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DETERMINATION OF Q OF SMALL ANTENNAS

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Hyvyysluku eli Q-arvo on antennin tärkeä ominaisuus. Teorian pienten antennien Q-arvoista kehitti alunperin L.J. Chu jo 1940-luvulla, mutta teoriaa on päivitetty ajan kuluessa. Sitä ovat korjanneet mm. Collin, Fante ja McLean 1960-1990 –luvuilla, sekä hiljattain Yaghjian, Thal ja Gustafsson 2000-luvulla. Viime aikoina on yritetty määrittää Q-arvon ja antennin kaistanleveyden suhdetta.

Toteutuneen Q-arvon ja pienimmän mahdollisen Q-arvon suhde kuvaa, kuinka tehokkaasti antenni käyttää tilavuutensa. Antennin tarkan Q-arvon voi määrittää antennin tuottamista sähkömagneettisista kentistä tai sen voi approksimoida antennin sisäänmenoimpedanssista. Pienimmän mahdollisen Q-arvon voi laskea suoraan antennin dimensioista. Tässä opinnäytetyössä on tehty kirjallisuuskatsaus erilaisiin tapoihin määrittää sekä antennin tarkkaa Q-arvoa että pienintä mahdollista Q-arvoa.

Avainsanat: Hyvyysluku, Q-arvo, säteilyhyvyysluku, pienten antennien fundamentaalit rajoitukset, pienten antennien fyysiset rajoitukset, sähköisesti pieni antenni, minimihyvyysluku, Chu-raja, Chu-Harrington-raja, Gustafsson-raja, antennin kaistanleveys.

Aalto University School of Electrical Engineering abstract of Bachelor's Thesis

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The Q factor or quality factor is an important characterization of an antenna. The theory on Q of small antenna was originally conceived by L.J Chu as early as 1940, but it has been changing with time, it was revised by Collin, Fante, and Mclean in the 1960's to 1990's and recently by Yaghjian, Gustafsson, Thal and Best in the 2000s. There have been attempts to recently to determine the relationship between Q and the frequency bandwidth of an antenna.
The relation between the realized Q and the minimum Q describes how efficiently an antenna uses its volume. The exact Q of an antenna can be determined by either from the electromagnetic fields produced by the antenna, or from the impedance of the antenna. The Minimum Q can be calculated directly from the dimensions of the antenna.
In this thesis work, a literature survey on the different methods used in the determining the exact Q and minimum Q is made.

Keywords: Quality factor, Q value, radiation Q, fundamental limits of small antennas, physical limitations of small antennas, electrically small antenna, minimum Q, exact Q of antenna, Chu limit, Chu-Harrington limit, Gustafsson limit, antenna bandwidth.

Preface

I would like to thank my instructor, Risto Valkonen for the support, guidance and patience during this work.

I would also like to thank my mother and all my friends for the great support and friendship you have given me through all these years. Without you my life wouldn't be as happy.

Otaniemi,

Akshay Sridharan

Contents

List of symbols and abbreviations

Symbols

α	Real Frequency
α_l, β_l	Coefficients of the spherical Henkel function
Γ	Reflection Coefficient
γ_e	Electric polarizability dyadic
γ_m	Magnetic polarizability dyadic
ε	permittivity
η	antenna radiation efficiency factor
σ_a	Absorption cross section of an unmatched antenna
σ_{ext}	Extinction cross section
θ	Polar Angle
λ	Wave length
μ	permeability
ν	Function of frequency
σ_a	Absorption cross section of an unmatched antenna
σ_{ext}	Extinction cross section
φ	Azimuthal angle
χ_e	Electric susceptibility dyadic
χ_m	Magnetic susceptibility dyadic
Ω	Solid angle
ω	Angular frequency
ω_0	Angular frequency at resonance
A_n	Coefficient
a	radius of Chu Sphere
B	Bandwidth
C	Capacitor

C_n	Coefficient
D_n	Coefficient
D	directivity
$\vec{E}$	Electric field vector
f	Frequency
G_Λ	Minimum partial realized gain
$G_l(x), H_l(x)$	Spherical Henkel functions of second kind
$\vec{H}$	Magnetic field vector
I	Current
i_R	Current in the antenna
j	complex number square root of -1
$j_n(kr)$	Spherical Bessel wave function of second kind
k	Wave number
N_R	Real valued function of E and H
$\hat{p}_e$	Electric Polarization
$\hat{p}_m^*$	Magnetic Polarization
P_A	Received antenna power
Q	Quality factor
R	Resistance
R_{rad}	Radiation Resistance of antenna
R_{loss}	Lossy Radiation Resistance of antenna
r	radius of sphere
S	Complex frequency
T	coupling coefficient
U_e	Electric energy density
U_m	Magnetic energy density
V	voltage

W_E	Time average stored electric energy
W_M	Time average stored magnetic energy
X	Reactance
$y_n(kr)$	Spherical Bessel wave function of first kind
Z	Impedance

Operators

LHS	Left hand side
$max(W_E, W_M)$	Maximum of two values $W_E and\ W_M$
$\frac{\partial}{\partial\omega}$	Partial Differentiator with respect to angular frequency
$\oint_S ds$	Surface integral over a surface S
$\sum_{m=0}^{n} X$	Summation of a function X (m) for values of m between 0 and n
$\vec{H}.\vec{\vec{H}}$	Vector Dot product
$\vec{E} \times \vec{\vec{H}}$	Vector Cross product

Abbreviations

ESA	Electrically small antenna
TE	Transverse Electric Field
TM	Transverse Magnetic Field
VSWR	Voltage Standing Wave Ratio

1 Introduction

Antenna miniaturization has been the most significant and interesting subject in antenna and related fields. Today the need for smaller antenna has increased rapidly as the demand for mobiles, hand held portables, RFIDs, GPS systems and other wireless equipment are quite huge. These devices have high market penetration and have become a common commodity now. The ever growing applications nurtured by wireless devices require the engineers to create smaller and multifunctional antennas. This work presents a chronological review of the theoretical work crucial to miniaturization.

The concept of Q was envisioned by Chu [4] as early as the in 1940's who evaluates the Q of an ideal antenna enclosed in an imaginary sphere. Collin and Rothschild [8] introduced their own method of evaluating Q based on the total reactive energy stored obtained by radiated field energy from the total energy of the fields. This method was successfully extended by Fante [13] and Mclean [9]. The concept of Q since its proposal was controversial.

This thesis discusses the various such attempts and provides descriptions of the various methods employed in Section 2, a comparison is also provided under Section 3 and finally concludes in the Section 4 by explaining why the concept of Q was warranted in the first place. First let's begin with the overview of the important characteristics.

1.1 Definition of a Small Antenna

Small antennas are referred to as "electrically small antennas" or ESAs. The name is an implication that their physical size is much smaller than a wavelength at the operational frequency.

