novel theoretical tools.

Aalto-DD 102/2014

ISBN 978-952-60-5767-5
ISBN 978-952-60-5768-2 (pdf)
ISSN-L 1799-4934
ISSN 1799-4934
ISSN 1799-4942 (pdf)

Aalto University
School of Science
Department of Applied Physics
www.aalto.fi

BUSINESS + ECONOMY

ART + DESIGN + ARCHITECTURE

SCIENCE + TECHNOLOGY

CROSSOVER

DOCTORAL DISSERTATIONS