The first definition was proposed by Wheeler [1] as an antenna whose maximum dimension is less than $\lambda/2\pi$ (radian length). Where λ is the wavelength.

Another common and an equivalent definition, ESA is an antenna that satisfies the condition

$$ka < 0.5 \tag{1}$$

Where k is the wave number $=2\pi/\lambda$

And 'a' is the radius of the minimum size sphere that encloses the antenna. The sphere is termed as 'Chu sphere'.

Another definition given by Hansen [2] is

$$ka < 1 \tag{2}$$

This is interpreted as an antenna enclosed inside a sphere of radius equal to one radian length and the sphere is called 'radian sphere'. This represents the boundary between the near and far field radiation for a Hertzian dipole.

1.2 Small antenna Parameters

The following characteristic parameters are the most important for a small antenna.

1.2.1 Directivity

Small antennas are often believed to have doughnut shaped Omni directional pattern of a Hertzian dipole of directivity D=1.5. The pattern is due to the radiation of TE_{10} or TM_{10} spherical modes. But Harrington [3], Kwon [5] and Pozar [6] have demonstrated unidirectional and bi directional patterns with D from 1 to 3 theoretically. Antennas with higher spherical TE_{mn} and TM_{nm} radiations are not of the small type. Small antennas are also termed super directive, since the directivity D remains unaltered with decrease in size *ka* [2, 7].

1.2.2 Radiation Efficiency

Antenna radiation efficiency factor η is the ratio of power radiated P_{rad} to the power delivered to an antenna P_A. The losses other than radiation are modeled as a resistor of value R_{loss}.
Mathematically is given as

$$\eta = \frac{R_{rad}}{R_{rad} + R_{loss}} = \frac{R_{rad}}{R_A} \tag{3}$$

The efficiency gets reduced as the antenna dimension *ka* is reduced for the fact that R_{loss} dominates. The reduction can be attributed to frequency dependent conduction and di electric losses.

1.2.3 Quality factor

The Quality factor Q is used to describe the high input reactance and narrow bandwidth of small antennas [15] and is defined in [4] as

$$Q = \frac{2\omega_0 max(W_E, W_M)}{P_A} \tag{4}$$

W_E and W_M are the time average stored electric and magnetic energies and P_A is the received antenna power. The Q value the quantity of our interest is inversely proportional to the antenna bandwidth [1].

2 Evolution of Theory of Small Antennas

2.1 Development in 1940s

2.1.1 Chu's contribution

Chu [4] determined the optimum performance of an antenna in free space and the corresponding relation between its gain and the bandwidth. He [4] enclosed the whole antenna structure of dimension '*2a*' inside a sphere of radius '*a*' and expressed the field outside the sphere due to an arbitrary current or source distribution inside the sphere as a complete set of spherical vectors waves that represented a spherical wave propagating radially outward. The difficulty faced was that, the current or source distribution inside the sphere could not be uniquely determined by the field distribution outside the sphere so Chu [4] created the field distribution outside the sphere mathematically with an infinite number of different source distributions. Chu [4] determined the radiation characteristics of the system from the expressions for the fields and found the directivity gain. He then could equate the directivity gain to the power gain, by neglecting conduction loss in the antenna structure. Then by utilizing the conventional concept of Q, Chu [4] obtained the frequency characteristics of the input impedance by extrapolation.

His [4] interpretation was that the Q so computed became vague whenever the value of Q was low. He [4] then determined the maximum and minimum Q through the process of maximization and minimization.

2.1.2 Analysis of Chu's method

Chu's [4] focus was mainly on Omni directional antenna with vertical polarization (shown in Fig. 1), so field pattern was also Omni directional.

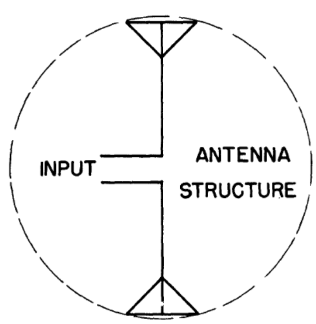

Figure 1.vertically polarized omnidirectional antenna

For an arbitrary current distribution and antenna structure, Chu [4] then expressed the field outside the sphere in terms of a complete set of orthogonal, spherical waves, propagating radially outward. He [4] deduced that for the Omni directional antenna only TM_{no} waves were required to describe the circularly symmetrical field with the specified polarization.

Finally with the field outside the sphere known he [4] computed the total complex power at the surface of the sphere as the integral of the complex Poynting vector over the same sphere:

2.1.2.1 Chu's Equivalent circuit for TM_n waves

Chu [4] understood from the power expression that it was difficult to separate the energy of the local field in the neighborhood of the antenna from the remainder as the energy was not linear in the field components and so the law of linear superposition could not be applied directly. The imaginary part of the integral of the complex Poynting vector was proportional only to the difference of the electrical and magnetic energy stored outside the sphere. In order to separate the energies associated with radiation and local field, he [4] converted the field problem to a circuit problem where the radiation loss was modeled by an equivalent conduction loss.

Chu [4] inferred that because of the orthogonal properties of the spherical wave functions, the total energy, electric or magnetic, stored outside the sphere was equal to the sum of the corresponding energies associated with each spherical wave, and the complex power transmitted across a closed spherical surface was equal to the sum of the complex powers associated with each spherical wave. The total energies and power remained unchanged as there was no coupling between any two of the spherical waves outside the sphere. Consequently, Chu [4] replaced the space outside the sphere by a number of independent equivalent circuits; each with a pair of terminals connected to a box that represented the inside of the sphere.

Chu [4] then calculated the impedance Z_n of the equivalent circuit of each spherical TM_n wave as a continued fraction using the recurrence formulas of the spherical Bessel functions as:

$$Z_n = \frac{n}{jka} + \cfrac{1}{\cfrac{2n-1}{jka} + \cfrac{1}{\cfrac{2n-3}{jka} \ddots \cfrac{1}{\cfrac{3}{jka} + \cfrac{1}{\cfrac{1}{jka}+1}}}} \tag{5}$$

This he [4] interpreted as a cascade of series capacitances and shunt inductances terminated with a unit resistance as shown in figure 2 and for smallest value of n, the impedance consisted of the simply three elements and it represented a wave that could be generated by an infinitesimally small dipole.

It can be inferred from his [4] work that **at low frequencies** the voltage applied appears across the capacitance as its impedance is high and the unit resistance is short-circuited by the inductance that has a low impedance, based on similar considerations **at high frequencies**, the impedance Z_n is resistive and a**t intermediate frequencies**, the reactance of Z_n is capacitive.

The circuit, for all values of n, behaved as a high-pass filter. As the dissipative element is hidden at the very end of the cascade, the difficulty of feeding average power into the dissipative element at a single frequency increases with the order of the wave.

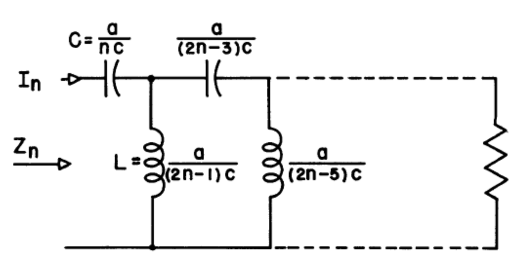

Figure 2.Equivalent circuit of TM_n spherical wave

Chu [4] then approximated the equivalent circuit to a simple series RLC circuit to obtain Z_n for the electric dipole as the complexity of Z_n increased rapidly with n.
With Z_n known the average electric energy was obtained.
He [4] then could calculate the power dissipation as well as the average energy stored in Z for the simplified circuit of the TM_n wave.
He [4] defined Q as

$$Q_n = \frac{2\omega W_n}{Pn} = \frac{1}{2}|kah_n|^2 \left[ka\frac{\partial X_n}{\partial \omega} - X_n\right] \tag{6}$$

The bandwidth of the equivalent circuit of the TM_n wave was deudced to be the reciprocal of Q_n when matched externally .
The Q expression was then defined to be

$$\boldsymbol{Q} = \frac{\boldsymbol{2\omega x\ mean\ electric\ energy\ stored}}{\boldsymbol{power\ dissipated\ in\ radiation}} \tag{7}$$

If this Q is high, it can be interpreted as the reciprocal of the fractional frequency bandwidth of the antenna. If it is low, the input impedance of the antenna varies slowly with frequency and the antenna has potentially a broad bandwidth. The ratio Q is therefore used as a crude indication for a broadband.
Q of the Ideal antenna was found upon adding the electric energy stored in all the equivalent circuits and the total power radiated as

$$Q = \frac{\sum A_n^2 \frac{n(n+1)}{2n+1} Q_n(ka)}{\sum A_n^2 \frac{n(n+1)}{2n+1}} \tag{8}$$

Additional conditions were required on G and Q to determine the limits of antenna performance as the coefficients A_n remained unknown. These were Maximum gain, Minimum Q can be obtained from Chu's [4] paper and stated by Mclean [8] as

$$Q = \frac{1 + 2(ka)^2}{(ka)^3[1 + (ka)^2]} \tag{9}$$

2.2 Development in 1960s

2.2.1 Collin and Rothschild's Contribution

Collin and Rothschild [8] defined the Q of an antenna as the Resonance Q in circuit theory:

$$Q = \frac{\omega W}{P}$$

(10)

Where the energy stored and dissipated were given by W and P

The non-resonance impedance was found to be proportional to $P + 2j\omega(W_m - W_e)$. Where W_m and W_e denote the magnetic and electric energy in the network. They [8] considered an ideal lossless reactive element that may be used for tuning the antenna, there by the Q was deduced as

$$Q = \frac{2\omega W_{max}}{P}$$

Where W_{max}= max (W_m, W_e)

(11)

The Q value reduced with a lossy tuning element.

Collin and Rothschild [8] believed that the Q was an important over-all parameter that specified antenna's performance and physical limitations of its size on gain. High Q value implied a high storage of reactive energy in the near field, large current, large ohmic losses, narrow bandwidth and high frequency sensitivity.

Collin and Rothschild [8] realized that a general method for the evaluation of Q was impossible as the energies stored in, the localized reactive field and the radiating field could not be separated due to their non-zero interactions. They [8] also realized that the integral of the Poynting vector over a surface yielded the difference between the magnetic and electric energies and also these were infinite as field is Omni present.

They [8] calculated Q from the power flow of the antenna, which was the product of energy density ($U_e + U_m$) and velocity of energy flow. They easily evaluated U_e, U_m and the power flow at infinity by integrating the complex Poynting vector was real over a surface S at infinity. Then the energy density in the reactive field was obtained by subtracting U_e and U_m from the total energy density in the field.

$$\frac{1}{2}\oint_S \vec{E} \times \vec{\bar{H}}.ds = P + 2j\omega(W_m - W_e)$$

(12)

2.2.2 Analysis of Collin and Rothschild's method

They [8] used a much simplified approach from Chu [4] wherein they [8] evaluated only the energy stored in the fields for a TM_{n0} wave as they realized that Q was independent of the azimuthal number m. Also they [8] realized that TM_{mn} and TE_{nm} were duals with the same Q values and the only difference was storage of greater magnetic or electric energy.

Integrating the equation (12) over sphere of radius $r=a$ with the field components of a TM_{n0} mode the complex Poynting vector was found as

$$P + 2j\omega(W_m - W_e)$$

$$= \frac{k\pi}{\omega\epsilon_0} * \frac{2n(n+1)}{2n+1} + j\frac{k}{\omega\epsilon_0} * \frac{2n(n+1)}{2n+1} * \{j_n(kr)[krj_n(kr)]' - y_n(kr)[kry_n(kr)]\}$$

(13)

The radiated power in [15] i.e. the real part was then subtracted from the energy density $(W_m - W_e)$ to obtain the energy in the evanescent fields. With this known now the Q was obtained by using equation (13) as

$$Q_n = ka - \left(\frac{ka}{2} + \frac{n+1}{ka}\right).(C_n^2 + D_n^2) + \left(n + \frac{3}{2}\right)(C_n D_{n+1} - C_{n+1} D_n) - \frac{(ka)^2}{2}(C_{n+1}^2 + D_{n+1}^2)$$

(14)

For the lowest mode Q was

$$Q_1 = \frac{1}{ka} + \frac{1}{(ka)^3}$$

(15)

The results were in agreement with Chu and stressed the fact that Q is large for small values of ka. This method of evaluation of Q was general as compared to Chu [4] and could be applied to any antenna with the field values known.

2.2.3 Fante's Contribution

Fante's [13] work was along the works of Collin and Rothschild [8]. He [13] found the Q of the antenna excited in both TE and TM modes. He [13] proposed an additional factor Q_n and derived its expression. Fante showed that Q and **fractional bandwidth** are inversely and approximately proportional when Q was very large.

2.2.4 Analysis of Fante's work

He [13] uses Chu's [4] sphere technique and Collin and Rothschild's [8] expressions for calculating the energy density stored in the evanescent electric and magnetic fields outside the Chu sphere. They were

$$W_e + W_m \cong \int_0^\infty dr \left[\int_0^{2\pi} d\varphi \int_0^\pi sin\theta d\theta r^2.\{\frac{\varepsilon_0}{4}|E|^2 + \frac{\mu_0}{4}|H_0|^2\} - \frac{N_R}{2C}\right]$$

(16)

$$N_R = Re\int_{s_\infty} (E \times H^*).dS$$

(17)

Where $s_\infty$is a sphere of infinite radius. In order to calculate the exact values of $W_e\ and\ W_m$ he resorts to Poynting theorem

$$W_e - W_m = \frac{1}{4\omega} Im \int_0^{2\pi} d\varphi \int_0^\pi d\theta sin\theta a^2 (E \times H^*).i_R$$

(18)

The values of W_e *and* W_m were then calculated in terms of the fields expressed in vector spherical harmonics and then Q was found by using the definition of Harrington [3] as

$$Q = \max \begin{cases} \dfrac{\sum_{n=1}^{\infty}[a_n^2 Q_n + b_n^2 Q_n']}{\sum_{n=1}^{\infty}[a_n^2 + b_n^2]} \\ \dfrac{\sum_{n=1}^{\infty}[a_n^2 Q_n' + b_n^2 Q_n]}{\sum_{n=1}^{\infty}[a_n^2 + b_n^2]} \end{cases}$$

(19)

where

$$a_n^2 = \sum_{m=0}^{n} \lambda_{nm} |A_{nm}|^2$$

(20)

$$b_n^2 = \sum_{m=0}^{n} \lambda_{nm} |B_{nm}|^2$$

(21)

$$\lambda_{nm} = \frac{2\pi\varepsilon_m}{2n+1} n(n+1) \frac{(n+m)!}{(n-m)!}$$

(22)

Q_n *is the same as in* [4]

$$Q_n' = ka - \frac{(ka)^3}{2}[|h_n(ka)|^2 - j_{n-1}(ka) j_{n+1}(ka) - y_{n-1}(ka) y_{n+1}(ka)]$$

(23)

Fante's [13] deduces that $Q_n \gg Q_n'$ for small *ka* values Q_n and Q_n' are comparable when *ka* is in the order of n. He [13] also notes that (19) simples to Collin and Rothschild's [4] Q if either a_n^2 *or* b_n^2 is 0. His [13] value of Q is higher than that of Chu [4]

$$Q_n - Q_n^{CHU} = \left[ka - \frac{1}{ka|h_n(ka)|^2}\right] = \begin{cases} 0 \; for \; ka \gg n \\ ka - \dfrac{(ka)^{2n+1}}{3^2.5^2 \ldots (2n-1)^2} \end{cases}$$

(24)

2.3 Development in 1980s

2.3.1 Pues' contribution

Pues [16] models the impedance of the microstrip antennas by a series resonant or parallel resonant RLC circuit and expresses the impedance as

$$Z_{in} = R_0(1 + jQv)\ or\ \frac{R_0}{1 + jQv}$$

(25)

Where $v = \frac{f}{f_r} - \frac{f_r}{f}$ also from the VSWR at the input gives

$$\Gamma = \left|\frac{z_{in}(f) - Z_0}{z_{in}(f) + Z_0}\right| = \left|\frac{VSWR(f) - 1}{VSWR(f) + 1}\right|$$

(26)

Γ is the reflection coefficient

The VSWR Bandwidth $B = \frac{f_2 - f_1}{f_r}$ where $VSWR(f_1) = VSWR(f_2) = S$

(27)

$$B = \frac{1}{Q}\sqrt{\frac{(TS - 1)(S - T)}{S}}$$

B is VSWR bandwidth and T is the coupling coefficient

(28)

2.4 Development in 1990s

2.4.1 McLean's contribution.

McLean [9] deduced the minimum attainable radiation *Q* of a linearly polarized antenna precisely. His [9] value of *Q* was different from the approximate values obtained by Chu [4]. He [9] found that the result he obtained was in tandem with Chu [4] when the value of *ka* was small but for higher values, close to but less than unity the expression was very different. The value of *Q* was larger than the previously determined values of *Q* for the relatively bigger sized small antennas. The bandwidth was therefore lesser than that previously calculated.

2.4.2 Analysis of McLean's method

The exact expression of *Q* was obtained from magnetic vector potential A_r. The magnetic and electric densities stored then were calculated from

$$w_e = \frac{1}{2}\varepsilon\vec{E}.\vec{\vec{E}}$$

$$w_m = \frac{1}{2}\mu\vec{H}.\vec{\vec{H}}$$

(29)

(30)

He [9] then calculated the electric and magnetic energy density in the travelling wave from the field expressions using

$$w_e^{rad} = \frac{1}{2}\varepsilon\left|E_\theta^{rad}\right|^2$$

(31)

The total non-propagating energy was then obtained by obtaining w_e' as

$$w_e' = w_e - w_e^{rad}$$

(32)

And then by integrating (32).

The radiation power was obtained by integrating the real part of the Poynting vector as in equation (12). The Quality factor then was obtained by using the expression (11) as

$$Q = \frac{1}{(ka)^3} + \frac{1}{ka}$$

(33)

Alternatively another method employed by him [9] was use Chu's [4] equivalent ladder network for a TM_{01} mode to calculate the energy stores in the capacitor and the energy dissipated in the resistor. These were

$$W_e' = \frac{1}{2}C|V_c|^2 = \frac{1}{2\omega}\cdot\frac{1}{ka}$$

(34)

$$P_r = |I_r|^2 R = \frac{(ka)^2}{1+(ka)^2}$$

(35)

Q then simply was obtained by using equations (11), (34) and (35) and was the same as obtained in (20).

2.5 Development in 2000s

2.5.1 Hansen and Collin's contribution

Hansen and Collin [10] derived the *Q* for the TE and TM mode and presented an approximate form for the lowest order Tm mode. They modified Chu's [4] *Q* by calculating the total energy stored inside the sphere by using Thal's [11] technique.

2.5.1.1 Analysis of Hansen and Collin's method

Based on the work of Collin, Rothschild and Thal [11] they [10] calculated the internal energy stored inside by integrating the inward complex Poynting vector over the spherical surface.

This energy was neglected by Chu [4] as a result his Q value was much lower than the actual value. They [10] modified Chu's [4] Q as

$$Q_{NEW} = Q_{Chu} + \Delta Q \tag{36}$$

After numerical manipulations the ΔQ was obtained as

$$\Delta Q = SF_{TM}\{\frac{(ka)^3}{2}[D_n^2 - D_{n-1}D_{n+1}] + \frac{(ka)^2}{2n+1}[(n+1)D_nD_{n-1} - nD_nD_{n+1}]\} \tag{37}$$

$$Q_{Chu} = ka - \left(\frac{(ka)^3}{2} + n(n+1)ka\right).(C_n^2 + D_n^2) - \frac{(ka)^3}{2}(C_{n+1}^2 + D_{n+1}^2) + \frac{2n+3}{2}(ka)^2(C_nC_{n+1} + D_nD_{n+1}) \tag{38}$$

The above equation (37) was the same for a TE mode except for the scale factor. The scale factor for a TE mode was larger than that of TM mode so the Minimum value of Q obtained was found to be higher than the value of Chu [4]. It was shown that for TE_1 mode and small *ka* values less than 0.7

$$\Delta Q \approx 2Q_{Chu} \tag{39}$$

This theoretical value was in total tandem with the numerical values of Thal [11].

They [10] then made an attempt to simplify the value of Q with up to 2 terms as

$$Q_{NEW} = \frac{A}{ka} + \frac{B}{(ka)^3} \tag{40}$$

The factors *A* and *B* using ρth error approximations were found and the Q was then

$$Q_{NEW} = \frac{0.71327}{ka} + \frac{1.49589}{(ka)^3} \tag{41}$$

The above result had an rms error of 0.153% and were irrational and an easy approximate for A and B were found to be 0.707 and 1.5 (rms error of 0.286%) that lead to

$$Q_{NEW} \approx \frac{1}{\sqrt{2}ka} + \frac{3}{2(ka)^3} \tag{42}$$

These Q values obtained by Hansen and Collin [10] are closer to the actual Q of small antennas. The two terms least ρth approximation of Q produced a value that was simple and accurate.

2.5.2 Yaghjian and Best's contribution

Yaghjian and Best [12] precisely obtained the expressions and relationship between the bandwidth and quality factor Q, these were more accurate than the previous work of authors [3], [4], [8], [9], [13] and [15]. They [12] also showed that the Q was inversely proportional to the bandwidth. The bandwidth expression derived was valid for all frequency ranges and this was a notable achievement. They [12] expressed the exact Q of the antenna as function of the dispersion energies and the frequency derivative of the input reactance. The internal energy expressions were similar but different from previous authors and were dependent on the choice of the coordinate systems. A general method was devised that was different from the previous authors and the ambiguity in the definition of Q was eliminated as asymmetric far field radiation patterns were dealt.

Yaghjian and Best [12] defined the VSWR bandwidth of an antenna as they believed that it is more fundamental than the conductance bandwidth as it existed for all frequencies. Q was re expressed in terms of the internal energy integrals of electric and magnetic fields. The ambiguity concerning the far field and field within the antenna were removed and so the Q value was more precise. They [12] also clearly proved that Q increased rapidly when the dimension of the antenna was reduced and when the frequency efficiency and far field pattern was constant. They [12] also discussed the Q, bandwidth relation for negative values of μ and ε.

2.5.2.1 Analysis of Yaghjian and Best's method

2.5.2.1.1 Exact Q from Maxwell's Equations

They [12] defined the quality factor Q as

$$Q(\omega_0) = \frac{\omega_0 |W(\omega_0)|}{P_A(\omega_0)} \tag{43}$$

The internal energy was then expressed in terms of the sum of electric, magnetic and magnetoelectric energies as

$$W(\omega_0) = W_m(\omega_0) + W_e(\omega_0) + W_{me}(\omega_0) \tag{44}$$

$$W(\omega_0) = \frac{I_0^2}{4} X_0'(\omega_0) - W_L(\omega_0) + W_R(\omega_0) \tag{45}$$

P_A the accepted power was then

$$P_A = \frac{I_0^2 R_0}{2}$$

(46)

Where I_0 and R_0 are the current and resistance at the input of the antenna

So the Q can be exactly reduced to

$$Q(\omega_0) = \left| \frac{\omega_0}{2R_0(\omega_0)} X_0'(\omega_0) - \frac{2\omega_0}{I_0^2 R_0(\omega_0)} [W_L(\omega_0) + W_R(\omega_0)] \right|$$

(47)

Here $W_L and\ W_R$are the energies lost and radiated by the antenna.

This Q value in (47) was approximated by them [12] in a different and more precise way than preciously done by Chu [4] as

$$Q(\omega_0) = \frac{\omega_0}{2R_0(\omega_0)} Z_0'(\omega_0) = \frac{\omega_0}{2R_0(\omega_0)} \sqrt{[R'(\omega_0)]^2 + \left[X_0'(\omega_0) + \frac{|X_0(\omega_0)|}{\omega_0}\right]^2}$$

(48)

The approximation used by Chu was

$$Q(\omega_0) = \frac{\omega_0}{2R_0(\omega_0)} X_0'(\omega_0)$$

(49)

They [12] state that the inverse relationship between bandwidth and Q may not be satisfied of the antenna contains nonlinear or active materials and tuning elements as the bandwidth can be widened without changing the internal energy or Q.

2.5.2.1.2 Q as a function of antenna size

They [12] proved that Q increases rapidly with decreasing antenna size.

From equation (30) and from the definition of antenna efficiency

$$\eta = \frac{P_{rad}}{P_{rad} + P_{loss}}$$

(50)

$$Q(\omega_0) = \frac{\omega_0 \eta(\omega_0) |W(\omega_0)|}{P_{rad}(\omega_0)}$$

(51)

$P_R(\omega_0)$ is the power radiated and is given as

$$P_R(\omega_0) = \frac{1}{2Z_f}\int_{4\pi} |F(\theta,\varphi)|^2 d\Omega \tag{52}$$

Where $F(\theta,\varphi) = \lim_{r\to\infty} re^{jkr}E(r)$

So from equations (31), (40) and (41) we have

$$Q(\omega_0) = \frac{\eta k}{2}\frac{\lim_{r\to\infty}\left[\int_a^r \int_{4\pi}(|E|^2 + Z_f^2|H|^2)r^2 d\Omega dr - 2r\int_{4\pi}|F|^2 d\Omega\right]}{\int_{4\pi}|F|^2 d\Omega} + Q^{in}(\omega_0) \tag{53}$$

$$Q^{in}(\omega_0) = \frac{\eta\omega_0 Z_f}{2}\frac{Re\int_{v_0(a)}\{E^*.(\omega_0\varepsilon)'.E + H^*.(\omega_0\mu)'.H + E.[(\omega_0(v_t+\tau^*)]'.H\}}{\int_{4\pi}|F|^2 d\Omega} \tag{54}$$

The volume $v_0(r)$ was divided into $v_0(a)$ and $v_0(r) - v_0(a)$ and a is the radius of Chu's sphere.

$$Q(\omega_0) = |Q_e(\omega_0) + Q_m(\omega_0) + Q^{in}(\omega_0) - \eta k_0 a| \tag{55}$$

Where $Q_e(\omega_0) = \frac{\eta k}{2}\frac{\lim_{r\to\infty}\left[\int_a^r \int_{4\pi}(|E|^2)r^2 d\Omega dr - (r-r_0)\int_{4\pi}|F|^2 d\Omega\right]}{\int_{4\pi}|F|^2 d\Omega}$ (56)

Similar equations are available for $Q_m(\omega_0)$ and denote the electric and magnetic reactive energies outside the sphere r_0 also defined by Fante.

Then the reactive energies can be expressed by Hankel functions as

$$Q_m(\omega_0) \approx \eta \int_{k_0 a}^{\infty}\{\sum_{l=1}^{L}[\,|\alpha_l|^2 \begin{Bmatrix} H_l(x) \\ G_l(x) \end{Bmatrix} + |\beta_l|^2]\ \}dx \tag{57}$$

As seen by them [12] $k_0 a$ becomes smaller than L the Hankel function increase rapidly $Q_m and\ Q_e$ become much greater than. Practical antennas cannot have enormously high reactive fields so the gain above few dB is not realizable.

2.5.3 Thal's Contribution

Thal [14] states that Chu's [4] approach can be too approximate in cases like helical antenna as the method does not account for the energy within the antenna and between the antenna envelope and the Chu's sphere and in some cases they may be different. As a result Chu's [4] Q is a lower limit of the actual Q and therefore a much larger value is attainable. Thal [14] thereby derives the Q for the helical antenna with the help of computation by using FORTRAN codes.

Thal [14] then compares the values obtained from Chu's [4] method of computation (Q_n) and the Q of the antenna when excited by TE and TM modes by including the internal energy (${}^{TM}Q_n$ *and* ${}^{TE}Q_n$)

$$\frac{{}^{TM}Q_n}{Q_n} \rightarrow 2 - \frac{1}{(n+1)}$$

(58)

$$\frac{{}^{TE}Q_n}{Q_n} \rightarrow 2 + \frac{1}{(n)}$$

(59)

$$as\ ka \rightarrow 0$$

The values of ${}^{TM}Q_n$ *and* ${}^{TE}Q_n$ were obtained easily from the energy stored in the capacitors and inductors of the equivalent circuit (fig.5 [14]) so the value of Q was expressed as

$$Q \approx \frac{2\omega W}{(P_{TE} + P_{TM})}$$

(60)

$$\frac{1}{Q} \approx \frac{P_{TE}}{2\omega W} + \frac{P_{TM}}{2\omega W}$$

(61)

$$Q \approx \frac{Q_{TM} Q_{TE}}{Q_{TM} + Q_{TE}}$$

(62)

2.5.4 Geyi's Contribution

Geyi [15] gave a complete description of the powers associated with the electromagnetic fields in a complex form of the Poynting vector. He [15] also extended the Foster theorem to the antenna which was essentially a lossy 2 port device and proved its validity as with a lossless device. He also proved the inverse relation between Q and bandwidth.

2.5.4.1 Analysis of Geyi's work

2.5.4.1.1 Expression for Q

Geyi [15] used the Poynting vector to express complex power as

$$\frac{1}{2}I^*(s).V(s) = P^{rad} + 2\alpha\left[W_m + W_e - \frac{r_\infty}{c}P_{rad}\right] + 2j\omega(W_m - W_e)$$

(63)

Here the complex frequency is $s = \alpha + j\omega \ and \ V(s), I(s)$ are voltage and current vectors at the antenna terminal. The complex impedance was

$$Z_A(s) = R_A(\alpha,\omega) + jX_A(\alpha,\omega) = \frac{V(s)}{I(s)}$$

(64)

So form (62) and (63) $Z_A(s)$ was then

$$Z_A(s) = \frac{2P^{rad}}{|I(s)|^2} + \frac{4\alpha\left[W_m + W_e - \frac{r_\infty}{c}P_{rad}\right]}{|I(s)|^2} + \frac{4j\omega(W_m - W_e)}{|I(s)|^2}$$

(65)

Therefore $R_A(\alpha,\omega) = \frac{2P^{rad}}{|I(s)|^2} + \frac{4\alpha\left[W_m+W_e-\frac{r_\infty}{c}P_{rad}\right]}{|I(s)|^2}$

(66)

And $X_A(\alpha,\omega) = \frac{4j\omega(W_m-W_e)}{|I(s)|^2}$

(67)

Differentiating equation (65) and solving for $W_m \ and \ W_e$ and using $|I(s)|^2 = [I^*]^t[I]$ we get

$$W_m = \frac{1}{8}[I^*]^t[I](\frac{d[X_A]}{d\omega} + \frac{X_A}{\omega})$$

(68)

$$W_e = \frac{1}{8}[I^*]^t[I](\frac{d[X_A]}{d\omega} - \frac{X_A}{\omega})$$

(69)

$$P^{rad} = \frac{1}{2}Re\{[I^*]^t.V\} = \frac{1}{2}Re\{[I^*]^t.[Z_A + [Z^*]^t\ ].I\}$$

(70)

And using Q as in (11) we have

$$Q = \frac{[I^*]^t \left[\omega \frac{d[X_A]}{d\omega} \pm [X_A]\right] I}{[I^*]^t [Z_A + [Z^*]^t\ ].I} = \frac{\left[\omega \frac{d[X_A]}{d\omega} \pm [X_A]\right]}{2\ R_A^{rad}}$$

(71)

2.5.4.1.2 Relation between Q and Bandwidth

By expressing radiation resistance using Taylor series with the assumption that α is small the half power bandwidth was found by Geyi [15] as

$$B = \frac{4P^{rad}}{\omega_r |I|^2 |\frac{d[X_A]}{d\omega}|} = \frac{4P^{rad}}{|8\omega_r W \pm |I|^2 X_A|_{\omega_r}} = \frac{4P^{rad}}{|8\omega_r W|_{\omega_r}} = \frac{1}{Q}$$

(72)

Where $Q \gg 1\ and\ W = \max\ (W_m, W_e)$

2.5.5 Gustafsson's Contribution

Gustafsson [17] extended the theory of particle scattering to antennas. Gustafsson [17] used the polarizabilty dyadics and successfully separated the electric and magnetic energies of an antenna of an arbitrary shape. His [17] method was better than Chu [4] as the smallest circumscribing Chu's sphere was far from optimum for arbitrary shaped antennas. He [17] calculates the absorption cross section of the antenna. He [17] used an equivalent model for an antenna obtained from scattering theory. He [17] then uses the optical theorem to find out the absorption cross section of the antenna

2.5.5.1 Analysis of Gustafsson's method

Gustafsson [17] considers an antenna of arbitrary shape in free space with a plane wave incident on it. The principles of linearity causality and ideality of the antenna are assumed to hold. The characteristics are modeled form Maxwell's equation and the anisotropic relations are expressed in the form of electric and magnetic susceptibility dyadics $\chi_e\ and\ \chi_m$. He [17] then considers a bounding volume of V of arbitrary shape such that complete absorption of the incident wave is within the volume V. He [17] also considers the reflection at the port of the antenna

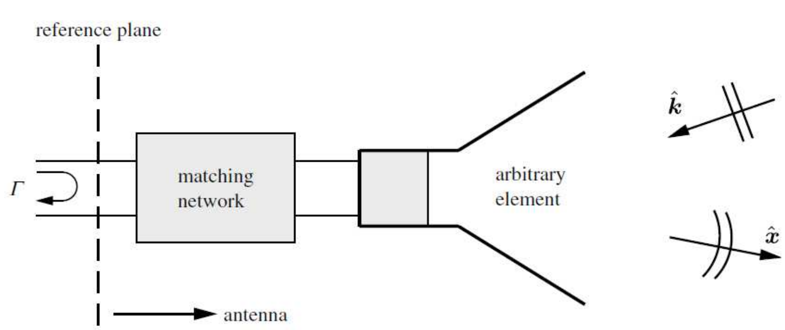

Figure 2 A hypothetical antenna subject to a plane wave in the K direction as in [17]

He [17] derived the dispersion relation for the extinction cross section from the electric and magnetic polarizability dyadics γ_e, γ_m as

$$\int_0^\infty \sigma_{ext}(\lambda)d\lambda = \pi^2(\hat{p}_e{}^*.\gamma_e.\hat{p}_e + \hat{p}_m^*.\gamma_m.\hat{p}_m)$$

(73)

Where $\hat{p}_m = \hat{k} \times \hat{p}_e$ and σ_{ext} is the exctinction cross section, $\hat{p}_e$is the electric polarization =$\hat{p}_e = E/|E|$

The absorption cross section σ_a for an unmatched antenna is decreased by a factor of $(1 - |\Gamma|^2)$ and the σ_{ext} is bounded by it therefore

$$\sigma_{ext} \geq \sigma_a = (1 - |\Gamma|^2)\sigma_{a0}$$

(74)

σ_{ao} is the absorption cross section of the matched antenna

From the definition of gain and bandwidth of an antenna one can write the LHS of (72) also as

$$\int_0^\infty \sigma_{ext}(\lambda)d\lambda \geq \int_\Lambda \sigma_a(\lambda)d\lambda = \frac{1}{4\pi}\int_\Lambda (1 - |\Gamma|^2)\, \lambda^2 G(\lambda)d\lambda$$

(75)

Then the minimum partial realized gain is written as $G_\Lambda = \inf_{\lambda\epsilon\Lambda}(1 - |\Gamma|^2)G$ so (74) now becomes

$$\int_\Lambda (1 - |\Gamma|^2)\, \lambda^2 G(\lambda)d\lambda \;\geq\; G_\Lambda \int_\Lambda \lambda^2 d\lambda = \lambda_0^3 G_\Lambda B(1 + \frac{B^2}{12})$$

(76)

As B<<2 and from (73), (74) and from (75) we get

$$G_\Lambda B \leq \frac{4\pi}{\lambda_0^3}(\hat{p}_e{}^*.\gamma_e.\hat{p}_e + \hat{p}_m^*.\gamma_m.\hat{p}_m)$$

(77)

The absorption cross section is then extended to the case of N non interfering resonances cases as $\sigma_a = 4\pi k Im\rho_a$ where

$$\rho_a(k) = \sum_{n=1}^{N} \rho_n \frac{iQ_n k_n}{1 - \frac{iQ_n\left(\frac{k}{k_n} - \frac{k_n}{k}\right)}{2}}$$

(78)

Where ρ_n are positive weight functions that satisfy $\sum \rho_n = \rho(0)$, k is real Q_n denotes the quality factor of the resonance at k_n. Then for a matched antenna matched at $k = k_0$ the following inequality was obtained.

$$\frac{D}{Q} \leq \frac{K_0^3}{2\pi} (\hat{p}_e{}^{*} . \gamma_e . \hat{p}_e + \hat{p}_m^{*} . \gamma_m . \hat{p}_m)$$

(79)

Where D is the directivity of the antenna

2.5.5.2 Comparisons of limitation of Q and Directivity with Chu

Gustafsson [17] then compares the ratio of *D/Q* obtained by his method and that of Chu [4] according his results for the omnidirectional antenna in the Chu sphere.

According to Chu's [4] technique we have

For TE mode

$$\sup_{\hat{p}_e . \hat{p}_m = 0} \frac{D}{Q} \leq \frac{3}{2} \frac{k_0^3 a^3}{k_0^3 a^2 + 1} = \frac{3}{2} k_0^3 a^3 + O(k_0^5 a^5) \text{ as } k_0 a \to 0$$

(80)

Where $\sup\limits_{\hat{p}_e . \hat{p}_m = 0}$ is the supremum operator which is the polarization matching operator i.e. the polarization of the incident wave matches the polarization of the antenna

For TE and TM mode

$$\sup_{\hat{p}_e . \hat{p}_m = 0} \frac{D}{Q} \leq \frac{6 k_0^3 a^3}{2 k_0^3 a^2 + 1} = 6 k_0^3 a^3 + O(k_0^5 a^5) \text{ as } k_0 a \to 0$$

(81)

And according to Gustafsson [17] technique we have

For TE mode

$$\sup_{\hat{p}_e . \hat{p}_m = 0} \frac{D}{Q} \leq 2 k_0^3 a^3$$

(82)

For TE and TM

$$\sup_{\hat{p}_e.\hat{p}_m=0} \frac{D}{Q} \leq 4k_0^3a^3$$

(83)

2.5.5.3 Comparisons of bandwidth and gain

According to Gustafsson [17] the Gain and the bandwidth product was

For TE mode

$$\sup_{\hat{p}_e.\hat{p}_m=0} \frac{D}{Q} \leq 2\pi k_0^3a^3$$

(84)

For TE and TM

$$\sup_{\hat{p}_e.\hat{p}_m=0} \frac{D}{Q} \leq 4\pi k_0^3a^3$$

(85)

And that of Chu [4] is

$$\sup_{\hat{p}_e.\hat{p}_m=0} G_\Lambda B \leq \frac{3\pi}{2}\frac{(1-|\Gamma|^2)}{\ln 1/|\Gamma|} k_0^3a^3$$

(86)

3 Comparisons and Results

The work in small antennas is traceable to the work of Wheeler [1] in 1947. Wheeler discussed the limitation of small antennas using a simple model that approximated the antenna with a lumped capacitor or inductor and a radiating resistance. He [1] defined the precursor of *Q* the Radiation power factor *RPF* that gave the ratio of radiated power to reactive power. His [1] was the first attempt to find a relationship between antenna size and radiation. *RPF* was the inverse of *Q* or was equal to bandwidth. His [1] work was too approximate and was applicable only to extremely small antennas. He [1] did not account for the radiated spherical wave modes.

Chu [4] in 1948 derived the minimum *Q* for an omnidirectional antenna which he enclosed in a sphere named after him as Chu sphere. He [4] used spherical wave functions to represent the radiated fields as a sum of spherical modes. Each mode then was represented with an equivalent lumped circuit and then *Q* for each mode was found. Chu's work spurred a great interest and many authors later refined his limit. Hansen [2] simplified Chu's expression in 1981. Harrington [3] later considered the antenna radiating both TE and TM modes and obtained lower *Q* values. Both Harrington [3] and Chu [4] used circuit approximations while Collin and Rothschild [8] developed a field based approach to calculate the exact *Q* for an antenna radiating TE or TM modes. Their [8] method was then generalized by Fante [13] and included both TE and TM modes. He obtained the exact *Q* with both TE and TM modes.

Mclean [9] in 1996 used a new method to calculate the *Q* as he felt Wheeler's and Chu's work to be inaccurate and approximates. Foltz and McLean realized that their values were far away from Chu and therefore repeated Chu's work by employing prolate spheroidal wave functions.

Thiele [7] in 2003 believed that the current distribution had a strong influence on the value of *Q*. He [7] determined *Q* from the super directive ratio concept. His [7] results were closer to a practical dipole antenna.

Geyi [15] in 2003 pointed out that Collins' and Rothschild's analysis was not feasible for many antennas and his re investigation produced another approximate method for the calculation of *Q* that involved lesser rigorous integral computations.

Yaghjian and Best [12] in 2003-08 carried out an extensive work and obtained approximate expressions for the *Q* in terms of the fields, impedance and its relation to bandwidth. They [12] explored the effect of wire geometry, wire folding and volume utilization on radiation resistance and *Q*.

Kwon and Pozar [5] in 2005-09 pointed out the inconsistencies in previous results and did an extensive work in defining the TE and TM modes, antenna gain, *Q* and directionality.

Thal [11] in 2006-09 then considered the surface current distribution of the antenna over a sphere radiating both TE and TM modes. He [11] again extended the work of Chu [4] and introduced additional equivalent circuit to account for the energy stored inside the Chu's sphere. He [11] also found the relationship between gain, Q and concludes that they were all dependent quantities.

Gustafsson [17] then in 2007 presented an expression for Q of small antennas of arbitrary shapes. This was a totally different approach and used scattering theory and represented the antenna in terms of material dyadics.

The minimum Q as obtained by the different authors discussed is summarized in table 1. The modes are specified corresponding to their values. They are for a dipole antenna.

Minimum *Q*	**Author**	**Mode**
$\frac{1}{ka}+\frac{1}{(ka)^3}$	Mclean [9]	TE or TM mode
$\frac{1}{2}\left(\frac{2}{ka}+\frac{1}{(ka)^3}\right)$	Mclean [9]	TE and TM mode
$\frac{1.5}{(ka)^3}$ (for ka→ 0)	Thal [11]	TM mode
$\frac{3}{(ka)^3}$(for ka→ 0)	Thal [11]	TE Mode
$\frac{1}{(ka)^3}$(for ka→ 0)	Thal [11]	TE and TM mode
$\frac{G}{\eta}.\frac{1}{2(ka)^3}=\frac{1.5}{(ka)^3}$	Gustafsson [17] et al.	TM mode
$\frac{1}{\sqrt{2}ka}+\frac{3}{2(ka)^3}$	Hansen and Collin [10]	TM Mode
$\frac{1+2(ka)^2}{(ka)^3[1+(ka)^2]}$	Chu [4] (omni directional antenna)	TE or TM Mode

Table1-Q limit as obtained by authors

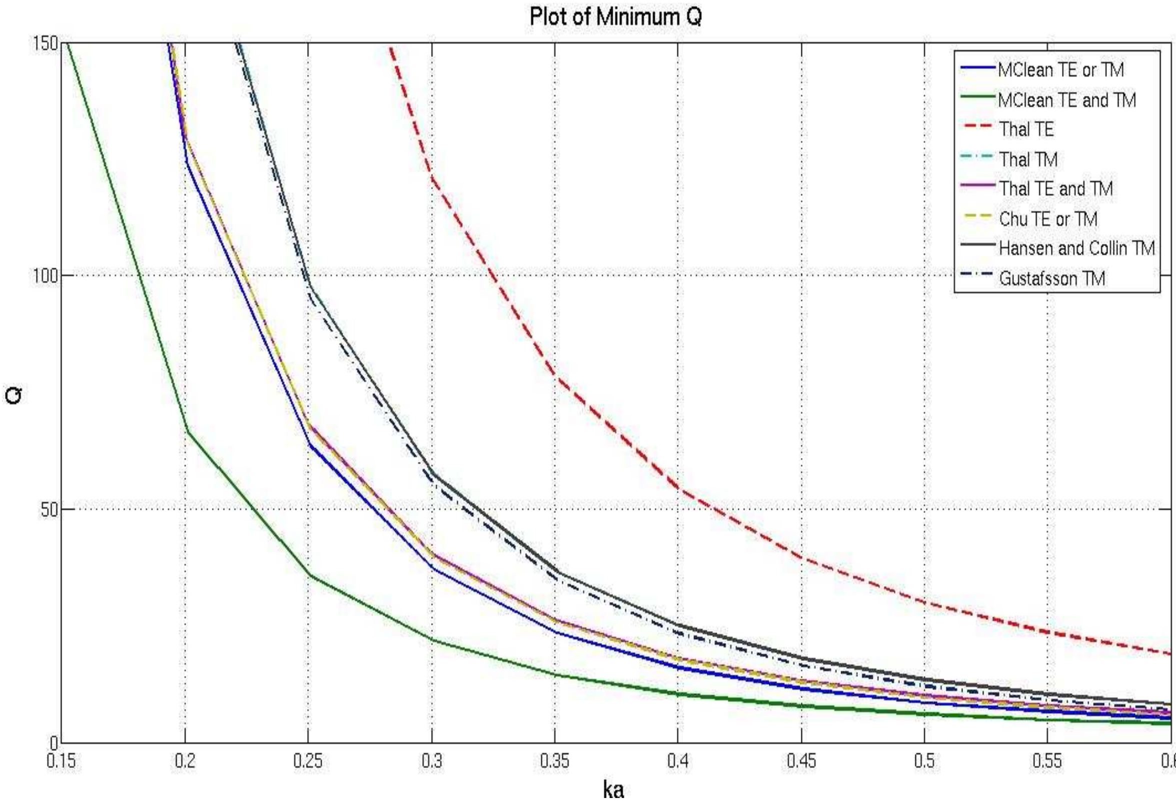

Figure 3- A plot comparing the Minimum *Q* vs *ka*

The figure gives the plot of the minimum *Q* values vs *ka* obtained for TE, TM and both TE and TM modes as obtained by Chu [4], Thal [14], Gustafsson [17], Hansen and Collin [10] and Mclean [9]. It is notable that the *Q* values are very close for *ka* values close to 1, but differ very much when they tend to zero.

4 Conclusions

Why was the concept of *Q* introduced?

The concept of *Q* seems unnecessary as the bandwidth, gain etc. are more important practical parameters of an antenna. The answer is however that the bandwidth is difficult to be computed, measured or estimated directly. The *Q* is a more fundamental quantity that is defined in terms of antenna fields and its inverse readily gives the bandwidth in most cases. So the bandwidth can be altered by easily restructuring the antenna to modify its interior fields and therefore it's *Q*.

How it was calculated?

The value of *Q* was obtained by various authors using different techniques each unique to itself. Some authors corrected the previous values by re working on it and others used a radically new approach. Various approximation techniques were also employed to obtain minimum *Q* values. All the results are in tandem for ka values close or greater than 1 but diverge significantly when ka is very small.

A through literature survey in the determination of *Q* was thus presented in this work, along with the various limitations and its relationship to bandwidth.

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